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THE VELOCITY OF ONE DIMENSION COSMOS

Nenad Dj. Lazarov

Department of Theoretical Physics and Condensed Matter Physics
Institute of Nuclear Sciences Vinca - National Institute of the Republic of Serbia
University of Belgrade, Belgrade, Serbia

ORCID ID: Nenad Dj. Lazarov https://orcid.org/0000-0003-3173-9636

Abstract. In this work we consider some properties which have been generated by well-known FRLW metric in the last hundred years. Geodesic equations in one space dimension will be presented. After solving geodesic equations in one space dimension and obtained expression for velocity of tuba cosmos, the theoretical results will be compared with astronomical observation to estimate constant k which is parameter of geometric tensor in FRLW metric.
Keywords: FRLW metric, geodesic equations, tuba cosmos, geometric tensor.

1. Introduction

The question is why Newton didn't get the Einstain equations. The problem lies in the fact that Newtonian mechanics is a global theory, and includes a gravitational potential that diverges in a homogeneous and isotropic cosmos. General relativity is a local theory, involving differential geometry, as opposed to differential calculus in Newtonian mechanics. The distance in the homogeneous and isotropic spacetime of the dynamic cosmos is expressed by the Friedman Robertson Lemaitre Walker metric.

$$ds^2 = -c^2dt^2 + \frac{a^2}{1-kr^2}dr^2 + a^2r^2d\theta^2 + a^2r^2sin^2\theta d\varphi^2, \tag{1.1}$$

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Corresponding Author: Nenad Dj. Lazarov. E-mail addresses: lazarov@vinca.rs
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where a is function of time t, c is velocity of light and r is radius of cosmos. Metric elements of metric tensor is given:

$$g_{00} = -1, \; g_{11} = \frac{a^2}{1-kr^2}, \;\; g_{22} = a^2r^2, \; g_{33} = a^2r^2sin^2\theta. \tag{1.2}$$

Determinant is: $g = g_{00}g_{11}g_{22}g_{33} = -\frac{a^6}{1-kr^2}r^4sin^2\theta$ where a(t) is a cosmic scaling factor that depends on time and describes the evolution of the universe as relation between two points in the universe. The constant k describes the curvature of space. The FLRW metric describes an isotropic cosmos. As can be seen, there are no members that are mixed, i.e. which contain both time and space coordinates, so there is no privileged direction. Since the metric is spherically symmetric, it describes a homogeneous cosmos. At present, a(t) is defined as 1. For simplicity, we will let a(t)=a. If k=0 and $a^2(t) = 1$ we have an ordinary Euclidean metric in spherical coordinates, if $k > 0$ we have a closed cosmos (the volume integral converges), if $k < 0$ we have an open cosmos (the volume integral diverges). If $k = 0$, we have flat metric [3]

$$ds^2 = -c^2dt^2 + a^2dx^2 + +a^2dy^2 + a^2dz^2. \tag{1.3}$$

The FLRW model was made by four authors Alexander Friedmann, Georges Lemaitre, Howard P. Robertson and Arthur Geoffrey Walker. From 1920s to 1930s as Friedmann, Friedmann Robertson Walker (FRW), Robertson Walker (RW), or Friedmann Lemaitre (FL) model [4–7, 9]. This model is associated with the further developed Lambda-CDM model.

To determine the cosmic scaling factor a(t), we used for density of langragian f(R) function of Ricci scalar R which generalizes Einstein's general relativity. The simplest case is f(R)=R; this is general relativity. As a consequence of introducing an arbitrary function, there may be freedom to explain the accelerated expansion and structure formation of the Universe. First, f(R) gravity was first proposed in 1970 by Hans Adolph Buchdahl [2]. After that, Buchdahl f(R) gravity has become a research work by Starobinsky on cosmic inflation [10]. Following that, many authors worked on different shapes of f(R) gravity. From many exotic forms of function f (R) we can get different shapes scaling factor $a(t)$. It should be noted that in paper of authors E. Pachlaner and R. Sexl, the gravitation is as follows $f(R) = R^2$ [8].

The equation of modified gravitation *f(R)* are given by H. A. Buchdahl in his paper 1970 [2]:

$$R_{\mu\nu} \; - \frac{1}{2}g_{\mu\nu}\frac{f}{h} = \frac{h_{;\mu\nu}}{h} - g_{\mu\nu}\frac{h^{;\lambda}_{;\lambda}}{h}, \tag{1.4}$$

and trace equation is

$$R = \frac{2f}{h} - \frac{3}{h}h^{;\lambda}_{;\lambda}, \tag{1.5}$$

where f is a function of Ricci scalar R where are $h = \frac{df}{dR}$ and $h_{;\mu\nu} = \frac{\partial^2 h}{\partial x^\mu \partial x^\nu} - \Gamma^\lambda_{\mu\nu}\frac{\partial h}{\partial x^\lambda}$, $h_{;\mu}$ is a covariant derivative and $\Gamma^\lambda_{\mu\nu}$ is Cristoffel symbol of second order. Covariant dalamberian is $h^{;\lambda}_{;\lambda} = \frac{1}{\sqrt{-g}}\partial_\mu\left[\sqrt{-g}g^{\mu\nu}\partial_\nu h\right]$

As Ricci scalar is function of time t, we have following relation for covariant dalamberian.

Lemma [3]: if h is function of time the covariant dalamberian has next form $h^{;\lambda}_{;\lambda} = -\frac{d^2h}{dt^2} - 3H\frac{dh}{dt}$, where $H = \frac{1}{a}\frac{da}{dt}$ is Hubble parameter and h is function of time.

In next sections we get the velocity of spread cosmos in one dimension and determine the scaling factor $a(t)$ from some models of $f(R)$ function.

2. Geodesic equation of FRLW metric

Geodesic equation of Friedmann–Lemaitre–Robertson–Walker metric in four dimensions are [3]:

$$c^2\frac{d^2t}{dp^2} + \frac{a}{1-kr^2}\frac{da}{dt}\left(\frac{dr}{dp}\right)^2 + ar^2\frac{da}{dt}\left(\frac{d\theta}{dp}\right)^2 + ar^2\sin^2\theta\frac{da}{dt}\left(\frac{d\varphi}{dp}\right)^2 = 0, \tag{2.1}$$

$$\frac{d^2r}{dp^2} + \frac{2}{a}\frac{da}{dt}\frac{dr}{dp}\frac{dt}{dp} - r(1-kr^2)\left(\frac{d\theta}{dp}\right)^2 + \frac{kr}{1-kr^2}\left(\frac{dr}{dp}\right)^2 - r(1-kr^2)sin^2\theta\left(\frac{d\varphi}{dp}\right)^2 = 0, \tag{2.2}$$

$$\frac{d^2\theta}{dp^2} + \frac{2}{a}\frac{da}{dt}\frac{d\theta}{dp}\frac{dt}{dp} + \frac{2}{r}\frac{dr}{dp}\frac{d\theta}{dp} - sin\theta cos\theta\left(\frac{d\varphi}{dp}\right)^2 = 0, \tag{2.3}$$

$$\frac{d^2\varphi}{dp^2} + \frac{2}{a}\frac{da}{dt}\frac{d\varphi}{dp}\frac{dt}{dp} + \frac{2}{r}\frac{dr}{dp}\frac{d\varphi}{dp} + 2\text{ctg}\theta\frac{d\varphi}{dp}\frac{d\theta}{dp} = 0. \tag{2.4}$$

In one space dimension we get following geodesic equations

$$c^2\frac{d^2t}{dp^2} + \frac{a}{1-kr^2}\frac{da}{dt}\left(\frac{dr}{dp}\right)^2 = 0, \tag{2.5}$$

$$\frac{d^2r}{dp^2} + \frac{2}{a}\frac{da}{dt}\frac{dr}{dp}\frac{dt}{dp} + \frac{kr}{1-kr^2}\left(\frac{dr}{dp}\right)^2 = 0. \tag{2.6}$$

Equation (2.6) can be written in the following shape:

$$\frac{d}{dp}\left[ln\frac{dr}{dp} + \ln a^2 - \frac{1}{2}\ln(1-kr^2)\right] = 0, \tag{2.7}$$

and we get

$$\frac{dr}{dp} = \frac{\sqrt{1-kr^2}}{a^2}, \tag{2.8}$$

From equation (2.5) we get

$$\frac{d}{dp}\left[c\left(\frac{dt}{dp}\right)^2 - \frac{1}{ca^2}\right] = 0. \tag{2.9}$$

It is equal

$$\frac{dt}{dp} = \frac{1}{ca}. \tag{2.10}$$

When we divide (2.8) equation with (2.10) equation we get for velocity along radius r:

$$v = \frac{dr}{dt} = c\frac{\sqrt{1-kr^2}}{a}, \tag{2.11}$$

$$\frac{dr}{\sqrt{1-kr^2}} = c\frac{dt}{a}, \tag{2.12}$$

Finally we get radius which depends of time via scaling factor $a(t)$:

$$r = \pm\frac{1}{\sqrt{k}}\sin\left(\sqrt{k}c\int_0^t \frac{dt}{a(t)}\right). \tag{2.13}$$

3. Geodesic equation of FRLW flat metric

Geodesic equation of Friedmann–Lemaitre–Robertson–Walker flat metric in four dimensions are [3]:

$$c^2\frac{d^2t}{dp^2} + a\frac{da}{dt}\left(\frac{dx}{dp}\right)^2 + a\frac{da}{dt}\left(\frac{dy}{dp}\right)^2 + a\frac{da}{dt}\left(\frac{dz}{dp}\right)^2 = 0, \tag{3.1}$$

$$\frac{d^2x}{dp^2} + \frac{2}{a}\frac{da}{dt}\frac{dx}{dp}\frac{dt}{dp} = 0, \frac{d^2y}{dp^2} + \frac{2}{a}\frac{da}{dt}\frac{dy}{dp}\frac{dt}{dp} = 0, \frac{d^2z}{dp^2} + \frac{2}{a}\frac{da}{dt}\frac{dz}{dp}\frac{dt}{dp} = 0. \tag{3.2}$$

After solving equations we get that velocity of four dimensions flat metric is:

$$v = 3\frac{c}{a}. \tag{3.3}$$

4. Determination scaling factor $a(t)$ from some models of function $f(R)$

In FRLW metric Ricci scalar is given by following relations [10]:

$$\mathrm{R} = g^{\mu\upsilon}R_{\mu\upsilon} = -\frac{6}{c^2a}\frac{d^2a}{dt^2} - \frac{6}{c^2a^2}\left(\frac{da}{dt}\right)^2 - \frac{6k}{a^2}. \tag{4.1}$$

THE FIRST MODEL is the Einstain model. Trace equation is:

$$R\frac{df(R)}{dR} - 2f(R) + 3\frac{df(R)}{dR}{}^{;\lambda}{}_{;\lambda} = 0. \tag{4.2}$$

If $f(R) = R$, we get for trace equation $R = 0$, i.e. $R = -\frac{6}{c^2}\left(\frac{d^2p}{2pdt^2} + \frac{kc^2}{2p}\right) = 0$, where $p = \frac{a^2}{2}$.

The solution is $a^2 = -kc^2t^2 + 2C_1t$ where C_1 is some constant.

THE SECOND MODEL is: $f(R) = R^2$. From that model we get the trace equation $R^{;\lambda}_{;\lambda} = \frac{d^2R}{dt^2} + 3H\frac{dR}{dt} = 0$. After that we get the following equation:

$$\frac{dR}{dt}a^3 = K_a, \quad \frac{d^3a}{dt^3}a^2 + a\frac{da}{dt}\frac{d^2a}{dt^2} - 2\left(\frac{da}{dt}\right)^3 - 2kc^2\frac{da}{dt} = -\frac{K_a c^2}{6}, \tag{4.3}$$

where K_a is some constant. Simple solution is $a = \alpha t$ and $-2\alpha^3 - 2kc^2\alpha = -\frac{k_a c^2}{6}$ If $-2\alpha^3 - 2\alpha = -1$, we get for $\alpha = 0.38$.

THE THIRD MODEL is $f(R) = lnR$ and trace equation is $1 - 2lnR + 3\frac{1}{R}^{;\lambda}_{;\lambda} = 0$. We use next substitution $q = \frac{1}{R}$ and get equation $1 + 2lnq + 3\frac{d^2q}{dt^2} + 9H\frac{dq}{dt} = 0$ which has simple solution $q = \frac{1}{\sqrt{e}}$ and $R = \sqrt{e}$. We get following differential equation $R = -\frac{6}{c^2}\left(\frac{d^2p}{2pdt^2} + \frac{kc^2}{2p}\right) = \sqrt{e}$, where $p = \frac{a^2}{2}$. After that we get differential equation and solution for scaling factor $a(t)$:

$$\frac{d^2p}{dt^2} + \frac{c^2\sqrt{e}}{3}p = -kc^2, \quad p_p = -\frac{3k}{\sqrt{e}}, \quad \lambda^2 + \frac{c^2\sqrt{e}}{3} = 0. \tag{4.4}$$

$$\lambda = \pm i\sqrt{\frac{c^2\sqrt{e}}{3}}, \tag{4.5}$$

And finally solution is:

$$\frac{a^2}{2} = p = p_0 \sin\left(\sqrt{\frac{c^2\sqrt{e}}{3}}t + \varphi\right) - \frac{3k}{\sqrt{e}}. \tag{4.6}$$

5. Conclusion

For the FIRST MODEL we get: $a^2 = -kc^2t^2 + 2C_1t$ where C_1 is some constant and $v = \frac{c}{\sqrt{-kc^2t^2+2C_1t}}\sqrt{1-kr^2}$ When $k = 0$, the velocity of dynamic cosmos is $v = \frac{c}{\sqrt{2C_1t}}$. At present time the radius and age of cosmos are: $r_{cosmos} = 86 \cdot 10^{25}m$ [1] and $T_{cosmos} = 4.41 \cdot 10^{17}s$ [1] In the first Einstain model, for radius and velocity of dynamic cosmos we have got the following expressions:

$r = c\sqrt{\frac{2t}{C_1}} = 4.3 \cdot 10^9\sqrt{second} \cdot 3 \cdot 10^8\sqrt{t} = 12.9 \cdot 10^{17}\sqrt{t}\ metara,$

$v = \frac{c}{\sqrt{2C_1t}} = 1.5 \cdot 10^8 \cdot 4.3 \cdot 10^9\frac{1}{\sqrt{t}} = 6.5 \cdot 10^{17}\frac{1}{\sqrt{t}}\frac{m}{s}.$

In the SECOND MODEL, the velocity is given by following expression:

$$v = \frac{c}{\alpha t}\sqrt{1-kr^2}. \tag{5.1}$$

If we take for $\alpha = \sqrt{k}c = H_0 = 22.68 \cdot 10^{-19} s$ for velocity we have next expression $v = \frac{c}{H_0 t}\sqrt{1-kr^2}$. Also for radius of cosmos we get:

$$(5.2) r = \pm\frac{1}{\sqrt{k}}\sin\left(\sqrt{k}c\int_0^t \frac{dt}{a(t)}\right) = \pm\frac{1}{\sqrt{k}}\sin\left(\frac{\sqrt{k}c}{\alpha} ln\frac{t}{t_0}\right) = \pm\frac{1}{\sqrt{k}}\sin\left(ln\frac{t}{t_0}\right).$$

If k=0 then $v = \frac{c}{H_0 t}$ and $r = \frac{c}{H_0} ln\frac{t}{t_0}$.

In the THIRD MODEL, the velocity of dynamic cosmos is given by the following relation:

$$(5.3) \qquad v = \frac{c}{\sqrt{2p_0 \sin\left(\sqrt{\frac{c^2\sqrt{e}}{3}}t + \varphi\right) - \frac{6k}{\sqrt{e}}}}\sqrt{1-kr^2},$$

and for $k = 0$ the velocity is:

$$v = \frac{c}{\sqrt{2p_0 \mathrm{abs}\left(\sin\left(\sqrt{\frac{c^2\sqrt{e}}{3}}t+\varphi\right)\right)}}.$$

If $t = 0$ then $v = c$ and $2p_0 \sin(\varphi) = 1$ for velocity we get for $\varphi = \frac{\pi}{2}$:

$$v = \frac{c}{\sqrt{\mathrm{abs}\left(\sin\left(\sqrt{\frac{c^2\sqrt{e}}{3}}t+\frac{\pi}{2}\right)\right)}}.$$

It can be seen in the Einstain model that radius increases with square root of time and the velocity decreases as the reciprocal of the square root of time. Also, in the second model, radius increases with logarithmic value of time, and velocity decreases as the inverse of time. It is very interesting that in the third model the velocity of dynamic cosmos is greater than the velocity of light. Also, we can see that the velocity of four dimensions flat cosmos is three times greater than in two dimensions.

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