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SOME REMARKS ON LOCALLY NOETHERIAN AND LOCALLY ARTINIAN S-ACTS OVER MONOIDS

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Abstract. In the category of S-acts, artinian S-acts are introduced as the acts that satisfy the descending chain condition on its congruences. Noetherian S-acts are also the acts that satisfy the ascending chain condition on its congruences. Rees noetherian and Rees artinian case is defined using Rees congruence. This paper studies the classes of locally artinian, locally artinian, locally Rees artinian and locally Rees artinian S-acts. An S-act is said to be locally artinian if all its finitely generated subacts are artinian. The noetherian, Rees noetherian and Rees artinian cases are defined similarly. We give some general properties of these classes of S-acts, specially discuss the behaviour of such acts under Rees short exact sequence and coproduct. Finally, we establish some connections between some classes of S-acts such as projectivity and injectivity with the notions related to locally artinian and locally noetherian.
Keywords: S-acts, algebras, categories.

1. Introduction

An important and interesting discussion of module theory is to study the properties of noetherian and artinian modules which have been extensively studied since 1959. For noetherian and artinian modules we refer, for example, to [1]. A module

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is called *noetherian* if every ascending chain of submodules stabilizes. Additionally, an artinian module can be defined. The study of chain conditions for semigroups was initiated in [4]. Then the structure of some special semigroups satisfying ACC or DCC were studied by Kozhukhov in [9]. Then, descending and ascending chain conditions on commutative monoids with zero are discussed in [3]. Also, in [11], a semigroup S is introduced *right noetherian* if every right congruence on S is finitely generated.

In the category of S-acts, the concepts of finitely cogenerated and finitely Rees cogenerated S-acts are investigated in [7]. In particular, noetherian, artinian, Rees noetherian and Rees artinian S-acts are studied in [8]. An S-act is said to be *artinian* if it satisfies descending chain condition on its congruences, and it is called *Rees artinian* if it satisfies descending chain condition on its Rees congruences. An S-act is said to be *Rees noetherian* if it satisfies ascending chain condition on its congruences, and similarly *Rees noetherian* is defined using Rees congruences. It is also shown that an S-act is artinian if and only if each of its factor acts is finitely cogenerated.

Recall that, from [12], a module is called *locally noetherian* if all its finitely generated submodules are noetherian, and it is called *locally artinian* if all its finitely generated submodules are artinian. Motivated by this, we restrict our attention to finitely generated subacts, and we introduce the concepts of locally noetherian and locally artinian S-acts. It is structured as follows. In Section 2, we review all concepts and basic properties of S-acts which will be needed in this paper. In Section 3, we introduce the notions of locally artinian, locally noetherian, locally Rees artinian and locally Rees noetherian S-acts. We say that an S-act is *locally artinian* if every its finitely generated subact is artinian. The other notions can be defined similarly. Then, we study some general properties of such acts. Moreover, we show that these notions are carried over under Rees short exact sequences. In Section 4, we establish some homological classifications on monoids using these classes of S-acts.

2. Preliminaries

In this section we are going to recall some concepts and known results concerning chain conditions in the category of S-act.

Throughout the paper, S and A_S are used to denote a monoid and a right S-act, respectively. A nonempty set A is called a *right S-act*, usually denoted A_S, if S acts on A unitarily from the right; that is, there exists a mapping $A\times S \to A$, $(a, s) \mapsto as$, satisfying the conditions $(as)t = a(st)$ and $a1 = a$, for all $a \in A$ and all $s, t \in S$. A nonempty subset B of a right S-act A_S is called a *subact* of A if $bs \in B$ for all $s \in S$ and $b \in B$. Recall that $\Theta = \{\theta\}$ with the action $\theta s = \theta$ for all $s \in S$ is called *the one element S-act*. An S-act is said to be *simple* if it has no subacts other than itself, and it is called *θ-simple* if it has no subacts other than itself and the one-element subact Θ. Also, by [5, Proposition 2.1.8], coproducts of non-empty families of S-acts are their disjoint unions.

Recall that for groups and rings, the consideration of normal subgroups and ideals, respectively, avoids the direct use of congruences. However, in the category of S-acts their congruences are not defined by special subacts, and so we have to use congruence for the desired characterizations. Recall from [5, Definition 2.4.18.] that an equivalence relation ρ on an S-act A_S is said to be a *congruence* on A_S if $a\ \rho\ a'$ implies $as\ \rho\ a's$ for any $a, a' \in A_S$ and $s \in S$. The set of all congruences on A_S is denoted by $Con(A_S)$. In the category of S-acts only Rees congruences are defined by subacts. For a subact B_S of A_S defines the *Rees congruence* ρ_B on A, by setting $a\ \rho_B\ a'$ if $a, a' \in B$ or $a = a'$. Example 2.5.3 of [5] shows that not all cyclic acts are Rees factor acts.

For more basic information on S-acts, we refer the reader to [5].

Now, we recall from [7, 8] some preliminaries related to finitely (Rees) cogenerated, (Rees) artinian and (Rees) noetherian S-acts which will be needed in the characterizations of locally (Rees) artinian and locally (Rees) noetherian S-acts.

Definition 2.1. ([7]) Let A_S be an S-act.

(i) We say that A_S is *finitely cogenerated* if for every monomorphism $A_S \xrightarrow{f} \prod_{i\in I} A_i$,
$$A_S \xrightarrow{f} \prod_{i\in I} A_i \xrightarrow{\pi} \prod_{j\in J} A_j$$
is also a monomorphism for some finite subset J of I. Equivalently, for any family of congruences $\{\rho_i |\ i \in I\}$ on A_S, if $\cap_{i\in I}\rho_i = \Delta_A$, then $\cap_{j\in J}\rho_j = \Delta_A$ for some finite subset J of I.

(ii) We call A_S *finitely Rees cogenerated* whenever for any family of Rees congruences $\{\rho_{B_i} |\ i \in I\}$ on A_S, if $\cap_{i\in I}\rho_{B_i} = \Delta_A$, then $\cap_{j\in J}\rho_{B_j} = \Delta_A$ for some finite subset J of I.

Definition 2.2. ([8]) Let A_S be an S-act.

(i) We call A_S *artinian* (*noetherian*) if $Con(A_S)$ satisfies the descending (ascending) chain condition.

(ii) We say that A_S is *Rees artinian* (*Rees noetherian*) if it satisfies the descending (ascending) chain condition on its Rees congruences.

Theorem 2.1. ([8]) *For an S-act A_S, the following statements are equivalent.*

(i) *The S-act A_S is noetherian.*

(ii) *Every congruence of A_S is finitely generated.*

(iii) *Every non-empty subset of $Con(A_S)$ contains a maximal element.*

The following result characterizes artinian S-acts.

Theorem 2.2. ([8]) *For an S-act A_S, the following statements are equivalent.*

(i) *The S-act A_S is artinian.*

(ii) *Every factor act of A_S is finitely cogenerated.*

(iii) *Every non-empty subset of $Con(A_S)$ contains a minimal element.*

3. General Properties locally (Rees) noetherian and locally (Rees) artinian S-acts

In this section, we introduce locally (Rees) noetherian and locally (Rees) artinian S-acts and study several properties of such S-acts.

Definition 3.1. Let A_S be an S-act.

(i) We call A_S *locally noetherian* (*locally artinian*) if every finitely generated subact of A_S is noetherian (artinian).

(ii) We say that A_S is *locally Rees noetherian* (*locally Rees artinian*) if every finitely generated subact of A_S is Rees noetherian (Rees artinian).

(iii) S is called a *right locally* (*Rees*) *noetherian* monoid if the S-act S_S as a right S-act is locally (Rees) noetherian. Dually, *right locally* (*Rees*) *artinian* can be defined.

Clearly, all (Rees) noetherian and (Rees) artinian S-acts are locally (Rees) noetherian and locally (Rees) artinian, respectively. But the following example shows that the converse is not true. However, every finitely generated locally (Rees) noetherian or locally (Rees) artinian S-act is (Rees) noetherian or (Rees) artinian, respectively. So for a monoid S, the notions of right locally (Rees) noetherian and right (Rees) noetherian are the same, similarly, locally (Rees) artinian and (Rees) artinian.

Example 3.1. Let $S = (\mathbb{N}, \min)^{\varepsilon} = (\mathbb{N}, \min)\dot{\cup}\{\varepsilon\}$, where ε denotes the externally adjoint identity. Then, $K_S = S \setminus \{\varepsilon\}$ is a right ideal of S. The subacts of K_S are $1S \subseteq 2S \subseteq 3S \subseteq \ldots$. Hence, K_S is locally noetherian (locally artinian). In [7, Example 3.4], it is shown that K is not finitely cogenerated. Hence, using Theorem 3.4, K is neither artinian nor noetherian.

Let A_S be an act and $a \in A_S$. Then by λ_a we denote the homomorphism from S_S into A_S defined by $\lambda_a(s) = as$.

Proposition 3.1. *For an S-act A_S the following assertions are equivalent.*

(i) *A_S is locally* (*Rees*) *noetherian.*

(ii) *Every cyclic subact of A_S is* (*Rees*) *noetherian.*

(iii) *S/λ_a is (Rees) noetherian for each $a \in A$.*

Proof. (i) $\Rightarrow$ (ii). This is clear.

The implication (ii) $\Rightarrow$ (iii) comes from the isomorphism of S-acts $S/\lambda_a \cong aS$ for each $a \in A$.

(ii) $\Rightarrow$ (i). Let B_S be a finitely generated subact of A_S. Let $B_S = < b_1, b_2, ..., b_n >$. By assumption, b_iS is noetherian for each $1 \leq i \leq n$. Take $B_i = b_1S \cup ... \cup b_iS$. Define $f_i : b_iS \to B_i/B_{i-1}$ by $f(b_is) = [b_is]_\rho$ where $\rho = \rho_{B_{i-1}}$ and $i = 2, ..., n$. Clearly, f_i is an epimorphism, and so by [8, Lemma 2.1], B_i/B_{i-1} is noetherian. Then $B_1 \subseteq B_2 \subseteq ... \subseteq B_n = B_S$ such that B_1 and the factor S-acts B_{i+1}/B_i are noetherian for all $1 \leq i \leq n-1$. Therefore, using [8, Lemma 2.2], B_S is noetherian. The locally Rees noetherian case is proved similarly. □

Now, using Theorems 3.2 and 3.4 and the previous proposition we have the following results.

Proposition 3.2. *For an S-act A_S, the following statements are equivalent.*

(i) *The S-act A_S is locally noetherian.*

(ii) *For every $a \in A_S$ and every congruence $\rho \subseteq \rho_{aS}$, ρ is finitely generated.*

(iii) *For every $a \in A_S$, every non-empty subset of $Con(aS)$ contains a maximal element.*

By a similar argument, the following result characterizes locally (Rees) artinian S-acts.

Proposition 3.3. *For an S-act A_S the following assertions are equivalent.*

(i) *A_S is locally (Rees) artinian.*

(ii) *Every cyclic subact of A_S is (Rees) artinian.*

(iii) *S/λ_a is (Rees) artinian for each $a \in A$.*

Proposition 3.4. *For an S-act A_S, the following statements are equivalent.*

(i) *The S-act A_S is locally artinian.*

(ii) *For every $a \in A_S$, every factor act of aS is finitely cogenerated.*

(iii) *For every $a \in A_S$, every non-empty subset of $Con(aS)$ contains a minimal element.*

The following result presents that the class of locally (Rees) noetherian (locally (Rees) artinian) S-acts is closed under subacts and homomorphic image.

Lemma 3.1. *For a monoid S, the following statements are true.*

(i) *Every subact of a (Rees) locally noetherian (artinian) S-act is (Rees) locally noetherian (artinian).*

(ii) *Every factor act of a (Rees) locally noetherian (artinian) S-act is (Rees) locally noetherian (artinian).*

Recall that the class of all S-acts generated by A_S is denoted by $Gen(A_S)$.

Proposition 3.5. *For an S-act A_S the following assertions are equivalent.*

(i) *A_S is locally noetherian (locally artinian).*

(ii) *$\coprod_I A$ is locally noetherian (locally artinian), for an arbitrary set I.*

(iii) *Every S-act in $Gen(A_S)$ is locally noetherian (locally artinian).*

(iv) *Every finitely generated S-act in $Gen(A_S)$ is locally noetherian (locally artinian).*

(v) *$A \coprod \Theta$ is locally noetherian (locally artinian).*

The previous proposition is also valid for the cases of locally Rees noetherian and locally Rees artinian.

Let $f : A_S \longrightarrow B_S$ and $g : B_S \longrightarrow C_S$ be S-morphisms. Recall from [2, 10] that the sequence $A_S \xrightarrow{f} B_S \xrightarrow{g} C_S$ is said to be a *Rees short exact sequence* if f is one-to-one, g is onto, and $\ker g = \mathcal{K}_{\mathrm{Imf}}$, where $\mathcal{K}_{\mathrm{Imf}} = (f(A) \times f(A)) \cup \Delta_B$. The following theorem discusses the behavior of the properties of being (Rees) locally artinian and noetherian for Rees short exact sequences.

Theorem 3.1. *Let $A_S \xrightarrow{f} B_S \xrightarrow{g} C_S$ be a Rees short exact sequence of S-acts. The following holds.*

(i) *If B_S is locally (Rees) noetherian, then both A_S and C_S are locally (Rees) noetherian.*

(ii) *If A_S is (Rees) noetherian and C_S is locally (Rees) noetherian, then B_S is (Rees) locally noetherian.*

Proof. Part (i) is clear by Lemma 3.1.

(ii). Let A_S be (Rees) noetherian, C_S is (Rees) locally noetherian, and B'_S be a finitely generated subact of B_S. Then, $A'_S \longrightarrow B'_S \longrightarrow C'_S$ is a Rees short exact sequence of S-acts, where $A'_S = \{a \in A |\ f(a) \in B'_S\}$ and $C'_S = g(B')$. Since A'_S and C'_S are (Rees) noetherian, by [8, Theorem 2.3], B'_S is also (Rees) noetherian. □

Similarly, one can prove the following theorem.

Theorem 3.2. *Let $A_S \xrightarrow{f} B_S \xrightarrow{g} C_S$ be a Rees short exact sequence of S-acts. The following hold.*

(i) *If B_S is locally (Rees) artinian, then both A_S and C_S are locally (Rees) artinian.*

(ii) *If A_S is (Rees) artinian and C_S is locally (Rees) artinian, then B_S is locally (Rees) artinian.*

We know from [2] that for a monoid S with zero, $A_1 \longrightarrow A_1 \coprod A_2 \longrightarrow A_2$ is a Rees short exact sequences of S-acts. Note that in the category S-Act_0, $A_1 \coprod A_2 = A_1 \cup A_2$, where $A_1 \cap A_2 = \Theta$. In the next proposition, using [8, Proposition 2.4], we discuss the behavior of locally (Rees) artinian and noetherian S-acts with zero under coproducts.

Proposition 3.6. *Suppose S is a monoid which contains a zero. If A_i, $i \in I$ are S-acts, then $A = \coprod_{i\in I} A_i$ is locally (Rees) noetherian if and only if each A_i, $1 \leq i \leq n$, is locally (Rees) noetherian.*

The above result is also valid for locally artinian and locally Rees noetherian.

4. Homological Classifications

In this section, we provide some classifications of monoids over which each of (Rees) locally noetherian and (Rees) locally artinian implies a specific property, and vice versa. Subacts of a finitely generated S-act need not be finitely generated. For example, let $S = (\mathbb{N}, .)$, $A_S = \mathbb{N} \setminus \{1\}$. Clearly, S_S is cyclic but $A_S \subseteq S_S$ and A_S is not finitely generated. We prove in the next result that when S is a right (Rees) noetherian monoid, being finitely generated implies all of its subacts are finitely generated.

Proposition 4.1. *For a monoid S the following are equivalent.*

(i) *S is a right (Rees) noetherian monoid.*

(ii) *Every finitely generated S-act is (Rees) noetherian.*

(iii) *Every subact of a finitely generated S-act is finitely generated.*

(iv) *Every S-act is (Rees) locally noetherian.*

Proof. (i) $\Rightarrow$ (ii) is clear.

(i) $\Rightarrow$ (ii) Suppose an S-act A_S is finitely generated, so A_S is a factor S-act of some $\coprod_{i=1}^{i=n} S$ for some $n \in \mathbb{N}$. The S-act $\coprod_{i=1}^{i=n} S$ is (Rees) noetherian by [8, Proposition 2.2] and every factor S-act of $\coprod_{i=1}^{i=n} S$ is (Rees) noetherian by [8, Lemma 2.1]. Thus A_S is (Rees) noetherian. The other implications are clear. $\square$

Analogously to the way of the previous proof, we provide [8, Proposition 3.1] as follows.

Proposition 4.2. *For a monoid S, the following are equivalent.*

(i) *S is a right (Rees) artinian monoid.*

(ii) *Every finitely generated S-act is (Rees) artinian.*

(iii) *Every S-act is (Rees) locally artinian.*

Now, we give some characterizations of several classes of monoids in terms of locally noetherian S-acts and locally artinian S-acts.

Theorem 4.1. *For a monoid S, the following statements are equivalent.*

(i) *Every injective S-act is locally noetherian (locally artinian).*

(ii) *Every projective S-act is locally noetherian (locally artinian).*

(iii) *Every S-act is locally noetherian (locally artinian).*

(iv) *The monoid S is a right noetherian (artinian).*

(v) *There exists a generator locally noetherian (locally artinian) S-act.*

Proof. (v)$\Rightarrow$ (iii). Suppose that a generator G is (Rees) locally noetherian (artinian), and that B is an S-act. Then there exists an epimorphism $f : \coprod_{i\in I} G \to B$. Now, applying part (ii) of Proposition 3.5 and Lemma 3.1, we deduce that B is (Rees) locally noetherian (artinian).

The other implications are clear. □

The previous theorem is also valid for locally Rees artinian and locally Rees noetherian.

Definition 4.1. Over a monoid S an S-act A_S is called *unfaithful* if $\cap_{a\in A} \ker \lambda_a$ is a non-diagonal congruence on S.

The *right annihilator* of A_S is defined by

$$R_S(A) = \{(s,t) \in S \times S |\ as = at,\ \text{for all a} \in \text{A}\} = \cap_{\text{a}\in\text{A}} \ker \lambda_\text{a},$$

which is a two-sided congruence on S. So A_S a unfaithful S-act in case $R_S(A) \neq \Delta_S$.

Proposition 4.3. *For a monoid S, the following statements are equivalent.*

(i) *Every unfaithful S-act is (Rees) locally noetherian.*

(ii) *The monoid S is a right (Rees) noetherian.*

Proof. (i) $\Rightarrow$ (ii). Let $\rho \neq \Delta_S$ be a right congruence of S and let $v \neq u \in I$. Let $(u, v) \in \rho$ and $u \neq v$, so $\sigma = \rho(u, v) \subseteq \rho$. Since $(u, v) \in R_S(S/\sigma) \neq \Delta_S$, then S/σ is a locally noetherian S-act. Take $\rho/\sigma = \{([s]_\sigma, [t]_\sigma)|\ [s]_\rho = [t]_\rho\}$ which is obviously a congruence on S/σ. Using [8, Theorem 2.1.], ρ/σ is a finitely generated congruence. Now, since σ and ρ/σ are finitely generated, it is easy to see that ρ is also finitely generated. Therefore, by [8, Theorem 2.1.], S is a right noetherian monoid.

For the case Rees noetherian, it is enough to let I be a non-zero right ideal of S and let $v \neq u \in I$. Take $J = uS \cup vS$ instead of σ, and I instead of ρ.

(ii) $\Rightarrow$ (i). This follows from Theorem 4.1. □

Proposition 4.4. *For a monoid S, the following statements are equivalent.*

(i) *Every unfaithful S-act is* (*Rees*) *locally artinian.*

(ii) *The monoid S is a right* (*Rees*) *artinian.*

Proof. (i) $\Rightarrow$ (ii). Let $\rho \neq \Delta_S$ be a right congruence of S and let $v \neq u \in I$. Let $(u, v) \in \rho$ and $u \neq v$, so $\sigma = \rho(u, v) \subseteq \rho$. Since $(u, v) \in R_S(S/\sigma) \neq \Delta_S$, then $B_S = S/\sigma$ is a locally artinian, and so artinian S-act. Take $\theta = \rho/\sigma = \{([s]_\sigma, [t]_\sigma)|s\ \rho\ t\}$ which is obviously a congruence on B. Using [8, Theorem 2.2.], B/θ is a finitely cogenerated. Now, to show that S/ρ is finitely cogenerated, suppose $\cap_{i\in I}\rho_i = \rho$. Let $\theta_i = \rho_i/\sigma$. Clearly, $\cap_{i\in I}\theta_i = \theta$, and since B/θ is finitely cogenerated, $\cap_{j\in J}\theta_j = \theta$ for some finite subset J of I. Now, $\cap_{j\in J}\rho_j = \rho$ for some finite subset J of I. Therefore, by [8, Theorem 2.2.], S is a right artinian monoid.

(ii) $\Rightarrow$ (i). This follows from Theorem 4.1. □

The following example illustrates that in general noetherian S-acts need not be projective.

Example 4.1. Let $S = (\mathbb{N}, max)$. Then Θ_S is noetherian. But it is not projective since using [5, Proposition 3.17.2(4)], S does not contain a left zero.

The following theorem discusses when Rees noetherian or Rees artinian imply projective. Recall that a right S-act is called *local* if it contains exactly one maximal subact. A monoid S is also called *right* (*left*) *local* if it contains exactly one maximal right (left) ideal.

Theorem 4.2. *For a monoid S, the following statements are equivalent.*

(i) *Any Rees noetherian S-act is projective.*

(ii) *Any Rees artinian S-act is projective.*

(iii) $S = \Theta$ *or* $S = G \coprod \Theta$ *where G is a group.*

Proof. (i) $\Rightarrow$ (iii) and (ii) $\Rightarrow$ (iii). By [6, Remark 4.2], a monoid S is a group or right local. Since Θ is Rees noetherian and artinian, then Θ is projective. So S contains a left zero. If S is not one element, S is right local, then $M = \{s \in S \mid s \text{ is not right invertible}\}$ is the maximal right ideal of S. Then S/M is a θ-simple S-act, and so it is Rees noetherian and artinian. By assumption S/M is projective. Now, the following diagram

$$\begin{array}{ccccc} & & S/M & & \\ & & \big\downarrow{\scriptstyle \iota} & & \\ S & \xrightarrow{\pi} & S/M & \longrightarrow & 0 \end{array}$$

can be extended commutatively by an S-morphism $h : S/M \to S$. Now, from $\pi h = \iota$, we imply that $M = \Theta$, and so S is a group with zero.

(iii) $\Rightarrow$ (i). It follows by [5, Theorem IV.11.13]. $\square$

5. Conclusion

Locally artinian, locally noetherian S-acts and other related concepts were discussed in this paper. Some properties of S-acts are introduced. We also discuss when a subact of a finitely generated S-act is finitely generated. Then, we studied some relations with others concepts in general. Specially, we studied the relationships between locally artinian or locally noetherian S-acts and two important concepts projective and injective S-acts.

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