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SEMIPRIME IDEALS AND $P-$COMMUTING HOMODERIVATIONS ON IDEALS

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Abstract. The first purpose of this article is to examine the structure of an S/P quotient ring, where S is any ring and P is the semiprime ideal of S. More specifically, we look at differential identities in the semiprime ideal of an arbitrary ring using the P-commuting homoderivations.
Keywords: quotient ring, semiprime ideal, P-commuting homoderivations.

1. Introduction

Let S will be an associative ring with center Z. For any $a_1, a_2 \in S$ the symbol $[a_1, a_2]$ represents the Lie commutator $a_1a_2 - a_2a_1$ and the Jordan product $a_1oa_2 = a_1a_2 + a_2a_1$. Recall that an ideal P of S is said to be prime if $P \neq S$ and for all $a_1, a_2 \in S$, $a_1Sa_2 \subseteq P$ implies that $a_1 \in P$ or $a_2 \in P$. Therefore, S is called a prime ring if the ideal (0) is prime. P is a semiprime ideal if $P \neq S$ and for all $a_1 \in S$, $a_1Sa_1 \subseteq P$ implies that $a_1 \in P$ and S is a semiprime ring if $P = 0$ is a semiprime ideal of S.

Let P be a nonempty subset of S. A mapping F from S to S is called commuting on P if $[F(a_1), a_1] = 0$, for all $a_1 \in P$. This definition has been generalized such as: A map $F : S \to S$ is called a $P-$commuting map on P if $[F(a_1), a_1] \in P$, for all $x \in P$ and some $P \subset S$. In particular, if $P = 0$, then F is called a commuting map on S if $[F(a_1), a_1] = 0$. Note that every commuting map is a $P-$commuting map (put $0 = P$). But the converse is not true in general.

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The first well-known result on commuting maps is Posner's Second Theorem in [10]. This theorem states that the existence of a nonzero commuting derivation on a prime ring S implies S to be commutative. By a derivation, we mean an additive mapping $d : S \to S$ such that $d(a_1a_2) = d(a_1)a_2 + a_1d(a_2)$ for all $a_1, a_2 \in S$. Over the last several years, a number of authors studied commutativity theorems for prime rings admitting automorphisms or derivations on appropriate subsets of S.

In 2000, M. M. El Sofy Aly [7] defined a homoderivation on S as an additive mapping $\hbar : S \to S$ satisfying $\hbar(a_1a_2) = \hbar(a_1)\hbar(a_2) + \hbar(a_1)a_2 + a_1\hbar(a_2)$ for all $a_1, a_2 \in S$. An example of such mapping is to let $\hbar(a_1) = f(a_1) - a_1$, for all $a_1, a_2 \in S$ where f is an endomorphism on S. Another example can be given as follows: The additive mapping $\hbar : S \to S$ defined by $\hbar(a_1) = -a_1$ is a homoderivation of S.

In [6], Daif and Bell proved that S is semiprime ring, I is a nonzero ideal of S and d is a derivation of S such that $d([a_1, a_2]) = \pm[a_1, a_2]$, for all $a_1, a_2 \in I$, then S contains a nonzero central ideal. Further, Hongan [8] extended this theorem as follows: Let S be a 2-torsion free semiprime ring and I a nonzero ideal of S and d a derivation of S. If $d([a_1, a_2]) \pm [a_1, a_2] \in Z$, for all $a_1, a_2 \in I$, then $I \subseteq Z$.

Recently, Quadri et al. [11] generalized this result replacing derivation d with a generalized derivation in a prime ring S. More precisely, they proved the following:

Let S be a prime ring and I a nonzero ideal of S. If S admits a generalized derivation F associated with a nonzero derivation d such that any one of the following holds : (i) $F([a_1, a_2]) = [a_1, a_2]$ for all $a_1,\ a_2 \in I$; (ii)$F([a_1, a_2]) = -[a_1, a_2]$ for all $a_1, a_2 \in I$; (iii) $F(a_1oa_2) = (a_1oa_2)$ for all $a_1, a_2 \in I$; (iv) $F(a_1oa_2) = -(a_1oa_2)$ for all $a_1, a_2 \in I$; then S is commutative.

M. Ashraf et al. [4] proved that a prime ring S must be commutative, if S satisfies any one of the following conditions: $(i)f(a_1a_2) = a_1a_2$, $(ii)f(a_1)f(a_2) = a_1a_2$, where is a generalized derivation of S and I is a nonzero two-sided ideal of S. In [5], M. Ashraf and N. Rehman showed that a prime ring S with a nonzero ideal I must be commutative if it admits a derivation d satisfying either of the properties $d(a_1a_2) + a_1a_2 \in Z$ or $d(a_1a_2) - a_1a_2 \in Z$; for all $a_1, a_2 \in I$.

In 2020, to extend the theory of derivations rings, which has been studied for years, Almahdi et al set out to examine the derivations of a random ring S satisfying some P-valued conditions where P is the prime ideal of S. As an important development in this work, they stated the theorem known as Posner's Second Theorem as follows:

Let S be a ring, P is a prime ideal of S and d a derivation of S. If $[[d\left(a_1\right), a_1], a_2] \in P$ for all $a_1, a_2 \in S$, then $d\left(S\right) \subseteq P$ or S/P is commutative.

Our aim in this study is to examine the differential identities in the semiprime ideal of an arbitrary ring by using homoderivations and to generalize some results from previous articles with the help of homoderivations.

2. Results

For any $a_1, a_2, a_3 \in S$, as usual $[a_1, a_2] = a_1a_2 - a_2a_1$ and $a_1oa_2 = a_1a_2 + a_2a_1$ will denote the well-known Lie and Jordan product, respectively and make extensive use of basic commutator identities:

$$\begin{aligned}
[a_1, a_2a_3] &= a_2[a_1, a_3] + [a_1, a_2]a_3 \\
[a_1a_2, a_3] &= [a_1, a_3]a_2 + a_1[a_2, a_3] \\
a_1o(a_2a_3) &= (a_1oa_2)a_3 - a_2[a_1, a_3] = a_2(a_1oa_3) + [a_1, a_2]a_3 \\
(a_1a_2)oa_3 &= a_1(a_2oa_3) - [a_1, a_3]a_2 = (a_1oa_3)a_2 + a_1[a_2, a_3].
\end{aligned}$$

Remark 2.1. For all $a_1, a_2 \in S$, we get

$$\begin{aligned}
\hbar([a_1, a_2]) &= \hbar(a_1a_2 - a_2a_1) = \hbar(a_1a_2) - \hbar(a_2a_1) \\
&= \hbar(a_1)\hbar(a_2) + \hbar(a_1)a_2 + a_1\hbar(a_2) - \hbar(a_2)\hbar(a_1) - \hbar(a_2)a_1 - a_2\hbar(a_1) \\
&= [\hbar(a_1), \hbar(a_2)] + [\hbar(a_1), a_2] + [a_1, \hbar(a_2)].
\end{aligned}$$

Every prime ideal is a semiprime ideal, but the reverse is not true. For this reason, it is more important to examine identities containing derivations in semiprime ideals. In this study, which is an extension and generalization of the existing findings in the literature, we will conduct a new research. We will make use of homoderivations when examining differential properties in the semiprime ideal of an arbitrary ring.

Lemma 2.1. *[3, Lemma 1] Let S be a ring with P a semiprime ideal of S such that $P \subsetneq I$, I an ideal of S, and $a \in I$ such that $axa \in P$ for all $x \in I$, then $a \in P$.*

Theorem 2.1. *Let I be a nonzero ideal, P a semiprime ideal of a ring S such that $P \subsetneq I$. If $\hbar$ a nonzero homoderivation of S satisfies any one of the conditions*

1. *$\hbar([a_1, a_2]) \in P$,*
2. *$\hbar(a_1oa_2) \in P$ for all $a_1, a_2 \in I$, then $\hbar$ is P-commuting on I.*

Proof. (1) By the hypothesis, we have

$$\hbar([a_1, a_2]) \in P \text{ for all } a_1, a_2 \in I. \tag{2.1}$$

Writing a_2a_1 instead of a_2 in (2.1), we get

$$\hbar([a_1, a_2])\hbar(a_1) + \hbar([a_1, a_2])a_1 + [a_1, a_2]\hbar(a_1) \in P \text{ for all } a_1, a_2 \in I.$$

By using the hypothesis, we obtain that

$$[a_1, a_2]\hbar(a_1) \in P \text{ for all } a_1, a_2 \in I. \tag{2.2}$$

Writing ra_2 for $a_2, r \in S$ in last relation, we arrive at

$$[a_1, r]a_2\hbar(a_1) \in P \text{ for all } a_1, a_2 \in I, r \in S. \tag{2.3}$$

Replacing a_2a_1 instead of a_2 in (2.3), we get

$$[a_1, r]a_2a_1\hbar(a_1) \in P \text{ for all } a_1, a_2 \in I, r \in S. \tag{2.4}$$

Multiplying (2.3) by a_1 from the right, we get

$$[a_1, r]a_2\hbar(a_1)a_1 \in P \text{ for all } a_1, a_2 \in I, r \in S. \tag{2.5}$$

Subtracting expressions (2.4) and (2.5) we get that

$$[a_1, r]a_2 [\hbar(a_1), a_1] \in P \text{ for all } a_1, a_2 \in I, r \in S. \tag{2.6}$$

Writing $\hbar(a_1)$ instead of r in last expression, we get

$$[\hbar(a_1), a_1]a_2 [\hbar(a_1), a_1] \in P \text{ for all } a_1, a_2 \in I.$$

It follows that

$$[\hbar(a_1), a_1]I [\hbar(a_1), a_1] \in P \text{ for all } a_1, a_2 \in I.$$

By Lemma 1, we arrive at $[\hbar(a_1), a_1] \in P$ for all $a_1 \in I$. Hence $\hbar$ is P-commuting on I.

(2) By the hypothesis, we have

$$\hbar(a_1 o a_2) \in P \text{ for all } a_1, a_2 \in I. \tag{2.7}$$

Writing a_2a_1 instead of a_2 in (2.7) and by expanding the expression, we get

$$\hbar(a_1 o a_2)\hbar(a_1) + \hbar(a_1 o a_2)a_1 + (a_1 o a_2)\, \hbar(a_1) \in P \text{ for all } a_1, a_2 \in I.$$

Using (2.7) in last relation, we obtain that

$$(a_1 o a_2)\, \hbar(a_1) \in P \text{ for all } a_1, a_2 \in I. \tag{2.8}$$

Substituting ra_2 for a_2,where $r \in S$ in (2.8), we find that

$$r\, (a_1 o a_2)\, \hbar(a_1) + [a_1, r]\, a_2\hbar(a_1) \in P \text{ for all } a_1, a_2 \in I, r \in S.$$

Using (2.8), we get

$$[a_1, r]\, a_2\hbar(a_1) \in P \text{ for all } a_1, a_2 \in I, r \in S. \tag{2.9}$$

Replace a_2 by a_2a_1 in (2.9), we get

$$[a_1, r]\, a_2a_1\hbar(a_1) \in P \text{ for all } a_1, a_2 \in I, r \in S. \tag{2.10}$$

Multiplying (2.9) by a_1 from the right, we have

$$[a_1, r]\, a_2\hbar(a_1)a_1 \in P \text{ for all } a_1, a_2 \in I, r \in S. \tag{2.11}$$

Combining (2.10) and (2.11), we obtain that

$$[a_1, r]\, a_2\, [\hbar(a_1), a_1] \in P \text{ for all } a_1, a_2 \in I, r \in S$$

that is

$$[a_1, r]I\,[\hbar(a_1), a_1] \in P \text{ for all } a_1, a_2 \in I, r \in S.$$

Writing r by $\hbar(a_1)$ in this relation, we find that

$$[\hbar(a_1), a_1]I\,[\hbar(a_1), a_1] \in P \text{ for all } a_1 \in I.$$

By Lemma 1, we obtain that $[\hbar(a_1), a_1] \in P$ for all $a_1 \in I$. This means $\hbar$ is P-commuting on I. □

Theorem 2.2. *Let I be a nonzero ideal, P a semiprime ideal of a ring S such that $P \subsetneq I$ and $char(S/P) \neq 2$. If $\hbar$ a nonzero homoderivation of S satisfies condition $\hbar([a_1, a_2]) - [a_1, a_2] \in P$, for all $a_1, a_2 \in I$, then $\hbar$ is P-commuting on I.*

Proof. By the hypothesis, we have

$$\hbar([a_1, a_2]) - [a_1, a_2] \in P, \text{ for all } a_1, a_2 \in I. \tag{2.12}$$

Writing a_2 by $[a_1, a_2]$ in (2.12) and using the remark given above, we get

$$[\hbar(a_1), \hbar([a_1, a_2])] + [\hbar(a_1), [a_1, a_2]] + [a_1, \hbar([a_1, a_2])] - [a_1, [a_1, a_2]] \in P, \text{ for all } a_1, a_2 \in I.$$

Using (2.12), we arrive at

$$[\hbar(a_1), \hbar([a_1, a_2])] + [\hbar(a_1), [a_1, a_2]] \in P, \text{ for all } a_1, a_2 \in I. \tag{2.13}$$

Editing the last expression, we have

$$[\hbar(a_1), \hbar([a_1, a_2])] + [\hbar(a_1), [a_1, a_2]] - [\hbar(a_1), [a_1, a_2]] + [\hbar(a_1), [a_1, a_2]] \in P, \text{ for all } a_1, a_2 \in I.$$

Using the (2.12), we have

$$2\,[\hbar(a_1), [a_1, a_2]] \in P, \text{ for all } a_1, a_2 \in I.$$

Since $char(S/P) \neq 2$, we have

$$[\hbar(a_1), [a_1, a_2]] \in P, \text{ for all } a_1, a_2 \in I. \tag{2.14}$$

Writing a_2a_1 instead of a_2 in (2.14) and using (2.14), we obtain that

$$[a_1, a_2][\hbar(a_1), a_1] \in P, \text{ for all } a_1, a_2 \in I. \tag{2.15}$$

Taking $ra_2, r \in S$ for a_2 in (2.15) and using this, we have

$$[a_1, r]a_2[\hbar(a_1), a_1] \in P \text{ for all } a_1, a_2 \in I, r \in S.$$

Writing $\hbar(a_1)$ instead of r this relation

$$[a_1, \hbar(a_1)]a_2[\hbar(a_1), a_1] \in P \text{ for all } a_1 \in I$$

and so

$$[\hbar(a_1), a_1]I[\hbar(a_1), a_1] \in P \text{ for all } a_1 \in I.$$

By Lemma 1, we get that $[\hbar(a_1), a_1] \in P$ for all $a_1 \in I$. As a result $\hbar$ is P-commuting on I. □

Theorem 2.3. *Let I be a nonzero ideal, P a semiprime ideal of a ring S such that $P \subsetneq I$ and $char(S/P) \neq 2$. If $\hbar$ a nonzero homoderivation of S satisfies condition $\hbar(a_1 o a_2) - a_1 o a_2 \in P$, for all $a_1, a_2 \in I$, then $\hbar$ is P-commuting on I.*

Proof. By the hypothesis, we have

$$\hbar(a_1 o a_2) - a_1 o a_2 \in P, \text{ for all } a_1, a_2 \in I. \tag{2.16}$$

Writing a_2 by $a_2 a_1$ in (2.16) and using it, we get

$$\hbar\left(\left(a_1 o a_2\right) a_1\right) - \left(a_1 o a_2\right) a_1 = \hbar(a_1 o a_2)\hbar\left(a_1\right) + \hbar(a_1 o a_2) a_1 + \left(a_1 o a_2\right)\hbar\left(a_1\right) - \left(a_1 o a_2\right) a_1 \in P$$

and so

$$\hbar(a_1 o a_2)\hbar\left(a_1\right) + \left(a_1 o a_2\right)\hbar\left(a_1\right) \in P, \text{ for all } a_1, a_2 \in I. \tag{2.17}$$

Editing the last expression, we obtain that

$$\hbar(a_1 o a_2)\hbar\left(a_1\right) + \left(a_1 o a_2\right)\hbar\left(a_1\right) - \left(a_1 o a_2\right)\hbar\left(a_1\right) + \left(a_1 o a_2\right)\hbar\left(a_1\right) \in P, \text{ for all } a_1, a_2 \in I.$$

Using the (2.16), we have

$$2\left(a_1 o a_2\right)\hbar\left(a_1\right) \in P, \text{ for all } a_1, a_2 \in I.$$

Since $char(S/P) \neq 2$, we obtain that

$$\left(a_1 o a_2\right)\hbar\left(a_1\right) \in P, \text{ for all } a_1, a_2 \in I. \tag{2.18}$$

This last expression obtained is similar to expression (2.8) in the proof of Theorem 1(2). Therefore, by applying similar techniques, the desired result is achieved. □

Theorem 2.4. *Let I be a nonzero ideal, P a semiprime ideal of a ring S such that , $P \subsetneq I$ and $char(S/P) \neq 2$. If $\hbar$ a nonzero homoderivation of S satisfies any one of the condition*

1. *$\hbar(a_1 a_2) - a_1 a_2 \in P$,*
2. *$\hbar(a_1 a_2) - a_2 a_1 \in P$, for all $a_1, a_2 \in I$, then $\hbar$ is P-commuting on I.*

Proof. (1) By the hypothesis, we have

$$\hbar(a_1 a_2) - a_1 a_2 \in P, \text{ for all } a_1, a_2 \in I. \tag{2.19}$$

Swapping the roles of a_1 and a_2 in (2.19), we get

$$\hbar(a_2 a_1) - a_2 a_1 \in P, \text{ for all } a_1, a_2 \in I. \tag{2.20}$$

The expressions for (2.19) and (2.20) together give that

$$\hbar([a_1, a_2]) - [a_1, a_2] \in P, \text{ for all } a_1, a_2 \in I.$$

This expression is same as the assertion (1) of Theorem 2. Using the same arguments in there, we get the required result.

(2) By the hypothesis, we get

$$\hbar(a_1a_2) - a_2a_1 \in P, \text{ for all } a_1, a_2 \in I. \tag{2.21}$$

Swapping the roles of a_1 and a_2 in (2.21), we have

$$\hbar(a_2a_1) - a_1a_2 \in P, \text{ for all } a_1, a_2 \in I. \tag{2.22}$$

Combining (2.21) and (2.22), we get

$$\hbar\,(a_1oa_2) - (a_1oa_2) \in P, \text{ for all } a_1, a_2 \in I.$$

The last expression is the same as Theorem 3 (1). Therefore, the desired result is obtained by the proof of Theorem 3 (1). □

Theorem 2.5. *Let I be a nonzero ideal, P a semiprime ideal of a ring S such that , $P \subsetneq I$. If $\hbar$ a nonzero homoderivation of S satisfies any one of the conditio*

1. $\hbar(a_1a_2) + [a_1, a_2] \in P$,
2. $\hbar(a_1a_2) + (a_1oa_2) \in P$, *for all $a_1, a_2 \in I$, then $\hbar$ is P-commuting on I.*

Proof. By the hypothesis, we have

$$\hbar(a_1a_2) + [a_1, a_2] \in P, \text{ for all } a_1, a_2 \in I. \tag{2.23}$$

Writing a_2a_1 instead of a_2 in (2.23), we have

$$\hbar(a_1a_2)\hbar(a_1) + \hbar(a_1a_2)a_1 + a_1a_2\hbar(a_1) + [a_1, a_2]\,a_1 \in P, \text{ for all } a_1, a_2 \in I.$$

Using (2.23), we get

$$\hbar(a_1a_2)\hbar(a_1) + a_1a_2\hbar(a_1) \in P, \text{ for all } a_1, a_2 \in I. \tag{2.24}$$

This expression can be written as

$$\begin{aligned} & \hbar(a_1a_2)\hbar(a_1) + a_1a_2\hbar(a_1) - a_2a_1\hbar(a_1) + a_2a_1\hbar(a_1) \\ = \; & \hbar(a_1a_2)\hbar(a_1) + [a_1, a_2]\,\hbar(a_1) + a_2a_1\hbar(a_1) \in P. \end{aligned} \tag{2.25}$$

Using the hypothesis, we get

$$a_2a_1\hbar(a_1) \in P, \text{ for all } a_1, a_2 \in I.$$

Replacing a_2 by $a_1\hbar(a_1)a_3$ in last relation, we have

$$a_1\hbar(a_1)a_3a_1\hbar(a_1) \in P, \text{ for all } a_1, a_2, a_3 \in I$$

and so

$$a_1\hbar(a_1)Ia_1\hbar(a_1) \in P, \text{ for all } a_1, a_2 \in I$$

By Lemma 1, we obtain that

$$a_1\hbar(a_1) \in P, \text{ for all } a_1 \in I.$$

Now, substituting a_1a_2 for a_1 in (2.23) and applying similar operations above, we get

$$\hbar(a_1)a_1 \in P, \text{ for all } a_1 \in I.$$

From the last two expressions we get that

$$[\hbar(a_1), a_1] \in P, \text{ for all } a_1 \in I.$$

This means $\hbar$ is P-commuting on I. So the proof is complete.

(2) By the hypothesis, we get

$$\hbar(a_1a_2) + (a_1 o a_2) \in P, \text{ for all } a_1 \in I. \tag{2.26}$$

Replacing a_2 by a_2a_1 in (2.26), and using (2.26), we obtain that

$$\hbar(a_1a_2)\hbar(a_1) + a_1a_2\hbar(a_1) \in P, \text{ for all } a_1, a_2 \in I.$$

Last expression is same as (2.24). Now by following the same steps as after (2.24), we get required result. □

Example 2.1. Consider the ring $S = \left\{ \begin{pmatrix} 0 & 0 & 0 \\ a & 0 & b \\ c & 0 & 0 \end{pmatrix} \mid a, c, b \in \mathbb{R} \right\}$. Let $P = \left\{ \begin{pmatrix} 0 & 0 & 0 \\ a & 0 & 0 \\ c & 0 & 0 \end{pmatrix} \mid a, c \in \mathbb{R} \right\}$ be an ideal of S and $\hbar : S \to S$ be a map defined by

$$\hbar \begin{pmatrix} 0 & 0 & 0 \\ a & 0 & b \\ c & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ a & 0 & 0 \\ c & 0 & 0 \end{pmatrix}.$$

Here, $\hbar$ is a nonzero homoderivation of the ring S. Also, it is clear that the ideal P is not semiprime ideal. The conditions in the above theorems are satisfied but $\hbar$ is not P-commuting.

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