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THE CHARNES, COOPER AND RHODES MODEL AND ITS APPLICATION

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Abstract. The paper is about application of the Data Envelopment Analysis method for combined heat and power plants. The methodology of analyzing complex systems is developed. The efficiency of complex systems based on the Charnes, Cooper and Rhodes model is evaluated. An algorithm for estimating efficiency using the CCR model is presented. Experiments on the use of this algorithm on the model are carried out. Efficiency coefficients of the analyzed combined heat and power plants using the CCR model with input and output orientation have been calculated. The inputs and outputs for achieving the efficiency of the CCPPs facilities with efficiency coefficients less than 1 are calculated.

Keywords: Data Envelopment Analysis, CCR model, model orientation, method, heating system, combined heat and power plants, boiler houses.

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1. Introduction

Therefore, modern methods of evaluation and efficiency improvement become quite popular and in demand among owners, managers and engineers of advanced companies. Often, especially acute issue of complexity of evaluation and efficiency improvement arises for technical systems. So they are characterised by the complexity of functioning of technical processes of such systems, as well as a large set of technical indicators of the system for its evaluation. Therefore, it seems quite relevant to us to consider the application of the modern method of performance evaluation Data Envelopment Analysis (DEA) to evaluate the efficiency of technical systems, in particular, heat supply systems.

The articles [5, 10] consider the basic models of efficiency research. Additional models of efficiency research are considered in [2]. The practical use of basic models of efficiency research is considered in [7, 9]. Description, analysis, differences, disadvantages and advantages of efficiency research methods are presented by us in articles [16, 17]. In this paper, we will consider the application of the basic Charnes, Cooper and Rhodes (CCR) model in improving the efficiency of central heating system firms. The article contains 5 sections. This one spilled the Introduction. The second section is "Basic concepts", which gives a brief idea of the DEA method and the CCR model. The third section "Methodology" will reveal the methodology and algorithm for adapting the CCR model for the combined heat and power plants (CHPPs)

The «Experimental part» will describe the conducted experiments of using the DEA method for CHPPs. The fifth section «Findings» will describe the results of the conducted experiments. In the conclusion of the paper the main conclusions of the research will be presented.

2. Basic concepts

The first to measure the efficiency of a complex system based on input and output in science is considered to be the scientist Farell in 1957 [10]. The model was first described by Charnes, Cooper and Rhodes in 1978 [5]. The model was used to evaluate both technical and economic efficiency [13, 20, 21]. The essence of the model is to evaluate the effectiveness based on the construction of the efficiency boundary of the sample under study due to mathematical programming [9, 18].

Efficient units act as a benchmark for inefficient units and according to this, the method gives settings to improve the efficiency of inefficient units [11]. Based on the efficient units, an efficiency frontier is formed, which is an envelope hyper-surface that is constructed based on the performance of the system [12].

The efficiency line is formed in the multidimensional space based on the solution of the optimization linear programming problem. This boundary contains optimal objects, i.e., those objects that give the best result when compared with other objects in the sample [14, 15]. A more detailed description of the basic concepts can be found in our published articles [16, 17].

3. Methodology

The CCR model was developed one of the first and takes into account the constant return to scale.

In the CCR model, all possible input and output parameters of the studied objects are all possible combinations of scalar input and output parameters [8]. The CCR model considers the objects under study at constant returns to scale. Also this model can have 2 types of orientation: input orientated or output orientated.

Thus, the formula of the input-oriented CCR model taking into account the slack movement is as follows [5]:

$$\phi_0 + \epsilon \left(\sum\nolimits_{j=1}^{S} S_{i=1}^{+} + \sum\nolimits_{i=1}^{r} S_{i=1}^{-} \right) \tag{3.1}$$

ϵ is a non-Archimedean constant (for practical calculations it is replaced by 10^6) to account for the slacks problem during optimization.

By:

$$Y_{j0}\phi_0 - \sum_{m=1}^{n} Y_{jm}\lambda_m + S_r^{+};$$

for all investigated objects $j = 1, \ldots, s$

$$\sum_{m=1}^{n} X_{im}\lambda_m + S_i^{-} = X_{i0};$$

for all investigated objects $m = 1, \ldots, n$:

$$\lambda_m, S_j^{+}, S_i^{-} \geq, m = 1, 2, \ldots, n.$$

The formula of the output-oriented CCR model considering the slack movement is given below [5]:

$$\min \quad \theta_0, \lambda_j - \epsilon \left(\sum_{j=1}^{S} S_j^{+} + \sum_{i=1}^{r} S_i^{-} \right) \tag{3.2}$$

By:

$$\sum_{m=1}^{n} Y_{jm}\lambda_m - S_j^{+} = Y_{x0}$$

for all investigated objects j=1,. . . ,s;

$$X_{i0}\theta_0 - \sum_{m=1}^{n} X_{im}\lambda_m + S_i^{-} = 0$$

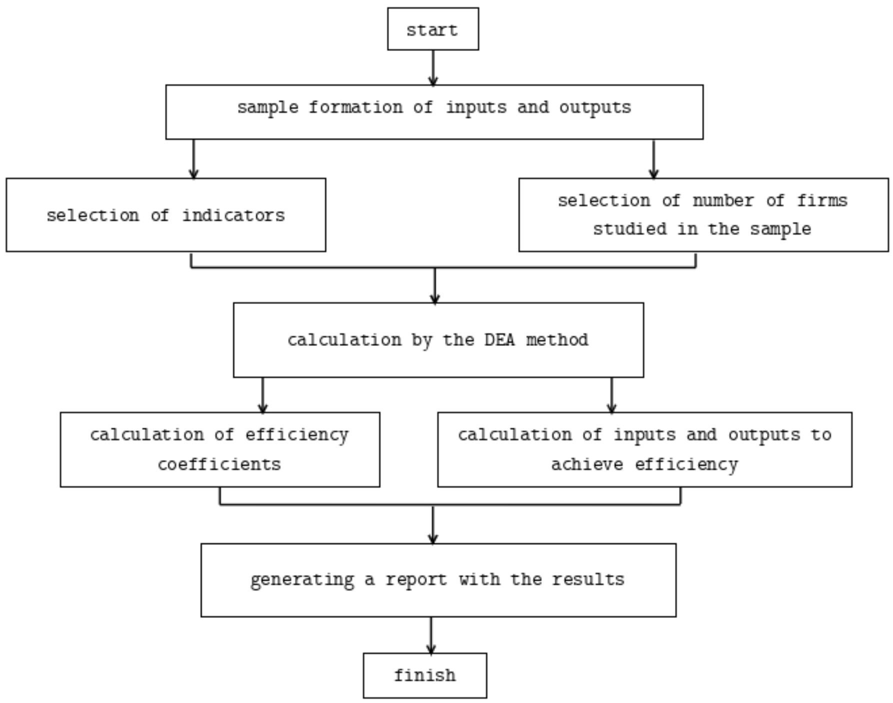

Fig. 3.1: AACCR

for all investigated objects $m = 1, \ldots, n$.

$$\lambda_m, S_j^+, S_i^- \geq 0, \quad m = 1, 2, \ldots, n.$$

These formulas will be used to carry out calculations in the experimental part. Based on the CCR model, the DEA method was further supplemented with many new models and was widely developed and applied [1, 6, 22].

We turn directly to adaptation this model in practice. We present the CCR methodology of the DEA model for the study and improvement of the efficiency of CHPPs. Below, we describe in stages the "algorithm for applying the CCR model" (AACCR). The AACCR is shown in Figure 1.

In this study, we will present the input and output parameters of CHPPs operation, which have not been considered in other papers. We will evaluate the efficiency of CHPP operation, not only from the point of view of heat generation, but also in relation to electricity generation. In this paper we consider CHPPs for which it is reasonable to increase electricity generation.

Further,we will present input and output parameters, which will be used for efficiency calculations in the experimental part (hereinafter referred to as inputs and outputs).

Outputs:

1. heat energy supply to the network (thousand Gcal) - output (Y1);
2. electricity supply (mln kWh) - output (Y2).

Inputs:

1. available heat capacity of the equipment (Gcal/hour) - input (X1);
2. conditional fuel consumption for the supplied heat energy (thousand tonnes of fuel equivalent per year) - input (X2).

The "output-oriented CCR model" (OUTCCR) is used for the purpose of the decision maker (DM) to increase outputs. If the goal of the DM is to reduce inputs, then the "input-oriented CCR model" (INCCR) is used.

4. The experimental part

In this section, we present experiments on the use of OUTCRY and INCCR for different purposes of DM in the management of firms in central heating. The objects of the study are CHPPs. The sample includes 15 CHPPs.

We build the basic CCR model [19]. For the experiments, we have a combination of inputs and outputs: 2 inputs and 2 1 outputs. We perform calculations demonstrating the feasibility of the method in the study area. We present input and output parameters (hereinafter referred to as inputs and outputs). We have described the inputs and outputs in the "Methodology" section. Next, we present calculations using the indicators of these two inputs and two outputs. We used DEEP software for calculations [4]. The initial data of the inputs and outputs are presented in Table 1.

Inputs:

1) available heat capacity of the equipment (Gcal/hour) - input (X1);

2) conditional fuel consumption for the supplied heat energy (thousand tonnes of fuel equivalent per year) - input (X2).

Output:

1) heat energy supply to the network (thousand Gcal) - output (Y1).

If it is important for the decision maker to increase the heat supply to the network, an output-oriented DEA model is used. If the goal for the decision maker is to reduce the input, then an input-oriented DEA model is used.

Next, we conduct experiments under different conditions and objectives for the decision maker. We calculate the efficiency by DEA method in DEAP software [4]. Table 4.1 shows the initial data of input and output indicators.

Table 4.1: Initial data

No. firm	Y1	Y2	X1	X2
1	3208	970	1445	567
2	3726	968	1412	673
3	1123	802	725	345
4	3378	890	1456	567
5	3912	1010	1445	694
6	1786	840	789	376
7	2856	890	1421	435
8	3334	910	1432	589
9	1204	806	712	330
10	2920	930	1254	442
11	3187	1005	1405	462
12	762	810	572	273
13	3302	1040	1254	501
14	3248	1020	1405	470
15	1311	910	572	292

The essence of the experiment 1. The purpose of the DM is to increase the values of the outputs. To do this, we use the OUTCCR model. We are conducting an experiment based on AACCR.

The essence of the experiment 2. The purpose of the DM is to reduce the values of the inputs. To do this, we use the INCCR model. We are conducting an experiment based on AACCR.

5. Findings

We present experiment 1.

We calculate the efficiency coefficients of the studied sample of thermal power plants. We present them in Table 5.1.

This table shows that DMU5, DMU11, DMU13, DMU14 and DMU15 are efficient. This means that their efficiency coefficients are maximised and equal to 1. The efficiency coefficient of the other DMUs is less than 1, respectively, they are not efficient. DMU9 has the lowest efficiency - its efficiency coefficient is 0.794. For the rest of the DMUs, the calculated efficiency coefficients are also shown in Table5.1.

We now calculate the input and output performance of the analyzed DMUs when they achieve efficiency, i.e., an efficiency coefficient equal to 1. The calculations will be presented in Table 5.2.

The table shows the inputs and outputs of the investigated CHPPs in achieving efficiency. These calculations allow us to carry out the OUTCCR model. That is, at the given values of inputs and outputs CHPPs reach the maximum coefficient of

Table 5.1: Efficiency coefficients in Experiment 1.

No. firm	efficiency coefficient
1	0.854
2	0.971
3	0.859
4	0.872
5	1.000
6	0.959
7	0.965
8	0.894
9	0.794
10	0.974
11	1.000
12	0.952
13	1.000
14	1.000
15	1.000

Table 5.2: The calculation

No. firm	Y1	Y2	X1	X2
1	3872.218	1219.135	1497.000	584
2	3940.669	1034.571	1405	673
3	1448.853	934.067	633.006	308
4	3846.667	1180.299	1447	595
5	3986.000	1010.000	1405	694
6	1792.414	875.874	712	358.597
7	2999.217	941.872	1297.383	434
8	3778.771	1122.323	1405	598
9	1516.116	1014.942	661.943	330
10	2997.373	954.643	1254	442
11	3192,715	1005	1381,085	462
12	1225.695	850.788	534.781	273
13	3302	1040	1254	501
14	3248	1020	1405	470
15	1311	910	572	292

efficiency equal to one. Let us compare these data with the data in Table 4.1. The unchanged input and output values remained for CHPPs: 5, 11, 13, 14, 15. Their input and output indicators do not change because they are efficient CHPPs. For all other CHPPs, the output indicators have changed. For this example, consider

Table 5.3: The efficiency coefficients. Experiment 2

No. firm	efficiency coefficient
1	0.854
2	0.971
3	0.859
4	0.872
5	1.000
6	0.959
7	0.965
8	0.894
9	0.794
10	0.974
11	1.000
12	0.952
13	1,000
14	1,000
15	1.000

CHPP 2 and its changes in outputs. The initial output indicators (Table 4.1):Y1 =3726, Y2=968. The efficiency coefficient of CHPP 2 is 0.971 (Table 5.1). To achieve an efficiency coefficient of CHPP 2 equal to one, we need to increase Y1 by 214.669 and Y2 by 66.571. Thus CHPP 2 will become efficient at the output indicators: Y1 =3940.669, Y2=1034.571. The input indicators do not change because we applied OUTCCR model.

Results for experiment 2.

Calculations on the efficiency coefficients are presented in Table 5.3.

The efficiency coefficients for both experiments are the same. This is because the same efficiency estimation model was used for the calculations. In this case, we only changed the orientation of the model.

In this experiment, the DEA model minimizes the input performance without changing the output performance.

We now calculate the input and output performance of the analyzed DMUs when they achieve efficiency, i.e. an efficiency coefficient equal to 1. We present the calculations in Table 5.4.

The table shows the inputs and outputs of the investigated CHPPs in achieving efficiency. These calculations allow us to carry out the INCCR model. The efficiency coefficients in the first (Table 5.1) and in the second experiment (Table 5.2) are the same. This is because we used the same model, just with different types of orientation. In this experiment, we used input orientation. Therefore, in this experiment, INCCR model will calculate the reduction of inputs to achieve efficiency (efficiency coefficient = 1).

Table 5.4: Calculation of input and output indicators at efficiency coefficient equal to 1

No. firm	Y1	Y2	X1	X2
1	3305	1040.551	1277.713	498.453
2	3825	968	1363.760	653.246
3	1244	802	543.505	264.452
4	3355	1029.438	1262.050	518.949
5	3986	1010	1405	694
6	1719	840	682.838	343.910
7	2894	890	1251.869	418.775
8	3379	910	1256.359	534.735
9	1204	806	525.672	262.064
10	2920	930	1221.630	430.590
11	3187	1005	1405	462
12	1166.934	810	509.143	259.912
13	3302	1040	1254	501
14	3248	1020	1405	470
15	1311	910	572	292

Let us compare the data in Table 5.4 with the original data in Table 4.1. The unchanged input and output remained for CHPPs: 5, 11, 13, 14, 15. Their inputs and outputs do not change because they are efficient CHPPs. For all other CHPPs, the inputs have changed. For this example, consider CHPP 2 and its changes in inputs. The initial data (Table 4.1) show: X1 =1412, X2=673. The efficiency coefficient of CHPP 2 is 0.971 (Table 5.1). To achieve an efficiency coefficient of CHPP 2 is equal to one, we need to reduce X1 by 48.24 and X2 by 19.754. Thus, CHPP 2 will become efficient at the X1 =1363.760 and X2 =653.246. The outputs do not change because we applied INCCR model.

Thus, in this study the application of DEA method in the heating system was shown using the presented methodology on the example of CHPPs. Experiments have been carried out using the CCR model with input orientation and output orientation. The change of model orientation depending on the goal of the decision maker was considered. Also, the performance indicators for the investigated sample objects were calculated.

The presented methodology, the obtained data on the presented experiments will allow using DEA method to improve the efficiency of enterprises of the heating system. In further research on the basis of the obtained results an automated decision support system will be developed.

6. Conclusion

The presented methodology, algorithm and the obtained from the presented experiments allow us to use the CCR model for the study of CHPPs. This methodology and the algorithm of using the CCR model allowed calculations of input and output data to improve the efficiency of the analyzed CHPPs.

Based on the results obtained, a method for evaluating and improving the efficiency of technical systems will be developed in the future. The developed method will be implemented as an automated decision support system.

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