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THE IMPACT OF SEMI-SYMMETRIC NON-METRIC CONNECTION ON A 3-DIMENSIONAL RIEMANNIAN MANIFOLD ADMITTING SOLITTONS

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Abstract. This article carries out the investigation of a 3-dimensional Riemannian manifold M^3 endowed with a semi-symmetric type of non-metric connection. After the introduction, we provide the basic results of a semi-symmetric non-metric connection. Next, we investigate gradient η-Ricci solitons and gradient Ricci-Yamabe solitons with respect to semi-symmetric non-metric connection and obtain several interesting results.
Keywords: Riemannian manifolds, gradient $\eta-$Ricci solitons, gradient Ricci-Yamabe solitons.

1. Introduction

In this paper, we study the impact of a semi-symmetric non-metric connection (shortly, $SSNMC$) on a 3-dimensional Riemannian manifold admitting solitons. Many years ago, on a differential manifold, Friedman and Schouten[14] presented the concept of semi-symmetric linear connection. In 1932, Hayden [19] introduced the notion of metric connection with torsion on the Riemannian manifold. Later, in 1970 the idea of semi-symmetric metric connection on a Riemannian manifold was further developed by Yano [23].

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Let $\bigtriangledown$ denote the Riemannian connection corresponding to the Riemannian metric g on M^3. A linear connection $\tilde{\bigtriangledown}$ defined on M^3 is said to be semi-symmetric if the torsion tensor $\tilde{T}$ of $\tilde{\bigtriangledown}$ defined by

$$\tilde{T}(U_1, V_1) = \tilde{\nabla}_{U_1} V_1 - \tilde{\nabla}_{V_1} U_1 - [U_1, V_1] \tag{1.1}$$

satisfies

$$\tilde{T}(U_1, V_1) = A(V_1)U_1 - A(U_1)V_1, \tag{1.2}$$

for all vector fields U_1 , V_1 on M^3, where A is a 1-form associated with the fixed vector field ξ_1 and satisfies $A(U_1) = g(U_1, \xi_1)$.

In the equation (1.2), if we replace the independent vector fields U_1 and V_1 respectively by ϕU_1 and ϕV_1 where ϕ is a (1,1) tensor field then the connection $\tilde{\bigtriangledown}$ becomes a quarter-symmetric connection ([15], [20]).

Again, a linear connection $\tilde{\bigtriangledown}$ on M^3 is said to be a metric connection if $\tilde{\bigtriangledown}g$=0 and if $\tilde{\bigtriangledown}g \neq 0$, then it is said to be non-metric [19]. Here,we consider $SSNMC$, that is $\bigtriangledown g \neq 0$ and the connection satisfies the equation (1.2). Agache and Chafle [1] introduced the idea of $SSNMC$ on a Riemannian manifold. After that, several researchers studied the properties of $SSNMC$ on manifolds with different structures ([4],[5],[6], [9], [12],[13],[22]).

In 1982, the notion of Ricci flow was introduced by Hamilton [17] to find the canonical metric on a smooth manifold. The Ricci flow is an evolution equation for metrics on a Riemannian manifold M^3 defined as: $\frac{\partial}{\partial t} g = -2\mathcal{S}$,

where $\mathcal{S}$ and g are the Ricci tensor and the metric tensor respectively.

A Ricci soliton [18] on a Riemannian manifold (M^3, g), is defined as a triple (g,W_1,λ), satisfying

$$\mathfrak{L}_{W_1} g + 2\lambda g + 2\mathcal{S} = 0, \tag{1.3}$$

where λ a real constant, W_1 is a potential vector field, $\mathfrak{L}$ is the Lie derivative, S is the Ricci tensor of the manifold.If W_1 is a Killing vector field or zero, then the Ricci soliton becomes trivial or reduces to an Einstein manifold respectively. The Ricci soliton is steady, expanding, or shrinking according to $\lambda = 0$,λ>0, or λ<0.

If $W_1 = Df$, where D indicates the gradient operator and f is a smooth function on M^3, then g is called gradient Ricci soliton and in such case equation (1.3) becomes

$$\nabla^2 f + \mathcal{S} + \lambda g = 0. \tag{1.4}$$

Cho and Kimura [3] introduced a more general notion of Ricci soliton, named η-Ricci soliton. An η-Ricci soliton is a Riemannian manifold (M^3,g) together with a vector field W_1 and two real constants λ , μ such that

$$\mathfrak{L}_{W_1} g + 2\mathcal{S} + 2\lambda g + 2\mu\eta \otimes \eta = 0, \tag{1.5}$$

where η is a g dual 1-form of W_1 and it is closed.

Again if $W_1 = Df$, then it is a gradient η-Ricci soliton and (1.5) takes the form

$$\nabla^2 f + \mathcal{S} + \lambda g + \mu\eta \otimes \eta = 0. \tag{1.6}$$

If λ and μ are smooth functions on M^3,then η-Ricci soliton is called almost η-Ricci soliton.

If $\mu = 0$, then η-Ricci soliton(or gradient η-Ricci soliton) turns into Ricci soliton (or gradient Ricci soliton) respectively.

Hamilton [17] proposed the idea of Yamabe flow, defined as follows:

$$\frac{\partial}{\partial t} g(t) + r g(t) = 0, \quad g_0 = g(t), \tag{1.7}$$

where t indicates the time and r being the scalar curvature of M^3.

On M^3, a Riemannian manifold equipped with a Riemannian metric g is named a Yamabe soliton if it satisfies,

$$\mathfrak{L}_{W_1} g - 2(r - \lambda) g = 0, \tag{1.8}$$

where λ is a real constant, r is the scalar curvature of the manifold .Here W_1 is the potential vector field. The Ricci soliton and Yamabe soliton are the same in 2-dimensional manifolds, but they are basically different in higher dimensions.

In recent years, the theory of geometric flows such as Ricci flow and Yamabe flow and their solitons have been the focus of attraction of many geometers. In 2019, Guler and Crasmareanu [16] introduced a new geometric flow which is a scalar combination of Ricci and Yamabe flow. This is additionally named (α, β) type Ricci-Yamabe flow. The Ricci-Yamabe flow is defined as follows:[16]

$$\frac{\partial}{\partial t} g(t) = \beta r(t) g(t) - 2\alpha \mathcal{S}(t), \quad g_0 = g(0), \tag{1.9}$$

where $\mathcal{S}$ is the Ricci tensor, r denotes the scalar curvature and $\lambda, \alpha, \beta \in \mathbb{R}$.

A Ricci-Yamabe soliton on (M^3, g) satisfies

$$\pounds_{W_1} g = -2\alpha S - (2\lambda - \beta r) g. \tag{1.10}$$

If $W_1 = Df$, then the Ricci-Yamabe soliton is called a gradient Ricci-Yamabe soliton and the equation becomes

$$\nabla^2 f = -\alpha \mathcal{S} - (\lambda - \frac{1}{2}\beta r) g. \tag{1.11}$$

The Ricci-Yamabe soliton (or gradient Ricci-Yamabe soliton) is said to be expanding for $\lambda > 0$, steady for $\lambda = 0$ and shrinking when $\lambda < 0$. If λ, β and α are smooth functions on M^3, then a Ricci-Yamabe soliton (or gradient Ricci-Yamabe soliton) is called an almost Ricci-Yamabe soliton (or gradient Ricci-Yamabe soliton). If $\beta = 0$, $\alpha = 1$, then Ricci-Yamabe soliton (or gradient Ricci-Yamabe soliton) becomes Ricci soliton (or gradient Ricci soliton). Similarly, it becomes Yamabe soliton (or gradient Yamabe soliton) if $\beta = 1$, $\alpha = 0$. In this connection, we mention the works of Blaga [2], Özgür [21] and De et. al.([7],[8], [11]).

In the present paper after the introduction in section 2, we provide the basic properties of $SSNMC$.Then in the next section, we characterize gradient η-Ricci solitons on M^3. Finally, we study gradient Ricci-Yamabe solitons on M^3.

2. Semi-symmetric non-metric connection

A linear connection $\tilde{\nabla}$ on M^3, defined by

$$\tilde{\nabla}_{U_1} V_1 = \nabla_{U_1} V_1 + A(V_1) U_1, \tag{2.1}$$

where ∇ is the Riemannian connection on M^3, is a semi-symmetric non-metric connection. It satisfies [1]

$$(\tilde{\nabla}_{U_1} g)(V_1, Y_1) = -A(V_1) g(U_1, Y_1) - A(Y_1) g(U_1, V_1). \tag{2.2}$$

Let $\tilde{R}$ and R denote the curvature tensor with respect to the semi-symmetric non-metric connection $\tilde{\nabla}$ and Riemannian connection ∇ respectively. Then $\tilde{R}$ and R are connected by [1]

$$\tilde{R}(U_1, V_1) Y_1 = R(U_1, V_1) Y_1 - \alpha^*(V_1, Y_1) U_1 + \alpha^*(U_1, Y_1) V_1, \tag{2.3}$$

for all U_1, V_1, Y_1 on M^3, where α^* is a $(0, 2)$- tensor field defined as follows:

$$\alpha^*(U_1, V_1) = (\nabla_{U_1} A)(V_1) - A(U_1) A(V_1). \tag{2.4}$$

Throughout this article, we consider the vector field ξ_1 is a unit parallel vector field with respect to the Levi-Civita connection ∇. Then $\nabla_{U_1} \xi_1 = 0$, which implies

$$R(U_1, V_1) \xi_1 = 0 \tag{2.5}$$

and

$$\mathcal{S}(U_1, \xi_1) = 0. \tag{2.6}$$

Also, using $\nabla_{U_1} \xi_1 = 0$, we get

$$(\nabla_{U_1} A) V_1 = 0. \tag{2.7}$$

Using (2.7) in (2.4) infers $\alpha^*(U_1, V_1) = -A(U_1) A(V_1)$. Then using the above equation in (2.3) we obtain ,

$$\tilde{R}(U_1, V_1) Y_1 = R(U_1, V_1) Y_1 + A(Y_1)[A(V_1) U_1 - A(U_1) V_1]. \tag{2.8}$$

From the foregoing equation, we have

$$\tilde{S}(U_1, V_1) = S(U_1, V_1) + 2A(U_1) A(V_1), \tag{2.9}$$

where $\tilde{S}$ and S denote the Ricci tensor of $\tilde{\nabla}$ and ∇ respectively.

Contracting the above equation, we provide

$$\tilde{r} = r + 2, \tag{2.10}$$

$\tilde{r}$ and r are the scalar curvature of $\tilde{\nabla}$ and ∇ respectively, since $A(\xi_1) = g(\xi_1, \xi_1) = 1$.

Using (2.5), we have from (2.8)

$$\tilde{R}(U_1, V_1)\xi_1 = A(V_1)U_1 - A(U_1)V_1. \tag{2.11}$$

So, we get relations

$$A(\tilde{R}(U_1, V_1)Y_1) = 0, \tag{2.12}$$

and

$$\tilde{S}(U_1, \xi_1) = 2A(U_1), \quad \tilde{Q}\xi_1 = 2\xi_1. \tag{2.13}$$

where $\tilde{Q}$ is the Ricci operator with respect to the semi-symmetric non-metric connection $\tilde{\nabla}$ defined by $g(\tilde{Q}U_1, V_1) = \tilde{S}(U_1, V_1)$.

We first establish the subsequent Lemma:

Lemma 2.1. *Let M^3 be a Riemannian manifold with a semi-symmetric non-metric connection $\tilde{\nabla}$. Then we have*

$$\xi_1 \tilde{r} = 0. \tag{2.14}$$

Proof: In M^3, the Riemannian curvature tensor can be written as

$$\begin{aligned} R(U_1, V_1)Y_1 \quad &= g(V_1, Y_1)QU_1 - g(U_1, Y_1)QV_1 + \mathcal{S}(V_1, Y_1)U_1 \\ &-\mathcal{S}(U_1, Y_1)V_1 - \frac{r}{2}[g(V_1, Y_1)U_1 - g(U_1, Y_1)V_1], \end{aligned} \tag{2.15}$$

Using (2.9) and (2.8), we get

$$\begin{aligned} \tilde{R}(U_1, V_1)Y_1 \quad & -A(Y_1)[A(V_1)U_1 - A(U_1)V_1] = g(V_1, Y_1)[\tilde{Q}U_1 - 2\xi_1 A(U_1)] \\ & -g(U_1, Y_1)[\tilde{Q}V_1 - 2\xi_1 A(V_1)] + [\tilde{\mathcal{S}}(V_1, Y_1) - 2A(V_1)A(Y_1)]U_1 \\ & -[\tilde{\mathcal{S}}(U_1, Y_1) - 2A(U_1)A(Y_1)]V_1 - \frac{r}{2}[g(V_1, Y_1)U_1 - g(U_1, Y_1)V_1], \end{aligned} \tag{2.16}$$

Put $V_1 = Y_1 = \xi_1$, in the above equation we get

$$\tilde{Q}U_1 = \left(\frac{\tilde{r}}{2} + 1\right)U_1 - \left(\frac{\tilde{r}}{2} - 1\right)A(U_1)\xi_1. \tag{2.17}$$

Taking covariant derivative along V_1, we infer

$$(\nabla_{V_1}\tilde{Q})U_1 = \frac{(V_1\tilde{r})}{2}[U_1 - A(U_1)\xi_1]. \tag{2.18}$$

Contracting the above equation we obtain the required result.

3. Gradient $\eta-$Ricci solitons

The soliton equation (1.6) can be written as

$$\tilde{\nabla}_{U_1} Df = -\tilde{Q}U_1 - \lambda U_1 - \mu\eta(U_1)\xi_1, \tag{3.1}$$

for all $U_1 \in \mathfrak{X}(M^3)$. Using (3.1) and the definition

$$\tilde{R}(U_1, V_1)Df = \tilde{\nabla}_{U_1}\tilde{\nabla}_{V_1} Df - \tilde{\nabla}_{V_1}\tilde{\nabla}_{U_1} Df - \tilde{\nabla}_{[U_1,V_1]} Df \tag{3.2}$$

we reveal

$$\tilde{R}(U_1, V_1)Df = -(\tilde{\nabla}_{U_1}\tilde{Q})(V_1) + (\tilde{\nabla}_{V_1}\tilde{Q})(U_1) - \mu[\eta(V_1)U_1 - \eta(U_1)V_1]. \tag{3.3}$$

Contracting the above equation, we have

$$\tilde{\mathcal{S}}(V_1, Df) = \frac{1}{2}(V_1\tilde{r}) - 2\mu\eta(V_1). \tag{3.4}$$

Again, from (2.17) we obtain,

$$\tilde{\mathcal{S}}(V_1, Df) = \left(\frac{\tilde{r}}{2} + 1\right)(V_1 f) - \left(\frac{\tilde{r}}{2} - 1\right)A(V_1)(\xi_1 f). \tag{3.5}$$

Comparing the equations (3.4) and (3.5) we get

$$\left(\frac{\tilde{r}}{2} + 1\right)(V_1 f) - \left(\frac{\tilde{r}}{2} - 1\right)A(V_1)(\xi_1 f) = \frac{1}{2}(V_1\tilde{r}) - 2\mu(V_1 f). \tag{3.6}$$

Now, putting $V_1 = \xi_1$ in (3.6) and using $\xi_1\tilde{r} = 0$ and $A(\xi_1) = 1$, we find

$$(\mu + 1)\xi_1 f = 0, \tag{3.7}$$

which implies $\xi_1 f = 0$, provided $\mu + 1 \neq 0$.

From (2.11), we find

$$g(\tilde{R}(U_1, V_1)\xi_1, Df) = A(V_1)g(U_1, Df) - A(U_1)g(V_1, Df). \tag{3.8}$$

Again from (3.3)

$$g(\tilde{R}(U_1, V_1)\xi_1, Df) = \mu[(V_1 f)A(U_1) - (U_1 f)A(V_1)]. \tag{3.9}$$

Combining the equation (3.8) and (3.9) we get

$$A(V_1)(U_1 f) - A(U_1)(V_1 f) = \mu[(V_1 f)A(U_1) - (U_1 f)A(V_1)]. \tag{3.10}$$

Setting $V_1 = \xi_1$ and using $\xi_1 f = 0$, $A(\xi_1) = 1$ we find

$$(\mu + 1)U_1 f = 0, \tag{3.11}$$

For $\mu \neq -1$, f is constant. Using $f = \textit{constant}$ the equation (3.1)) infers that the manifold reduces to a generalized Quasi-Einstein manifold [10].

Thus we can state the following:

Theorem 3.1. *If M^3 with a SSNMC admits the gradient η-Ricci soliton with $\mu \neq -1$, then M^3 reduces to a generalized Quasi-Einstein manifold.*

If $\mu = 0$, then the gradient η-Ricci soliton becomes gradient Ricci soliton. Then (3.11) gives $f = constant$ and hence the manifold becomes an Einstein manifold. Therefore, M^3 is of constant sectional curvature, since the manifold is of dimension 3.

Corollary 3.1. *A 3-dimensional Riemannian manifold with a SSNMC admitting gradient Ricci soliton is a manifold of constant sectional curvature.*

4. Gradient Ricci-Yamabe solitons

Let M^3 admit a gradient Ricci-Yamabe soliton.Then (1.11) implies

$$\tilde{\nabla}_{U_1} Df = -\alpha\tilde{Q}U_1 - (\lambda - \frac{\beta}{2}\tilde{r})U_1. \tag{4.1}$$

Taking covariant derivative of (4.1) along the vector field V_1, we obtain

$$\tilde{\nabla}_{V_1}\tilde{\nabla}_{U_1} Df = -\alpha\tilde{\nabla}_{V_1}\tilde{Q}U_1 + \frac{\beta}{2}(V_1\tilde{r})U_1 - (\lambda - \frac{\beta}{2}\tilde{r})\tilde{\nabla}_{V_1}U_1. \tag{4.2}$$

Interchanging U_1 and V_1 in the above equation we get

$$\tilde{\nabla}_{U_1}\tilde{\nabla}_{V_1} Df = -\alpha\tilde{\nabla}_{U_1}\tilde{Q}V_1 + \frac{\beta}{2}(U_1\tilde{r})V_1 - (\lambda - \frac{\beta}{2}\tilde{r})\tilde{\nabla}_{U_1}V_1, \tag{4.3}$$

and equation (4.1) yields

$$\tilde{\nabla}_{[U_1,V_1]} = -\alpha\tilde{Q}([U_1, V_1]) - (\lambda - \frac{\beta}{2}\tilde{r})[U_1, V_1]. \tag{4.4}$$

Using (4.2), (4.3) and (4.4) we have

$$\tilde{R}(U_1, V_1)Df = -\alpha(\tilde{\nabla}_{U_1}\tilde{Q})V_1 + \alpha(\tilde{\nabla}_{V_1}\tilde{Q})U_1 + \frac{\beta}{2}[(U_1\tilde{r})V_1 - (V_1\tilde{r})U_1]. \tag{4.5}$$

Contracting the previous equation over U_1, we get

$$\tilde{\mathcal{S}}(V_1, Df) = (\frac{\alpha}{2} - \beta)(V_1\tilde{r}). \tag{4.6}$$

Again from (2.17) we have

$$\tilde{\mathcal{S}}(U_1, Df) = (\frac{\tilde{r}}{2} + 1)U_1 f - (\frac{\tilde{r}}{2} - 1)A(U_1)\xi_1 f. \tag{4.7}$$

Combining (4.6) and (4.7), we infer

$$(\frac{\alpha}{2} - \beta)(U_1\tilde{r}) = (\frac{\tilde{r}}{2} + 1)U_1 f - (\frac{\tilde{r}}{2} - 1)A(U_1)\xi_1 f. \tag{4.8}$$

Setting $U_1 = \xi_1$ and using $\xi_1 \tilde{r} = 0$ and $A(\xi_1) = 1$ in the foregoing equation, we have

$$\xi_1 f = 0. \tag{4.9}$$

Thus from (4.8), we obtain

$$(\frac{\alpha}{2} - \beta)(U_1 \tilde{r}) = (\frac{\tilde{r}}{2} + 1) U_1 f. \tag{4.10}$$

Now, from equation (4.5) we find that

$$g(\tilde{R}(U_1, V_1)\xi_1, Df) = -\frac{\beta}{2}[(U_1 \tilde{r})A(V_1) - (V_1 \tilde{r})A(U_1)]. \tag{4.11}$$

Again from equation (2.11), we have

$$g(\tilde{R}(U_1, V_1)\xi_1, Df) = A(V_1)(U_1 f) - A(U_1)(V_1 f). \tag{4.12}$$

Combining the equation (4.11) and (4.12) we get

$$-\frac{\beta}{2}[(U_1 \tilde{r})A(V_1) - (V_1 \tilde{r})A(U_1)] = A(V_1)(U_1 f) - A(U_1)(V_1 f). \tag{4.13}$$

Setting $V_1 = \xi_1$ in the previous equation and using (4.9) gives

$$\frac{\beta}{2}[(U_1 \tilde{r}) = -(U_1 f). \tag{4.14}$$

Utilizing (4.14) in (4.10) we infer that

$$(2\alpha - 2\beta + \beta \tilde{r}) U_1 \tilde{r} = 0, \tag{4.15}$$

which entails that either $(U_1 \tilde{r}) = 0$ or $(U_1 \tilde{r}) \neq 0$.

In both the cases $\tilde{r} = constant$, since α, β are non-zero constants. Hence from (2.10), we infer that $r = constant$ i.e M^3 is of constant scalar curvature. Again, using the fact $\tilde{r} = constant$, equation (4.14) gives f is a constant and hence the gradient Ricci-Yamabe soliton becomes trivial.

Hence, we state the result as:

Theorem 4.1. *Let M^3 be a 3-dimensional Riemannian manifold with a $SSNMC$ admitting gradient Ricci-Yamabe solitons.*

Then the following cases occur :

(i) The scalar curvature is constant in M^3.

(ii) The soliton becomes trivial.

If $\alpha = 1$, $\beta = 0$, then the gradient Ricci-Yamabe soliton becomes gradient Ricci soliton.Then (4.15) gives $\tilde{r} = constant$ and the equation (4.14) infers that $f = constant$.Using this fact equation (4.1) leads that M^3 is an Einstein manifold. Therefore M^3 is of constant sectional curvature.

Corollary 4.1. *A 3-dimensional Riemannian manifold with a $SSNMC$ admitting gradient Ricci soliton is a manifold of constant sectional curvature.*

If $\alpha = 0$, $\beta = 1$, then the gradient Ricci-Yamabe soliton becomes gradient Yamabe soliton.Then equation (4.15) gives $\tilde{r} = 2$ and hence from (2.10), we obtain $r = 0$ i.e the scalar curvature vanishes in M^3.

Corollary 4.2. *If a 3-dimensional Riemannian manifold with a $SSNMC$ admitting gradient Yamabe soliton, then the scalar curvature of the manifold vanishes.*

REFERENCES

1. S. Agashe and M.R. Chafle: *A semi-symmetric non-metric connection on a Riemannian manifold*. Indian J. Pure Appl. Math. **23** (1992), 399–409.
2. A. M. Blaga: *η-Ricci solitons on Lorentzian para-Sasakian manifolds*. Filomat **30** (2016), 489-496.
3. J. T. Cho and M. Kimura: *Ricci Solutions and real hypersurfaces in a complex space form*. Tohoku Math. J. **61**, no. 2 (2009), 205-212.
4. L. Das, U. C. De , R.N. Singh and M. K. Pandey: *Loretzian nmanifold admitting a type of semi-symmetric non-metric connection.* **70** (2008), 78-85.
5. A. De: *On almost pseudo symmetric manifolds admitting a type of semi-symmetric non-metric connection*. Acta Math. Hungar **125** (1-2) (2009), 183–190.
6. K. De, U. C. De and A. Gezer: *Investigation on a Riemannian manifold with a semi-symmetric non-metric connection and gradient solitons* . Kraguevac Journal of Mathematics **49** (3)(2025), 387–400.
7. K. De and U. C. De: *A note on gradient Solitons on two classes of almost Kenmotsu Manifolds*. International Journal of Geometric Methods in Modern Physics, **19** (2022) 2250213 (12 pages).
8. K. De and U. C. De: *δ-almost Yamabe solitons in paracontact metric manifolds*. Mediterr. J. Math. **18**, 218 (2021).
9. U. C. De, K. De and S. Güler: *Characterizations of a Lorentzian Manifold with a semi-symmetric metric connection*. Publ. Math. Debrecen, **104** 3-4 (2024), 329–341.
10. U. C. De and G. C. Ghosh: *On generalized quasi-Einstein manifolds*. Kyung-pook Math. J. **44** (2004), 607-615.
11. U. C. De, A. Sardar and K. De: *Ricci-Yamabe solitons and 3-dimensional Riemannian manifolds* . Turk. J. Math. **2022**(46), 1078-1088.
12. Y. Dogru: *η-Bourguignon solitons with a semi-symmetric metric and semi-symmetric metric non-metric connection* . AIMS Math. **8**(2023), no. 5, 11943-11952.
13. Y. Dogru, C. Özgür, and C. Murathan: *Riemannian manifolds with a semi-symmetric non-metric connection satisfying some semisymmetry conditions*. Bull. Math. Anal. Appl. **3** (2)(2011), 206–212.
14. A. Friedmann and J. A. Schouten: *Über die geometric der holbsymmetrischen Ubertragurgen*. Math. Zeitschr. **21** (1924), 211–233.

15. S. Golab: *On semi-symmetric and quarter-symmetric linear connections.* Tensor (N. S.) **29** (1975), 249–254.

16. S. Guler and M. Crasmareanu: *Ricci-Yamabe maps for Riemannian flow and their volume variation and volume entropy.* Turkish Journal of Mathematics **43** (2019), 2631-2641.

17. R. S. Hamilton: *The Ricci flow on surfaces.* Contemp. Math. **71** (1988), 237–261.

18. R. S. Hamilton: *Three-manifolds with positive Ricci curvature.* J. Differ. Geom. **17** (1982), 255-306.

19. H. A. Hayden: *Subspace of space with torsion.* Proc. Lon. Math. Soc. **34** (1932), 27–50.

20. M. A. Lone and I. F. Harry: *On Ricci solitons with quarter-symmetric connection.* Ricerche di Matematica **(2024)**, 1-15.

21. C. Özgür: *On Ricci solitons with a semi-symmetric metric connection.* Filomat **35** (2021), no. 11,3635–3641.

22. C. Özgür and S. Sular: *Warped products with a semi-symmetric non-metric connection.* Arab J. Math. Sci. **36** (2011), 461–473.

23. K. Yano: *On semi-symmetric metric connection.* Revue Roumaine de Math. Pures et Appliques. **15** (1970), 1579–1586.