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**Editorial**

# XXII GEOMETRICAL SEMINAR

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## 1. Introduction

The Geometrical Seminar is traditional conference that began during the eighties in the last century, under the name Yugoslav Geometrical Seminar. The aim of these meetings is to bring together mathematicians, physicists and engineers interested in geometry and its applications, to give lectures on new results, exchange ideas, problems and conjectures. The last conference, XXI Geometrical Seminar 2022, was held at Belgrade 2022, and involved around 130 participants from all over the world. The international conference XXII Geometrical Seminar was held in Vrnjačka Banja, Serbia from May 25 to May 31, 2024. XXII Geometrical seminar is organized by Faculty of Science and Mathematics, University of Niš, and Faculty of Mathematics, University of Belgrade, in collaboration with Faculty of Science, University of Kragujevac, and Mathematical Institute of the Serbian Academy of Sciences and Arts (SANU), Belgrade.

Conference topics are: Differential Geometry, Topology, Lie Groups, Mathematical Physics, Discrete Geometry, Integrable Systems, Visualization, Teaching Geometry, Geometric Education, as well as other subjects related to the main themes.

The XXII Geometrical seminar is dedicated to professor Stana Nikčević.

This special issue of journal Facta Universitatis, Series: Mathematics and Informatics complies peer-reviewed papers covering various geometric topics presented at

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the conference. The collection of papers spans from theoretical studies to practical applications across multiple scientific disciplines. The XXII Geometrical Seminar hosted than 90 participants from all over the world. This is traditional meeting bringing together mathematicians, physicists and engineers with an interest in Geometry and its applications. The aim of the Geometrical Seminar is to enable researches to give lectures on new results, exchange ideas, problems and conjectures. The papers in this issue were accepted for presentation after having been subjected to the usual strict reviewing process by the members of the conference committee. The editor thanks the members of the Committees of Geometrical Seminar and to the Faculty of Sciences and Mathematics, Niš, Faculty of Mathematics, Belgrade, Faculty of Science, Kragujevac and Mathematical Institute SANU for their great effort in organizing this conference.

**The members of the International Advisory Committee of the conference are:**

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## 2. Overview of the Published Papers

This Special Issue contains 15 papers which were accepted for publication after a rigorous reviewing process.

In the paper [1] the author investigates the properties of 3-triangulation of polyhedra, when possible. Namely, it is known that 3-triangulation of convex polyhedra is always possible, but this is not the case with all non-convex ones. This is the reason to consider the decomposition of non-convex polyhedra into convex pieces if possible. After that, the author introduces the connection graph for the 3-triangulable polyhedron in such a way that these pieces are represented by the nodes of the graph. First, our attention shall be focused on toroids, a special class of non-convex polyhedra, and the minimal number of tetrahedra necessary to 3-triangulate them.

The main purpose of the paper [2] is to derive the conditions that ensure the stability of the generalized $S$-space forms with two structure vector fields. In addition, some particular conditions under which a generalized $S$-space form with two structure vector fields is unstable are obtained. Several consequences are also discussed at the end of the paper.

In the article [3], the author classifies general relativistic vacuum constraint equations on a Riemannian manifold using a method based on the pointwise decomposition of tensor products (reducible with respect to the action of the orthogonal group) into irreducible components. Each selected class of equations is described.

In the paper [4] the author considers some properties which have been generated by well-known FRLW metric in the last hundred years. Geodesic equations in one space dimension are presented. After solving geodesic equations in one space dimension and obtaining an expression for velocity of tuba cosmos, the theoretical results are compared with astronomical observation to estimate constant k which is parameter of geometric tensor in FRLW metric.

The paper [5] surveys recent results (concerning geodesic and Killing fields, rigidity and splitting theorems, Ricci-type solitons and Einstein-type metrics, etc.) in this new field of Riemannian geometry.

In the paper [6], the authors present two new variants of Homeier's iterative method for finding simple, real or complex, solutions of nonlinear equations. Increasing the order of convergence from three to four is achieved by one additional term. Through many numerical examples, by classical and criteria based on basins of attraction, it is shown that the new methods can be competitive to other fourth-order methods.

In the paper [7], canonical biholomorphically projective and equitorsion canonical biholomorphically projective mappings are defined. Some relations between corresponding curvature tensors of the generalized Riemannian spaces $GR_N$ and $G\overline{R}_N$ are obtained. At the end, invariant geometric object of equitorsion canonical biholomorphically projective mapping is found.

In the paper[8] the authors study two kinds of curvature invariants of Riemannian manifold equipped with a complex distribution $D$ related to sets of pairwise orthogonal subspaces of the distribution. One kind of invariant is based on the mutual curvature of the subspaces and another is similar to Chen's $\delta$-invariants. They compare the mutual curvature invariants with Chen-type invariants and prove geometric inequalities with intermediate mean curvature squared for CR-submanifolds in almost Hermitian spaces. In the case of a set of complex planes, the authors introduce and study curvature invariants based on the concept of holomorphic bisectional curvature. As applications, they give consequences of the absence of some $D$-minimal CR-submanifolds in almost Hermitian manifolds.

In the paper [9] the authors consider conformal and projective mappings of generalized Riemannian spaces in Eisenhart's sense and find necessary and sufficient conditions for these mappings to preserve curvature, Ricci and traceless Ricci tensors and some of their linear combinations. Particularly, they find that the following result holds in the case of Riemannian spaces: if a conformal mapping $f_1 : M \to \hat{M}$ is preserving the traceless Ricci tensor and a projective mapping $f_2 : \hat{M} \to \overline{M}$ is preserving the traceless Ricci tensor then the Yano tensor of concircular curvature is invariant with respect to the composition $f_3 = f_1 \circ f_2 : M \to \overline{M}$.

In this paper [10] the authors study infinitesimal bending of dual spherical curves using the Blaschke frame. They give the necessary and sufficient conditions for the

infinitesimal bending field. Also, they consider the hyperbolic paraboloid as a ruled surface corresponding to a dual spherical curve.

In the paper [11], the creation of the human organs' geometrical models was studied by including different relations between Regions of Interest (RGIs) models and specific properties, like functional, materials, and topological. Enhancing existing methodologies in CAS aims to offer a more comprehensive geometrical description of human organs, leading to the development of more precise and anatomically accurate personalized geometrical models.

In the paper [12] the author discusses properties of geodesic orbit Riemannian metrics on nilpotent Lie groups and some recent examples of such metrics. In particular, the author explains the construction of continuous families of pairwise non-isomorphic connected and simply connected nilpotent Lie groups of dimension $4k + 6$, $k \geq 1$, every of which admits geodesic orbit metrics.

In view of the meaning of ruled surfaces in aesthetics, statics, scale and manufacturing technologies, the authors in the paper [13] point out the possibility of a mathematical analysis in the case of infinitesimal deformations by considering the variations of the Willmore energy on Gaudi surface under infinitesimal bending in $\mathbb{R}^3$. Application could be connected with energy-efficient building design.

In the paper [14] the author proves the non-existence of Hopf real hypersurfaces in the nearly Kähler sphere $S^6(1)$ whose structure Jacobi operator is of Codazzi type.

The paper [15] studies a generalization of the classical Fermat–Torricelli problem to normed spaces of arbitrary finite dimension. Given integer $n$, we describe all normed spaces such that the solution of the Fermat–Torricelli problem is unique for any $n$ points in this space. More precise conditions for normed planes and three-dimensional spaces are presented. In addition, the author applies the criterion to norms whose unit balls are regular polyhedra.

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5. V. Rovenski: *Weak Almost Contact Structures: a Survey*. Facta Universitatis, Series: Mathematics and Informatics **39**(5) (2024), 821–841.
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