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ON SEMI-INVARIANT SUBMANIFOLDS OF ALMOST COMPLEX CONTACT METRIC MANIFOLDS

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Abstract. In this article, we study semi-invariant submanifolds of almost complex contact metric manifolds.We defined and investigated semi-invariant submanifolds of almost complex contact metric manifolds. We found necessary and sufficient conditions to be integrable and totally geodesic for distributions D defined on M. Also we obtained necessary and sufficient conditions to be integrable and totally geodesic for distributions $D^{\perp}$ defined on M.

Keywords: Complex contact metric manifolds invariant submanifolds, anti-invariant submanifolds

1. Introduction

Contact manifolds were first worked by W. M. Boothby and H. C. Wang [1], and J.W. Gray [5] described an almost contact manifold by reducing the structural group of the tangent bundle to $U(n)X1$. Later S. Sasaki[15] showed the existence of four tensor fields, and introduced the Riemannian metric with regard to the almost contact structure. Tensor calculus has been a powerful and prominent method since the study of contact manifold were initiated to use by these four tensor fields. Two special contact Riemannian manifolds are K-contact Riemannian manifolds and Sasakian manifolds.It can be said that a Sasakian manifold can be regarded as an odd-dimensional analogue of a Kahlerian manifold. Differentiable manifolds were worked by Y. Hatakeyama [6] with almost contact metric structure in 1963. In this work contact metric structure was called with vanishing N^i_j or N^i_{jk} K-contact metric structure or normal contact metric structure respectively. In 1976, D. E. Blair [4] provided necessary and sufficient conditions for normality on almost contact metric manifolds. Although complex contact manifolds are almost as old as real contact manifolds. In modern theory, this subject attracts less attention but recently many examples about this subject have been studied in the literature. B. Korkmaz [10] showed a complex analogue of real contact metric manifolds in her PhD thesis.

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The concept of complex contact manifold was found as a result of the works of Kobayashi and Boothby in late 1950s and the early 1960s. This is just shortly after the Boothby-Wang fibration in real contact geometry. Then in 1965, J. A. Wolf studied homogeneous complex contact manifolds. Ishihara and Konishi introduced a notion of normality for complex contact structures. In this development however, the notion of normality seems too strong since it precludes the complex Heisenberg group as one of the canonical examples, although it does include complex projective spaces as odd complex dimension as one would expect. Then B. Korkmaz [11] give a new condition for the normality. As a subject the Riemannian Geometry of complex contact manifolds have just made it debut and it tends to be studied on it. In the literature work we have done, the submanifolds of complex contact metric manifolds have defected not to be studied on and that's why we decided to work on this issue. Based on these studies, we defined semi-invariant submanifolds of almost complex contact metric manifold and we have investigated semi-invariant submanifolds of almost complex contact metric manifolds. We found necessary and sufficient conditions to be integrable and totally geodesic of distribution D defined on M. Also we obtained necessary and sufficient conditions to be integrable and totally geodesic of distribution $D^{\perp}$ defined on M .

2. Some fundamental concepts and definitions

2.1. Contact Manifolds

Firstly let us present definition of contact manifold. A C^{∞} manifold M^{2n+1} is called a contact manifold if there is a 1-form μ such that

$$\mu \wedge (d\mu)^{n} \neq 0. \tag{2.1}$$

In particulary, a contact manifold is routable when (2.1) inequality is provided. Since $d\mu$ has rank $2n$ on Grassmann algebra $\wedge T_{m}^{*}M$ at each point $m \in M$, it is obtained a 1-dimensional subspace,

$$\{W \in T_{m}M \mid d\mu\,(W, T_{m}M) = 0\}\,,$$

when $\mu \neq 0$. On the other hand if μ is zero, it is obtained complementary of that subspace. Also, we get a global vector field ξ satisfying

$$d\mu\,(\xi, W) = 0, \mu\,(\xi) = 1.$$

taking ξ_{m} in this subspace normalized by $\mu\,(\xi_{m}) = 1$. ξ is called the characterstic vector field of the contact structure [4].

Theorem 2.1. *Let M^{2n+1} be a contact manifold in widersense. If μ is odd, M^{2n+1} is routable, then M^{2n+1} is contact manifold* [4].

2.2. Almost Complex and Almost Contact structures

A tensor field J of type $(1,1)$ is called an almost complex structure, where $J^2 = -I$. A Riemannian manifold endowed with an almost complex structure is called an almost complex manifold. A Hermitian metric on an almost complex manifold (M,J) is an invariant Riemannian metric under J,i.e.,

$$g(JW, JZ) = g(W, Z).$$

Pointing out that J is negative-self-adjoint with respect to g, i.e.,

$$\begin{aligned} g(W, JZ) &= -g(JW, Z), \\ \Omega(W, Z) &= g(W, JZ) \end{aligned}$$

defines a 2-form called the fundamental 2-form of the Hermitian structure (M, J, g). A complex manifold M together with J the corresponding almost complex structure is called (M, J, g) Hermitian manifold. If $d\Omega = 0$, the structure is almost Keahlerian. Also note that, every almost complex manifold receives a Hermitian metric g defined by

$$g(W, Z) = k(W, Z) + k(JW, JZ),$$

where k is any Riemaniann metric.

In terms of structure tensors, we can say that M^{2n+1} has an almost contact structure or sometimes (ϕ, ξ, μ) −structure if it admits a tensor field ϕ of type $(1,1)$, a vector field ξ and a 1-form μ satisfying

$$\phi^2 = -I + \mu \otimes \xi, \mu(\xi) = 1[4].$$

Theorem 2.2. *Let M^{2n+1} be a (ϕ, ξ, μ) −structure. Then $\phi\xi = 0$ and $\mu \circ \phi = 0$. Moreover the endomorphism ϕ has rank $2n$[4].*

Definition 2.1. Let g be a Riemaniann metric providing

$$g(\phi W, \phi Z) = g(W, Z) - \mu(W)\mu(Z).$$

A manifold M^{2n+1} with a (ϕ, ξ, μ) −structure taking a Riemannann metric g is called an almost contact metric structure and we say that g is compatible metric[4].

2.3. Complex Contact Manifolds

Let us recall main notation about complex contact manifold for this subject, main reference is B.Korkmaz.

Definition 2.2. A complex contact manifold is called a complex manifold of odd complex dimension $2n+1$ together with an open covering $\{\mathbf{O}_\alpha\}$ by coordinate neighborhoods such that:

1. On each $\mathbf{O}_\alpha$ there is a holomorphic 1-form Ψ_α such that

$$\Psi_\alpha \wedge (d\Psi_\alpha)^n \neq 0;$$

2. On $\mathbf{O}_\alpha \cap \mathbf{O}_\beta \neq \varnothing$ there is a non-vanishing holomorphic function $f_{\alpha\beta}$ such that $\Psi_\alpha = f_{\alpha\beta}\Psi_\beta$.

The subspaces $\{W \in T_m\mathbf{O}_\alpha : \Psi_\alpha(W) = 0\}$ define a non-integrable holomorphic subbundle $\mathcal{H}$ of complex dimension 2n called the complex contact subbundle or horizontal subbundle. The quotient $L = T\ M/\mathcal{H}$ is a complex line bundle over M. For sake of brevity, we will often neglect the subscripts on local tensor fields. Define a local section X of TM, i.e., a section of $T\mathbf{O}$, by $dx(X, W) = 0$ for every $W \in \mathcal{H}, x(X) = 0$. Then such local sections define a global subbundle ϑ by $\vartheta\ |_{\mathbf{O}} = span\{X, JX\}$. Now we get $TM = \mathcal{H} \oplus \vartheta$ and we denote the projection map p from TM to $\mathcal{H}$. We suppose throughout in this study that ϑ is integrable and we call ϑ the vertical subbundle or characteristic subbundle.

Otherwise if M is a complex manifold with almost complex structure J, Hermitian metric g and open covering by coordinate neighborhoods $\{\mathbf{O}_\alpha\}$, M is called a complex almost contact metric manifold, if it provides the following two properties:

1. On each $\mathbf{O}_\alpha$ there exist 1-forms x_α and $y_\alpha = x_\alpha \circ J$ with orthogonal dual vector fields X_α and $Y_\alpha = -JX_\alpha$ and $(1,1)$ tensor fields G_α and $H_\alpha = G_\alpha J$ such that

$$\begin{aligned}
G_\alpha^2 &= H_\alpha^2 = -I + x_\alpha \otimes X_\alpha + y_\alpha \otimes Y_\alpha, && (2.2)\\
G_\alpha J &= -JG_\alpha, && (2.3)\\
G_\alpha X &= 0, && (2.4)\\
g(W, G_\alpha Z) &= -g(G_\alpha W, Z), && (2.5)\\
g(X_\alpha, W) &= x_\alpha(W), && (2.6)\\
x_\alpha(X_\alpha) &= 1 && (2.7)
\end{aligned}$$

2. On $\mathbf{O}_\alpha \cap \mathbf{O}_\beta \neq \varnothing$,

$$\begin{aligned}
x_\beta &= ax_\alpha - by_\alpha,\\
y_\beta &= bx_\alpha + ay_\alpha,\\
G_\beta &= aG_\alpha - bH_\alpha,\\
H_\beta &= bG_\alpha + aH_\alpha
\end{aligned}$$

where a and b are functions providing the equality $a^2 + b^2 = 1$[4]

Consequently, on a complex almost contact metric manifold M, the following identities hold:

$$H_\alpha G_\alpha = -G_\alpha H_\alpha = J + x_\alpha \otimes Y_\alpha - y_\alpha \otimes X_\alpha$$

$$JH_\alpha = -H_\alpha J = G_\alpha$$
$$g(H_\alpha W, Z) = -g(W, H_\alpha Z)$$

$$\begin{aligned} G_\alpha Y_\alpha &= H_\alpha X_\alpha = H_\alpha Y_\alpha = 0 \\ x_\alpha G_\alpha &= y_\alpha G_\alpha = x_\alpha H_\alpha = y_\alpha H_\alpha = 0 \\ JY_\alpha &= X_\alpha, g(X_\alpha, Y_\alpha) = 0. \end{aligned}$$

Let $(M, \{\omega_\alpha\})$ be a complex contact manifold. We can find a non-vanishing, complex-valued function multiple π_α of ω_α such that on $\mathbf{O}_\alpha \cap \mathbf{O}_\beta$, $\pi_\alpha = h_{\alpha\beta}\pi_\beta$ with $h_{\alpha\beta} : \mathbf{O}_\alpha \cap \mathbf{O}_\beta \to S^1$. Let $\pi_\alpha = x_\alpha - iy_\alpha$. Since ω_α is holomorphic $y_\alpha = x_\alpha J$.

We can locally descriptive of a vector field X providing following properties

1 for all W in $\mathcal{H}$, $du(X, W) = 0$

2 $x(X) = 1, y(X) = 0$.

Then we have a global subbundle ϑ locally spanned by X and $Y = -JX$ with $TM = \mathcal{H} \oplus \vartheta$. We say ϑ the vertical subbundle on contact structure. Here we can obtain a local $(1,1)$ tensor G from a complex almost contact metric structure on M such that $(x, y, X, Y, G, H = GJ, g)$[11].

Definition 2.3. Let $(M, \{\omega\})$ be a complex contact manifold with the complex structure J and hermitian metric g. (M, x, y, X, Y, g) is called a complex contact metric manifold if

1. There is a local $(1,1)$ tensor g such that $(x, y, X, Y, G, H = GJ, g)$ is a complex almost contact metric structure on M, and
2. $g(W, GZ) = dx(W, Z)$ and $g(W, HZ) = dy(W, Z)$ for all W, Z in $\mathcal{H}$.

Now, let us define 2-forms $\hat{G}$ and $\hat{H}$ by

$$\begin{aligned} \hat{G}(W, Z) &= g(W, GZ) \\ \hat{H}(W, Z) &= g(W, HZ). \end{aligned}$$

Then

$$\begin{aligned} \hat{G}(W, Z) &= dx(W, Z), \\ \hat{H}(W, Z) &= dy(W, Z), \end{aligned}$$

where W, Z are horizontal vector fields. Generally, for $\sigma(W) = g(\nabla_W X, Y)$, we get

$$\hat{G} = dx - \sigma \wedge y \tag{2.8}$$
$$\hat{H} = dx + \sigma \wedge x. \tag{2.9}$$

[11].

Let p be projection map from TM to $\mathcal{H}$. There is a symmetric operator $h = \frac{1}{2}\mathcal{L}_\xi \phi$ playing an important role in real contact geometry, where ξ is the characteristic vector field, ϕ is the structure tensor of the real contact metric structure and $\mathcal{L}$ represents the Lie- differentiation. Especially, we obtain

$$\nabla_W \xi = -\phi W - \phi h W$$

on a real contact manifold. We define symmetric operators h_X, h_Y from TM to $\mathcal{H}$ in the same way and as follows:

$$\begin{aligned} h_X &= \frac{1}{2} sym\left(\mathcal{L}_X G\right) \circ p \\ h_Y &= \frac{1}{2} sym\left(\mathcal{L}_Y G\right) \circ p, \end{aligned}$$

where sym represents the symmetrization. Then for levi-civita connection ∇ of g, we have

$$\begin{aligned} h_X G &= -G h_X, h_Y H = -H h_Y, \\ h_X\left(X\right) &= h_X\left(Y\right) = h_Y\left(X\right) = h_Y\left(Y\right) = 0, \end{aligned}$$

and

$$\nabla_W X = -GW - G h_X W + \sigma\left(W\right) Y, \tag{2.10}$$

$$\nabla_W Y = -HW - H h_Y W - \sigma\left(W\right) X, \tag{2.11}$$

where ∇ is Levi-Civita connection of g[11].

Therefore

$$\nabla_X X = \sigma\left(X\right) Y, \nabla_Y X = \sigma\left(Y\right) Y, \nabla_X Y = -\sigma\left(X\right) X, \nabla_Y Y = -\sigma\left(Y\right) X. \tag{2.12}$$

Lemma 2.1. $\nabla_X G = \sigma\left(X\right) H$ *and* $\nabla_Y H = -\sigma\left(Y\right) G$.

Let M be a complex contact metric manifold. the authors in [8] defined $(1,2)$ tensors S and T on a complex almost contact manifolds as follows:

$$\begin{aligned} S\left(W,Z\right) = {} & \left[G,G\right]\left(W,Z\right) + 2y(Z)HW - 2y(W)HZ \\ & + 2g(W,GZ)X - 2g(W,HZ)Y - \sigma\left(GW\right)HZ \\ & + \sigma\left(GZ\right)HW + \sigma\left(W\right)GHZ - \sigma\left(Z\right)GHW \end{aligned} \tag{2.13}$$

$$\begin{aligned} T\left(W,Z\right) = {} & \left[H,H\right]\left(W,Z\right) + 2u(Z)GW - 2x(W)GZ \\ & + 2g(W,HZ)Y - 2g(W,GZ)X + \sigma(HW)GZ \\ & - \sigma(HZ)GW - \sigma(W)HGZ + \sigma(Z)HGW \end{aligned} \tag{2.14}$$

where

$$[G,G](W,Z) = (\nabla_{GW} G) Z - (\nabla_{GZ} G) W - G(\nabla_W G) Z + G(\nabla_Z G) W$$

is the Nijenhuis torsion of G. In [8] the authors introduced concept of normality in which case the two tensor S and T are vanish. One of the important their result is that if M is normal then it is Keahlerian.

Definition 2.4. [11] A complex contact metric manifold M is called normal if

1. $S(W,Z) = T(W,Z) = 0$ for all W, Z in $\mathcal{H}$, and
2. $S(X,W) = T(Y,W) = 0$ for all W.

In real contact geometry, normality means the vanishing of the operator h. The following proposition gives the parallel result for complex contact geometry.

Proposition 2.1. *If M is normal, then $h_x = h_y = 0$[11].*

By the above proposition, on a normal complex contact metric manifold we have

$$\nabla_W X = -GW + \sigma(W) Y \tag{2.15}$$

and

$$\nabla_W Y = -HW - \sigma(W) X \tag{2.16}$$

3. Semi-Invariant Submanifolds Of Almost Complex Contact Metric Manifolds

Assume that complex contact structure of $\bar{M}$ is defined by $\left(\bar{M}, \bar{X}, \bar{Y}, \bar{x}, \bar{y}, \bar{g}, \bar{H} = \bar{G}J\right)$. A submanifold M is a semi-invariant submanifold of almost complex contact metric manifold $\bar{M}$, if there is $\left(D, D^{\perp}\right)$ orthogonal distribution on M providing the following conditions: 1- $TM = D \oplus D^{\perp}$

2- D is invariant according to $\bar{G}$, that is, $\bar{G}D_z = D_z$ for any $z \in M$

3- $D^{\perp}$ is anti-invariant according to $\bar{G}$, that is $\bar{G}D_z^{\perp} \subset T_z M^{\perp}$ for any $z \in M$,

where D and $D^{\perp}$ distributions are horizontal and vertical distributions respectively.

If $D^{\perp} = \{0\}\,(D = \{0\})$, semi-invariant submanifold M is invariant(anti-invariant) submanifold of almost complex contact metric manifold $\bar{M}$. If M is neither invariant submanifold nor anti-invariant submanifold of almost complex contact metric manifold $\bar{M}$, then such a submanifold be proper semi-invariant submanifold.

Let T and R is defined as projection morphisms for D and $D^{\perp}$, respectively. In this case, we can write

$$W = RW + TW + X + Y \tag{3.1}$$

for $W \in \Gamma(TM)$ and $N \in \Gamma\left(TM^{\perp}\right)$. If $\bar{G}$ is applied to (3.1), we can find

$$\bar{G}W = SW + LW$$

where $SW \in \Gamma(D)$ and $LW \in \Gamma\left(D^{\perp}\right)$ and

$$\bar{G}N = BN + CN$$

where $BN \in \Gamma(TM)$ and $CN \in \Gamma(T\mu)$. Also, μ is subvector fibre that is orthogonal complemant to D' and it is expressed as

$$TM^{\perp} = D' \perp \mu.$$

Proposition 3.1. *M is a semi-invariant submanifold of almost complex contact metric manifold $\bar{M}$. In this case, distribution μ is invariant according to G.*

Proof. For $K \in \Gamma(\mu), W \in \Gamma(D)$ and $Z \in \Gamma\left(D^{\perp}\right)$, we obtain

$$\begin{aligned} \bar{g}(GK, W) &= g(G^2K, GW) = 0, \\ \bar{g}(GK, Z) &= g(G^2K, GZ) = 0, \\ \bar{g}(GK, X) &= 0, \\ \bar{g}(GK, Y) &= 0. \end{aligned}$$

Thus, there are not component of $D, D^{\perp}$ and $GD^{\perp}$ of KW. Therefore, μ is invariant. □

Theorem 3.1. *Let M be a semi-invariant submanifold of an almost complex contact metric manifold $\bar{M}$.In this case, D is integrable if and only if there is not component of $h(W, GZ) - h(Z, GW)$ on $GD^{\perp}$, where $W, Z \in \Gamma(D)$.*

Proof. For any $W, Z \in \Gamma(D)$ and $C \in \Gamma\left(D^{\perp}\right)$, we have

$$g([W, Z], C) = g\left(\bar{\nabla}_W Z, C\right) - g\left(\bar{\nabla}_Z W, C\right). \tag{3.2}$$

Here,we can find

$$\begin{aligned} g\left(\bar{\nabla}_W Z, C\right) &= g\left(G\bar{\nabla}_W Z, CR\right) \\ &= -g((\bar{\nabla}_W G)Z, GC) + g(\bar{\nabla}_W GZ, GC) \\ &= g(h(W, GZ), GC) \end{aligned} \tag{3.3}$$

$$g\left(\bar{\nabla}_Z W, C\right) = g(h(Z, GW), GC). \tag{3.4}$$

By using (3.3) and (3.4) in (3.2),we have

$$g([W, Z], R) = g(h(W, GZ) - h(Z, GW), GR) = 0.$$

This completes the proof. □

Theorem 3.2. *Let M be a semi-invariant submanifold of an almost complex contact manifold $\bar{M}$, In this case, $D^{\perp}$ is integrable if and only if*

$$g(A_{GW}Z - A_{GZ}W, C) = 0,$$

where for any $W, Z \in \Gamma(D^{\perp})$ and $C \in \Gamma(D)$.

Proof. For any $W, Z \in \Gamma(D^{\perp})$ and $C \in \Gamma(D)$,

$$g([W,Z],C) = g\left(\bar{\nabla}_W Z, C\right) - g\left(\bar{\nabla}_Z W, C\right). \tag{3.5}$$

From here, we can find

$$g\left(\bar{\nabla}_W Z, C\right) = -g(A_{GZ}W, GC) \tag{3.6}$$

and

$$g\left(\bar{\nabla}_Z W, C\right) = -g(A_{GW}Z, GC). \tag{3.7}$$

By using (3.6) and (3.7) in (3.5), we have $g(A_{GW}Z - A_{GZ}W, C) = 0$. This completes the proof. □

Theorem 3.3. *Let M be a semi-invariant submanifold of an almost contact metric manifold $\bar{M}$. In this case, D defines by totally geodesic foliations if and only if $h(W, GZ)$ hasn't got a component on $GD^{\perp}$.*

Proof. For any $W, Z \in \Gamma(D)$ and $C \in \Gamma(D^{\perp})$,we can find

$$g(\nabla_W Z, C) = g\left(\bar{\nabla}_W Z, C\right) = g(G\bar{\nabla}_W Z, GC) = g(\bar{\nabla}_W GZ, GC) = g(h(W, GZ), GC).$$

Therefore, the proof is completed. □

Theorem 3.4. *Let M be semi invariant submanifold with almost complex contact metric structure of almost contact metric manifold $\bar{M}$. In this case, $D^{\perp}$ defines by totally geodesic foliations if and only if $A_{GZ}W$ hasn't got a component on D.*

Proof. For any $W, Z \in \Gamma(D^{\perp})$ and $C \in \Gamma(D)$

$$g(\nabla_W Z, C) = g(\bar{\nabla}_W Z, C) = g(G\bar{\nabla}_W Z, GC) = g(\bar{\nabla}_W GZ, GC) = -g(A_{GZ}W, GC).$$

Hence, the proof is completed. □

Lemma 3.1. *Let M be semi-invariant submanifold with almost complex contact metric structure of almost contact metric manifold $\bar{M}$. Then we have*

$$\nabla_W X = -GW + \sigma(W)Y, \tag{3.8}$$

$$h(W, X) = 0, \tag{3.9}$$

for any $W \in \Gamma(D)$.

$$\nabla_Z X = \sigma(Z)Y, \tag{3.10}$$

$$h(X,Z) = -GZ, \tag{3.11}$$

for any $Z \in \Gamma(D^{\perp})$. *Also we find*

$$\nabla_X X = \sigma(X)Y, \tag{3.12}$$

$$h(X,X) = 0. \tag{3.13}$$

Proof. (3.8) and (3.9) are obtained by using

$$\bar{\nabla}_W X = \nabla_W X + h(W,X) = -GW + \sigma(W)Y,$$

As M is normal, we have

$$\bar{\nabla}_Z X = \nabla_Z X + h(X,Z) = -GZ + \sigma(Z)Y,$$

which gives (3.10) and (3.11). Also from

$$\bar{\nabla}_X X = -GX + \sigma(X)Y,$$

we find (3.12) and (3.13). □

Lemma 3.2. *Let M be a semi-invariant submanifold of a normal complex contact metric manifold $\bar{M}$. Then we have*

$$A_{GW}Z = A_{GZ}W$$

for all $W, Z \in \Gamma(D^{\perp})$.

Proof.

$$\begin{aligned} g(A_{GW}Z, C) &= g(GW, h(Z,C)) \\ &= -g(W, G\bar{\nabla}_C Z) \\ &= g(W, (\bar{\nabla}_C G)Z) - g(W, \bar{\nabla}_C GZ) \end{aligned} \tag{3.14}$$

for $C \in \Gamma(TM)$.

$$\begin{aligned} g(W, (\bar{\nabla}_C G)Z) &= \sigma(C)\, g(HZ, W) + \nu(C)\, \Omega(GW, GZ) \\ &\quad -2\nu(C)\, g(HGZ, W) - \upsilon(Z)\, g(C, W) \\ &\quad -\nu(Z)\, g(JC, W) + \upsilon(W)\, g(C, Z) - \nu(W)\, g(C, JZ) \\ &= 0. \end{aligned}$$

Also we have

$$g(A_{GW}Z, C) = -g(W, \bar{\nabla}_C GZ) = g(A_{GZ}W, C).$$

Therefore the proof is completed. □

Conclusion In this study, semi invariant submanifolds of almost complex contact metric manifolds are investigated and defined. We found necessary and sufficient conditions to be integrable and totally geodesic for distributions D defined on M. Further we found necessary and sufficient conditions to be integrable and totally geodesic for distributions $D^{\perp}$ defined on M.

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