

ON A TYPE OF SEMI-GENERALIZED RECURRENT P -SASAKIAN MANIFOLDS

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Abstract. In the present paper we study some geometrical properties of semi-generalized recurrent P -Sasakian manifolds.

keywords: Semi-generalized recurrent manifold, P -Sasakian manifolds, Conircular curvature tensor, Einstein manifold, M -projective curvature tensor.

1. Introduction

The idea of recurrent manifolds was introduced by A.G. Walker in 1950 [16]. On the other hand, De and Guha [3] introduced generalized recurrent manifold with the non-zero 1-form A and another non-zero associated 1-form B . Such a manifold has been denoted by GK_n . If the associated 1-form B becomes zero, then the manifold GK_n reduces to a recurrent manifold introduced by Ruse [12] which is denoted by K_n . In 1977, Adati and Matsumoto [1] defined P -Sasakian and Special Para Sasakian manifolds, which are special classes of an almost para-contact manifold introduced by Sato [13]. Para Sasakian manifolds have been studied by De and Pathak [4], Matsumoto et.al. [8], Matsumoto [9], Shukla and Shukla [14], Singh [15], De and Sarkar [5] and many others.

A Riemannian manifold (M^n, g) is called a semi-generalized recurrent manifold if its curvature tensor R satisfies the condition

$$(1.1) \quad (\nabla_X R)(Y, Z)W = A(X)R(Y, Z)W + B(X)g(Z, W)Y,$$

where A and B are two 1-forms, B is non-zero, P_1 and P_2 are two vector fields such that

$$(1.2) \quad g(X, P_1) = A(X), \quad g(X, P_2) = B(X),$$

for any vector field X and ∇ denotes the operator of covariant differentiation with respect to the metric g .

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Generalizing the notion of recurrency the author Khan [6] introduced the notion of generalized recurrent Sasakian manifold. In the paper [11], B. Prasad introduced the notion of semi-generalized recurrent manifold and obtained some interesting results. Recently Rajesh Kumar et.al. [7] studied semi generalized recurrent LP -Sasakian manifolds. Motivated by the above studies, in this paper we extend the study of semi-generalized recurrent to Para-Sasakian manifolds. The paper is organized as follows: Section 2, consist the basic definitions of P -Sasakian and Einstein manifolds. In Section 3, we studied semi-generalized recurrent P -Sasakian manifolds. In section 4, we prove that a semi-generalized ϕ -recurrent P -Sasakian manifold is an Einstein manifold. Section 5 is devoted to the study of semi-generalized concircular ϕ -recurrent P -Sasakian manifolds. Section 6 is about the study of M -projective ϕ -recurrent P -Sasakian manifolds respectively. In the last section we studied the three-dimensional locally semi-generalized ϕ -recurrent P -Sasakian manifolds.

2. Introduction

An n -dimensional differentiable manifold M^n is a Para-Sasakian (briefly P -Sasakian) manifold if it admits a $(1,1)$ tensor field ϕ , a contravariant vector field ξ , a covariant vector field η , and a Riemannian metric g , which satisfy

$$(2.1) \quad \phi^2 X = X - \eta(X)\xi, \quad g(X, \xi) = \eta(X), \quad \phi\xi = 0,$$

$$(2.2) \quad g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y),$$

$$(2.3) \quad (\nabla_X \phi)Y = -g(X, Y)\xi - \eta(Y)X + 2\eta(X)\eta(Y)\xi,$$

$$(2.4) \quad \nabla_X \xi = \phi X,$$

$$(2.5) \quad (\nabla_X \eta)(Y) = g(\phi X, Y) = g(\phi Y, X),$$

for any vector fields X and Y , where ∇ denotes covariant differentiation with respect to g ([1],[13]).

It can be seen that in a P -Sasakian manifold M^n with the structure (ϕ, ξ, η, g) , the following relations hold:

$$(2.6) \quad \eta(\xi) = 1, \quad \eta(\phi X) = 0,$$

$$(2.7) \quad \text{rank}(\phi) = (n - 1).$$

Further in a P -Sasakian manifold the following relations also hold:

$$(2.8) \quad \eta(R(X, Y)Z) = g(X, Z)\eta(Y) - g(Y, Z)\eta(X),$$

$$(2.9) \quad R(X, Y)\xi = \eta(X)Y - \eta(Y)X,$$

$$(2.10) \quad R(\xi, X)Y = \eta(Y)X - g(X, Y)\xi,$$

$$(2.11) \quad R(\xi, X)\xi = X - \eta(X)\xi,$$

$$(2.12) \quad Q\xi = -(n-1)\xi,$$

$$(2.13) \quad S(X, \xi) = -(n-1)\eta(X),$$

$$(2.14) \quad S(\phi X, \phi Y) = S(X, Y) + (n-1)\eta(X)\eta(Y),$$

for any vector fields X, Y, Z , where R and S are the Riemannian curvature tensor and Ricci tensor of the manifold respectively.

A P -Sasakian manifold M^n is said to be Einstein if the Ricci tensor S is of the form

$$(2.15) \quad S(X, Y) = \lambda g(X, Y),$$

where λ is a constant.

3. Semi-Generalized Recurrent P -Sasakian manifolds

Definition 3.1. A Riemannian manifold (M^n, g) is semi-generalized Ricci recurrent manifold ([3],[2]) if

$$(3.1) \quad (\nabla_X S)(Y, Z) = A(X)S(Y, Z) + nB(X)g(Y, Z).$$

Theorem 3.1. The scalar curvature r of a semi-generalized recurrent P -Sasakian manifold is related in terms of contact forms $\eta(P_1)$ and $\eta(P_2)$ as given by

$$r = -\frac{1}{\eta(P_1)} \left[(n^2 + 2)\eta(P_2) + 2(n-1)\eta(P_1) \right].$$

Proof. Permutting equation (1.1) twice with respect to X, Y, Z ; adding the three equations and using Bianchi's second identity, we have

$$(3.2) \quad \begin{aligned} & A(X)R(Y, Z)W + B(X)g(Z, W)Y + A(Y)R(Z, X)W \\ & + B(Y)g(X, W)Z + A(Z)R(X, Y)W + B(Z)g(Y, W)X = 0. \end{aligned}$$

Contracting (3.2) with respect to Y , we get

$$(3.3) \quad \begin{aligned} & A(X)S(Z, W) + nB(X)g(Z, W) - g(R(Z, X)P_1, W) \\ & + B(Z)g(X, W) - A(Z)S(X, W) + B(Z)g(X, W) = 0. \end{aligned}$$

In view of $S(Y, Z) = g(QY, Z)$, the equation (3.3) reduces to

$$(3.4) \quad \begin{aligned} & A(X)g(QZ, W) + nB(X)g(Z, W) - g(R(Z, X)P_1, W) \\ & + B(Z)g(X, W) - A(Z)g(QX, W) + B(Z)g(X, W) = 0. \end{aligned}$$

Factoring off W , we get from (3.4)

$$(3.5) \quad \begin{aligned} & A(X)QZ + nB(X)Z - R(Z, X)P_1 \\ & + B(Z)X - A(Z)QX + B(Z)X = 0. \end{aligned}$$

Contracting (3.5) with respect to Z , we get

$$(3.6) \quad A(X)r + (n^2 + 2)B(X) - 2S(X, P_1) = 0.$$

Putting $X = \xi$ in the equation (3.6) and using the equations (1.2) and (2.13), we get

$$r = -\frac{1}{\eta(P_1)} \left[(n^2 + 2)\eta(P_2) + 2(n - 1)\eta(P_1) \right].$$

This completes the proof. \square

Theorem 3.2. *In a semi-generalized Ricci-recurrent P-Sasakian manifold, the 1-form A and B are related as*

$$-(n - 1)A(X) + nB(X) = 0.$$

Proof. Taking $Z = \xi$ in (3.1), we have

$$(3.7) \quad (\nabla_X S)(Y, \xi) = A(X)S(Y, \xi) + nB(X)g(Y, \xi).$$

The left hand side of (3.7), clearly can be written in the form

$$(\nabla_X S)(Y, \xi) = \nabla_X S(Y, \xi) - S(\nabla_X Y, \xi) - S(Y, \nabla_X \xi),$$

which in view of (2.4), (2.5) and (2.13) gives

$$-(n - 1)g(Y, \phi X) - S(Y, \phi X).$$

While the right hand side of (3.7) equals

$$A(X)S(Y, \xi) + nB(X)g(Y, \xi) = -(n - 1)A(X)\eta(Y) + nB(X)\eta(Y).$$

Hence,

$$(3.8) \quad -(n - 1)g(Y, \phi X) - S(Y, \phi X) = -(n - 1)A(X)\eta(Y) + nB(X)\eta(Y).$$

Putting $Y = \xi$ in (3.8) and then using (2.1), (2.6) and (2.13), we get

$$-(n - 1)\eta(\phi X) + (n - 1)\eta(\phi X) = -(n - 1)A(X) + nB(X),$$

Or,

$$(3.9) \quad -(n - 1)A(X) + nB(X) = 0.$$

\square

Theorem 3.3. *If a semi-generalized Ricci-recurrent P -Sasakian manifold is an Einstein manifold then 1-forms A and B are related as $\lambda A(Y) + nB(Y) = 0$.*

Proof. For an Einstein manifold, we have $S(Y, Z) = \lambda g(Y, Z)$ and $(\nabla_U S) = 0$, where λ is constant.

Hence from (3.6) we have

$$(3.10) \quad \begin{aligned} & [\lambda A(X) + nB(X)]g(Y, Z) + [\lambda A(Y) + nB(Y)]g(Z, X) \\ & + [\lambda A(Z) + nB(Z)]g(X, Y) = 0. \end{aligned}$$

Replacing Z by ξ in (3.10) and using (1.2) and (2.1), we have

$$(3.11) \quad \begin{aligned} & [\lambda A(X) + nB(X)]\eta(Y) + [\lambda A(Y) + nB(Y)]\eta(X) \\ & + [\lambda\eta(P_1) + n\eta(P_2)]g(X, Y) = 0. \end{aligned}$$

Again, taking $X=Y=\xi$ in (3.11) and using (1.2), (2.1) and (2.6), we have

$$(3.12) \quad [\lambda\eta(P_1) + n\eta(P_2)] = 0.$$

Using (1.2) and (2.1) in the above relation, it follows that

$$\lambda A(Y) + nB(Y) = 0.$$

□

4. Semi-Generalized ϕ -Recurrent P -Sasakian manifolds

Definition 4.1. A P -Sasakian manifold (M^n, g) is called semi-generalized ϕ recurrent if its curvature tensor R satisfies the condition

$$(4.1) \quad \phi^2((\nabla_W R)(X, Y)Z) = A(W)R(X, Y)Z + B(W)g(Y, Z)X,$$

where A and B are two 1-forms, B is non-zero and these are defined by

$$(4.2) \quad A(W) = g(W, P_1), \quad B(W) = g(W, P_2)$$

and P_1 and P_2 are vector fields associated with 1-forms A and B , respectively.

Theorem 4.1. *A semi generalized ϕ -recurrent P -Sasakian manifold (M^n, g) is an Einstein manifold and moreover; the 1-forms A and B are related as $(n - 1)A(W) = nB(W)$.*

Proof. Let us consider a semi-generalized ϕ -recurrent P -Sasakian manifold. Then by virtue of (2.1) and (4.1) we have

$$(4.3) \quad \begin{aligned} & (\nabla_W R)(X, Y)Z - \eta((\nabla_W R)(X, Y)Z)\xi \\ & = A(W)R(X, Y)Z + B(W)g(Y, Z)X. \end{aligned}$$

From which it follows that

$$(4.4) \quad \begin{aligned} & g((\nabla_W R)(X, Y)Z, U) - \eta((\nabla_W R)(X, Y)Z)\eta(U) \\ &= A(W)g(R(X, Y)Z, U) + B(W)g(Y, Z)g(X, U). \end{aligned}$$

Let $\{e_i\}, i = 1, 2, \dots, n$ be an orthonormal basis of the tangent space at any point of the manifold. Then putting $X = U = e_i$ in (4.4) and taking summation over $i, 1 \leq i \leq n$, we get

$$(4.5) \quad \begin{aligned} (\nabla_W S)(Y, Z) &= \sum_{i=1}^n \eta((\nabla_W R)(e_i, Y)Z)\eta(e_i) \\ &= A(W)S(Y, Z) + nB(W)g(Y, Z). \end{aligned}$$

The second term of left hand side of (4.5) by putting $Z = \xi$ takes the form $g((\nabla_W R)(e_i, Y)\xi, \xi)$, which is zero in this case. So, by replacing Z by ξ in (4.5) and using (2.13), we get

$$(4.6) \quad (\nabla_W S)(Y, \xi) = -(n-1)A(W)\eta(Y) + nB(W)\eta(Y).$$

We know that

$$(\nabla_W S)(Y, \xi) = \nabla_W S(Y, \xi) - S(\nabla_W Y, \xi) - S(Y, \nabla_W \xi),$$

using (2.4), (2.5) and (2.13) in the above relation, it follows

$$(4.7) \quad (\nabla_W S)(Y, \xi) = -(n-1)g(\phi W, Y) - S(\phi W, Y).$$

From (4.6) and (4.7) we obtain

$$(4.8) \quad -(n-1)g(\phi W, Y) - S(\phi W, Y) = -(n-1)A(W)\eta(Y) + nB(W)\eta(Y).$$

Replacing $Y = \xi$ in (4.8) then using (2.1) and (2.6), we get

$$(4.9) \quad (n-1)A(W) = nB(W).$$

Using (4.9) in (4.8), we obtain

$$(4.10) \quad S(Y, \phi W) = -(n-1)g(Y, \phi W).$$

Again, replacing Y by ϕY both sides in the above equation (4.10) and using the equations (2.2) and (2.14), we obtain

$$S(Y, W) = -(n-1)g(Y, W),$$

i.e., the manifold is an Einstein manifold. \square

5. Semi-Generalized Concircular ϕ -Recurrent P -Sasakian manifolds

Definition 5.1. A P -Sasakian manifold (M^n, g) is called semi-generalized concircular ϕ -recurrent if its concircular curvature tensor

$$(5.1) \quad C(X, Y)Z = R(X, Y)Z - \frac{r}{n(n-1)} [g(Y, Z)X - g(X, Z)Y]$$

satisfies the condition

$$(5.2) \quad \phi^2((\nabla_W C)(X, Y)Z) = A(W)C(X, Y)Z + B(W)g(Y, Z)X,$$

where A and B are defined as (4.2) and r is the scalar curvature of the manifold (M^n, g) .

Theorem 5.1. Let (M^n, g) be a semi-generalized concircular ϕ -recurrent P -Sasakian manifold then

$$\left[-(n-1) - \frac{r}{n} \right] A(W) + nB(W) = 0.$$

Proof. Let us consider a semi-generalized ϕ -recurrent P -Sasakian manifold. Then by virtue of (2.1) and (5.2), we have

$$(5.3) \quad \begin{aligned} & (\nabla_W C)(X, Y)Z - \eta((\nabla_W C)(X, Y)Z)\xi \\ & = A(W)C(X, Y)Z + B(W)g(Y, Z)X, \end{aligned}$$

from which it follows that

$$(5.4) \quad \begin{aligned} & g((\nabla_W C)(X, Y)Z, U) - \eta((\nabla_W C)(X, Y)Z)\eta(U) \\ & = A(W)g(C(X, Y)Z, U) + B(W)g(Y, Z)g(X, U). \end{aligned}$$

Let $\{e_i\}, i = 1, 2, \dots, n$ be an orthonormal basis of the tangent space at any point of the manifold. Then putting $Y = Z = e_i$ in (5.4) and taking summation over $i, 1 \leq i \leq n$, we get

$$(5.5) \quad \begin{aligned} (\nabla_W S)(X, U) &= \frac{W(r)}{n} g(X, U) - \frac{W(r)}{n} \eta(X)\eta(U) \\ &+ (\nabla_W S)(X, \xi)\eta(U) + nB(W)g(X, U) \\ &+ \left[S(X, U) - \frac{r}{n} g(X, U) \right] A(W). \end{aligned}$$

Replacing U by ξ in (5.5) and using (2.1) and (2.13), we have

$$(5.6) \quad \left[-(n-1) - \frac{r}{n} \right] A(W)\eta(X) + nB(W)\eta(X) = 0.$$

Putting $X = \xi$ in (5.6), we obtain

$$\left[-(n-1) - \frac{r}{n} \right] A(W) + nB(W) = 0.$$

This completes the proof. \square

Theorem 5.2. *A semi-generalized concircular φ -recurrent P-Sasakian manifold is an Einstein manifold.*

Proof. Putting $X = U = e_i$ in (5.4) and taking summation over i , $1 \leq i \leq n$, we get

$$\begin{aligned}
 (\nabla_W S)(Y, Z) &= \sum_{i=1}^n g((\nabla_W R)(e_i, Y)Z, \xi)g(e_i, \xi) \\
 &+ \frac{W(r)}{n} g(Y, Z) - \frac{W(r)}{n(n-1)} [g(Y, Z) - \eta(Y)\eta(Z)] \\
 (5.7) \quad &+ \left[S(Y, Z) - \frac{r}{n} g(Y, Z) \right] A(W) + nB(W)g(Y, Z).
 \end{aligned}$$

Replacing Z by ξ in (5.7) and using (5.6), we have

$$(5.8) \quad (\nabla_W S)(Y, Z) = \frac{W(r)}{n} \eta(Y).$$

We know that

$$(\nabla_W S)(Y, \xi) = \nabla_W S(Y, \xi) - S(\nabla_W Y, \xi) - S(Y, \nabla_W \xi),$$

using (2.4), (2.5) and (2.13) in the above relation, it follows that

$$(5.9) \quad (\nabla_W S)(Y, \xi) = -(n-1)g(Y, \phi W) - S(Y, \phi W).$$

In view of (5.8) and (5.9), we obtain

$$(5.10) \quad S(Y, \phi W) = -(n-1)g(Y, \phi W) - \frac{W(r)}{n} \eta(Y).$$

Replacing Y by φY in (5.10) then using (2.2), (2.6) and (2.14), we obtain

$$S(Y, W) = -(n-1)g(Y, W).$$

□

6. Semi-generalized M-Projective φ -recurrent P-Sasakian manifolds

Definition 6.1. A P-Sasakian manifold (M^n, g) is called semi-generalized M-projective curvature tensor [10]

$$\begin{aligned}
 W^*(X, Y)Z &= R(X, Y)Z - \frac{1}{2(n-1)} [S(Y, Z)X - S(X, Z)Y \\
 (6.1) \quad &+ g(Y, Z)QX - g(X, Z)QY].
 \end{aligned}$$

satisfies the condition

$$\begin{aligned}
 &((\nabla_V W^*)(X, Y)Z) - \eta((\nabla_V W^*)(X, Y)Z)\xi \\
 (6.2) \quad &= A(V)W^*(X, Y)Z + B(V)g(Y, Z)X,
 \end{aligned}$$

where A and B are defined as (4.2) and r is a scalar curvature of the manifold (M^n, g) .

Theorem 6.1. *Let (M^n, g) be a semi-generalized M-Projective φ -recurrent P-Sasakian manifold then*

$$-\left[\frac{n^2 - n + r}{2(n - 1)}\right]A(V) + nB(V) = 0.$$

Proof. Let us consider a semi-generalized φ -recurrent P-Sasakian manifold. Then by virtue of (2.1) and (6.2), we have

$$(6.3) \quad \begin{aligned} & (\nabla_V W^*)(X, Y)Z - \eta((\nabla_V W^*)(X, Y)Z)\xi \\ &= A(V)W^*(X, Y)Z + B(V)g(Y, Z)X, \end{aligned}$$

from which it follows that

$$(6.4) \quad \begin{aligned} & g((\nabla_V W^*)(X, Y)Z, U) - \eta((\nabla_V W^*)(X, Y)Z)\eta(U) \\ &= A(V)g(W^*(X, Y)Z, U) + B(V)g(Y, Z)g(X, U). \end{aligned}$$

Let $\{e_i\}, i = 1, 2, \dots, n$ be an orthonormal basis of the tangent space at any point of the manifold. Then putting $Y = Z = e_i$ in (6.4) and taking summation over $i, 1 \leq i \leq n$, we get

$$(6.5) \quad \begin{aligned} & \frac{n}{2(n - 1)} (\nabla_V S)(X, U) - \frac{1}{2(n - 1)} V(r)g(X, U) \\ & - \frac{n}{2(n - 1)} (\nabla_V S)(X, \xi)\eta(U) + \frac{1}{2(n - 1)} V(r)\eta(X)\eta(U) \\ &= \left[\frac{n}{2(n - 1)} S(X, U) - \frac{r}{2(n - 1)} g(X, U) \right] A(V) \\ &+ nB(V)g(X, U). \end{aligned}$$

Replacing U by ξ in (6.5) and using (2.1) and (2.6), we have

$$(6.6) \quad -A(V) \left[\frac{n^2 - n + r}{2(n - 1)} \right] \eta(X) + nB(V)\eta(X) = 0.$$

Putting $X = \xi$ in (6.6), we obtain

$$-\left[\frac{n^2 - n + r}{2(n - 1)}\right]A(V) + nB(V) = 0.$$

This completes the proof. \square

Theorem 6.2. *A semi-generalized M-Projective φ -recurrent P-Sasakian manifold is an Einstein manifold.*

Proof. Putting $X = U = e_i$ in (6.4) and taking summation over $i, 1 \leq i \leq n$, we get

$$\begin{aligned}
 \frac{n}{2(n-1)} (\nabla_V S)(Y, Z) &= \sum_{i=1}^n g((\nabla_V R)(e_i, Y)Z, \xi)g(e_i, \xi) \\
 &+ \frac{1}{2(n-1)} V(r)g(Y, Z) \\
 &- \frac{1}{2(n-1)} [(\nabla_V S)(Y, Z)g(\xi, \xi) - (\nabla_V S)(\xi, Z)\eta(Y) \\
 &+ g(Y, Z)(\nabla_V S)(\xi, \xi) - (\nabla_V S)(Y, \xi)\eta(Z)] \\
 &+ \left[\frac{n}{2(n-1)} S(Y, Z) - \frac{r}{2(n-1)} g(Y, Z) \right] A(V) \\
 (6.7) \quad &+ nB(V)g(Y, Z).
 \end{aligned}$$

Replacing Z by ξ in (6.7) and using (2.1), (2.6) and (2.13), we have

$$(6.8) \quad (\nabla_V S)(Y, \xi) = -\frac{1}{n} V(r)\eta(Y).$$

We know that

$$(\nabla_V S)(Y, \xi) = \nabla_V S(Y, \xi) - S(\nabla_V Y, \xi) - S(Y, \nabla_V \xi),$$

using (2.4), (2.5) and (2.13) in above relation, it follows that

$$(6.9) \quad (\nabla_V S)(Y, \xi) = -(n-1)g(Y, \varphi V) - S(Y, \varphi V).$$

In view of (6.8) and (6.9)

$$(6.10) \quad S(Y, \varphi V) = -(n-1)g(Y, \varphi V) + \frac{1}{n} V(r)\eta(Y).$$

Replacing Y by φY in (6.10) then using (2.2) and (2.14), we get

$$S(Y, V) = -(n-1)g(Y, V).$$

This completes the proof. \square

7. Three Dimensional Locally Semi-Generalized φ -recurrent P -Sasakian manifolds

Theorem 7.1. *The curvature tensor of three dimensional semi-generalized φ -recurrent P -Sasakian manifold is given by*

$$R(X, Y, Z) = \left[\frac{dr(e_i)}{2A(e_i)} - \frac{B(e_i)}{A(e_i)} \right] g(Y, Z)X - \frac{dr(e_i)}{2A(e_i)} g(X, Z)Y.$$

Proof. In a three-dimensional Riemannian manifold (M^3, g) , we have

$$(7.1) \quad \begin{aligned} R(X, Y)Z &= g(Y, Z)QX - g(X, Z)QY + S(Y, Z)X \\ &- S(X, Z)Y + \frac{r}{2} [g(X, Z)Y - g(Y, Z)X], \end{aligned}$$

where Q is the Ricci operator, i.e., $S(X, Y) = g(QX, Y)$ and r is the scalar curvature of the manifold. Now putting $Z = \xi$ in (7.1) and using (2.13), we get

$$(7.2) \quad \begin{aligned} R(X, Y)\xi &= \eta(Y)QX - \eta(X)QY + (n-1)[\eta(X)Y - \eta(Y)X] \\ &+ \frac{r}{2} [\eta(X)Y - \eta(Y)X]. \end{aligned}$$

Using (2.9) in (7.2), we have

$$(7.3) \quad \left[(2-n) - \frac{r}{2} \right] [\eta(X)Y - \eta(Y)X] = \eta(Y)QX - \eta(X)QY.$$

Putting $Y = \xi$ in the equation (7.3) and using the equations (2.6) and (2.12), we get

$$(7.4) \quad QX = \left[(3-2n) - \frac{r}{2} \right] \eta(X)\xi - \left[(2-n) - \frac{r}{2} \right] X.$$

Therefore, it follows from (7.4) that

$$(7.5) \quad S(X, Y) = \left[(3-2n) - \frac{r}{2} \right] \eta(X)\eta(Y) - \left[(2-n) - \frac{r}{2} \right] g(X, Y).$$

Thus from (7.1), (7.4) and (7.5), we get

$$(7.6) \quad \begin{aligned} R(X, Y)Z &= \left[2(2-n) - \frac{r}{2} \right] [g(X, Z)Y - g(Y, Z)X] \\ &+ \left[(3-2n) - \frac{r}{2} \right] [g(Y, Z)\eta(X)\xi - g(X, Z)\eta(Y)\xi] \\ &+ \eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y. \end{aligned}$$

Taking the covariant differentiation to the both sides of the equation (7.6), we get

$$(7.7) \quad \begin{aligned} (\nabla_W R)(X, Y)Z &= -\frac{dr(W)}{2} [g(X, Z)Y - g(Y, Z)X + g(Y, Z)\eta(X)\xi \\ &- g(X, Z)\eta(Y)\xi + \eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y] \\ &+ \left[(3-2n) - \frac{r}{2} \right] [g(Y, Z)\eta(X) - g(X, Z)\eta(Y)] (\nabla_W \xi) \\ &+ \left[(3-2n) - \frac{r}{2} \right] [g(Y, Z)\xi - \eta(Z)Y] (\nabla_W \eta)(X) \\ &+ \left[(3-2n) - \frac{r}{2} \right] [\eta(Y)X - \eta(X)Y] (\nabla_W \eta)(Z) \\ &- \left[(3-2n) - \frac{r}{2} \right] [g(X, Z)\xi - \eta(Z)X] (\nabla_W \eta)(Y). \end{aligned}$$

Noting, that we may assume that all vector fields X, Y, Z, W are orthogonal to ξ and using (2.1), we get

$$(7.8) \quad \begin{aligned} (\nabla_W R)(X, Y)Z &= -\frac{dr(W)}{2}[g(X, Z)Y - g(Y, Z)X] \\ &+ \left[(3 - 2n) - \frac{r}{2} \right] [g(Y, Z)(\nabla_W \eta)(X) \\ &- g(X, Z)(\nabla_W \eta)(Y)]\xi. \end{aligned}$$

Applying φ^2 to the both side of (7.8) and using (2.1) and (2.6), we get

$$(7.9) \quad \varphi^2((\nabla_W R)(X, Y)Z) = \frac{dr(W)}{2} [g(Y, Z)X - g(X, Z)Y].$$

By (4.1), the equation (7.9) reduces to

$$A(W)R(X, Y)Z = \left[\frac{dr(W)}{2} - B(W) \right] g(Y, Z)X - \frac{dr(W)}{2} g(X, Z)Y.$$

Putting $W = \{e_i\}$, where $i=1, 2, 3$ is an orthonormal basis of the tangent space at any point of the manifold and taking summation over $i, 1 \leq i \leq 3$, we obtain

$$R(X, Y)Z = \left[\frac{dr(e_i)}{2A(e_i)} - \frac{B(e_i)}{A(e_i)} \right] g(Y, Z)X - \frac{dr(e_i)}{2A(e_i)} g(X, Z)Y.$$

□

8. Conclusion

This paper is all about the study of geometrical properties of a semi generalized recurrent Para-Sasakian manifold. We prove that a semi generalized ϕ -recurrent Para-Sasakian manifold is an Einstein manifold. It is also stabilised that a semi generalized M projective ϕ -recurrent Para-Sasakian manifold and semi generalized concircular ϕ -recurrent Para-Sasakian manifolds are also Einstein manifold.

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