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P−CONNECTEDNESS BETWEEN SOFT SETS

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Abstract. In the present paper, the concepts of soft P-connectedness between soft sets and soft set P-connected mappings in soft topological spaces have been introduced and studied.
Keywords: P-connectednes, soft sets, soft topological spaces

1. Introduction

Most of our real life problems in engineering, social and medical science, economics, environment, etc. involve imprecise data and their solutions involve the use of mathematical principles based on uncertainty and imprecision. To handle such uncertainties, a number of theories have been proposed. Some of these are vague sets [8], fuzzy sets [23], intuitionistic fuzzy sets [3], and rough sets [17]. Recently, soft set have played an important role in this field. The concept of soft set theory has been initiated by Molodtsov [15] in 1999 as a general mathematical tool to deal with uncertainties while modeling problems in engineering physics, computer science, economics, social sciences and medical sciences. In ([15],[16]), Molodtsov successfully applied the soft theory in several directions such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability, and theory of measurement. Maji et al. ([6],[14]) gave the first practical application of soft sets in decision making problems. In 2003, Maji et al. [13] defined and studied several basic notions of soft set theory. In 2005, Pei and Miao [18] and Chen [7] improved the work of Maji et al. ([13],[14]). In recent years, an increasing number of papers have been written about soft sets theory and its applications in various fields ([5],[16]). Shabir and Naz [20] introduced the notion of soft topological spaces which are defined to be over an initial universe with a fixed set of parameters and the notions of soft open sets, soft closed sets, soft closure, soft interior and soft separation axioms. After the publication of Shabir and Naz [20] paper, many topological concepts such as connectedness [19] , compactness [24], semi-open ([4],[7]) , $\alpha-$open [11], b−open [11] , preopen[11], clopen [11], $\beta-$open [11],$g-$closed [21], g$\beta-$closed [2] and gs$\beta-$closed [2] sets have been extended in soft topological spaces. Recently J. Subhashinin , Dr. C. Sekar. [21] introduced the concepts of soft P-connectedness in soft topological spaces.

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In this paper we introduce the concepts of soft P-connectedness between the soft sets and soft P-connected mappings in soft topological spaces and study some of their properties.

2. Preliminaries

Let U be an initial universe set, E be a set of parameters, P(U) be the power set of U and A ⊆ E.

Definition 2.1. [15] A pair (F, A) is called a soft set over U , where F is a mapping given by F: A → P(U).
In other words, a soft set over U is a parameterized family of subsets of the universe U. For all e ∈ A, F(e) may be considered as the set of e−approximate elements of the soft set (F, A).

Definition 2.2. [13] For two soft sets (F, A) and (G, B) over a common universe U, we say that (F, A) is a soft subset of (G, B) denoted by (F, A) ⊆ (G, B), if
(a) A ⊆ B and
(b) F (e) ⊆ G (e) for all e ∈ E.

Definition 2.3. [13] Two soft sets (F, A) and (G, B) over a common universe U are said to be soft equal denoted by (F, A) = (G, B) if (F, A) ⊆ (G, B) and (G, B) ⊆ (F, A).

Definition 2.4. [1] The complement of a soft set (F, A) denoted by $(F, A)^c$, is defined by: $(F, A)^c = (F^c$, A), where, F^c : A → P(U) is a mapping given by, F^c(e) = U − F(e), for all e ∈ E.

Definition 2.5. [13] Let a soft set (F, A) over U.
(a) Null soft set denoted by ϕ if for all e ∈ A , F(e) = ϕ.
(b) Absolute soft set denoted by $\widetilde{U}$, if for each e ∈ A , F(e) = U.
Clearly, $\widetilde{U}^c = \phi$ and $\phi^c = \widetilde{U}$.

Definition 2.6. [1] Union of two sets (F, A) and (G, B) over the common universe U is the soft (H, C), where C = A ∪ B, and for all e ∈ C,

$$H(e) = \begin{cases} F(e), & ife \in A - B \\ G(e), & ife \in B - A \\ H(e), & \text{if } e \in A \cap B \end{cases}$$

Definition 2.7. [1] Intersection of two soft sets (F, A) and (G, B) over a common universe U, is the soft set (H, C) where C = A ∩ B and H (e) = F(e) ∩ G(e) for each e ∈ E.

Let X and Y be the initial universe sets and E and K be the nonempty sets of parameters, S(X, E) denotes the family of all soft sets over X, and S(Y, K) denotes the family of all soft sets over Y.

Definition 2.8. [12] Let S(X, E) and S(Y, K) be families of soft sets. Let u: X → Y and p: E → K be mappings. Then a mapping f_{pu}: S(X, E) → S(Y, K) is defined as:

(a) Let (F, A) be a soft set in S(X, E). The image of (F, A) under f_{pu}, Written as f_{pu} (F, A) = (f_{pu}(F), p(A)), is a soft set in S(Y, K) such that

$$f_{pu}(F)(k) = \begin{cases} \bigcup_{e\in p^{-1}(k)\bigcap A} u(F(e)) & ,p^{-}1(k)\bigcap A \neq\phi \\ \phi & ,p^{-1}(k)\bigcap A =\phi \end{cases}$$

For all k ∈ K.

(b) Let (G, B) be a soft set in S(Y, K). The inverse image of (G, B) under f_{pu}, written as

$$f_{pu}^{-1}(G, B) = \begin{cases} u^{-1}G(p(e)) & ,p(e)\in B \\ \phi & ,otherwise \end{cases}$$

For all e ∈ E.

The soft mapping f_{pu} is called surjective if p and u are surjective. The soft mapping f_{pu} is called injective if p and u are injective.

Definition 2.9. [20]A subfamily τ of S(X, E) is called a soft topology on X if:

1. $\widetilde{\phi}$, $\widetilde{X}$ belong to τ.
2. The union of any number of soft sets in τ belongs to τ.
3. The intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E) is called a soft topological space over X. The members of τ are called soft open sets in X and their complements called soft closed sets in X.

Definition 2.10. If (X, τ, E) is a soft topological space and (F, E) ∈ S(X, E).
(a) The soft closure of (F, E) is denoted by Cl(F, E), is defined as the intersection of all soft closed super sets of (F, E) [20].
(b) The soft interior of (F, E) is denoted by Int(F, E), is defined as a soft union of all soft open subsets of (F, E) [24].

Theorem 2.1. *Let (X, τ, E) be a soft topological space and let (F, E), (G, E) ∈ S(X, E). Then :*
(a)(F, E) is soft closed iff (F, E) = Cl (F, E) [20].
(b) If (F, E) ⊆ (G, E), then Cl(F, E) ⊆ Cl(G, E) [20].
(c) (F, E) is soft open iff (F, E) = Int(F, E) [24].
(d) If (F, E) ⊆ (G, E), then Int(F, E) ⊆ Int(G, E) [24].
(e) $(Cl(F, E)^c) = Int((F, E)^c)$ *[24].*
(f) $(Int(F, E))^c = Cl((F, E)^c)$ *[24].*

Definition 2.11. Let (X, τ, E) and (Y, υ, K) be soft topological spaces. A soft mapping f_{pu} : (X, τ, E)→(Y, υ, K) is called :
(a) Soft continuous if f_{pu}^{-1}(G, K) is soft open in X, for every soft open set (G, K) in Y. [24]
(b) Soft open if f_{pu} (F, E) is soft open in Y, for all soft open sets (F, E) in X. [25]
(c) Soft closed if f_{pu} (F, E) is soft closed in Y, for all soft closed sets (F, E) in X. [25]

Definition 2.12. [9] A soft topological space (X, τ, E) is soft connected if and only if no nonempty soft subset of (X, τ, E) which is both soft open and soft closed in (X, τ, E) .

Definition 2.13. [22] A soft topological space (X ,τ, E) is said to be soft connected between its soft subsets (F_1, E) and (F_2, E) if and only if there is no soft clopen subset (F, E) over X such that (F_1,E) $\subset$ (F, E) and (F, E) $\cap$ (F_2,E) = ϕ.

Definition 2.14. [21] Let (X, τ, E) be a soft topological space, a soft set (F, A) is said to be soft preopen set if there exists a soft open set (F, O) such that (F, A) $\subseteq$ (F, O) $\subseteq$ Cl(F, A). Then $(F, A)^c$ is said to be soft preclosed .

Remark 2.1. [21] A soft set (F, A) which is both soft preopen and soft preclosed is known as soft preclopen set. Clearly, ϕ and $\tilde{X}$ are soft preclopen sets.

Remark 2.2. [21] Every soft open set (respectively soft closed set) is a soft preopen set (respectively soft preclosed set) but the converse may not be true.

Theorem 2.2. *[21] (i) Arbitrary soft union of soft preopen sets is a soft preopen set. (ii) Arbitrary soft intersection of soft preclosed sets is a soft preclosed set.*

Definition 2.15. [21] The intersection of all soft preclosed sets containing a soft set (F, A) in a soft topological space (X, τ, E) is called the soft preclosure of (F, A) and is denoted by Pcl(F, A).

Definition 2.16. [21] The union of all soft preopen sets contained in a soft set (F, A) in a soft topological space (X, τ, E) is called the soft preinterior of (F, A) and is denoted by Pint(F, A).

Definition 2.17. [21] Let (X, τ, E) be a soft topological space. Two nonempty soft sub sets (F, A) and (F, B) of S(X, E) are called soft preseparated iff Pcl(F, A) $\cap$ (F, B) = (F, A) $\cap$ Pcl(F, B) = ϕ .

Definition 2.18. [21] Let (X, τ, E) be a soft topological space. If there does not exist a soft preseparation of X, then it is said to be soft P-connected .

Lemma 2.1. *[21] A soft topological space (X, τ, E) is said to be soft P-connectedness if there is no nonempty proper soft set over X which is both soft preopen and soft preclosed.*

Lemma 2.2. *[10] If (F, E) is soft open and (G, E) is soft preopen then (G, E) $\cap$ (F, E) is soft preopen .*

Definition 2.19. [24] The soft set (F, E) $\in$ S(X, E) is called a soft point if there exist x $\in$ X and e $\in$ E such that F(e) = {x} and F(e') = ϕ for each e' $\in$ E – {e}, and the soft point (F, E) is denoted by $(x_e)_E$.

Remark 2.3. [21] Every soft P-connected space is soft connected space but the converse is not true .

3. P-Connectedness Between Soft Sets

Definition 3.1. A soft topological space (X, τ, E) is said to be soft P-connected between between soft sets (F_1, E) and (F_2, E) if and only if there is no soft preclopen set (F, E) over X such that (F_1, E) $\subset$ (F, E) and (F, E) $\cap$ (F_2, E) = ϕ.

Theorem 3.1. *Every soft topological space which is soft P-connected between soft sets (F_1, E) and (F_2, E) is soft connected between (F_1, E) and (F_2, E).*

Proof. *Suppose soft topological space (X, τ, E) is not soft connected between (F_1, E) and (F_2, E). Then, there is a soft clopen set (F, E) over X such that (F_1, E) $\subset$ (F, E) and (F, E) $\cap$ (F_2, E) = ϕ. Since, every soft clopen set is soft preclopen, it follows that (X, τ, E) is not soft P-connected between (F_1, E) and (F_2, E). This is a contradiction.* □

Remark 3.1. The converse of Theorem 3.1 may not be true.

Example 3.1. Let X = $\{x_1, x_2, x_3, x_4\}$,E = $\{e_1, e_2\}$ and soft sets are defined as : (F, E) = $\{(e_1,\{x_1, x_3\}),(e_2, \{x_1 ,x_3\})\}$, (F_1, E) = $\{(e_1 ,\{x_1\}) ,(e_2,\{x_1\})\}$ and (F_2, E) = $\{(e_1 ,\{x_3\}),(e_2 ,\{x_3\})\}$. Let $\tau = \{\phi$, (F ,E) , $\tilde{X}\}$ is topology on X. Then, soft topological space (X, τ, E) is soft connected between the soft sets (F_1, E) and (F_2, E) but not soft P-connectedness between (F_1, E) and (F_2, E)

Theorem 3.2. *A soft topological space (X, τ, E) is soft P-connected between soft sets (F_1, E) and (F_2, E) if and only if there is no soft preclopen set (F, E) over X such that (F_1, E) $\subset$ (F, E) $\subset$ $(F_2, E)^C$.*

Proof. *It follows from definition 3.1.* □

Theorem 3.3. *If a soft topological space (X, τ, E) is soft P-connected between soft sets (F_1, E) and (F_2, E) then (F_1, E) $\neq \phi \neq$ (F_2, E) .*

Proof. *If any soft set (F_1, E) = ϕ , then ϕ being soft preclopen set over X,(X, τ, E) cannot be soft P-connected between soft sets (F_1, E) and (F_2,E).* □

Theorem 3.4. *If a soft topological space (X, τ, E) is soft P-connected between soft sets (F_1, E) and (F_2, E) and (F_1, E) $\subset$ (F_3, E) and (F_2, E) $\subset$ (F_4, E) then (X, τ, E) is soft P-connected between soft sets (F_3, E) and (F_4, E).*

Proof. *Suppose a soft topological space (X, τ, E) is not soft P-connected between soft sets (F_3, E) and (F_4, E) then there is a soft P-clopen set (F, E) over X such that (F_3,E) $\subset$ (F, E) and (F, E) $\cap$ (F_4, E) = ϕ. Consequently, (X, τ, E) is not soft P-connected between soft sets (F_1, E) and (F_2, E) .* □

Lemma 3.1. *A soft point $(x_e)_E \in$ Pcl(F, E) if and only if (F, E) $\cap$ (G,E) $\neq \phi$ for all soft preopen set (G,E) containing $(x_e)_E$ over X .*

Proof. *It is obvious .* □

Theorem 3.5. *If a soft topological space (X, τ, E) is soft P-connected between soft sets (F_1, E) and (F_2, E) if and only if (X, τ, E) is soft P-connected between soft sets Pcl(F_1, E) and Pcl(F_2, E).*

Proof. *Necessity: It follows from Theorem (7)(ii) [10] and Theorem 3.4 .*
Sufficiency: If a soft topological space (X, τ, E) is not soft P-connected between soft sets (F_1, E) and (F_2, E) then there exists a soft preclopen set (F, E) over X such that (F_1, E) $\subset$ (F, E) and (F, E) $\cap$ (F_2, E) = ϕ. Since (F, E) is soft preclosed Pcl(F_1,E) $\subset$ Pcl(F, E) = (F, E). Clearly, (F, E) cap Pcl(F_2,E) = ϕ. For if $(x_e)_E \in$ (F, E) $\cap$ Pcl(F_2, E) then by lemma 3.1, (F, E) $\cap$ (F_2, E) $\neq \phi$, because (F, E) is soft preopen. Hence, (X, τ, E) is not soft P-connected between soft sets Pcl(F_1, E) and Pcl(F_2, E). □

Theorem 3.6. *If a soft topological space (X, τ, E) is soft P-connected between soft sets (F_1, E) and (F_2, E) then (X, τ, E) is soft P-connected between soft sets Cl(F_1, E) and Cl(F_2, E).*

Proof. *Follows from theorems 3.4 .* □

Remark 3.2. The converse of theorem 3.6 is not true in general .

Example 3.2. Let X = $\{x_1, x_2, x_3, x_4\}$, E = $\{e_1, e_2\}$ and soft sets are defined as: (F, E) = $\{(e_1,\{x_1, x_3\}),(e_2, \{x_1, x_3\})\}$, (F_1, E) = $\{(e_1, \{x_1\}), (e_2,\{x_1\})\}$ and (F_2, E) = $\{(e_1, \{x_2\}),(e_2, \{x_2\})\}$. Let $\tau = \{\phi$, (F ,E) , $\tilde{X}\}$ is topology on X. Then, soft topological space (X,τ ,E) is soft P-connected between the soft sets Cl(F_1,E) and Cl(F_2,E) but not soft P-connected between (F_1, E) and (F_2, E) .

Theorem 3.7. *A soft topological space (X, τ, E) is not soft P-connected between (F_{A_0}, E) and (F_{A_1}, E) if and only if there exist soft preclopen disjoint soft sets (F_0, E) and (F_1, E) such that X = (F_0,E) $\cup$ (F_1, E) and (F_{A_i}, E) $\subset$ (F_i,E), i = 0,1.*

Proof. *This immediately follows from the definition of a space which is soft P-connected between two of its soft subsets.* □

Theorem 3.8. *If (F_1, E) and (F_2, E) are soft sets over X and (F_1, E) $\cap$ (F_2, E) $\neq \phi$, then the soft topological space (X, τ, E) is soft P-connected between (F_1, E) and (F_2, E).*

Proof. *If (F, E) is any soft preclopen set over X such that (F_1, E) $\subset$ (F, E) ,then (F_1, E) $\cap$ (F_2, E) $\neq \phi \Rightarrow$ (F, E) $\cap$ (F_2, E) $\neq \phi$. This proves the theorem .* □

Remark 3.3. The converse of theorem 3.5 need not be true .

Example 3.3. Let X = $\{x_1, x_2, x_3, x_4\}$,E = $\{e_1, e_2\}$ and soft sets are defined as : (F, E) = $\{(e_1,\{x_1, x_3\}),(e_2, \{x_1, x_3\})\}$ and (F_1,E) = $\{(e_1, \{x_2, x_4\}),(e_2, \{x_2, x_4\})\}$. Let $\tau = \{\phi$, (F ,E) , $\tilde{X}\}$ is topology on X. Then, a soft topological space (X, τ, E) is soft P-connected between the soft sets (F, E) and (F_1, E) whereas, (F, E) $\cap$ (F_1,E) = ϕ .

Theorem 3.9. *If a soft topological space (X, τ, E) is neither soft connected between (A ,E) and (B_0,E) nor soft P-connected between (A ,E) and (B_1,E) then it is not soft P-connected between (A ,E) and (B_0,E) $\cup$ (B_1,E).*

Proof. *Since a soft topological space (X, τ, E) is not soft connected between (A ,E) and (B_0,E) ,there is a soft clopen set (F_0,E) over X such that (A,E) $\subset$ (F_0,E) and (F_0,E) $\cap$ (B_0,E) = ϕ. Also, since (X, τ, E) is not soft P-connected between (A ,E) and (B_1,E) there exists a soft preclopen set (F_1,E) over X such that (A ,E) $\subset$ (F_1,E) and (B_1,E) $\cap$ (F_1,E) = ϕ .Put (F ,E) = (F_0,E) $\cap$ (F_1,E). Since, each soft closed set is soft preclosed and any intersection of soft preclosed set is soft preclosed, (F, E) is soft preclosed . Also, by lemma 2.2 (F, E) is soft preopen . Therefore (F, E) is soft preclopen over X such that (A,E) $\subset$ (F, E) and (F, E) $\cap$ ((B_0,E) $\cup$ (B_1, E)) = ϕ. Hence, (X, τ, E) is not soft P-connected between (A,E) and (B_0 ,E) $\cup$ (B_1 ,E) .* □

Remark 3.4. If a soft topological space (X, τ, E) is soft P-connected neither between (A,E) and (B_0,E), nor between (A ,E) and (B_1,E) then it is not necessarily true that (X, τ, E) is not soft P-connected between (A ,E) and (B_0,E) $\cup$ (B_1,E).

Example 3.4. Let X = $\{x_1 , x_2 ,x_3 ,x_4 \}$,E = $\{e_1, e_2\}$ and soft sets are defined as : (F, E) = $\{(e_1,\{x_1 ,x_3 \}),(e_2, \{x_1 ,x_3 \})\}$, (F_1,E) = $\{(e_1 ,\{x_2\}) ,(e_2,\{x_2 \})\}$, (F_2,E) = $\{(e_1 ,\{x_1 \}),(e_2 ,\{x_1 \})\}$ and (F_3,E) = $\{(e_1 ,\{x_3 \}) , (e_2,\{x_3\})\}$ Let $\tau = \{\phi$, (F ,E) , $\tilde{X}$ } is topology on X . Then soft topological space (X,τ ,E) is soft P-connected neither between the soft sets (F_1,E) and (F_2,E) nor (F_1,E) and (F_3,E) but it is soft P-connected between (F_1,E) and (F, E).

Theorem 3.10. *A soft topological space (X, τ, E) is soft P-connected if and only if it soft P-connected between every pair of its nonempty soft sets.*

Proof. *Let (A, E) and (B, E) be a pair of nonempty soft sets over X. Suppose (X, τ, E) is not soft P-connected between (A, E) and (B, E). Then, there is a soft preclopen set (F, E) over X such that (A, E) $\subset$ (F, E) and (B, E) $\cap$ (F, E) = ϕ .Since, (A, E) and (B, E) are nonempty it follows that (F, E) is a nonempty soft proper preclopen set over X. Hence, by lemma 2.1 (X, τ, E) is not soft P-connected.*
Conversely, suppose that (X, τ, E) is not soft P-connected. Then, there exists a nonempty soft proper (F, E) over X which is both soft preopen and preclosed. Consequently, (X, τ, E) is not soft P-connected between (F, E) and $(F, E)^c$. Thus (X, τ, E) is not soft P-connected between arbitrary pair of its nonempty soft sets. □

Remark 3.5. If a soft topological space (X, τ, E) is soft P-connected between a pair of its soft sets, then it is not necessarily soft P-connected between each pair of its soft sets and so is not necessarily soft P-connected .

Example 3.5. Let X = $\{x_1 , x_2 ,x_3 ,x_4 \}$,E = $\{e_1, e_2\}$ and soft sets are defined as: (F, E) = $\{(e_1,\{x_1 ,x_3 \}),(e_2, \{x_1 ,x_3 \})\}$, (F_1,E) = $\{(e_1 ,\{x_2\}) ,(e_2,\{x_2 \})\}$, (F_2,E) = $\{(e_1 ,\{x_1 \}),(e_2 ,\{x_1 \})\}$ and (F_3,E) = $\{(e_1 ,\{ x_3 \}) , (e_2,\{x_3\})\}$. Let $\tau = \{\phi$, (F ,E) , $\tilde{X}$ } is topology on X. Then, soft topological space (X,τ ,E)is soft P-connected between soft sets (F_1,E) and (F, E) but it is not soft P-connected between (F_2,E) and (F_3,E). Also a soft topological space (X, τ, E) is not soft P-connected.

Lemma 3.2. *Let a soft set (A, E) of a subspace (Y, τ_Y , E) of a soft topological space (X, τ , E) and (A, E) is soft preopen set over X, then (A, E) is a soft preopen set over Y.*

Proof. *It is obvious .* □

Lemma 3.3. *Let (Y, τ_Y , E) be a soft topological subspace of soft topological (X, τ , E) and (F, E) be a soft preopen set over X, then (F, E) $\cap$ $\tilde{Y}$ is soft preopen in Y.*

Proof. Follows from lemma 2.2 and lemma 3.2 . □

Lemma 3.4. *Let (Y, τ_Y , E) be a soft subspace of a soft topological space (X, τ , E) and (, E) be a soft preopen set over Y then (F, E) is soft preopen in X.*

Proof. It is obvious . □

Lemma 3.5. *If (Y, τ_Y, E) is soft open in (X, τ, E) and (B,E) is a soft preclosed set over X then $\tilde{Y}$ $\cap$ (B, E) is soft preclosed in (Y, τ_Y, E) .*

Proof. We have , (B, E) is soft preclosed over X
$\Rightarrow (B, E)^C$ is soft preopen over X
$\Rightarrow \tilde{Y} \cap (B, E)^c = (\tilde{Y} \cap (B, E))^c$ is soft preopen in Y (by lemma 3.3)
$\Rightarrow \tilde{Y}$ $\cap$ (B,E) is soft preclosed over Y. □

Theorem 3.11. *Let (Y, τ_Y, E) be an open soft subspace of a space (X, τ, E). If (Y,τ_Y,E) is soft P-connected between the soft sets (A_Y, E) and (B_Y, E) over Y, then the whole space (X, τ, E) is soft P-connected between (A_Y, E) and (B_Y, E) .*

Proof. Suppose (X, τ, E) is not soft P-connected between (A_Y , E) and (B_Y , E). Then, there is soft Preopen set (F ,E) over X such that (A_Y , E) $\subset$ (F, E) and (F, E) $\cap$ (B_Y, E) = ϕ .Hence, By lemma 3.3 and lemma 3.5 , $\tilde{Y}$ $\cap$ (F ,E) is soft preclopen over Y such that (A_Y , E) $\subset$ (F, E) $\cap$ $\tilde{Y}$ and ((F, E) $\cap$ $\tilde{Y}$) $\cap$ (B_Y , E) = ϕ .
Consequently, (Y,τ_Y,E) is not soft P-connected between (A_Y , E) and (B_Y , E), a contradiction. □

Lemma 3.6. *A soft set (F ,E) over X is soft preclosed if and only if (F ,E) $\supset$ Cl(Int(F ,E)).*

Proof. We have , (F ,E) is soft preclosed over X
$\Leftrightarrow (F, E)^c$ in X .
$\Leftrightarrow (F, E)^c \subset Int(Cl((F, E)^c)$
$\Leftrightarrow (F ,E) \supset (Int(Cl((F, E)^c)))^c = Cl(Int(F ,E))$. □

Lemma 3.7. *Let (Y, τ_Y, E) be a soft subspace of a space (X, τ, E). If a soft set (F_Y, E) is soft preclosed over Y and $\tilde{Y}$ is soft preclosed over X then (F_Y, E) is soft preclosed in X.*

Proof. If a soft set (F_Y, E) is soft preclosed over Y and $\tilde{Y}$ is soft preclosed over X, then by lemma 3.6 (F_Y , E) $\supset$ $Cl_Y(Int_Y$ (F_Y , E)) and $\tilde{Y} \supset Cl(Int(\tilde{Y}))$,where Int_Y (resp. Cl_Y) denotes the interior (resp. closure) operators relative over Y. Clearly Int_Y (F_Y, E) $\subset$ $\tilde{Y}$ .Therefore, $Cl_Y(Int_Y(F_Y$, E)) = Cl($Int_Y(F_Y$, E)) $\cap$ $\tilde{Y}$. But, $Int_Y(F_Y$, E) $\supset$ $Int_Y(F_Y$, E) $\cap$ Int($\tilde{Y}$) = Int(F_Y, E) . Thus we have Cl(Int_Y (F_Y, E)) $\cap$ $\tilde{Y}$ $\supset$ Cl(Int(F_Y, E)) $\cap$ $\tilde{Y}$ $\supset$ Cl(Int(F_Y, E)) $\cap$ Cl(Int($\tilde{Y}$)) $\supset$ Cl(Int((F_Y, E) $\cap$ $\tilde{Y}$)). Consequently, (F_Y, E) $\supset$ Cl(Int(F_Y, E)) .Hence, by lemma 3.6 (F_Y, E) is soft preclosed over X. □

Theorem 3.12. *Let (Y, τ_Y, E) be a soft preclopen subspace of a soft topological space (X, τ, E) and soft subsets (A_Y, E) and (B_Y, E) over Y .If (X, τ, E) is soft P-connected between (A_Y, E) and (B_Y, E) then (Y,τ_Y,E) is soft P-connected between (A_Y, E) and (B_Y, E).*

Proof. *Suppose (Y, τ_Y, E) is not soft P-connected between (A_Y, E) and (B_Y, E). Then, there is a soft preclopen set (F_Y, E) of Y such that (A_Y, E) $\subset$ (F_Y, E) and (F_Y, E) $\cap$ (B_Y, E) $= \phi$ .Since, $\tilde{Y}$ is soft preclopen in X, by lemma 3.4 and lemma 3.7, (F_Y, E) is soft preclopen set of X such that (A_Y, E) $\subset$ (F_Y, E) and (F_Y, E) $\cap$ (B_Y, E) $= \phi$. Consequently, (X, τ, E) is not soft P-connected between (A_Y, E) and (B_Y, E), a contradiction .* □

Lemma 3.8. *If (C, E) is soft closed and (T, E) is soft preclosed in X, then (C, E) $\cup$ (T, E) is soft preclosed in X.*

Proof. *We have (C, E) is soft closed and (T, E) is soft preclosed in X Rightarrow $(C, E)^c$ is soft open and $(T, E)^c$ is soft preopen in X Rightarrow $(C, E)^c \cap (T, E)^c = ((C, E) \cap (T, E)^c$ is soft preopen in X [by lemma 2.2] Rightarrow (C, E) $\cap$ (T, E) is soft preclosed in X.* □

Remark 3.6. [22] Every soft connected space is connected between any pair of its nonempty subsets .

Example 3.6. Let X=$\{x_1, x_2,x_3\}$ and E = $\{e_1,e_2\}$. Soft sets (F_1,E), (F_2,E),(F_3,E),(F_4,E) are defined as : (F_1,E) = $\{(e_1,\{x_1\}), (e_2,\{x_1\})\}$, (F_2,E) = $\{(e_1,\{x_2\}), (e_2,\{x_2\})\}$,($F_3$,E) = $\{(e_1,\{x_1,x_2\}),(e_2, \{x_1,x_2\})\}$, ($F_4$,E) = $\{(e_1,\{x_2,x_3\}),(e_2, \{x_2,x_3\})\}$ and (F_5,E) = $\{(e_1,\{x_3\}),(e_2,\{x_3\})\}$ .Let $\tau = \{ \phi, \widetilde{X}$, ($F_1$,E), ($F_2$,E),($F_3$,E),($F_4$,E)$\}$ be soft topology on X . Then a soft topological space (X,τ,E) is soft P-connected between soft sets (F_2,E) and (F_5,E) and hence soft connected between (F_2,E) and (F_5,E). However, the soft topological space (X,τ,E) is neither soft connected nor soft P-connected.

Example 3.7. Let X = $\{x_1, x_2,x_3 ,x_4\}$, E = $\{e_1,e_2\}$ and soft sets are defined as: (F, E) = $\{(e_1,\{x_1,x_3\}),(e_2, \{x_1,x_3\})\}$,($F_1$,E) = $\{(e_1,\{x_1\}), (e_2,\{x_1\})\}$ and (F_2,E) = $\{(e_1,\{x_3\}),(e_2,\{x_3\})\}$ .Let $\tau = \{ \phi, \widetilde{X}$, (F, E) $\}$ is soft topology on X .Then soft topological space (X,τ,E) is soft connected but not soft P-connected between soft sets (F_1, E) and (F_2, E).

Thus we reach the following diagram of implications.

$$\begin{array}{ccc} \text{Soft P-connected} & \Rightarrow & \text{Soft connected} \\ \Downarrow & & \Downarrow \\ \text{Soft P-connectedness} & \Rightarrow & \text{Soft connectedness} \\ \text{between soft sets} & & \text{between soft sets} \end{array}$$

4. Soft Set P-Connected Mappings

Definition 4.1. A soft mapping f_{pu} : (X,τ,E) $\rightarrow$ (Y,ϑ,K) is said to be soft set P-connected provided, if a soft topological space (X,τ,E) is soft P-connected between sets (A,E) and (B,E) then a soft subspace (f_{pu}(X),$\vartheta_{f_{pu}(X)}$,K) is soft P-connected between f_{pu}(A,E) and f_{pu}(B,E) with respect to relative topology.

Theorem 4.1. *A soft mapping f_{pu} : (X,τ,E) $\rightarrow$ (Y,ϑ,K) is soft set P-connected mapping if and only if f_{pu}^{-1}(F,K) is a soft preclopen set over X for any soft preclopen set (F,K) of (f_{pu} (X), $\vartheta_{f_{pu}(X)}$, K).*

Proof. Necessity: Let f_{pu} be soft set P-connected mapping and (F,K) be a soft preclopen set in (f_{pu} (X),$\vartheta_{f_{pu}(X)}$,K). Suppose f_{pu}^{-1}(F,K) is not soft preclopen in (X,τ,E). Then, (X,τ,E) is soft P-connected between f_{pu}^{-1}(F,K) and $(f_{pu}^{-1}(F,K))^c$. Therefore, (f_{pu} (X),$\vartheta_{f_{pu}(X)}$,K) is soft P-connected between f_{pu} (f_{pu}^{-1}(F,K)) and $f_{pu}((f_{pu}^{-1}(F,K))^c)$ because f_{pu} is soft set P-connected. But, f_{pu} (f_{pu}^{-1}(F,K)) = (F,K) $\cap$ (f_{pu} (X),$\vartheta_{f_{pu}(X)}$,K) = (F,K) and $f_{pu}((f_{pu}^{-1}(F,K))^c = (F,K)^c$ imply that (F,K) is not soft preclopen in (f_{pu} (X),$\vartheta_{f_{pu}(X)}$,K), a contradiction. Hence, f_{pu}^{-1}(F,K) is soft preclopen in (X,τ,E) .
Sufficiency: Let (X,τ,E) be soft P-connected between (A, E) and (B, E). If (f_{pu} (X),$\vartheta_{f_{pu}(X)}$,K) is not soft P-connected between f_{pu}(A, E) and f_{pu}(B, E) then there exists a soft preclopen set (F, K) in (f_{pu}(X),$\vartheta_{f_{pu}(X)}$,K) such that f_{pu}(A, E) $\subset$ (F,K) $\subset$ $(f_{pu}(B,E))^c$. By hypothesis f_{pu}^{-1}(F,K) is a soft preclopen set over X and (A,E) $\subset$ f_{pu}^{-1}(F,K) $\subset$ $(B,E)^c$. Therefore, (X,τ, E) is not soft P-connected between (A, E) and (B, E). This is a contradiction. Hence, f_{pu} is soft set P-connected. □

Remark 4.1. The concepts of soft set P-connected mapping and soft set-connected mapping are independent.

Example 4.1. Let X = {x_1, x_2, x_3 }, E = {e_1, e_2 } and Y = {y_1, y_2, y_3}, K = {k_1, k_2 } . Let τ = {ϕ, (F_1,E),(F_2,E) , $\tilde{X}$ }, and υ = {ϕ, (G_1,K),(G_2 ,K), $\tilde{Y}$ } are topologies on X and Y respectively, where (F_1,E) = {(e_1,{x_1 }),(e_2, {x_1})}, (F_2,E) = {(e_1,{x_2,x_3 }),(e_2, {x_2,x_3 })} and (G_1,K) = {(k_1, {y_1 }) ,(k_2, {y_1 })}, (G_2,K) = {(k_1, {y_2,y_3 }),(k_2, {y_2,y_3 })}. Then soft mapping f_{pu} : (X, τ, E)$\rightarrow$ (Y,υ,K) defined by u(x_1) = y_3, u (x_2)= y_2, u(x_3) = y_1 and p(e_1)= k_1, p(e_2) = k_2 is soft set P-connected but it is not soft set-connected .

Example 4.2. Let X = {x_1, x_2, x_3 }, E = {e_1, e_2 } and Y = {y_1, y_2, y_3}, K = {k_1, k_2 } . Let τ = {ϕ, (F_1,E),(F_2,E),(F_3,E), $\tilde{X}$ }, and υ = {ϕ, (G_1 ,K),(G_2,K), $\tilde{Y}$ } are topologies on X and Y respectively, where (F_1,E) = {(e_1,{x_1 }),(e_2, {x_1})},(F_2,E) = {(e_1,{x_2 }),(e_2, {x_2})}, (F_3,E) = {(e_1,{x_2,x_3 }) ,(e_2, {x_2,x_3 })} and (G_1,K) = {(k_1, {y_1 }),(k_2, {y_1 })}, (G_2,K) = {(k_1, {y_2,y_3 }),(k_2, {y_2,y_3 })}. Then, soft mapping f_{pu} : (X, τ, E)$\rightarrow$ (Y,υ,K) defined by u(x_1) = y_3, u (x_2)= y_2, u(x_3) = y_2 and p(e_1)= k_1, p(e_2) = k_2 is soft set-connected but it is not soft set P-connected .

Theorem 4.2. *Every soft mapping f_{pu} : (X,τ,E) $\rightarrow$ (Y,ϑ,K) such that (f_{pu} (X),$\vartheta_{f_{pu}(X)}$,K) is a soft P-connected set is a soft set P-connected.*

Proof. Let (f_{pu} (X),$\vartheta_{f_{pu}(X)}$,K) be soft P-connected. Then, by lemma 2.1 no nonempty proper soft set of (f_{pu} (X),$\vartheta_{f_{pu}(X)}$,K) which is soft preclopen. Hence, f_{pu} is soft set P-connected. □

Theorem 4.3. *Let f_{pu} : (X,τ,E) $\rightarrow$ (Y,ϑ,K) be a soft set P-connected mapping. If (X,τ,E) is soft P-connected set, then (f_{pu} (X),$\vartheta_{f_{pu}(X)}$,K) is a soft P-connected set of (Y,ϑ,K).*

Proof. *Suppose (f_{pu} (X),$\vartheta_{f_{pu}(X)}$,K) is not soft P-connected in (Y,ϑ,K). Then, by lemma 2.1, there is a nonempty proper soft preclopen set (F,K) of (f_{pu} (X),$\vartheta_{f_{pu}(X)}$,K). Since f_{pu} is soft set P-connected, f_{pu}^{-1}(F,K) is a nonempty proper soft preclopen set over X. Consequently, (X,τ,E) is not soft P-connected.* □

Theorem 4.4. *Let f_{pu} : (X,τ,E) $\rightarrow$ (Y,ϑ,K) be a soft set P-connected mapping and (A,E) be a soft open set over X such that f_{pu}(A,E) is soft preclopen set of (f_{pu} (X),$\vartheta_{f_{pu}(X)}$,K). Then, f_{pu} /(A,E) : (A,E) $\rightarrow$ (Y,ϑ,K) is soft set P-connected mapping.*

Proof. *Let (A,E) be soft P-connected between (U,E) and (V,E). Then, by theorem 3.11, (X,τ,E) is soft P-connected between (U,E) and (V,E). Since f_{pu} is soft set P-connected,(f_{pu} (X) ,$\vartheta_{f_{pu}(X)}$,K) is soft P-connected between f_{pu}(U,E) and f_{pu}(V,E). Now, since f_{pu}(A,E) is a soft preclopen set of (f_{pu} (X),$\vartheta_{f_{pu}(X)}$,K), it follows by theorem 3.12 that f_{pu}(A,E) is soft P-connected between f_{pu}(U,E) and f_{pu}(V,E). This proves the theorem.* □

Theorem 4.5. *Let f_{pu} : (X,τ,E) $\rightarrow$ (Y,ϑ,K) be a soft set P-connected surjection. Then, for any soft preclopen set (F,K) of (Y,ϑ,K) is soft P-connected if $f_{pu}^{-1}(F,K)$ is soft P-connected in (X ,τ,E). In particular, if (X,τ,E) is soft P-connected then (Y,ϑ,K) is soft P-connected.*

Proof. *By theorem 4.4 f_{pu} / f_{pu}^{-1}(F,K): f_{pu}^{-1}(F,K) $\rightarrow$ (Y,ϑ,K) is soft set P-connected. And, since f_{pu}^{-1} (F,K) is soft P-connected by theorem 4.3, f_{pu} / f_{pu}^{-1}(F,K)(f_{pu}^{-1}(F,K)) = (F,K) is soft P-connected .* □

Theorem 4.6. *Let $f_{p_1u_1}$: (X,τ,E) $\rightarrow$ (Y,ϑ,K) be a surjective soft set P-connected and $g_{p_2u_2}$:(Y,ϑ,K) $\rightarrow$ (Z,η,T) a soft set P-connected mapping. Then, ($g_{p_2u_2} o f_{p_1u_1}$) : (X,τ,E) $\rightarrow$ (Z,η,T) is soft set P-connected.*

Proof. *Let (F,T) be a soft preclopen set in $g_{p_2u_2}(Y)$. Then, $g_{p_2u_2}^{-1}(F,T)$ is soft preclopen over Y = $f_{p_1u_1}$ (X) and so $f_{p_1u_1}^{-1}$ ($g_{p_2u_2}^{-1}(F,T)$) is soft preclopen in (X,τ,E) .Now, ($g_{p_2u_2} o f_{p_1u_1}$)(X) = $g_{p_2u_2}$ (Y) and $(g_{p_2u_2} o f_{p_1u_1})^{-1}$ (F,T) = $f_{p_1u_1}^{-1}$ ($g_{p_2u_2}^{-1}(F,T)$) is soft preclopen in (X,τ,E). Hence ($g_{p_2u_2} o f_{p_1u_1}$) is soft set P-connected.* □

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