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BLOW UP OF POSITIVE INITIAL-ENERGY SOLUTIONS FOR THE EXTENSIBLE BEAM EQUATION WITH NONLINEAR DAMPING AND SOURCE TERMS *

Erhan Pişkin and Nazlı Irkıl

Abstract. In this paper, we study the following extensible beam equation

$$u_{tt} + \triangle^2 u - M\left(\|\nabla u\|^2\right) \triangle u + |u_t|^{p-1} u_t = |u|^{q-1} u$$

with initial and boundary conditions. Under suitable conditions on the initial datum, we prove that the solution blows up in finite time with positive initial-energy.

Keywords: Extensible beam equation, blow up, nonlinear damping term

1. Introduction

In this paper, we study the following extensible beam equation

(1.1)
$$\begin{cases} u_{tt} + \triangle^2 u - M\left(\|\nabla u\|^2\right) \triangle u + |u_t|^{p-1} u_t = |u|^{q-1} u, & (x,t) \in \Omega \times (0,T), \\ u(x,0) = u_0(x), \ u_t(x,0) = u_1(x), & x \in \Omega, \\ u(x,t) = \frac{\partial}{\partial \nu} u(x,t) = 0, & x \in \partial\Omega, \end{cases}$$

where $p, q \geq 1$ are real numbers, Ω is a bounded domain with smooth boundary $\partial\Omega$ in R^n, ν is the outer normal, and $M(s) = \alpha + \beta s^\gamma$, $\alpha, \beta \geq 0$, $\gamma \geq 1$.

This kind of wave equation is obtained from the extensible beam equation of Woinowsky-Krieger [19]. This type of problem have been considered by many authors such as [16, 20, 21, 2, 4, 10, 3].

In the case of $M(s) = 1$ and without fourth order term $\triangle^2 u$, the equation (1.1) can be written in the following form

$$u_{tt} - \triangle u + |u_t|^{p-1} u_t = |u|^{q-1} u. \tag{1.2}$$

The existence and blow up in finite time of solutions for (1.2) were established in [7, 8, 9, 11, 12, 18]. Recently, the problem (1.1) was studied by Esquivel-Avila

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[5, 6], he proved blow up, unboundedness, convergence and global attractor. Very recently, Pişkin [17] studied the local and global existence, asymptotic behavior and blow up.

In this paper, we prove the blow up of solutions for the problem (1.1), with positive initial energy.

This paper is organized as follows. In section 2, we present some lemmas and notations needed later of this article. In section 3, blow up of the solution is discussed.

2. Preliminaries

In this section, we shall give some assumptions and lemmas which will be used throughout this paper. Let $\|.\|$ and $\|.\|_p$ denote the usual $L^2(\Omega)$ norm and $L^p(\Omega)$ norm, respectively.

Now, we state the general hypotheses

(H) For the nonlinearity, we suppose that

$$1 < p < \infty \text{ if } n \leq 2, \text{ and } 1 < p \leq \frac{n+2}{n-2} \text{ if } n > 2, \tag{2.1}$$

$$1 < q < \infty \text{ if } n \leq 2, \text{ and } 1 < q \leq \frac{n}{n-2} \text{ if } n > 2. \tag{2.2}$$

Lemma 2.1. *(Sobolev-Poincare inequality) [1]. Let p be a number with* $2 \leq p < \infty$ $(n = 1, 2)$ *or* $2 \leq p \leq \frac{2n}{n-2}$ $(n \geq 3)$, *then there is a constant* C *such that*

$$\|u\|_p \leq C \|\nabla u\| \text{ for } u \in H_0^1(\Omega).$$

Lemma 2.2. *[11]. Suppose that*

$$p \leq 2\frac{n-1}{n-2}, \ n \geq 3$$

holds. Then there exists a positive constant $C > 1$ *depending on* Ω *only such that*

$$\|u\|_p^s \leq C\left(\|\nabla u\|^2 + \|u\|_p^p\right)$$

for any $u \in H_0^1(\Omega)$, $2 \leq s \leq p$.

Next, we state the local existence theorem of problem (1.1), whose proof can be found in [17].

Theorem 2.1. *(Local existence). Assume that (H) holds, and that* $(u_0, u_1) \in H_0^2(\Omega) \times L^2(\Omega)$, *then there exists a unique solution* u *of (1.1) satisfying*

$$u \in C\left([0,T); H_0^2(\Omega)\right), \quad u_t \in C\left([0,T); L^2(\Omega)\right) \cap L^{p+1}(\Omega \times (0,T)).$$

Moreover, at least one of the following statements holds:

(i) $T = \infty$,

(ii) $\|u_t\|^2 + \|\triangle u\|^2 \longrightarrow \infty$ *as* $t \longrightarrow T^-$.

3. Blow up of solutions

In this section, we are going to consider the blow up of the solution for problem (1.1).

In our proof, without loss of generality and sake of simplicity we can take $\alpha = \beta = 1$. We set

$$\alpha_1 = B^{-\frac{q+1}{q}},\ B = \beta^{\frac{1}{q+1}},\ E_1 = \left(\frac{1}{2} + \frac{1}{q+1}\right)\alpha_1^2 \tag{3.1}$$

and

$$E(t) = \frac{1}{2}\|u_t\|^2 + \frac{1}{2}\left(\|\nabla u\|^2 + \|\Delta u\|^2\right) + \frac{1}{2(\gamma+1)}\|\nabla u\|^{2(\gamma+1)} - \frac{1}{q+1}\|u\|_{q+1}^{q+1}. \tag{3.2}$$

Lemma 3.1. *Let* u *be the solution of (1.1). Suppose that (H) holds. Assume further that* $E(0) < E_1$ *and*

$$\left(\|\nabla u_0\|^2 + \|\Delta u_0\|^2 + \frac{1}{\gamma+1}\|\nabla u_0\|^{2(\gamma+1)}\right)^{\frac{1}{2}} \geq \alpha_1 \tag{3.3}$$

Then there exists a constant $\alpha_2 > \alpha_1$ *such that*

$$\left(\|\nabla u\|^2 + \|\Delta u\|^2 + \frac{1}{\gamma+1}\|\nabla u\|^{2(\gamma+1)}\right)^{\frac{1}{2}} \geq \alpha_2, \tag{3.4}$$

and

$$\|u\|_{q+1} \geq B\alpha_2 \tag{3.5}$$

for $\forall t \in [0,t)$.

Proof. By $E(t)$, Sobolev embedding theorem and the definition of B, we have

$$\begin{aligned} E(t) &\geq \frac{1}{2}\left(\|\nabla u\|^2+\|\Delta u\|^2\right)+\frac{1}{2(\gamma+1)}\|\nabla u\|^{2(\gamma+1)}-\frac{1}{q+1}\|u\|_{q+1}^{q+1} \\ &\geq \frac{1}{2}\left(\|\nabla u\|^2+\|\Delta u\|^2\right)+\frac{1}{2(\gamma+1)}\|\nabla u\|^{2(\gamma+1)}-\frac{1}{q+1}\beta\|\nabla u\|^{q+1} \\ &= \frac{1}{2}\left(\|\nabla u\|^2+\|\Delta u\|^2\right)+\frac{1}{2(\gamma+1)}\|\nabla u\|^{2(\gamma+1)}-\frac{\beta}{q+1}\left(\|\nabla u\|^2\right)^{\frac{q+1}{2}} \\ &\geq \frac{1}{2}\left(\|\nabla u\|^2+\|\Delta u\|^2\right)+\frac{1}{2(\gamma+1)}\|\nabla u\|^{2(\gamma+1)} \\ &\quad -\frac{B^{q+1}}{q+1}\left(\|\nabla u\|^2+\|\Delta u\|^2+\frac{1}{(\gamma+1)}\|\nabla u\|^{2(\gamma+1)}\right)^{\frac{q+1}{2}} \\ &= \frac{1}{2}\left(\|\nabla u\|^2+\|\Delta u\|^2+\frac{1}{(\gamma+1)}\|\nabla u\|^{2(\gamma+1)}\right) \\ &\quad -\frac{B^{q+1}}{q+1}\left(\|\nabla u\|^2+\|\Delta u\|^2+\frac{1}{(\gamma+1)}\|\nabla u\|^{2(\gamma+1)}\right)^{\frac{q+1}{2}} \\ &= \frac{1}{2}\alpha^2-\frac{B^{q+1}}{q+1}\alpha^{q+1} \\ &= G(\alpha), \end{aligned} \tag{3.6}$$

where

$$\left(\|\nabla u\|^2+\|\Delta u\|^2+\frac{1}{\gamma+1}\|\nabla u\|^{2(\gamma+1)}\right)^{\frac{1}{2}}=\alpha.$$

It is easy to verify that G, is increasing for $0<\alpha<\alpha_1$, and decreasing for $\alpha>\alpha_1$. That is

$$\begin{aligned} G(\alpha_1) &> G(\alpha),\ 0<\alpha<\alpha_1 \\ G(\alpha) &< G(\alpha_1),\ \alpha>\alpha_1 \end{aligned}$$

For $\alpha\to\infty,\ G(\alpha)\to-\infty$ and

$$G(\alpha_1)=\frac{1}{2}\alpha_1^2-\frac{B^{q+1}}{q+1}\alpha_1^{q+1}=E_1 \tag{3.7}$$

where α_1 is given in (3.1). Since $E(0)<E_1$, there exists $\alpha_2>\alpha_1$ such that $E(0)=G(\alpha_2)$.

If we set $\left(\|\nabla u_0\|^2+\|\Delta u_0\|^2+\frac{1}{\gamma+1}\|\nabla u_0\|^{2(\gamma+1)}\right)^{\frac{1}{2}}=\alpha_0$. Then, because of $E(t)>G(\alpha)$, we can write $G(\alpha_0)\leq E(0)=G(\alpha_2)$, which implies that $\alpha_0\geq\alpha_2$.

To establish (3.4), we suppose by contradiction that

$$\left(\|\nabla u_0\|^2+\|\Delta u_0\|^2+\frac{1}{\gamma+1}\|\nabla u_0\|^{2(\gamma+1)}\right)^{\frac{1}{2}}<\alpha_2$$

for some $t_0 > 0$ and by the continuity of $\left(\|\nabla u\|^2 + \|\Delta u\|^2 + \frac{1}{\gamma+1}\|\nabla u\|^{2(\gamma+1)}\right)$ we can choose t_0 such that

$$\left(\|\nabla u_0\|^2 + \|\Delta u_0\|^2 + \frac{1}{\gamma+1}\|\nabla u_0\|^{2(\gamma+1)}\right)^{\frac{1}{2}} > \alpha_1.$$

Again, the use of (3.6) leads to

$$\begin{aligned} E(t_0) &\geq G\left(\|\nabla u_0\|^2 + \|\Delta u_0\|^2 + \frac{1}{\gamma+1}\|\nabla u_0\|^{2(\gamma+1)}\right) \\ &> G(\alpha_2) = E(0). \end{aligned}$$

This is impossible since $E(t) \leq E(0)$, for all $t \in [0, T)$. Thus, (3.4) is established.

Now, to prove (3.5) we can use of

$$\begin{aligned} \frac{1}{2}\left(\|\nabla u\|^2 + \|\Delta u\|^2 + \frac{1}{\gamma+1}\|\nabla u\|^{2(\gamma+1)}\right) &\leq E(t) + \frac{1}{q+1}\|u\|_{q+1}^{q+1} \\ &\leq E(0) + \frac{1}{q+1}\|u\|_{q+1}^{q+1}, \end{aligned}$$

since $E(t) \leq E(0)$.

Consequently, (3.4) yields

$$\begin{aligned} \frac{1}{q+1}\|u\|_{q+1}^{q+1} &\geq \frac{1}{2}\left(\|\nabla u\|^2 + \|\Delta u\|^2 + \frac{1}{\gamma+1}\|\nabla u\|^{2(\gamma+1)}\right) - E(0) \\ &\geq \frac{1}{2}\alpha_2^2 - E(0) \\ &\geq \frac{1}{2}\alpha_2^2 - G(\alpha_2) \\ &= \frac{1}{2}\alpha_2^2 - \left(\frac{1}{2}\alpha_2^2 - \frac{B^{q+1}}{q+1}\alpha_2^{q+1}\right) \\ &= \frac{B^{q+1}}{q+1}\alpha_2^{q+1}. \end{aligned}$$

□

Theorem 3.1. *Suppose that (H) holds. Assume further that* $q > \max\{2\gamma+1, p\}$. *Then any the solution of (1.1) with initial data satisfying*

$$\left(\|\nabla u_0\|^2 + \|\Delta u_0\|^2 + \frac{1}{\gamma+1}\|\nabla u_0\|^{2(\gamma+1)}\right)^{\frac{1}{2}} \geq \alpha_1 \text{ and } E(0) < E_1,$$

blow up in finite time.

Proof. We suppose that the solution exists for all time and we reach to a contradiction. For this purpose, we set

$$H(t) = E_1 - E(t). \tag{3.8}$$

By using (3.2), (3.8), we have

$$H'(t) = -E'(t) = -\|u_t\|_{p+1}^{p+1}. \tag{3.9}$$

$$\begin{aligned} 0 &< H(0) \leq H(t) = E_1 - \left[\frac{1}{2}\|u_t\|^2 + \frac{1}{2}\left(\|\nabla u\|^2 + \|\Delta u\|^2\right) \right. \\ &\quad \left. + \frac{1}{2(\gamma+1)}\|\nabla u\|^{2(\gamma+1)} - \frac{1}{q+1}\|u\|_{q+1}^{q+1} \right] \end{aligned} \tag{3.10}$$

From $\alpha_2 > \alpha_1$, we obtain

$$\begin{aligned} H(t) &\leq E_1 - \frac{1}{2}\left(\|\nabla u\|^2 + \|\Delta u\|^2 + \frac{1}{(\gamma+1)}\|\nabla u\|^{2(\gamma+1)}\right) + \frac{1}{q+1}\|u\|_{q+1}^{q+1} \\ &\leq E_1 - \frac{1}{2}\alpha_2^2 + \frac{1}{q+1}\|u\|_{q+1}^{q+1} \\ &\leq -\frac{1}{q+1}\alpha_1^{q+1} + \frac{1}{q+1}\|u\|_{q+1}^{q+1} \\ &\leq \frac{1}{q+1}\|u\|_{q+1}^{q+1}, \ \forall t \geq 0. \end{aligned} \tag{3.11}$$

We then define

$$\Psi(t) = H^{1-\sigma}(t) + \varepsilon \int_\Omega u u_t dx, \tag{3.12}$$

where ε small to be chosen later and

$$0 < \sigma \leq \min\left\{ \frac{q-p}{p(q+1)}, \frac{q-1}{2(q+1)} \right\}. \tag{3.13}$$

Our goal is to show that $\Psi(t)$ satisfies a differential inequality of the form

$$\Psi'(t) \geq \xi \Psi^{\zeta}(t), \ \zeta > 1.$$

This, of course, will lead to a blow up in finite time.

By taking a derivative of (3.12) and using Eq. (1.1) we obtain

$$\begin{aligned} \Psi'(t) &= (1-\sigma) H^{-\sigma}(t) H'(t) + \varepsilon \|u_t\|^2 - \varepsilon \|\triangle u\|^2 - \varepsilon \|\nabla u\|^2 \\ &\quad - \varepsilon \|\nabla u\|^{2(\gamma+1)} + \varepsilon \|u\|_{q+1}^{q+1} - \varepsilon \int_\Omega u u_t |u_t|^{p-1} dx. \end{aligned} \tag{3.14}$$

By using the definition of the $H(t)$, it follows that

$$
\begin{aligned}
-\|\nabla u\|^{2(\gamma+1)} &= 2(\gamma+1)H(t)+(\gamma+1)\left(\|u_t\|^2+\|\triangle u\|^2+\|\nabla u\|^2\right)\\
&\quad -\frac{2(\gamma+1)}{q+1}\|u\|_{q+1}^{q+1}. \qquad (3.15)
\end{aligned}
$$

Inserting (3.15) into (3.14), we obtain

$$
\begin{aligned}
\Psi'(t) &= (1-\sigma)H^{-\sigma}(t)H'(t)+\varepsilon(\gamma+2)\|u_t\|^2+\varepsilon\gamma\left(\|\triangle u\|^2+\|\nabla u\|^2\right)\\
&\quad +2\varepsilon(\gamma+1)H(t)+\varepsilon\left(\frac{q-2\gamma-1}{q+1}\right)\|u\|_{q+1}^{q+1}-\varepsilon\int_\Omega uu_t|u_t|^{p-1}dx. \qquad (3.16)
\end{aligned}
$$

In order to estimate the last term in (3.16), we make use of the following Young's inequality

$$XY \le \frac{\delta^k X^k}{k}+\frac{\delta^{-l}Y^l}{l},$$

where $X,Y\ge 0,\ \delta>0,\ k,l\in R^+$ such that $\frac{1}{k}+\frac{1}{l}=1$. Consequently, applying the previous we have

$$
\begin{aligned}
\int_\Omega uu_t|u_t|^{p-1}dx &\le \frac{\delta^{p+1}}{p+1}\|u\|_{p+1}^{p+1}+\frac{p\delta^{-\frac{p+1}{p}}}{p+1}\|u_t\|_{p+1}^{p+1}\\
&\le \frac{\delta^{p+1}}{p+1}\|u\|_{p+1}^{p+1}+\frac{p\delta^{-\frac{p+1}{p}}}{p+1}H'(t),
\end{aligned}
$$

where δ is constant depending on the time t and specified later. Therefore, (3.16) becomes

$$
\begin{aligned}
\Psi'(t) &\ge (1-\sigma)H^{-\sigma}(t)H'(t)+\varepsilon(\gamma+2)\|u_t\|^2+\varepsilon\gamma\left(\|\triangle u\|^2+\|\nabla u\|^2\right)\\
&\quad +2\varepsilon(\gamma+1)H(t)+\varepsilon\left(\frac{q-2\gamma-1}{q+1}\right)\|u\|_{q+1}^{q+1}\\
&\quad -\varepsilon\frac{p\delta^{-\frac{p+1}{p}}}{p+1}H'(t)-\varepsilon\frac{\delta^{p+1}}{p+1}\|u\|_{p+1}^{p+1}. \qquad (3.17)
\end{aligned}
$$

At this point we choose δ so that $\delta^{-\frac{p+1}{p}}=kH^{-\sigma}(t)$, where $k>0$ is specified later, we obtain

$$
\begin{aligned}
\Psi'(t) &\ge \left((1-\sigma)-\varepsilon\frac{pk}{p+1}\right)H^{-\sigma}(t)H'(t)+\varepsilon(\gamma+2)\|u_t\|^2+\varepsilon\gamma\left(\|\triangle u\|^2+\|\nabla u\|^2\right)\\
&\quad +2\varepsilon(\gamma+1)H(t)+\varepsilon\left(\frac{q-2\gamma-1}{q+1}\right)\|u\|_{q+1}^{q+1}-\varepsilon\frac{k^{-p}}{p+1}H^{\sigma p}(t)\|u\|_{p+1}^{p+1}. \qquad (3.18)
\end{aligned}
$$

Since $q>p$ and $H(t)\le\frac{1}{q+1}\|u\|_{q+1}^{q+1}$, we obtain

$$H^{\sigma p}(t)\|u\|_{p+1}^{p+1}\le C'\left(\frac{1}{q+1}\right)^{\sigma p}\|u\|_{q+1}^{p+1+\sigma p(q+1)}.$$

Thus, (3.18) yields

$$\Psi'(t) \geq \left((1-\sigma)-\varepsilon\frac{pk}{p+1}\right)H^{-\sigma}(t)H'(t)+\varepsilon(\gamma+2)\|u_t\|^2+\varepsilon\gamma\left(\|\triangle u\|^2+\|\nabla u\|^2\right)$$
$$+2\varepsilon(\gamma+1)H(t)+\varepsilon\left(\frac{q-2\gamma-1}{q+1}\right)\|u\|_{q+1}^{q+1}-\varepsilon\frac{k^{-p}}{p+1}C'\left(\frac{1}{q+1}\right)^{\sigma p}\|u\|_{q+1}^{p+1+\sigma p(q+1)}. \tag{3.19}$$

From (3.13), we have $2 \leq p+1+\sigma p(q+1) \leq q+1$. By using Lemma 2.2, we have

$$\begin{aligned}\|u\|_{q+1}^{p+1+\sigma p(q+1)} &\leq C\left(\|\nabla u\|^2+\|u\|_{q+1}^{q+1}\right)\\ &\leq C\left(\|\triangle u\|^2+\|\nabla u\|^2+\|u\|_{q+1}^{q+1}\right).\end{aligned}$$

Thus,

$$\Psi'(t) \geq \left((1-\sigma)-\varepsilon\frac{pk}{p+1}\right)H^{-\sigma}(t)H'(t)+\eta\left(\|u_t\|^2+\|\triangle u\|^2+\|\nabla u\|^2+H(t)+\|u\|_{q+1}^{q+1}\right) \tag{3.20}$$

where $\eta=\min\left\{\varepsilon(\gamma+2),\ \varepsilon\left(\gamma-\frac{k^{-p}}{p+1}C'C\left(\frac{1}{q+1}\right)^{\sigma p}\right),\ 2\varepsilon(\gamma+1),\ \varepsilon\left(\frac{q-2\gamma-1}{q+1}-\frac{k^{-p}}{p+1}C'C\left(\frac{1}{q+1}\right)^{\sigma p}\right)\right\}>0$, we pick ε small enough so that $(1-\sigma)-\varepsilon\frac{pk}{p+1}\geq 0$ and

$$\Psi(t) \geq \Psi(0)=H^{1-\sigma}(0)+\varepsilon\int_\Omega u_0u_1dx>0,\ \forall t\geq 0. \tag{3.21}$$

On the other hand, applying Hölder inequality, we obtain

$$\begin{aligned}\left|\int_\Omega uu_tdx\right|^{\frac{1}{1-\sigma}} &\leq \|u\|^{\frac{1}{1-\sigma}}\|u_t\|^{\frac{1}{1-\sigma}}\\ &\leq C\left(\|u\|_{q+1}^{\frac{1}{1-\sigma}}\|u_t\|^{\frac{1}{1-\sigma}}\right).\end{aligned}$$

Young's inequality gives

$$\left|\int_\Omega uu_tdx\right|^{\frac{1}{1-\sigma}} \leq C\left(\|u\|_{q+1}^{\frac{\mu}{1-\sigma}}+\|u_t\|^{\frac{\theta}{1-\sigma}}\right), \tag{3.22}$$

for $\frac{1}{\mu}+\frac{1}{\theta}=1$. We take $\theta=2(1-\sigma)$, to obtain $\mu=\frac{2(1-\sigma)}{1-2\sigma}\leq q+1$ by (3.13). Therefore, (3.22) becomes

$$\left|\int_\Omega uu_tdx\right|^{\frac{1}{1-\sigma}} \leq C\left(\|u_t\|^2+\|u\|_{q+1}^{\frac{2}{1-2\sigma}}\right).$$

By using Lemma 2.2, we obtain

$$\left|\int_\Omega uu_tdx\right|^{\frac{1}{1-\sigma}} \leq C\left(\|u_t\|^2+\|u\|_{q+1}^{q+1}+\|\nabla u\|^2\right).$$

Thus,

$$\begin{aligned}
\Psi^{\frac{1}{1-\sigma}}(t) &= \left[H^{1-\sigma}(t)+\varepsilon\int_\Omega uu_t dx\right]^{\frac{1}{1-\sigma}} \\
&\le 2^{\frac{\sigma}{1-\sigma}}\left(H(t)+\varepsilon^{\frac{1}{1-\sigma}}\left|\int_\Omega uu_t dx\right|^{\frac{1}{1-\sigma}}\right) \\
&\le C\left(\|u_t\|^2+H(t)+\|u\|_{q+1}^{q+1}+\|\nabla u\|^2\right) \\
&\le C\left(\|u_t\|^2+H(t)+\|u\|_{q+1}^{q+1}+\|\triangle u\|^2+\|\nabla u\|^2\right). \qquad (3.23)
\end{aligned}$$

By combining of (3.20) and (3.23), we find that

$$\Psi'(t)\ge \xi\Psi^{\frac{1}{1-\sigma}}(t), \qquad (3.24)$$

where ξ is a positive constant.

Integrating both sides of (3.24) over $(0,t)$ yields $\Psi^{\frac{\sigma}{1-\sigma}}(t) \ge \frac{1}{\Psi^{-\frac{\sigma}{1-\sigma}}(0)-\frac{\xi\sigma t}{1-\sigma}}$, which implies that the solution blows up in a finite time T^*, with

$$T^*\le \frac{1-\sigma}{\xi\sigma\Psi^{\frac{\sigma}{1-\sigma}}(0)}.$$

□

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Erhan Pişkin
Dicle University
Department of Mathematics
21280 Diyarbakır, Turkey
episkin@dicle.edu.tr

Nazlı Irkıl
Dicle University
Department of Mathematics
21280 Diyarbakır, Turkey
nazliirkil@gmail.com