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AN OUTSPREAD ON VALUED LOGIC SUPERHYPERALGEBRAS

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Abstract. In all classical logical algebras, only two elements can be equated to one element, and this is not possible when we want to equate more than two elements to one or more element. In this study, we are looking for a new idea to cover this defect. This paper considers the logic algebra structures and generalizes them to superhyper logic algebra. Indeed, we extended the axioms of logic algebra to neutrosophic superhyper logic algebras.
Keywords: algebra, logic, neutrosophic.

1. Introduction

The theory of logic algebra is one of the important branches of mathematics that is applied in other sciences. Some researchers and mathematical theorists introduced some type of logic algebra and extended these scopes of mathematics. Y. Imai and K. Iseki introduced two classes of abstract algebras: BCK-algebras and BCI-algebras, and also proved that the class of BCK-algebras is a proper subclass of the class of BCI-algebras [8, 9, 10]. Later Q. P. Hu and X. Li introduced

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a wide class of abstract algebras - BCH-algebras, and have shown that the class of BCI-algebras is a proper subclass of the class of BCH-algebras [2]. Florentin Smarandache introduced a new concept of neutro-algebra as a generalization of partial algebra. He proved that a neutro-algebra is a generalization of partial algebra, and introduced the neutro-function (and neutro-operation). Recently in the scope of neutro logical (hyper) algebra, Hamidi, et al. have introduced the concept of neutro BCK-subalgebras [5], neutro d-subalgebras [4] and single-valued neutro hyper BCK-subalgebras [6] as a generalization of BCK-algebras and hyper BCK-subalgebras, respectively and presented the main results in this regard. Also, Florentin Smarandache presented a novel concept as super hyperalgebra with its super hyperoperations and super hyperaxioms, then introduced some concepts such as super hypertopology and especially the super hyperfunction and neutrosophic super hyperfunction [16, 17]. In the continuation of the super hyperalgebra topics, Hamidi et. al presented the novel concepts of supervertices, superedges, and superhypergraph via the concept of flow. They computed the number of superedges of any given superhypergraphs and based on the numbers of superedges and partitions of an underlying set of superhypergraphs, obtained the number of all superhypergraphs on any nonempty set. They also introduced the incidence matrix of superhypergraph and computed the characteristic polynomial for the incidence matrix of superhypergraphs, so obtained the spectrum of superhypergraphs. The flow of superedges plays the main role in computing of spectrum of superhypergraphs, so they computed the spectrum of superhypergraphs in some types regular flow, regular reversed flow, and regular two-sided flow [3]. Recently, Hamidi has introduced the concept of super hyper BCK-algebras as a generalization of BCK-algebras and investigated some properties of this novel concept [7]. To see more content related to BE-algebras, BCK-algebras and superhyper algebras refer to the sources [1, 3, 14, 15, 18, 20].

Motivation and advantage: In the real world, communication is one of the most important principles of progress. Naturally, the wider the communication, the more impact it can have. In classical algebraic systems, two elements can only be equal to one element, allowing us to check the relationship of three elements at once. The problem becomes important when we want to find connections between more than three elements or a set of elements. According to the mentioned limitations, our main motivation in this study is to expand the principles of the subject in a way to creates connections between a set of elements based on systematic rules. Therefore, we have here studied the concept of a superhyper of logical algebras. Regarding these points, we consider some of the two-valued logic algebras such as BH-algebras, BE-algebras, and BCK-algebras and extended them to superhyper BH-algebras, superhyper BE-algebras and superhyper BCK-algebras, respectively. We investigated the properties of these superhyper algebras and proved that they have unique categorical properties. The basic comparison between these superhyperalgebras has been examined in detail and the relationship between them has been discussed. This study aims to extend logic algebras to superhyper algebras using the superhyper axioms. Since we can characterize factual, intermediate, and false problems in logic, we actually attempt to overspread the axiom of classical algebras to the superhyper axiom in logic algebras.

1.1. Preliminaries

Definition 1.1. [16, 17] Let X be a nonempty set and $0 \in X$. Then $(X, \circ^*_{(m,n)})$ is called an (m,n)-super hyperalgebra, where $\circ^*_{(m,n)} : X^m \to P^n_*(X)$ is called an (m,n)-super hyperoperation, $P^n_*(X)$ is the n^{th} powerset of the set $X, \emptyset \notin P^n_*(X)$, for any $A \in P^n_*(X)$, we identify $\{A\}$ with $A, m, \geq 2, n \geq 0, X^m = \underbrace{X \times X \times \ldots X}_{m-times}$ and $P^0_*(X) = X$.

Definition 1.2. [13] Let H be a nonempty set and $\varrho : H \times \ H \to P^*(H)$ be a hyperoperation. Then $(H; \varrho, 1)$ is called a hyper BE–algebra, if for all $x, y, z \in H$ it satisfies the following axioms:

(HBE_1) $x < 1$ and $x < x$,

(HBE_2) $\varrho(x, \varrho(y, z) = \varrho(y, \varrho(x, z)$,

(HBE_3) $x \in \varrho(1, x)$,

(HBE_4) $1 < x$ implies $x = 1$.

Where the relation " $<$ " is defined by $x < y \Leftrightarrow 1 \in \varrho(x, y)$.

Definition 1.3. [2] Let $X \neq \emptyset$. Then a universal algebra $(X, \vartheta, 0)$ of type $(2, 0)$ is called a *BCK-algebra*, if $\forall\ x, y, z \in X$:
$(BCI$-1) $((x\vartheta\ y)\vartheta\ (x\vartheta\ z))\vartheta\ (z\vartheta\ y) = 0$,
$(BCI$-2) $(x\vartheta\ (x\vartheta\ y))\vartheta\ y = 0$,
$(BCI$-3) $x\vartheta\ x = 0$,
$(BCI$-4) $x\vartheta\ y = 0$ and $y\vartheta\ x = 0$ imply $x = y$,
$(BCK$-5) $0\vartheta\ x = 0$,
where $\vartheta(x, y)$ is denoted by $x\vartheta\ y$.

Definition 1.4. [11] An algebra $(X, 0)$ of type $(2, 0)$ with the following axioms is called a BH-algebra, for all $x, y, z, \in X$,

(i) $x * x = 0$,

(ii) $x * y = 0$ and $y * x = 0$ imply $x = y$,

(iii) $x * 0 = x$ for all $x \in X$.

Definition 1.5. [1, 12] Let $X \neq \emptyset$ and $P^*(X) = \{Y \mid \emptyset \neq Y \subseteq X\}$. Then for a map $\varrho : X^2 \to P^*(X)$ a hyperalgebraic system $(X, \varrho, 0)$ is called a *hyper BCK-algebra*, if $\forall\ x, y, z \in X$:

$(H1)$ $(x \varrho z) \varrho (y \varrho z) \ll x \varrho y$,
$(H2)$ $(x \varrho y) \varrho z = (x \varrho z) \varrho y$,
$(H3)$ $x \varrho X \ll x$,
$(H4)$ $x \ll y$ and $y \ll x$ imply $x = y$,
where $x \ll y$ is defined by $0 \in x \varrho y$, $\forall W, Z \subseteq X$, $W \ll Z \Leftrightarrow \forall a \in W \exists b \in Z$ s.t $a \ll b$, $(W \varrho Z) = \bigcup_{a\in W, b\in Z} (a \varrho b)$ and $\varrho(x, y)$ is denoted by $x\varrho y$.

1.2. On superhyper BH-subalgebras

In this subsection, we make the concept of superhyper logic BH-subalgebras as an extension of logic subalgebras and seek some of their properties.

Definition 1.6. Let X be a nonempty set and $0 \in X$ and $\alpha = \underbrace{\epsilon, \epsilon, \dots \epsilon}_{(m-2)-times}$. Then $(X, \circ^*_{(m,n)}, \epsilon)$ is called an (m, n)-super hyper BH-subalgebra, if

(i) $\epsilon \in \circ^*_{(m,n)}\big(x, x, \dots, x\big)$,

(ii) if $\epsilon \in \circ^*_{(m,n)}\big(x_1, x_2, \dots, x_m\big)$ and $\epsilon \in \circ^*_{(m,n)}\big(x_m, x_{m-1}, \dots, x_1\big)$, then $x_i = x_j$, where $i + j = m + 1$,

(iii) $x \in \circ^*_{(m,n)}\big(x, \epsilon, \alpha\big)$.

Example 1.1. Let $X = \{\epsilon, a\}$. Then $(X, \circ^*)$ is a $(3, 3)$-super hyper BH-subalgebra as follows:

$$\circ^*_{(3,3)}(x, y, z) = \begin{cases} P^3_*(\{x, \epsilon\}) & \text{if } x = z = y \\ P^3_*(\{x, y, z\}) & \text{if } z = \epsilon \end{cases},$$

where

$$P_*(\{a\})P^2_*(\{a\}) = P^3_*(\{a\}) = \{a\}, P_*(\{1, a\}) = \{1, a, \{1, a\}\},$$
$$P^2_*(\{1, a\}) = \{1, a, \{1, a\}, \{1, \{1, a\}\}, \{a, \{1, a\}\}\},$$
$$P^3_*(\{1, a\}) = \big\{1, a, \{1, a\}, \{1, \{1, a\}\}, \{a, \{1, a\}\}, \{1, \{1, \{1, a\}\}, \{1, \{a, \{1, a\}\},$$
$$\{a, \{1, \{1, a\}\}, \{a, \{a, \{1, a\}\}, \{\{1, a\}, \{1, \{1, a\}\}\}, \{\{1, a\}, \{a, \{1, a\}\}, \{\{1, \{1, a\}\},$$
$$\{a, \{1, a\}\}\}\big\}.$$

(i) By definition, $\epsilon \in \circ^*_{(3,3)}(x, x, x) = P^3_*(\{\epsilon, x\})$.

(ii) By definition, $x \in \circ^*_{(3,3)}(x, \epsilon, \epsilon) = P^3_*(\{x\})$.

(iii) By definition, if $\epsilon \in \circ^*_{(3,3)}(x, y, z)$ and $\epsilon \in \circ^*_{(3,3)}(z, y, x)$, then $x = y = z$.

(ii) Then $(X, \circ^*)$ is a $(3, 0)$-super hyper BH-subalgebra as follows:

$$\circ^*_{(3,1)}(x, y, z) = \begin{cases} 1 & \text{if } x = y = z \\ z & o.w \end{cases},$$

Example 1.2. Let $\mathbb{R}$ be the set of all real numbers. Then $(\mathbb{R}, \circ^*)$ is a $(3,4)$-super hyper BH-subalgebra as follows:

$$\circ^*_{(3,4)}(x,y,z) = \begin{cases} P^4_*(\{x,0\}) & \text{if } x=z=y \\ P^4_*(\{\frac{(x-y)^2}{x}, \frac{(x-z)^2}{x}, \frac{(z-y)^2}{z}\}) & \text{o.w} \end{cases}.$$

Theorem 1.1. *Let $(X, \circ^*_{(m,n)})$ be a (m,n)-super hyper BH-subalgebra. If*

$$\circ^*_{(m,n)}\big(x, \underbrace{\epsilon,\dots,\epsilon}_{(m-2)-times}, \circ^*_{(m,n)}(x, \underbrace{\epsilon,\dots,\epsilon}_{(m-2)-times}, y)\big) = \circ^*_{(m,n)}\big(x, \underbrace{\epsilon,\dots,\epsilon}_{(m-2)-times}, y\big),$$

then there exists $U \subseteq X$ such that $\{x,\epsilon\} \subseteq U$.

Proof. Since $(X, \circ^*_{(m,n)})$ is a (m,n)-super hyper BH-subalgebra, by putting $x = y$, we get that

$$\circ^*_{(m,n)}\big(x, \underbrace{\epsilon,\dots,\epsilon}_{(m-2)-times}, \circ^*_{(m,n)}(x, \underbrace{\epsilon,\dots,\epsilon}_{(m-2)-times}, y)\big) =$$

$$= \circ^*_{(m,n)}\big(x, \underbrace{\epsilon,\dots,\epsilon}_{(m-2)-times}, \circ^*_{(m,n)}(x, \underbrace{\epsilon,\dots,\epsilon}_{(m-2)-times}, x)\big) = \circ^*_{(m,n)}\big(x, \underbrace{\epsilon,\dots,\epsilon}_{(m-2)-times}, x\big).$$

By definition,

$$\epsilon \in \circ^*_{(m,n)}\big(x, \underbrace{\epsilon,\dots,\epsilon}_{(m-2)-times}, x) \text{ and } x \in \circ^*_{(m,n)}\big(x, \underbrace{\epsilon,\dots,\epsilon}_{(m-2)-times}, \circ^*_{(m,n)}(x, \underbrace{\epsilon,\dots,\epsilon}_{(m-2)-times}, x)\big).$$

Hence there exists $U \subseteq X$ such that $\{x,\epsilon\} \subseteq U$. $\square$

Theorem 1.2. *Let $(X, \circ^*_{(m,n)})$ be a (m,n)-super hyper BH-subalgebra. If $\delta = \underbrace{\epsilon,\dots,\epsilon}_{(m-2)-times}$ and $\circ^*_{(m,n)}\big(\circ^*_{(m,n)}(x,\delta,y), z, \delta) = \circ^*_{(m,n)}(x, \delta, \circ^*_{(m,n)}(y,\delta), z)$, then $x \in \circ^*_{(m,n)}(\underbrace{\epsilon,\dots,\epsilon}_{(m-1)-times}, x)$.*

Proof. Since $(X, \circ^*_{(m,n)})$ is a (m,n)-super hyper BH-subalgebra, by putting $x = y = z$, we get that

$$\circ^*_{(m,n)}\big(\circ^*_{(m,n)}(x,\delta,x), x, \delta) = \circ^*_{(m,n)}(x, \delta, \circ^*_{(m,n)}(x,\delta), x)$$

It follows that

$$x \in \circ^*_{(m,n)}(x,\epsilon,\dots,\epsilon) \subseteq \circ^*_{(m,n)}(x, \underbrace{\epsilon,\dots,\epsilon}_{(m-2)-times}, \circ^*_{(m,n)}(x, \underbrace{\epsilon,\dots,\epsilon}_{(m-2)-times}), x)$$

$$= \circ^*_{(m,n)}\big(\circ^*_{(m,n)}(x, \underbrace{\epsilon,\dots,\epsilon}_{(m-2)-times}, x), x, \underbrace{\epsilon,\dots,\epsilon}_{(m-2)-times}).$$

Therefore, $x \in \circ^*_{(m,n)}(\underbrace{\epsilon,\dots,\epsilon}_{(m-1)-times}, x)$. $\square$

1.3. On superhyper BE-subalgebras

In this subsection, we make the concept of superhyper logic BE-subalgebras as an extension of logic subalgebras and seek some of their properties.

Definition 1.7. Let X be a nonempty set and $1 \in X$. Then $(X, \circ^*_{(m,n)}, 1)$ is called an (m,n)-super hyper BE-subalgebra, if $\beta = \underbrace{(1,1,\ldots,1)}_{(m-2)-times}$

(i) $1 \in \circ^*_{(m,n)} \underbrace{(x,x,\ldots,x)}_{m-times}$,

(ii) $1 \in \circ^*_{(m,n)}(x,1,\beta)$,

(iii) $x \in \circ^*_{(m,n)}(1,\beta,x)$,

(iv) $\circ^*_{(m,n)}(\beta,x,\circ^*_{(m,n)}(y,x_1,\ldots,x_{m-1})) = \circ^*_{(m,n)}(\beta,y,\circ^*_{(m,n)}(x,x_1,\ldots,x_{m-1}))$.

Theorem 1.3. *Let $(X, \circ^*_{(m,n)})$ be a distributive (m,n)-super hyper BE-subalgebra. If $\beta = \underbrace{(1,1,\ldots,1)}_{(m-2)-times}$, then*

(i) *If $x \leq y$, then $\circ^*_{(m,n)}(\beta,z,x) \leq \circ^*_{(m,n)}(\beta,z,y)$,*

(ii) $\circ^*_{(m,n)}(\beta,y,z) \leq \circ^*_{(m,n)}(\beta,x,y), \circ^*_{(m,n)}(\beta,x,z))$,

(iii) $\circ^*_{(m,n)}(\beta,y,x) \leq \circ^*_{(m,n)}(\circ^*_{(m,n)}(\beta,z,y), \circ^*_{(m,n)}(\beta,z,x))$.

Proof. Immediate by definition. □

Theorem 1.4. *Let $(X, \circ^*_{(m,n)})$ be a commutative (m,n)-super hyper BE-subalgebra. If $\beta = \underbrace{(1,1,\ldots,1)}_{(m-2)-times}$, then $\circ^*_{(m,n)}(\beta,x,y) \subseteq \circ^*_{(m,n)}\big(\beta, \circ^*_{(m,n)}(\beta, \circ^*_{(m,n)}(\beta,x,y),y),y\big)$.*

Proof. Let $x, y \in X$. Then

$$\begin{aligned}
&\circ^*_{(m,n)}(\underbrace{1,1,\ldots,1}_{(m-2)-times},x,y) \subseteq \circ^*_{(m,n)}(\underbrace{1,1,\ldots,1}_{(m-1)-times}, \circ^*_{(m,n)}(\underbrace{1,1,\ldots,1}_{(m-2)-times},x,y)) \\
\subseteq\ &\circ^*_{(m,n)}(\underbrace{1,1,\ldots,1}_{(m-2)-times}, \circ^*_{(m,n)}(\underbrace{1,1,\ldots,1}_{(m-1)-times}, y, \circ^*_{(m,n)}(\underbrace{1,1,\ldots,1}_{(m-2)-times},x,y), \\
&\circ^*_{(m,n)}(\underbrace{1,1,\ldots,1}_{(m-2)-times},x,y))) \subseteq \circ^*_{(m,n)}(\underbrace{1,1,\ldots,1}_{(m-2)-times}, \circ^*_{(m,n)}(\underbrace{1,1,\ldots,1}_{(m-2)-times}, \\
&\circ^*_{(m,n)}(\underbrace{1,1,\ldots,1}_{(m-2)-times},x,y),y),y).
\end{aligned}$$

Thus

$$\circ^*_{(m,n)}(\underbrace{1,1,\ldots,1}_{(m-2)-times},x,y) \subseteq \circ^*_{(m,n)}\big(\underbrace{1,1,\ldots,1}_{(m-2)-times},\circ^*_{(m,n)}(\underbrace{1,1,\ldots,1}_{(m-2)-times},$$
$$\circ^*_{(m,n)}(\underbrace{1,1,\ldots,1}_{(m-2)-times},x,y),y),y\big).$$

□

Theorem 1.5. *Let $(X,\circ^*_{(m,n)})$ be an (m,n)-super hyper BE-subalgebra. Then* $1 \in \circ^*_{(m,n)}(\underbrace{(1,1,\ldots,1)}_{(m-2)-times},x,\circ^*_{(m,n)}(y,x,\ldots,x))$.

Proof. Let $(X,\circ^*_{(m,n)})$ be an (m,n)-super hyper BE-subalgebra and $x,y \in X$. Then

$$1 \in \circ^*_{(m,n)}(\underbrace{(1,1,\ldots,1)}_{(m-2)-times},y,1) \subseteq \circ^*_{(m,n)}(\underbrace{(1,1,\ldots,1)}_{(m-2)-times},y,\circ^*_{(m,n)}(x,x,\ldots,x))$$
$$= \circ^*_{(m,n)}(\underbrace{(1,1,\ldots,1)}_{(m-2)-times},x,\circ^*_{(m,n)}(y,x,\ldots,x)).$$

□

Theorem 1.6. *Let $(X,\circ^*_{(m,n)})$ be an (m,n)-super hyper BE-subalgebra. Then*

$$1 \in \circ^*_{(m,n)}(\underbrace{1,1,\ldots,1}_{(m-2)-times},x,\circ^*_{(m,n)}(\circ^*_{(m,n)}(\underbrace{1,1,\ldots,1}_{(m-2)-times},x,y),\underbrace{1,1,\ldots,1}_{(m-2)-times},y).$$

Proof. Let $x,y \in X$. Then

$$1 \in \circ^*_{(m,n)}(\underbrace{1,1,\ldots,1}_{(m-2)-times},\circ^*_{(m,n)}(\underbrace{(1,1,\ldots,1)}_{(m-2)-times},x,y),\circ^*_{(m,n)}(\underbrace{(1,1,\ldots,1)}_{(m-2)-times},x,y))$$
$$= \circ^*_{(m,n)}(\underbrace{1,1,\ldots,1}_{(m-2)-times},x,\circ^*_{(m,n)}(\underbrace{1,1,\ldots,1}_{(m-2)-times},\circ^*_{(m,n)}(\underbrace{1,1,\ldots,1}_{(m-2)-times},x,y),y)).$$

It concludes that

$$1 \in \circ^*_{(m,n)}(\underbrace{1,1,\ldots,1}_{(m-2)-times},x,\circ^*_{(m,n)}(\circ^*_{(m,n)}(\underbrace{1,1,\ldots,1}_{(m-2)-times},x,y),\underbrace{1,1,\ldots,1}_{(m-2)-times},y). \quad \square$$

1.4. On superhyper BCK-subalgebras

In this subsection, we make the concept of superhyper logic BCK-subalgebras as an extension of logic subalgebras and seek some of their properties.

Definition 1.8. Let X be a nonempty set and $0 \in X$ and $\alpha = \underbrace{0, 0, \ldots 0}_{(m-2)-times}$. Then $(X, \circ^*_{(m,n)})$ is called an (m,n)-super hyper BCK-subalgebra, if

(i) $0 \in \circ^*_{(m,n)}\big(\circ^*_{(m,n)} (\circ^*_{(m,n)}(x^1_1, x^1_2, \ldots, x^1_m), \ldots, \circ^*_{(m,n)}(x^1_1, x^m_2, \ldots, x^m_m)), 0, \alpha, \circ^*_{(m,n)}(x^m_m, x^{m-1}_m, \ldots, x^1_m)\big)$,

(ii) $0 \in \circ^*_{(m,n)}\big(\circ^*_{(m,n)} (x^1_1, \underbrace{0, 0, \ldots 0}_{(m-2)-times}, \circ^*_{(m,n)}(x^1_1, x^1_2, \ldots, x^1_m)), \underbrace{0, 0, \ldots 0}_{(m-1)-times}, x^1_m)\big)$,

(iii) $0 \in \circ^*_{(m,n)}\big(x, x, \ldots, x\big)$,

(iv) if $0 \in \circ^*_{(m,n)}\big(x_1, x_2, \ldots, x_m\big)$ and $0 \in \circ^*_{(m,n)}\big(x_m, x_{m-1}, \ldots, x_1\big)$, then $x_i = x_j$, where $i + j = m + 1$,

(v) $0 \in \circ^*_{(m,n)}\big(0, 0, \ldots, x\big)$,

Theorem 1.7. *Let $(X, \circ^*_{(m,n)})$ be an (m,n)-super hyper BCK-subalgebra. Then for any $k \geq n$, $(X, \circ^*_{(m,n)})$ is an (m,k)-super hyper BCK-subalgebra.*

Proof. Let $(X, \circ^*_{(m,n)})$ be an (m,n)-super hyper BCK-subalgebra and $k \geq n$. Since $P^n_*(X) \subseteq P^k_*(X)$, for any $x_1, x_2, \ldots, x_m \in X, \circ^*_{(m,n)}(x_1, x_2, \ldots, x_m) \subseteq \circ^*_{(m,k)}(x_1, x_2, \ldots, x_m)$. Thus $0 \in \circ^*_{(m,n)}(x_1, x_2, \ldots, x_m)$ implies that $0 \in \circ^*_{(m,k)}(x_1, x_2, \ldots, x_m)$ and all axioms are valid. □

Theorem 1.8. *Let m be an even and $x_1, x_2, \ldots, x_m \in X$. Then $(X, \circ^*_{(m,n)})$ is an (m,n)-super hyper BCK-subalgebra if and only if*

(i) $\circ^*_{(m,n)}(\circ^*_{(m,n)}(x^1_1, x^1_2, \ldots, x^1_m), \ldots, \circ^*_{(m,n)}(x^1_1, x^m_2, \ldots, x^m_m))) \leq \circ^*_{(m,n)}(x^m_m, x^{m-1}_m, \ldots, x^1_m)$,

(ii) $\circ^*_{(m,n)}(x^1_1, \underbrace{0, 0, \ldots 0}_{(m-2)-times}, \circ^*_{(m,n)}(x^1_1, x^1_2, \ldots, x^1_m)) \leq \circ^*_{(m,n)}\big(\underbrace{0, 0, \ldots 0}_{(m-1)-times}, x^1_m)\big)$,

(iii) $(\underbrace{x, x, \ldots, x}_{(\frac{m}{2})-times}) \leq (\underbrace{x, x, \ldots, x}_{(\frac{m}{2})-times})$,

(iv) *if* $(x_1, x_2, \ldots, x_{\frac{m}{2}}) \leq (x_{\frac{m}{2}+1}, x_{\frac{m}{2}+2}, \ldots, x_m)$ *and* $(x_{\frac{m}{2}+1}, x_{\frac{m}{2}+2}, \ldots, x_m) \leq (x_1, x_2, \ldots, x_{\frac{m}{2}})$, *then* $x_i = x_j$, *where* $|i - j| = 2$,

(v) $(\underbrace{0, 0, \ldots, 0}_{(\frac{m}{2})-times}) \leq (x_{\frac{m}{2}+1}, x_{\frac{m}{2}+2}, \ldots, x_m)$,

(vi) $(x_1, x_2, \ldots, x_{\frac{m}{2}}) \leq (x_{\frac{m}{2}+1}, x_{\frac{m}{2}+2}, \ldots, x_m) \Leftrightarrow 0 \in \circ^*_{(m,n)}(x_1, x_2, \ldots, x_m)$.

Proof. It is obtained by definition. □

Theorem 1.9. *Let m be an even and $(X, \circ^*_{(m,n)})$ be an (m,n)-super hyper BCK-subalgebra and $x_1, x_2, \ldots, x_{\frac{m}{2}}, y_1, y_2, \ldots, y_{\frac{m}{2}}, z_1, z_2, \ldots, z_{\frac{m}{2}} \in X$. If $\alpha = \underbrace{0, \ldots, 0}_{(\frac{m}{2}-1)-times}$, then $\circ^*_{(m,n)}(\circ^*_{(m,n)}(x_1, \ldots, x_{\frac{m}{2}}, y_1, \ldots, y_{\frac{m}{2}}), \alpha, z_1, \ldots, z_{\frac{m}{2}}) \approx \circ^*_{(m,n)}(\circ^*_{(m,n)}(x_1, \ldots, x_{\frac{m}{2}}, z_1, \ldots, z_{\frac{m}{2}}), \alpha, y_1, \ldots, y_{\frac{m}{2}})$.*

Proof. Let $x_1, x_2, \ldots, x_{\frac{m}{2}}, y_1, y_2, \ldots, y_{\frac{m}{2}}, z_1, z_2, \ldots, z_{\frac{m}{2}} \in X$. Thus get that

$$0 \approx \circ^*_{(m,n)}(\circ^*_{(m,n)}(x_1, \ldots, x_{\frac{m}{2}}, y_1, \ldots, y_{\frac{m}{2}}), \underbrace{0, \ldots 0}_{(\frac{m}{2}-1)-times}, x_1, \ldots, x_{\frac{m}{2}})$$

and

$$0 \approx \circ^*_{(m,n)}(\circ^*_{(m,n)}(x_1, \ldots, x_{\frac{m}{2}}, x_1, \ldots, x_{\frac{m}{2}}), \underbrace{0, \ldots 0}_{(\frac{m}{2}-1)-times}, y_1, \ldots, y_{\frac{m}{2}}).$$

In addition, by definition we get that $0 \approx \circ^*_{(m,n)}(\underbrace{0, \ldots 0}_{(\frac{m}{2})-times}, y_1, \ldots, y_{\frac{m}{2}})$, hence the proof is completed. □

Theorem 1.10. *Let m be an even and $(X, \circ^*_{(m,n)})$ be an (m,n)-super hyper BCK-subalgebra and $x_1, x_2, \ldots, x_{m-1} \in X$. Then $(x_1, \ldots, x_{m-1}) \approx \circ^*_{(m,n)}(x_1, \ldots, x_{m-1}, 0)$.*

Proof. Let $x_1, x_2, \ldots, x_m \in X$. Then

$0 \approx \circ^*_{(m,n)}(x_1, x_2, \ldots, x_{m-1}, \circ^*_{(m,n)}(x_1, x_2, \ldots, x_{m-1}, 0))$. Moreover by above Theorem, we have $0 \approx \circ^*_{(m,n)}(\circ^*_{(m,n)}(x_1, \ldots, x_{m-1}, 0), x_1, \ldots, x_{m-1})$. Thus we conclude that $(x_1, \ldots, x_{m-1}) \approx \circ^*_{(m,n)}(x_1, \ldots, x_{m-1}, 0)$. □

Corollary 1.1. *The class of (m,n)-super hyper BCK-subalgebras is a subclass of (m,n)-super hyper BH-subalgebra.*

2. Conclusion and discussion

The current paper has defined and considered the notion of superhyperalgebras to logic algebras and has introduced the concepts of (m,n)-super hyper BH-subalgebras, (m,n)-super hyper BCK-subalgebras and (m,n)-super hyper BE-subalgebras. We investigated the important properties of these logic superhyperalgebras and found a relation between them. The advantage of(m,n)-super hyper BH-subalgebras, (m,n)-super hyper BCK-subalgebras and (m,n)-super hyper BE-subalgebras is that it removes all the limitations of connecting elements, and based on (m,n)-super hyper BH-subalgebras, (m, n)-super hyper BCK-subalgebras and (m, n)-super hyper BE-subalgebras, any number of elements and any number of sets can be linked together. On the other hand, limiting (m, n)-super hyper BH-subalgebras, (m, n)-super hyper BCK-subalgebras and (m, n)-super hyper BE-subalgebras leads to hyper BH-subalgebras, hyper BCK-subalgebras and hyper BE-subalgebras and BH-subalgebras, BCK-subalgebras and BE-subalgebras and covers all properties of hyper BH-subalgebras, hyper BCK-subalgebras and hyper BE-subalgebras and BH-subalgebras, BCK-subalgebras and BE-subalgebras. On the other hand, the complexity of hyperoperation calculations increases when the number of nested sets increases. As the order of power sets increases, the number of sets to which we associate elements becomes larger and larger. Another limitation we have in this matter is finding a simple algorithm based on which we can find the relationship of elements with a power set. We hope that these results are helpful for further studies in fuzzy logic superhyperalgebras. In our future studies, we hope to obtain more results regarding neutrosophic superhyperalgebras, categorical superhyperalgebras, fundamental relations on superhyperalgebras, and their applications.

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