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ON RELATIVE TYPE AND WEAK TYPE OF ENTIRE FUNCTIONS OF TWO COMPLEX VARIABLES

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Abstract. In this paper, we introduce the idea of relative type and relative weak type of entire functions of two complex variables with respect to another entire function of two complex variables and prove some related growth properties of it.

Keywords: Complex function, order, lower order, relative order, relative type, relative weak type.

1. Introduction, Definitions and Notations

Let f be any entire function of two complex variables holomorphic in the closed polydisc

$$U = \{(z_1, z_2) : \ |z_i| \leq r_i, \ i = 1, 2 \text{ for all } \ r_1 \geq 0, r_2 \geq 0\}$$

and $M_f(r_1, r_2) = \max\{|f(z_1, z_2)| : |z_i| \leq r_i, \ i = 1, 2\}$. Then in view of maximum principal and Hartogs's theorem {[10], p.2, p.51}, $M_f(r_1, r_2)$ is increasing function of r_1, r_2. For any two entire functions f and g of two complex variables, the ratio $\frac{M_f(r_1,r_2)}{M_g(r_1,r_2)}$ as $r_1, r_2 \to \infty$ is called the *growth* of f with respect to g. Taking this into account, the following definition is well known:

Definition 1.1. {[10], p.339, (see also [5])} The *order* ${}_{v_2}\rho_f$ of an entire function $f(z_1, z_2)$ is defined as

$${}_{v_2}\rho_f = \limsup_{r_1, r_2 \to \infty} \frac{\log\log M_f(r_1, r_2)}{\log M_{\exp(z_1 z_2)}(r_1, r_2)} = \limsup_{r_1, r_2 \to \infty} \frac{\log\log M_f(r_1, r_2)}{\log M_{\exp(z_1 z_2)}(r_1, r_2)} .$$

We see that the *order* ${}_{v_2}\rho_f$ of an entire function $f(z_1, z_2)$ is defined in terms of the growth of $f(z_1, z_2)$ with respect to the exponential function $\exp(z_1 z_2)$.

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However, In the same way one can define the *lower order* of $f(z_1, z_2)$ denoted by ${}_{v_2}\lambda_f$ as follows :

$$ {}_{v_2}\lambda_f = \liminf_{r_1,r_2\to\infty} \frac{\log\log M_f(r_1,r_2)}{\log M_{\exp(z_1 z_2)}(r_1,r_2)} = \liminf_{r_1,r_2\to\infty} \frac{\log\log M_f(r_1,r_2)}{\log M_{\exp(z_1 z_2)}(r_1,r_2)} . $$

An entire function of two complex variables for which *order* and *lower order* are the same is said to be of *regular growth.* Functions which are not of *regular growth* are said to be of *irregular growth.*

The rate of *growth* of an entire function generally depends upon the *order (lower order)* of it. The entire function with higher *order* is of faster growth than that of lesser *order.* But if *orders* of two entire functions are the same, then it is impossible to detect the function with faster growth. In that case, it is necessary to compute another class of growth indicators of entire functions called their *types* and thus one can define *type* of an entire function $f(z_1, z_2)$ denoted by ${}_{v_2}\sigma_f$ in the following way:

Definition 1.2. The *type* ${}_{v_2}\sigma_f$ of an entire function $f(z_1, z_2)$ is defined as

$$ {}_{v_2}\sigma_f = \limsup_{r_1,r_2\to\infty} \frac{\log M_f(r_1,r_2)}{[r_1 r_2]^{{}_{v_2}\rho_f}},\ 0 < {}_{v_2}\rho_f < \infty . $$

Similarly, the *lower type* ${}_{v_2}\overline{\sigma}_f$ of an entire function $f(z_1, z_2)$ may be defined as

$$ {}_{v_2}\overline{\sigma}_f = \liminf_{r_1,r_2\to\infty} \frac{\log M_f(r_1,r_2)}{[r_1 r_2]^{{}_{v_2}\rho_f}},\ 0 < {}_{v_2}\rho_f < \infty . $$

Analogously, to determine the relative growth of two entire functions of two complex variables having same non zero finite *lower order* one may introduce the definition of *weak type* ${}_{v_2}\tau_f$ of $f(z_1, z_2)$ of finite positive *lower order* ${}_{v_2}\lambda_f$ in the following way:

Definition 1.3. The *weak type* ${}_{v_2}\tau_f$ of an entire function $f(z_1, z_2)$ of finite positive *lower order* ${}_{v_2}\lambda_f$ is defined by

$$ {}_{v_2}\tau_f = \liminf_{r_1,r_2\to\infty} \frac{\log M_f(r_1,r_2)}{[r_1 r_2]^{{}_{v_2}\lambda_f}},\ 0 < {}_{v_2}\lambda_f < \infty . $$

Similarly, one may define the growth indicator ${}_{v_2}\overline{\tau}_f$ of an entire function $f(z_1, z_2)$ of finite positive *lower order* ${}_{v_2}\lambda_f$ in the following way:

$$ {}_{v_2}\overline{\tau}_f = \limsup_{r_1,r_2\to\infty} \frac{\log M_f(r_1,r_2)}{[r_1 r_2]^{{}_{v_2}\lambda_f}},\ 0 < {}_{v_2}\lambda_f < \infty . $$

Bernal (see [6], [7]) introduced the definition of *relative order* between two entire functions of single variable. During the past decades, several authors (see [11],[12],[13],[14]) made close investigations on the properties of *relative order* of entire functions of single variable. Using the idea of Bernal's *relative order* (see [6], [7]) of entire functions of single variable, Banerjee and Datta [8] introduced the definition of *relative order* of entire functions of two complex variables to avoid comparing growth just with $\exp(z_1 z_2)$ which is as follows:

$$\begin{aligned} {}_{v_2}\rho_g(f) &= \inf\left\{\mu > 0 : M_f(r_1, r_2) < M_g\left(r_1^{\mu}, r_2^{\mu}\right); r_1 \geq R(\mu), r_2 \geq R(\mu)\right\} \\ &= \limsup_{r_1, r_2 \to \infty} \frac{\log M_g^{-1} M_f(r_1, r_2)}{\log(r_1 r_2)} \end{aligned}$$

where g is also an entire function holomorphic in the closed polydisc

$$U = \{(z_1, z_2) : \ |z_i| \leq r_i, \ i = 1, 2 \text{ for all } \ r_1 \geq 0, r_2 \geq 0\}$$

and the definition coincides with the classical one [8] if $g(z_1, z_2) = \exp(z_1 z_2)$.

Likewise, one can define the *relative lower order* of f with respect to g denoted by ${}_{v_2}\lambda_g(f)$ as follows:

$${}_{v_2}\lambda_g(f) = \liminf_{r_1, r_2 \to \infty} \frac{\log M_g^{-1} M_f(r_1, r_2)}{\log(r_1 r_2)} .$$

Now, in the case of *relative order* of entire functions of two complex variables, it therefore seems reasonable to define suitably the *relative type* and *relative weak type* respectively in order to compare the relative growth of two entire functions of two complex variables having same non zero finite *relative order* or *relative lower order* with respect to another entire function of two complex variables. Their definitions are as follows:

Definition 1.4. Let $f(z_1, z_2)$ and $g(z_1, z_2)$ be any two entire functions such that $0 < {}_{v_2}\rho_g(f) < \infty$. Then the *relative type* ${}_{v_2}\sigma_g(f)$ of $f(z_1, z_2)$ with respect to $g(z_1, z_2)$ is defined as :

$$\begin{aligned} {}_{v_2}\sigma_g(f) &= \inf\Big\{k > 0 : M_f(r_1, r_2) < M_g(k r_1^{{}_{v_2}\rho_g(f)}, k r_2^{{}_{v_2}\rho_g(f)}) \\ &\qquad \text{for all sufficiently large values of } r_1 \text{ and } r_2\Big\} . \end{aligned}$$

The equivalent formula for ${}_{v_2}\sigma_g(f)$ is

$${}_{v_2}\sigma_g(f) = \limsup_{r_1, r_2 \to \infty} \frac{M_g^{-1} M_f(r_1, r_2)}{[r_1 r_2]^{{}_{v_2}\rho_g(f)}} .$$

Likewise, one can define the *relative lower type* of an entire function $f(z_1, z_2)$ with respect to an entire function $g(z_1, z_2)$ denoted by ${}_{v_2}\overline{\sigma}_g(f)$ as follows :

$${}_{v_2}\overline{\sigma}_g(f) = \liminf_{r_1, r_2 \to \infty} \frac{M_g^{-1} M_f(r_1, r_2)}{[r_1 r_2]^{{}_{v_2}\rho_g(f)}}, \ 0 < {}_{v_2}\rho_g(f) < \infty .$$

Definition 1.5. The *relative weak type* ${}_{v_2}\tau_g(f)$ of an entire function $f(z_1, z_2)$ with respect to another entire function $g(z_1, z_2)$ having finite positive *relative lower order* ${}_{v_2}\lambda_g(f)$ is defined as:

$${}_{v_2}\tau_g(f) = \liminf_{r_1, r_2 \to \infty} \frac{M_g^{-1} M_f(r_1, r_2)}{[r_1 r_2]^{{}_{v_2}\lambda_g(f)}} .$$

Also, one may define the growth indicator ${}_{v_2}\overline{\tau}_g(f)$ of an entire function f with respect to an entire function g in the following way:

$${}_{v_2}\overline{\tau}_g(f) = \limsup_{r_1, r_2 \to \infty} \frac{M_g^{-1} M_f(r_1, r_2)}{[r_1 r_2]^{{}_{v_2}\lambda_g(f)}}, \ 0 < {}_{v_2}\lambda_g(f) < \infty .$$

Considering $g(z_1, z_2) = \exp(z_1 z_2)$ one may easily verify that Definition 1.4 and Definition 1.5 coincide with Definition 1.2 and Definition 1.3, respectively.

In this paper, we study some relative growth properties of entire functions of two complex variables with respect to another entire function of two complex variables on the basis of *relative type* and *relative weak type* of two complex variables. We do not explain the standard definitions and notations in the theory of entire functions as those are available in [10].

2. Lemmas

In this section, we present some lemmas which will be needed in the sequel.

Lemma 2.1. *[9] Let $f(z_1, z_2)$ be an entire function with $0 \leq {}_{v_2}\lambda_f \leq {}_{v_2}\rho_f < \infty$ and $g(z_1, z_2)$ be entire of regular growth . Then*

$${}_{v_2}\lambda_g(f) = \frac{{}_{v_2}\lambda_f}{{}_{v_2}\lambda_g} \quad \textit{and} \quad {}_{v_2}\rho_g(f) = \frac{{}_{v_2}\rho_f}{{}_{v_2}\rho_g} .$$

Lemma 2.2. *[9] Let $f(z_1, z_2)$ be an entire function with regular growth and $g(z_1, z_2)$ be entire with $0 \leq {}_{v_2}\lambda_g \leq {}_{v_2}\rho_g < \infty$. Then*

$${}_{v_2}\lambda_g(f) = \frac{{}_{v_2}\rho_f}{{}_{v_2}\rho_g} \quad \textit{and} \quad {}_{v_2}\rho_g(f) = \frac{{}_{v_2}\lambda_f}{{}_{v_2}\lambda_g} .$$

3. Main Results

In this section, we present the main results of the paper.

Theorem 3.1. *Let $f(z_1, z_2)$ and $g(z_1, z_2)$ be any two entire functions with finite non-zero order. Also let $g(z_1, z_2)$ be of regular growth. Then*

$$\left[\frac{{}_{v_2}\overline{\sigma}_f}{{}_{v_2}\sigma_g}\right]^{\frac{1}{{}_{v_2}\rho_g}} \leq {}_{v_2}\overline{\sigma}_g(f) \leq \min\left\{\left[\frac{{}_{v_2}\overline{\sigma}_f}{{}_{v_2}\overline{\sigma}_g}\right]^{\frac{1}{{}_{v_2}\rho_g}}, \left[\frac{{}_{v_2}\sigma_f}{{}_{v_2}\sigma_g}\right]^{\frac{1}{{}_{v_2}\rho_g}}\right\}$$

$$\leq \max\left\{\left[\frac{{}_{v_2}\overline{\sigma}_f}{{}_{v_2}\overline{\sigma}_g}\right]^{\frac{1}{v_2\rho_g}}, \left[\frac{{}_{v_2}\sigma_f}{{}_{v_2}\sigma_g}\right]^{\frac{1}{v_2\rho_g}}\right\} \leq {}_{v_2}\sigma_g\left(f\right) \leq \left[\frac{{}_{v_2}\sigma_f}{{}_{v_2}\overline{\sigma}_g}\right]^{\frac{1}{v_2\rho_g}}.$$

Proof. From the definitions of ${}_{v_2}\sigma_f$ and ${}_{v_2}\overline{\sigma}_f$ we have for all sufficiently large values of r_1, r_2 that

$$M_f\left(r_1, r_2\right) \leq \exp\left\{\left({}_{v_2}\sigma_f + \varepsilon\right)\left[r_1 r_2\right]^{v_2\rho_f}\right\}, \tag{3.1}$$

$$M_f\left(r_1, r_2\right) \geq \exp\left\{\left({}_{v_2}\overline{\sigma}_f - \varepsilon\right)\left[r_1 r_2\right]^{v_2\rho_f}\right\} \tag{3.2}$$

and also for a sequence of values of r_1, r_2 tending to infinity we get that

$$M_f\left(r_1, r_2\right) \geq \exp\left\{\left({}_{v_2}\sigma_f - \varepsilon\right)\left[r_1 r_2\right]^{v_2\rho_f}\right\}, \tag{3.3}$$

$$M_f\left(r_1, r_2\right) \leq \exp\left\{\left({}_{v_2}\overline{\sigma}_f + \varepsilon\right)\left[r_1 r_2\right]^{v_2\rho_f}\right\}. \tag{3.4}$$

Similarly, from the definitions of ${}_{v_2}\sigma_g$ and ${}_{v_2}\overline{\sigma}_f$, it follows for all sufficiently large values of r_1, r_2 that

$$\begin{aligned} M_g\left(r_1, r_2\right) &\leq \exp\left\{\left({}_{v_2}\sigma_g + \varepsilon\right)\left[r_1 r_2\right]^{v_2\rho_g}\right\} \\ \text{i.e., } \left[r_1 r_2\right] &\leq M_g^{-1}\left[\exp\left\{\left({}_{v_2}\sigma_g + \varepsilon\right)\left[r_1 r_2\right]^{v_2\rho_g}\right\}\right] \\ \text{i.e., } M_g^{-1}\left(r_1, r_2\right) &\geq \left[\left(\frac{\log\left(r_1 r_2\right)}{\left({}_{v_2}\sigma_g + \varepsilon\right)}\right)^{\frac{1}{v_2\rho_g}}\right], \end{aligned} \tag{3.5}$$

$$\begin{aligned} M_g\left(r_1, r_2\right) &\geq \exp\left\{\left({}_{v_2}\overline{\sigma} - \varepsilon\right)\left[r_1 r_2\right]^{v_2\rho_g}\right\} \\ \text{i.e., } \left[r_1 r_2\right] &\geq M_g^{-1}\left[\exp\left\{\left({}_{v_2}\overline{\sigma} - \varepsilon\right)\left[r_1 r_2\right]^{v_2\rho_g}\right\}\right] \\ \text{i.e., } M_g^{-1}\left(r_1, r_2\right) &\leq \left[\left(\frac{\log\left(r_1 r_2\right)}{\left({}_{v_2}\overline{\sigma} - \varepsilon\right)}\right)^{\frac{1}{v_2\rho_g}}\right] \end{aligned} \tag{3.6}$$

and for a sequence of values of r_1, r_2 tending to infinity we obtain that

$$\begin{aligned} M_g\left(r_1, r_2\right) &\geq \exp\left\{\left({}_{v_2}\sigma_g - \varepsilon\right)\left[r_1 r_2\right]^{v_2\rho_g}\right\} \\ \text{i.e., } \left[r_1 r_2\right] &\geq M_g^{-1}\left[\exp\left\{\left({}_{v_2}\sigma_g - \varepsilon\right)\left[r_1 r_2\right]^{v_2\rho_g}\right\}\right] \\ \text{i.e., } M_g^{-1}\left(r_1, r_2\right) &\leq \left[\left(\frac{\log\left(r_1 r_2\right)}{\left({}_{v_2}\sigma_g - \varepsilon\right)}\right)^{\frac{1}{v_2\rho_g}}\right], \end{aligned} \tag{3.7}$$

$$\begin{aligned} M_g\left(r_1, r_2\right) &\leq \exp\left\{\left({}_{v_2}\overline{\sigma}_g + \varepsilon\right)\left[r_1 r_2\right]^{v_2\rho_g}\right\} \\ \text{i.e., } \left[r_1 r_2\right] &\leq M_g^{-1}\left[\exp\left\{\left({}_{v_2}\overline{\sigma}_g + \varepsilon\right)\left[r_1 r_2\right]^{v_2\rho_g}\right\}\right] \\ \text{i.e., } M_g^{-1}\left(r_1, r_2\right) &\geq \left[\left(\frac{\log\left(r_1 r_2\right)}{\left({}_{v_2}\overline{\sigma}_g - \varepsilon\right)}\right)^{\frac{1}{v_2\rho_g}}\right]. \end{aligned} \tag{3.8}$$

Now from (3.3) and in view of (3.5), we get for a sequence of values of r_1, r_2 tending to infinity we get that

$$M_g^{-1} M_f\left(r_1, r_2\right) \geq M_g^{-1}\left[\exp\left\{\left({}_{v_2}\sigma_f - \varepsilon\right)\left[r_1 r_2\right]^{v_2\rho_f}\right\}\right]$$

$$i.e.,\ M_g^{-1}M_f(r_1,r_2) \geq \left[\left(\frac{\log\exp\{({}_{v_2}\sigma_f-\varepsilon)[r_1r_2]^{v_2\rho_f}\}}{({}_{v_2}\sigma_g+\varepsilon)}\right)^{\frac{1}{v_2\rho_g}}\right]$$

$$i.e.,\ M_g^{-1}M_f(r_1,r_2) \geq \left[\frac{({}_{v_2}\sigma_f-\varepsilon)}{({}_{v_2}\sigma_g+\varepsilon)}\right]^{\frac{1}{v_2\rho_g}}\cdot[r_1r_2]^{\frac{v_2\rho_f}{v_2\rho_g}}$$

$$i.e.,\ \frac{M_g^{-1}M_f(r_1,r_2)}{[r_1r_2]^{\frac{v_2\rho_f}{v_2\rho_g}}} \geq \left[\frac{({}_{v_2}\sigma_f-\varepsilon)}{({}_{v_2}\sigma_g+\varepsilon)}\right]^{\frac{1}{v_2\rho_g}}.$$

As $\varepsilon\,(>0)$ is arbitrary, in view of Lemma 2.1 it follows that

$$\limsup_{r_1,r_2\to\infty}\frac{M_g^{-1}M_f(r_1,r_2)}{[r_1r_2]^{v_2\rho_g(f)}} \geq \left[\frac{{}_{v_2}\sigma_f}{{}_{v_2}\sigma_g}\right]^{\frac{1}{v_2\rho_g}}$$

$$i.e.,\ {}_{v_2}\sigma_g(f) \geq \left[\frac{{}_{v_2}\sigma_f}{{}_{v_2}\sigma_g}\right]^{\frac{1}{v_2\rho_g}}. \tag{3.9}$$

Analogously, from (3.2) and in view of (3.8), it follows for a sequence of values of r_1, r_2 tending to infinity we get that

$$M_g^{-1}M_f(r_1,r_2) \geq M_g^{-1}\left[\exp\left\{({}_{v_2}\overline{\sigma}_f-\varepsilon)[r_1r_2]^{v_2\rho_f}\right\}\right]$$

$$i.e.,\ M_g^{-1}M_f(r_1,r_2) \geq \left[\left(\frac{\log\exp\{({}_{v_2}\overline{\sigma}_f-\varepsilon)[r_1r_2]^{v_2\rho_f}\}}{({}_{v_2}\overline{\sigma}_g+\varepsilon)}\right)^{\frac{1}{v_2\rho_g}}\right]$$

$$i.e.,\ M_g^{-1}M_f(r_1,r_2) \geq \left[\frac{({}_{v_2}\overline{\sigma}_f-\varepsilon)}{({}_{v_2}\overline{\sigma}_g+\varepsilon)}\right]^{\frac{1}{v_2\rho_g}}\cdot[r_1r_2]^{\frac{v_2\rho_f}{v_2\rho_g}}$$

$$i.e.,\ \frac{M_g^{-1}M_f(r_1,r_2)}{[r_1r_2]^{\frac{v_2\rho_f}{v_2\rho_g}}} \geq \left[\frac{({}_{v_2}\overline{\sigma}_f-\varepsilon)}{({}_{v_2}\overline{\sigma}_g+\varepsilon)}\right]^{\frac{1}{v_2\rho_g}}.$$

Since $\varepsilon\,(>0)$ is arbitrary, we get from above and Lemma 2.1 that

$$\limsup_{r_1,r_2\to\infty}\frac{M_g^{-1}M_f(r_1,r_2)}{[r_1r_2]^{v_2\rho_g(f)}} \geq \left[\frac{{}_{v_2}\overline{\sigma}_f}{{}_{v_2}\overline{\sigma}_g}\right]^{\frac{1}{v_2\rho_g}}$$

$$i.e.,\ {}_{v_2}\sigma_g(f) \geq \left[\frac{{}_{v_2}\overline{\sigma}_f}{{}_{v_2}\overline{\sigma}_g}\right]^{\frac{1}{v_2\rho_g}}. \tag{3.10}$$

Again, in view of (3.6) we have from (3.1), for all sufficiently large values of r_1, r_2 that

$$M_g^{-1}M_f(r_1,r_2) \leq M_g^{-1}\left[\exp\{({}_{v_2}\sigma_f+\varepsilon)[r_1r_2]^{v_2\rho_f}\}\right]$$

$$i.e.,\ M_g^{-1}M_f(r_1,r_2) \leq \left[\left(\frac{\log\exp\{({}_{v_2}\sigma_f+\varepsilon)[r_1r_2]^{v_2\rho_f}\}}{({}_{v_2}\overline{\sigma}_g-\varepsilon)}\right)^{\frac{1}{v_2\rho_g}}\right]$$

$$i.e.,\ M_g^{-1}M_f(r_1,r_2) \leq \left[\frac{({}_{v_2}\sigma_f+\varepsilon)}{({}_{v_2}\overline{\sigma}_g-\varepsilon)}\right]^{\frac{1}{v_2\rho_g}} \cdot [r_1r_2]^{\frac{v_2\rho_f}{v_2\rho_g}}$$

$$i.e.,\ \frac{M_g^{-1}M_f(r_1,r_2)}{[r_1r_2]^{\frac{v_2\rho_f}{v_2\rho_g}}} \leq \left[\frac{({}_{v_2}\sigma_f+\varepsilon)}{({}_{v_2}\overline{\sigma}_g-\varepsilon)}\right]^{\frac{1}{v_2\rho_g}} .$$

Since $\varepsilon\,(>0)$ is arbitrary, we obtain in view of Lemma 2.1 that

$$\limsup_{r_1,r_2\to\infty}\frac{M_g^{-1}M_f(r_1,r_2)}{[r_1r_2]^{v_2\rho_g(f)}} \leq \left[\frac{{}_{v_2}\sigma_f}{{}_{v_2}\overline{\sigma}_g}\right]^{\frac{1}{v_2\rho_g}}$$

$$i.e.,\ {}_{v_2}\sigma_g(f) \leq \left[\frac{{}_{v_2}\sigma_f}{{}_{v_2}\overline{\sigma}_g}\right]^{\frac{1}{v_2\rho_g}} . \tag{3.11}$$

Again, from (3.2) and in view of (3.5), we get for all sufficiently large values of r_1, r_2 that

$$M_g^{-1}M_f(r_1,r_2) \geq M_g^{-1}\left[\exp\left\{({}_{v_2}\overline{\sigma}_f-\varepsilon)[r_1r_2]^{v_2\rho_f}\right\}\right]$$

$$i.e.,\ M_g^{-1}M_f(r_1,r_2) \geq \left[\left(\frac{\log\exp\left\{({}_{v_2}\overline{\sigma}_f-\varepsilon)[r_1r_2]^{v_2\rho_f}\right\}}{({}_{v_2}\sigma_g+\varepsilon)}\right)^{\frac{1}{v_2\rho_g}}\right]$$

$$i.e.,\ M_g^{-1}M_f(r_1,r_2) \geq \left[\frac{({}_{v_2}\overline{\sigma}_f-\varepsilon)}{({}_{v_2}\sigma_g+\varepsilon)}\right]^{\frac{1}{v_2\rho_g}} \cdot [r_1r_2]^{\frac{v_2\rho_f}{v_2\rho_g}}$$

$$i.e.,\ \frac{M_g^{-1}M_f(r_1,r_2)}{[r_1r_2]^{\frac{v_2\rho_f}{v_2\rho_g}}} \geq \left[\frac{({}_{v_2}\overline{\sigma}_f-\varepsilon)}{({}_{v_2}\sigma_g+\varepsilon)}\right]^{\frac{1}{v_2\rho_g}} .$$

As $\varepsilon\,(>0)$ is arbitrary, it follows from the above and Lemma 2.1 that

$$\liminf_{r_1,r_2\to\infty}\frac{M_g^{-1}M_f(r_1,r_2)}{[r_1r_2]^{v_2\rho_g(f)}} \geq \left[\frac{{}_{v_2}\overline{\sigma}_f}{{}_{v_2}\sigma_g}\right]^{\frac{1}{v_2\rho_g}}$$

$$i.e.,\ {}_{v_2}\overline{\sigma}_g(f) \geq \left[\frac{{}_{v_2}\overline{\sigma}_f}{{}_{v_2}\sigma_g}\right]^{\frac{1}{v_2\rho_g}} . \tag{3.12}$$

Also in view of (3.7), we get from (3.1) for a sequence of values of r_1, r_2 tending to infinity that

$$M_g^{-1}M_f(r_1,r_2) \leq M_g^{-1}\left[\exp\left\{({}_{v_2}\sigma_f+\varepsilon)[r_1r_2]^{v_2\rho_f}\right\}\right]$$

$$i.e.,\ M_g^{-1}M_f(r_1,r_2) \leq \left[\left(\frac{\log\exp\left\{({}_{v_2}\sigma_f+\varepsilon)[r_1r_2]^{v_2\rho_f}\right\}}{({}_{v_2}\sigma_g-\varepsilon)}\right)^{\frac{1}{v_2\rho_g}}\right]$$

$$i.e.,\ M_g^{-1}M_f(r_1,r_2) \leq \left[\frac{({}_{v_2}\sigma_f+\varepsilon)}{({}_{v_2}\sigma_g-\varepsilon)}\right]^{\frac{1}{v_2\rho_g}} \cdot [r_1r_2]^{\frac{v_2\rho_f}{v_2\rho_g}}$$

$$i.e.,\ \frac{M_g^{-1}M_f(r_1,r_2)}{[r_1r_2]^{\frac{v_2\rho_f}{v_2\rho_g}}} \leq \left[\frac{({}_{v_2}\sigma_f+\varepsilon)}{({}_{v_2}\sigma_g-\varepsilon)}\right]^{\frac{1}{v_2\rho_g}} .$$

Since $\varepsilon\,(>0)$ is arbitrary, we get from Lemma 2.1 and above that

$$\liminf_{r_1,r_2\to\infty}\frac{M_g^{-1}M_f(r_1,r_2)}{[r_1r_2]^{v_2\rho_g(f)}} \le \left[\frac{v_2\sigma_f}{v_2\sigma_g}\right]^{\frac{1}{v_2\rho_g}}$$

$$\text{(3.13)} \qquad i.e.,\ {}_{v_2}\overline{\sigma}_g(f) \le \left[\frac{v_2\sigma_f}{v_2\sigma_g}\right]^{\frac{1}{v_2\rho_g}}.$$

Similarly, from (3.4) and in view of (3.6), it follows for a sequence of values of r_1, r_2 tending to infinity we get that

$$M_g^{-1}M_f(r_1,r_2) \le M_g^{-1}\left[\exp\left\{\left({}_{v_2}\overline{\sigma}_f+\varepsilon\right)[r_1r_2]^{v_2\rho_f}\right\}\right]$$

$$i.e.,\ M_g^{-1}M_f(r_1,r_2) \le \left[\left(\frac{\log\exp\left\{\left({}_{v_2}\overline{\sigma}_f+\varepsilon\right)[r_1r_2]^{v_2\rho_f}\right\}}{\left({}_{v_2}\overline{\sigma}_g-\varepsilon\right)}\right)^{\frac{1}{v_2\rho_g}}\right]$$

$$i.e.,\ M_g^{-1}M_f(r_1,r_2) \le \left[\frac{\left({}_{v_2}\overline{\sigma}_f+\varepsilon\right)}{\left({}_{v_2}\overline{\sigma}_g-\varepsilon\right)}\right]^{\frac{1}{v_2\rho_g}}\cdot[r_1r_2]^{\frac{v_2\rho_f}{v_2\rho_g}}$$

$$i.e.,\ \frac{M_g^{-1}M_f(r_1,r_2)}{[r_1r_2]^{\frac{v_2\rho_f}{v_2\rho_g}}} \le \left[\frac{\left({}_{v_2}\overline{\sigma}_f+\varepsilon\right)}{\left({}_{v_2}\overline{\sigma}_g-\varepsilon\right)}\right]^{\frac{1}{v_2\rho_g}}.$$

As $\varepsilon\,(>0)$ is arbitrary, we obtain from Lemma 2.1 and above that

$$\liminf_{r_1,r_2\to\infty}\frac{M_g^{-1}M_f(r_1,r_2)}{[r_1r_2]^{\rho_g(f)}_{v_2}} \le \left[\frac{v_2\overline{\sigma}_f}{v_2\overline{\sigma}_g}\right]^{\frac{1}{v_2\rho_g}}$$

$$\text{(3.14)} \qquad i.e.,\ {}_{v_2}\overline{\sigma}_g(f) \le \left[\frac{v_2\overline{\sigma}_f}{v_2\overline{\sigma}_g}\right]^{\frac{1}{v_2\rho_g}}.$$

Thus the theorem follows from (3.9), (3.10), (3.11), (3.12), (3.13) and (3.14). □

In view of Theorem 3.1, one can easily verify the following corollaries :

Corollary 3.1. *Let $f(z_1,z_2)$ be an entire function such that ${}_{v_2}\sigma_f = {}_{v_2}\overline{\sigma}_f$ and $g(z_1,z_2)$ be an entire function of regular growth. Then*

$${}_{v_2}\sigma_g(f) = \left[\frac{v_2\sigma_f}{v_2\overline{\sigma}_g}\right]^{\frac{1}{v_2\rho_g}} \text{ and } {}_{v_2}\overline{\sigma}_g(f) = \left[\frac{v_2\sigma_f}{v_2\sigma_g}\right]^{\frac{1}{v_2\rho_g}}.$$

Corollary 3.2. *Let $f(z_1,z_2)$ be an entire function with non zero finite order and $g(z_1,z_2)$ be entire of regular growth with ${}_{v_2}\sigma_g = {}_{v_2}\overline{\sigma}_g$. Then*

$${}_{v_2}\sigma_g(f) = \left[\frac{v_2\sigma_f}{v_2\sigma_g}\right]^{\frac{1}{v_2\rho_g}} \text{ and } {}_{v_2}\overline{\sigma}_g(f) = \left[\frac{v_2\overline{\sigma}_f}{v_2\sigma_g}\right]^{\frac{1}{v_2\rho_g}}.$$

In addition, if ${}_{v_2}\sigma_f = {}_{v_2}\overline{\sigma}_f$ *then*

$$ {}_{v_2}\sigma_g(f) = {}_{v_2}\overline{\sigma}_g(f) = \left[\frac{{}_{v_2}\sigma_f}{{}_{v_2}\sigma_g}\right]^{\frac{1}{{}_{v_2}\rho_g}} . $$

Corollary 3.3. *Let $f(z_1, z_2)$ be an entire function with non zero finite order. Then for any entire function $g(z_1, z_2)$,*

$$ \begin{aligned} (i)\ {}_{v_2}\overline{\sigma}_g(f) &= \infty \text{ when } {}_{v_2}\sigma_g = 0 ,\\ (ii)\ {}_{v_2}\sigma_g(f) &= \infty \text{ when } {}_{v_2}\overline{\sigma}_g = 0 ,\\ (iii)\ {}_{v_2}\overline{\sigma}_g(f) &= 0 \text{ when } {}_{v_2}\sigma_g = \infty \end{aligned} $$

and

$$ (iv)\ {}_{v_2}\sigma_g(f) = \infty \text{ when } {}_{v_2}\overline{\sigma}_g = \infty, $$

where $g(z_1, z_2)$ is of regular growth.

Corollary 3.4. *Let $g(z_1, z_2)$ be an entire function with regular growth . Then for any entire function $f(z_1, z_2)$,*

$$ \begin{aligned} (i)\ {}_{v_2}\sigma_g(f) &= 0 \text{ when } {}_{v_2}\sigma_f = 0 ,\\ (ii)\ {}_{v_2}\overline{\sigma}_g(f) &= 0 \text{ when } {}_{v_2}\overline{\sigma}_f = 0 ,\\ (iii)\ {}_{v_2}\sigma_g(f) &= \infty \text{ when } {}_{v_2}\sigma_f = \infty \end{aligned} $$

and

$$ (iv)\ {}_{v_2}\overline{\sigma}_g(f) = \infty \text{ when } {}_{v_2}\overline{\sigma}_f = \infty . $$

In the line of Theorem 3.1 and with the help of Lemma 2.2 one can prove the following theorem and therefore its proof is omitted:

Theorem 3.2. *Let $f(z_1, z_2)$ and $g(z_1, z_2)$ be any two entire functions with finite non-zero order. Also let $f(z_1, z_2)$ be of regular growth. Then*

$$ \left[\frac{{}_{v_2}\overline{\sigma}_f}{{}_{v_2}\sigma_g}\right]^{\frac{1}{{}_{v_2}\rho_g}} \le {}_{v_2}\tau_g(f) \le \min\left\{\left[\frac{{}_{v_2}\overline{\sigma}_f}{{}_{v_2}\overline{\sigma}_g}\right]^{\frac{1}{{}_{v_2}\rho_g}}, \left[\frac{{}_{v_2}\sigma_f}{{}_{v_2}\sigma_g}\right]^{\frac{1}{{}_{v_2}\rho_g}}\right\} $$

$$ \le \max\left\{\left[\frac{{}_{v_2}\overline{\sigma}_f}{{}_{v_2}\overline{\sigma}_g}\right]^{\frac{1}{{}_{v_2}\rho_g}}, \left[\frac{{}_{v_2}\sigma_f}{{}_{v_2}\sigma_g}\right]^{\frac{1}{{}_{v_2}\rho_g}}\right\} \le {}_{v_2}\overline{\tau}_g(f) \le \left[\frac{{}_{v_2}\sigma_f}{{}_{v_2}\overline{\sigma}_g}\right]^{\frac{1}{{}_{v_2}\rho_g}} . $$

In view of Theorem 3.2 one can easily verify the following corollaries :

Corollary 3.5. *Let $f(z_1, z_2)$ be an entire function with regular growth and ${}_{v_2}\sigma_f = {}_{v_2}\overline{\sigma}_f$ and $g(z_1, z_2)$ be an entire function of non zero finite order. Then*

$$ {}_{v_2}\overline{\tau}_g(f) = \left[\frac{{}_{v_2}\sigma_f}{{}_{v_2}\overline{\sigma}_g}\right]^{\frac{1}{{}_{v_2}\rho_g}} \text{ and } {}_{v_2}\tau_g(f) = \left[\frac{{}_{v_2}\sigma_f}{{}_{v_2}\sigma_g}\right]^{\frac{1}{{}_{v_2}\rho_g}} . $$

Corollary 3.6. *Let $f(z_1, z_2)$ be an entire function with regular growth and $g(z_1, z_2)$ be an entire function with ${}_{v_2}\sigma_g = {}_{v_2}\overline{\sigma}_g$. Then*

$$_{v_2}\overline{\tau}_g(f) = \left[\frac{_{v_2}\sigma_f}{_{v_2}\sigma_g}\right]^{\frac{1}{_{v_2}\rho_g}} \text{ and } {}_{v_2}\tau_g(f) = \left[\frac{_{v_2}\overline{\sigma}_f}{_{v_2}\sigma_g}\right]^{\frac{1}{_{v_2}\rho_g}} .$$

In addition, if ${}_{v_2}\sigma_f = {}_{v_2}\overline{\sigma}_f$ then

$$_{v_2}\overline{\tau}_g(f) = {}_{v_2}\tau_g(f) = \left[\frac{_{v_2}\sigma_f}{_{v_2}\sigma_g}\right]^{\frac{1}{_{v_2}\rho_g}} .$$

Corollary 3.7. *Let $g(z_1, z_2)$ be an entire function with non zero finite order. Then for any entire function $f(z_1, z_2)$,*

$$\begin{aligned} (i)\ {}_{v_2}\tau_g(f) &= \infty \text{ when } {}_{v_2}\sigma_g = 0 , \\ (ii)\ {}_{v_2}\overline{\tau}_g(f) &= \infty \text{ when } {}_{v_2}\overline{\sigma}_g = 0 , \\ (iii)\ {}_{v_2}\tau_g(f) &= 0 \text{ when } {}_{v_2}\sigma_g = \infty \end{aligned}$$

and

$$(iv)\ {}_{v_2}\overline{\tau}_g(f) = \infty \text{ when } {}_{v_2}\overline{\sigma}_g = \infty,$$

where $f(z_1, z_2)$ is of regular growth.

Corollary 3.8. *Let $f(z_1, z_2)$ be an entire function with regular growth . Then for any entire function $g(z_1, z_2)$,*

$$\begin{aligned} (i)\ {}_{v_2}\overline{\tau}_g(f) &= 0 \text{ when } {}_{v_2}\sigma_f = 0 , \\ (ii)\ {}_{v_2}\tau_g(f) &= 0 \text{ when } {}_{v_2}\overline{\sigma}_f = 0 , \\ (iii)\ {}_{v_2}\overline{\tau}_g(f) &= \infty \text{ when } {}_{v_2}\sigma_f = \infty \end{aligned}$$

and

$$(iv)\ {}_{v_2}\tau_g(f) = \infty \text{ when } {}_{v_2}\overline{\sigma}_f = \infty .$$

Similarly, in the line of Theorem 3.1 and Theorem 3.2 and with the help of Lemma 2.1 and Lemma 2.2, one may easily prove the following two theorems and therefore their proofs are omitted:

Theorem 3.3. *Let $f(z_1, z_2)$ and $g(z_1, z_2)$ be any two entire functions with finite non-zero lower order. Also let $g(z_1, z_2)$ be of regular growth. Then*

$$\left[\frac{_{v_2}\tau_f}{_{v_2}\overline{\tau}_g}\right]^{\frac{1}{_{v_2}\lambda_g}} \leq {}_{v_2}\tau_g(f) \leq \min\left\{\left[\frac{_{v_2}\tau_f}{_{v_2}\tau_g}\right]^{\frac{1}{_{v_2}\lambda_g}}, \left[\frac{_{v_2}\overline{\tau}_f}{_{v_2}\overline{\tau}_g}\right]^{\frac{1}{_{v_2}\lambda_g}}\right\}$$

$$\leq \max\left\{\left[\frac{_{v_2}\tau_f}{_{v_2}\tau_g}\right]^{\frac{1}{_{v_2}\lambda_g}}, \left[\frac{_{v_2}\overline{\tau}_f}{_{v_2}\overline{\tau}_g}\right]^{\frac{1}{_{v_2}\lambda_g}}\right\} \leq {}_{v_2}\overline{\tau}_g(f) \leq \left[\frac{_{v_2}\overline{\tau}_f}{_{v_2}\tau_g}\right]^{\frac{1}{_{v_2}\lambda_g}} .$$

In view of Theorem 3.3, one can easily verify the following corollaries :

Corollary 3.9. *Let $f(z_1, z_2)$ be an entire function such that ${}_{v_2}\tau_f = {}_{v_2}\overline{\tau}_f$ and $g(z_1, z_2)$ be an entire function of regular growth. Then*

$$ {}_{v_2}\overline{\tau}_g(f) = \left[\frac{{}_{v_2}\tau_f}{{}_{v_2}\tau_g}\right]^{\frac{1}{{}_{v_2}\lambda_g}} \text{ and } {}_{v_2}\tau_g(f) = \left[\frac{{}_{v_2}\overline{\tau}_f}{{}_{v_2}\overline{\tau}_g}\right]^{\frac{1}{{}_{v_2}\lambda_g}} . $$

Corollary 3.10. *Let $f(z_1, z_2)$ be an entire function with non-zero finite lower order and $g(z_1, z_2)$ be entire of regular growth with ${}_{v_2}\tau_g = {}_{v_2}\overline{\tau}_g$. Then*

$$ {}_{v_2}\overline{\tau}_g(f) = \left[\frac{{}_{v_2}\overline{\tau}_f}{{}_{v_2}\overline{\tau}_g}\right]^{\frac{1}{{}_{v_2}\lambda_g}} \text{ and } \tau_g(f) = \left[\frac{{}_{v_2}\tau_f}{{}_{v_2}\overline{\tau}_g}\right]^{\frac{1}{{}_{v_2}\lambda_g}} . $$

In addition, if ${}_{v_2}\tau_f = {}_{v_2}\overline{\tau}_f$ then

$$ {}_{v_2}\tau_g(f) = {}_{v_2}\overline{\tau}_g(f) = \left[\frac{{}_{v_2}\overline{\tau}_f}{{}_{v_2}\overline{\tau}_g}\right]^{\frac{1}{{}_{v_2}\lambda_g}} . $$

Corollary 3.11. *Let $f(z_1, z_2)$ be an entire function with non-zero finite lower order. Then for any entire function $g(z_1, z_2)$,*

$$ \begin{aligned} (i)\ {}_{v_2}\tau_g(f) &= \infty \text{ when } {}_{v_2}\overline{\tau}_g = 0 , \\ (ii)\ {}_{v_2}\overline{\tau}_g(f) &= \infty \text{ when } {}_{v_2}\tau_g = 0 , \\ (iii)\ {}_{v_2}\tau_g(f) &= 0 \text{ when } {}_{v_2}\overline{\tau}_g = \infty \end{aligned} $$

and

$$ (iv)\ {}_{v_2}\overline{\tau}_g(f) = \infty \text{ when } {}_{v_2}\tau_g = \infty, $$

where $g(z_1, z_2)$ is of regular growth.

Corollary 3.12. *Let $g(z_1, z_2)$ be an entire function with regular growth . Then for any entire function $f(z_1, z_2)$,*

$$ \begin{aligned} (i)\ {}_{v_2}\overline{\tau}_g(f) &= 0 \text{ when } {}_{v_2}\overline{\tau}_f = 0 , \\ (ii)\ {}_{v_2}\tau_g(f) &= 0 \text{ when } {}_{v_2}\tau_f = 0 , \\ (iii)\ {}_{v_2}\overline{\tau}_g(f) &= \infty \text{ when } {}_{v_2}\overline{\tau}_f = \infty \end{aligned} $$

and

$$ (iv)\ {}_{v_2}\tau_g(f) = \infty \text{ when } {}_{v_2}\tau_f = \infty . $$

Theorem 3.4. *Let $f(z_1, z_2)$ and $g(z_1, z_2)$ be any two entire functions with finite non-zero order. Also let $f(z_1, z_2)$ be of regular growth. Then*

$$ \begin{aligned} \left[\frac{{}_{v_2}\tau_f}{{}_{v_2}\overline{\tau}_g}\right]^{\frac{1}{{}_{v_2}\lambda_g}} &\leq {}_{v_2}\overline{\sigma}_g(f) \leq \min\left\{\left[\frac{{}_{v_2}\tau_f}{{}_{v_2}\tau_g}\right]^{\frac{1}{{}_{v_2}\lambda_g}}, \left[\frac{{}_{v_2}\overline{\tau}_f}{{}_{v_2}\overline{\tau}_g}\right]^{\frac{1}{{}_{v_2}\lambda_g}}\right\} \\ &\leq \max\left\{\left[\frac{{}_{v_2}\tau_f}{{}_{v_2}\tau_g}\right]^{\frac{1}{{}_{v_2}\lambda_g}}, \left[\frac{{}_{v_2}\overline{\tau}_f}{{}_{v_2}\overline{\tau}_g}\right]^{\frac{1}{{}_{v_2}\lambda_g}}\right\} \leq {}_{v_2}\sigma_g(f) \leq \left[\frac{{}_{v_2}\overline{\tau}_f}{{}_{v_2}\tau_g}\right]^{\frac{1}{{}_{v_2}\lambda_g}} . \end{aligned} $$

In view of Theorem 3.4, one can easily verify the following corollaries :

Corollary 3.13. *Let $f(z_1, z_2)$ be an entire function with regular growth and ${}_{v_2}\overline{\tau}_f = {}_{v_2}\tau_f$ and $g(z_1, z_2)$ be an entire function of non zero finite lower order. Then*

$$ {}_{v_2}\sigma_g(f) = \left[\frac{{}_{v_2}\overline{\tau}_f}{{}_{v_2}\tau_g}\right]^{\frac{1}{{}_{v_2}\lambda_g}} \text{ and } {}_{v_2}\overline{\sigma}_g(f) = \left[\frac{{}_{v_2}\overline{\tau}_f}{{}_{v_2}\overline{\tau}_g}\right]^{\frac{1}{{}_{v_2}\lambda_g}} . $$

Corollary 3.14. *Let $f(z_1, z_2)$ be an entire function with regular growth and $g(z_1, z_2)$ be an entire function with ${}_{v_2}\tau_g = {}_{v_2}\overline{\tau}_g$. Then*

$$ {}_{v_2}\sigma_g(f) = \left[\frac{{}_{v_2}\overline{\tau}_f}{{}_{v_2}\overline{\tau}_g}\right]^{\frac{1}{{}_{v_2}\lambda_g}} \text{ and } {}_{v_2}\overline{\sigma}_g(f) = \left[\frac{{}_{v_2}\tau_f}{{}_{v_2}\overline{\tau}_g}\right]^{\frac{1}{{}_{v_2}\lambda_g}} . $$

In addition, if ${}_{v_2}\tau_f = {}_{v_2}\overline{\tau}_f$ then

$$ {}_{v_2}\sigma_g(f) = {}_{v_2}\overline{\sigma}_g(f) = \left[\frac{{}_{v_2}\overline{\tau}_f}{{}_{v_2}\overline{\tau}_g}\right]^{\frac{1}{{}_{v_2}\lambda_g}} . $$

Corollary 3.15. *Let $g(z_1, z_2)$ be an entire function with non zero finite lower order. Then for any entire function $f(z_1, z_2)$,*

$$ \begin{aligned} (i)\ {}_{v_2}\overline{\sigma}_g(f) &= \infty \text{ when } {}_{v_2}\overline{\tau}_g = 0 , \\ (ii)\ {}_{v_2}\sigma_g(f) &= \infty \text{ when } {}_{v_2}\tau_g = 0 , \\ (iii)\ {}_{v_2}\overline{\sigma}_g(f) &= 0 \text{ when } {}_{v_2}\overline{\tau}_g = \infty \end{aligned} $$

and

$$ (iv)\ {}_{v_2}\sigma_g(f) = \infty \text{ when } {}_{v_2}\tau_g = \infty, $$

where $f(z_1, z_2)$ is of regular growth.

Corollary 3.16. *Let $f(z_1, z_2)$ be an entire function with regular growth . Then for any entire function $g(z_1, z_2)$,*

$$ \begin{aligned} (i)\ {}_{v_2}\sigma_g(f) &= 0 \text{ when } {}_{v_2}\overline{\tau}_f = 0 , \\ (ii)\ {}_{v_2}\overline{\sigma}_g(f) &= 0 \text{ when } {}_{v_2}\tau_f = 0 , \\ (iii)\ {}_{v_2}\sigma_g(f) &= \infty \text{ when } {}_{v_2}\overline{\tau}_f = \infty \end{aligned} $$

and

$$ (iv)\ {}_{v_2}\overline{\sigma}_g(f) = \infty \text{ when } {}_{v_2}\tau_f = \infty . $$

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