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SOFT CONNECTED PROPERTIES AND IRRESOLUTE SOFT FUNCTIONS BASED ON B-OPEN SOFT SETS

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Abstract. In this paper, we introduce another application on the b-open soft sets [10], by studying the notion of connectedness based on it. We give basic definitions and theorems about it. Further, we introduce the notion of b-irresolute soft functions as a generalization of the b-continuous soft functions and study their properties in detail. Finally, we show that the surjective b-irresolute soft image of soft b-connected space is also soft b-connected.
Keywords: Soft b-connected; Soft b-component; Soft b-hyperconnected; Soft b-separated; Soft b-connected; b-irresolute soft functions

1. Introduction

Theories such as theory of vague sets and theory of rough sets are considered as mathematical tools for dealing with uncertainties. But these theories have their own difficulties. The concept of soft sets was first introduced by Molodtsov [25] in 1999 as a general mathematical tool for dealing with uncertain objects in order to solve complicated problems in economics, engineering and other disciplines.

After presentation of the operations of soft sets [24], the properties and applications of soft set theory have been studied increasingly [7, 26, 27]. To develop soft set theory, the operations of the soft sets are redefined and a uni-int decision making method was constructed by using these new operations [9].
Recently, in 2011, Shabir and Naz [29] initiated the study of soft topological spaces. They defined soft topology on the collection τ of soft sets over X. Consequently, they defined basic notions of soft topological spaces such as open soft and closed soft sets, soft subspace, soft closure, soft nbd of a point, soft separation axioms, soft regular spaces and soft normal spaces and established their several properties. Hussain and Ahmad [13] investigated the properties of open (closed) soft, soft nbd and soft closure. They also defined and discussed the properties of soft interior,

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soft exterior and soft boundary which are fundamental for further research on soft topology and will strengthen the foundations of the theory of soft topological spaces.

It got some stability only after the introduction of soft topology [29] in 2011. In [14], Kandil et al. introduced some soft operations such as semi-open soft, pre-open soft, α-open soft and β-open soft and investigated their properties in detail. Kandil et al. [21] introduced the notion of soft semi-separation axioms. The notion of soft ideal was initiated for the first time by Kandil et al. [17]. They also introduced the concept of soft local function. These concepts are discussed with a view to find new soft topologies from the original one, called soft topological spaces with soft ideal $(X, \tau, E, \tilde{I})$. Applications to various fields were further investigated by Kandil et al.[15, 16, 18, 19, 20, 22]. The notion of supra soft topological spaces was initiated for the first time by El-sheikh and Abd El-latif [11], which is extended in [3]. The notion of b-open soft sets was initiated in [6, 10] and extended in [28]. An applications on b-open soft sets were introduced in [1, 12].

In this paper, we introduce and study the notion of connectedness based on the notion of b-open soft sets and give basic definitions and theorems about it. Further, we introduce the notion of b-irresolute soft functions as a generalization to the b-continuous soft function and study their properties in detail. Finally, we show that the surjective b-irresolute soft image of soft b-connected space is also soft b-connected.

2. Preliminaries

Definition 2.1. [25] Let X be an initial universe and E be a set of parameters. Let $P(X)$ denote the power set of X and A be a non-empty subset of E. A pair (F, A) denoted by F_A is called a soft set over X , where F is a mapping given by $F : A \to P(X)$. In other words, a soft set over X is a parameterized family of subsets of the universe X. For a particular $e \in A$, $F(e)$ may be considered the set of e-approximate elements of the soft set (F, A) and if $e \notin A$, then $F(e) = \phi$ i.e $F_A = \{(e, F(e)) : e \in A \subseteq E,\ F : A \to P(X)\}$. The family of all these soft sets denoted by $SS(X)_A$.

Definition 2.2. [29] Let F_E be a soft set over X and $x \in X$. We say that $x \in F_E$ read as x belongs to the soft set F_E whenever $x \in F(e)$ for all $e \in E$.

Definition 2.3. [29] Let τ be a collection of soft sets over a universe X with a fixed set of parameters E, then $\tau \subseteq SS(X)_E$ is called a soft topology on X if

(1) $\tilde{X}, \tilde{\phi} \in \tau$, where $\tilde{\phi}(e) = \phi$ and $\tilde{X}(e) = X,\ \forall e \in E$,

(2) the union of any number of soft sets in τ belongs to τ,

(3) the intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E) is called a soft topological space over X.

Definition 2.4. [7, 13] The complement of a soft set F_A, denoted by F_A^c, $F^c : A \to P(X)$ is mapping given by $F^c(e) = X - F(e)$, $\forall\, e \in A$ and F^c is called the soft complement function of F. Also, F_A is said to be closed soft set in X, if its relative complement F_A^c is open soft set.

Definition 2.5. [13] Let(X, τ, E) be a soft topological space. The members of τ are said to be open soft sets in X. We denote the set of all open soft sets over X by $OS(X, \tau, E)$, or when there can be no confusion by $OS(X)$ and the set of all closed soft sets by $CS(X, \tau, E)$, or $CS(X)$.

Definition 2.6. [29] Let (X, τ, E) be a soft topological space and $F_A \in SS(X)_E$. The soft closure of F_A, denoted by $cl(F_A)$ is the intersection of all closed soft super sets of F_A. Clearly $cl(F_A)$ is the smallest closed soft set over X which contains F_A i.e
$cl(F_A) = \tilde{\cap}\{H_C : H_C \textit{ is closed soft set and } F_A \tilde{\subseteq} H_C\})$.

Definition 2.7. [30] Let (X, τ, E) be a soft topological space and $F_A \in SS(X)_E$. The soft interior of G_B, denoted by $int(G_B)$ is the union of all open soft subsets of G_B. Clearly $int(G_B)$ is the largest open soft set over X which contained in G_B i.e $int(G_B) = \tilde{\cup}\{H_C : H_C \textit{ is an open soft set and } H_C \tilde{\subseteq} G_B\})$.

Definition 2.8. [30] The soft set $F_E \in SS(X)_E$ is called a soft point in $\tilde{X}$ if there exist $x \in X$ and $\alpha \in E$ such that $F(\alpha) = \{x\}$ and $F(\alpha^c) = \phi$ for each $\alpha^c \in E - \{\alpha\}$, and the soft point F_E is denoted by x_α.

Definition 2.9. [30] The soft point x_α is said to be belonging to the soft set G_A, denoted by $x_\alpha \tilde{\in} G_A$, if for the element $\alpha \in A$, $F(\alpha) \subseteq G(\alpha)$.

Definition 2.10. [29] Let (X, τ, E) be a soft topological space, $F_E \in SS(X)_E$ and Y be a non-null subset of X. Then the sub-soft set of F_E over Y denoted by (F_Y, E), is defined as follows:

$$F_Y(e) = Y \cap F(e)\ \forall e \in E.$$

In other words $(F_Y, E) = \tilde{Y} \tilde{\cap} F_E$.

Definition 2.11. [29] Let (X, τ, E) be a soft topological space and Y be a non-null subset of X. Then,

$$\tau_Y = \{(F_Y, E) : F_E \in \tau\}$$

is said to be the soft relative topology on Y and (Y, τ_Y, E) is called a soft subspace of (X, τ, E).

Theorem 2.1. [29] *Let (Y, τ_Y, E) be a soft subspace of a soft topological space (X, τ, E) and $F_E \in SS(X)_E$. Then,*

(1) *If F_E is open soft set in Y and $\tilde{Y} \in \tau$, then $F_E \in \tau$.*

(2) *F_E is open soft set in Y if and only if $F_E = \tilde{Y} \tilde{\cap} G_E$ for some $G_E \in \tau$.*

(3) *F_E is closed soft set in Y if and only if $F_E = \tilde{Y} \tilde{\cap} H_E$ for some H_E is τ-closed soft set.*

Definition 2.12. [6, 10] Let (X, τ, E) be a soft topological space and $F_E \in SS(X)_E$. Then, F_E is called a b-open soft set if $F_E \tilde{\subseteq} cl(int(F_E)) \tilde{\cup} int(cl(F_E))$. The set of all b-open soft sets is denoted by $BOS(X, \tau, E)$, or $BOS(X)$ and the set of all b-closed soft sets is denoted by $BCS(X, \tau, E)$, or $BCS(X)$.

Definition 2.13. [6, 10] Let (X, τ, E) be a soft topological space and $F_E \in SS(X)_E$. Then, the b-soft interior of F_E is denoted by $bSint(F_E))$, where $bSint(F_E)) = \tilde{\bigcup}\{G_E : G_E \tilde{\subseteq} F_E,\ G_E \in BOS(X)\}$.
Also, the b-soft closure of F_E is denoted by $bScl(F_E))$, where $bScl(F_E)) = \tilde{\bigcap}\{H_E : H_E \in BCS(X),\ F_E \tilde{\subseteq} H_E\}$.

Definition 2.14. [28] The soft b-boundary of a soft set F_E, denoted by b-$Sbd(F_E)$, and is defined as b-$Sbd(F_E) = bScl(F_E) \tilde{\cap} bScl(F_E^c) = bScl(F_E) - bSint(F_E)$.

Definition 2.15. [5] Let $SS(X)_E$ and $SS(Y)_K$ be two soft classes, $u : X \to Y$ and $p : E \to K$ be mappings. Then, the mapping $f_{pu} : SS(X)_E \ \to \ SS(Y)_K$ is defined as: for a soft set (F, A) in $SS(X)_E$, $(f_{pu}(F, A), B)$, $B = p(A) \subseteq K$, is a soft set in $SS(Y)_K$ given by

$$f_{pu}(F, A)(b) = u(\textstyle\bigcup_{a \in p^{-1}(b) \cap A} \ F(a))$$

for $b \in B \subseteq K$. $(f_{pu}(F, A), B)$ is called a soft image of a soft set (F, A). If $B = K$, then we shall write $(f_{pu}(F, A), K)$ as $f_{pu}(F, A)$. The soft function f_{pu} is called surjective (resp. injective) if p and u are surjective (resp. injective).

Definition 2.16. [5] Let $f_{pu} : SS(X)_E \ \to \ SS(Y)_K$ be a mapping from a soft class $SS(X)_E$ to another soft class $SS(Y)_K$, and (G, C) be a soft set in the soft class $SS(Y)_K$, where $C \subseteq K$. Let $u : X \to Y$ and $p : E \to K$ be mappings. Then, $(f_{pu}^{-1}(G, C), D)$, $D = p^{-1}(C) \subseteq K$, is a soft set in the soft class $SS(X)_E$, defined as:

$$f_{pu}^{-1}(G, C)(b) = u^{-1}(G(p(a)))$$

for $a \in D \subseteq E$. $(f_{pu}^{-1}(G, C), D)$ is called a soft inverse image of (G, C). Hereafter we shall write $(f_{pu}^{-1}(G, C), E) = f_{pu}^{-1}(G, C)$.

Definition 2.17. [10, 30] Let (X, τ_1, E) and (Y, τ_2, K) be soft topological spaces and $f_{pu} : SS(X)_E \rightarrow SS(Y)_K$ be a function. Then, the function f_{pu} is called

(1) Continuous soft if $f_{pu}^{-1}(G_K) \in \tau_1 \ \forall \ G_K \in \tau_2$.

(2) Open soft if $f_{pu}(G_E) \in \tau_2 \forall \ G_E \in \tau_1$.

(3) Closed soft if $f_{pu}(G_E) \in \tau_2^c \forall \ G_E \in \tau_1^c$.

(4) b-Continuous soft if $f_{pu}^{-1}(G_K) \in BOS(X) \forall \ G_K \in \tau_2$.

Definition 2.18. [23] A non-null soft subsets F_E, G_E of a soft topological space (X, τ, E) are said to be soft separated sets if $cl(F_E)\tilde{\cap}G_E = F_E\tilde{\cap}cl(G_E) = \tilde{\phi}$. A soft topological space (X, τ, E) is said to be soft connected if and only if $\tilde{X}$ can not expressed as the soft union of two soft separated sets in (X, τ, E), which equivalent to, $\tilde{X}$ can not expressed as the soft union of two open soft sets in (X, τ, E). Otherwise, (X, τ, E) is said to be soft disconnected.

Definition 2.19. [16] A soft topological space (X, τ, E) is said to be soft hyperconnected if and only if every pair of non null proper open soft sets F_E, G_E, has a non null soft intersection, i.e (X, τ, E) is said to be soft hyperconnected iff $\forall F_E, G_E \in \tau$ we have $F_E\tilde{\cap}G_E \neq \tilde{\phi}$.

Definition 2.20. [8] Let (X, τ, E) be a soft topological space. A soft semi separation on $\tilde{X}$ is a pair of non-null proper semi open soft sets F_E, G_E such that $F_E\tilde{\cap}G_E = \tilde{\phi}$ and $\tilde{X} = F_E\tilde{\cup}G_E$. A soft topological space (X, τ, E) is said to be soft semi connected if and only if there is no soft semi separations on $\tilde{X}$. Otherwise, (X, τ, E) is said to be soft semi disconnected.

Definition 2.21. [2] Let (X, τ, E) be a soft topological space. A soft β- (resp. pre-) separation on $\tilde{X}$ is a pair of non-null proper β- (resp. pre-) open soft sets F_E, G_E such that $F_E\tilde{\cap}G_E = \tilde{\phi}$ and $\tilde{X} = F_E\tilde{\cup}G_E$. A soft topological space (X, τ, E) is said to be soft β- (resp. pre-) connected if and only if there is no soft β- (resp. pre-) separations on $\tilde{X}$. Otherwise, (X, τ, E) is said to be soft β- (resp. pre-) disconnected.

3. Soft b-separateness

In this section, we will research the notion of soft b-separated sets in soft topological spaces and study its basic properties in detail.

Definition 3.1. Two non-null soft sets G_E and H_E of a soft topological space (X, τ, E) are said to be soft b-separated sets if $G_E\tilde{\cap}bScl(H_E) = \tilde{\phi}$ and $bScl(G_E)\tilde{\cap}H_E = \tilde{\phi}$. Obviously, from the fact that $bcl(F_E)\tilde{\subseteq}cl(F_E)$ for each soft set $F_E \in SS(X)_E$, every soft separated set is soft b-separated. But, the converse may not be true as shown in the following example.

Example 3.1. Let $X = \{h_1, h_2, h_3, h_4\}$, $E = \{e_1, e_2\}$, $\tau = \{\tilde{X}, \tilde{\phi}, F_{1E}, F_{2E}, F_{3E}\}$, where F_{1E}, F_{2E}, F_{3E} are soft sets over X defined by:
$F_1(e_1) = \{h_1\}$, $F_1(e_2) = \{h_1\}$,
$F_2(e_1) = \{h_2\}$, $F_2(e_2) = \{h_2\}$,
$F_3(e_1) = \{h_1, h_2\}$, $F_3(e_2) = \{h_1, h_2\}$.
The soft sets H_E and G_E, which defined by:
$H(e_1) = \{h_1\}$, $H(e_2) = \{h_1\}$,
$G(e_1) = \{h_3, h_4\}$, $G(e_2) = \{h_3, h_4\}$, are soft b-separated but not soft separated.

Definition 3.2. A soft set F_E of a soft topological space (X, τ, E) is said to be b-clopen soft set if it is both b-open soft set and b-closed soft set.

Proposition 3.1. **(1)** *Each two soft b-separated sets are always disjoint*

(2) *Each two disjoint soft sets, in which both of them either b-open soft sets or b-closed soft sets, are soft b-separated.*

Proof. Clear from Definition 3.1.

Theorem 3.1. *Let G_E and H_E be non-null soft sets of a soft topological space (X, τ, E). Then, the following statements hold:*

(1) *If G_E and H_E are soft b-separated, $G_{1E} \tilde{\subseteq} G_E$ and $H_{1E} \tilde{\subseteq} H_E$, then G_{1E} and H_{1E} are soft b-separated sets.*

(2) *If G_E and H_E are b-open soft sets, $U_E = G_E \tilde{\cap} (\tilde{X} - H_E)$ and $V_E = H_E \tilde{\cap} (\tilde{X} - G_E)$, then U_E and V_E are soft b-separated sets.*

Proof.

(1) Since $G_{1E} \tilde{\subseteq} G_E$. Then, $bScl(G_{1E}) \tilde{\subseteq} bScl(G_E)$. Hence,

$$H_{1E} \tilde{\cap} bScl(G_{1E}) \tilde{\subseteq} H_E \tilde{\cap} bScl(G_E) = \tilde{\phi}.$$

Similarly, $G_{1E} \tilde{\cap} bScl(H_{1E}) = \tilde{\phi}$. Thus, G_{1E} and H_{1E} are soft b-separated sets.

(2) Let G_E and H_E be b-open soft sets. Then, ($\tilde{X}$ - G_E) and ($\tilde{X}$ - H_E) are b-closed soft sets. Assume that, $U_E = G_E \tilde{\cap} (\tilde{X} - H_E)$ and $V_E = H_E \tilde{\cap} (\tilde{X} - G_E)$. Then, $U_E \tilde{\subseteq} (\tilde{X} - H_E)$ and $V_E \tilde{\subseteq} (\tilde{X} - G_E)$. Hence, $bScl(U_E) \tilde{\subseteq} (\tilde{X} - H_E) \tilde{\subseteq} (\tilde{X} - V_E)$ and $bScl(V_E) \tilde{\subseteq} \tilde{X} - G_E \tilde{\subseteq} (\tilde{X} - U_E)$. Consequently, $bScl(U_E) \tilde{\cap} V_E = \tilde{\phi}$ and $bScl(V_E) \tilde{\cap} U_E = \tilde{\phi}$. Therefore, U_E and V_E are soft b-separated sets.

Theorem 3.2. *Any two soft sets G_E and H_E of a soft topological space (X, τ, E) are soft b-separated sets if and only if there exist b-open soft sets U_E and V_E such that $G_E \tilde{\subseteq} U_E$, $H_E \tilde{\subseteq} V_E$ and $G_E \tilde{\cap} V_E = \tilde{\phi}$, $H_E \tilde{\cap} U_E = \tilde{\phi}$.*

Proof. Let G_E and H_E be soft b-separated sets. Then, $G_E \tilde{\cap} bScl(H_E) = \tilde{\phi}$ and $bScl(G_E) \tilde{\cap} H_E = \tilde{\phi}$. Let $V_E = \tilde{X} - bScl(G_E)$ and $U_E = \tilde{X} - bScl(H_E)$. Thus, U_E and V_E are b-open soft sets such that $G_E \tilde{\subseteq} U_E$, $H_E \tilde{\subseteq} V_E$, $G_E \tilde{\cap} V_E = \tilde{\phi}$ and $H_E \tilde{\cap} U_E = \tilde{\phi}$.

On the other hand, let U_E, V_E be b-open soft sets such that $G_E\tilde{\subseteq}U_E$, $H_E\tilde{\subseteq}V_E$ and $G_E\tilde{\cap}V_E = \tilde{\phi}$, $H_E\tilde{\cap}U_E = \tilde{\phi}$. Since ($\tilde{X}$ - V_E) and ($\tilde{X}$ - U_E) are b-closed soft sets. Then, $bScl(G_E)\tilde{\subseteq}(X_E$ - $V_E)\tilde{\subseteq}(X_E$ - $H_E)$ and $bScl(H_E)\tilde{\subseteq}(X_E$ - $U_E)$ $\tilde{\subseteq}(X_E$ - $G_E)$. Thus, $bScl(G_E)\tilde{\cap}H_E = \tilde{\phi}$ and $bScl(H_E)\tilde{\cap}G_E = \tilde{\phi}$. This means that, G_E and H_E are soft b-separated sets.

4. Soft b-connectedness

In this section, we will research the notion of soft b-connectedness in soft topological spaces be means of b-open soft sets, b-closed soft sets, soft -separated sets and study its basic properties.

Definition 4.1. Let (X, τ, E) be a soft topological space. A soft b-separation of $\tilde{X}$ is a pair of non-null proper b-open soft sets in τ such that $F_E\tilde{\cap}G_E = \tilde{\phi}$ and $\tilde{X} = F_E\tilde{\cup}G_E$.

Definition 4.2. A soft topological space (X, τ, E) is said to be b-soft connected if and only if there is no soft b-separations of $\tilde{X}$. If (X, τ, E) has such soft b-separations, then (X, τ, E) is said to be soft b-disconnected.

Theorem 4.1. *Let (X, τ, E) be a soft topological space , then the following statements are equivalent:*

(1) *$\tilde{X}$ is soft b-connected.*

(2) *$\tilde{X}$ cannot be expressed as a soft union of two non-null disjoint b-open soft sets.*

(3) *$\tilde{X}$ cannot be expressed as a soft union of two non-null disjoint b-closed soft sets.*

(4) *There is no proper b-clopen soft set in (X, τ, E).*

(5) *$\tilde{X}$ cannot be expressed as a soft union of two non-null b-separated soft sets.*

Proof.

(1) $\Leftrightarrow$ (2) It is obvious from Definition 4.2.

(2) $\Rightarrow$ (3) Suppose that $\tilde{X} = F_E\tilde{\cup}G_E$ for some b-closed soft sets F_E and G_E such that $F_E\tilde{\cap}G_E = \tilde{\phi}$. Then, $F_E = (G_E)^c$ which is b-open soft set, $\tilde{X} = G_E\tilde{\cup}G_E^c$ and $G_E\tilde{\cap}G_E^c = \tilde{\phi}$, which is a contradiction with (2).

(3) $\Rightarrow$ (4) Suppose that there is a proper b-clopen soft subset F_E of $\tilde{X}$. Then, F_E^c is b-clopen soft set, where $\tilde{X} = F_E\tilde{\cup}F_E^c$ and $F_E\tilde{\cap}F_E^c = \tilde{\phi}$, which is a contradiction with (3).

(4) $\Rightarrow$ (3) Suppose that $\tilde{X} = F_E \tilde{\cup} G_E$ for some b-closed soft sets F_E and G_E such that $F_E \tilde{\cap} G_E = \tilde{\phi}$. Then, $F_E = G_E^c$ and $G_E = F_E^c$. Thus, F_E and G_E are proper b-clopen soft sets, which is a contradiction with (4).

(3) $\Rightarrow$ (5) Suppose that $\tilde{X} = H_E \tilde{\cup} G_E$ for some b-separated soft sets H_E and G_E. Then, $G_E \tilde{\cap} bScl(H_E) = \tilde{\phi}$ and $bScl(G_E) \tilde{\cap} H_E = \tilde{\phi}$. It follows that, $H_E \tilde{\cap} G_E = \tilde{\phi}$. Hence, $H_E = G_E^c$ and $G_E = H_E^c$. Therefore, $bScl(H_E) \tilde{\subseteq} G_E^c = H_E$ and $bScl(G_E) \tilde{\subseteq} H_E^c = G_E$. But, $H_E \tilde{\subseteq} bScl(H_E)$ and $G_E \tilde{\subseteq} bScl(G_E)$. Thus, H_E and G_E are b-closed soft sets, which is a contradiction with (3).

(5) $\Rightarrow$ (1) Suppose that $\tilde{X} = F_E \tilde{\cup} G_E$ for some b-open soft sets F_E and G_E such that $F_E \tilde{\cap} G_E = \tilde{\phi}$. Then, $F_E = G_E^c$ and $G_E = F_E^c$. Thus, F_E and G_E are b-clopen soft sets. Hence, F_E and G_E are b-separated soft sets, which is a contradiction with (5).

Corollary 4.1. *Let (X, τ, E) be a soft topological space, then the following statements are equivalent:*

(1) *$\tilde{X}$ is soft b-connected.*

(2) *If $\tilde{X} = F_E \tilde{\cup} G_E$ for some b-open soft sets F_E and G_E such that $F_E \tilde{\cap} G_E = \tilde{\phi}$. Then, either $F_E = \tilde{\phi}$ or $G_E = \tilde{\phi}$.*

(3) *If $\tilde{X} = F_E \tilde{\cup} G_E$ for some b-closed soft sets F_E and G_E such that $F_E \tilde{\cap} G_E = \tilde{\phi}$. Then, either $F_E = \tilde{\phi}$ or $G_E = \tilde{\phi}$.*

Proof. Obvious from Theorem 4.1.

Definition 4.3. A soft subset F_E of a soft topological space (X, τ, E) is soft b-connected, if it is soft b-connected as a soft subspace. In other words, a soft subset F_E of a soft topological space (X, τ, E) is said to be soft b-connected relative to $\tilde{X}$ if there is not exist two soft b-separated subsets H_E and G_E relative to $\tilde{X}$ and $F_E = H_E \tilde{\cup} G_E$. Otherwise, F_E is said to be soft b-disconnected.

Remark 4.1. *Each soft disconnected set is soft b-disconnected, but not conversely as shown in the following examples.*

Example 4.1. **(1)** In Example 3.1, the soft set G_E, where $G(e_1) = \{h_1, h_3, h_4\}$, $G(e_2) = \{h_1, h_3, h_4\}$, is soft connected set, but not soft b-connected.

(2) Any soft space with the indiscrete soft topology is soft connected, but not soft b-connected, because the family of b-open soft sets establish a discrete soft topology.

Corollary 4.2. *Let (X, τ_1, E) and (X, τ_2, E) be soft topological spaces and $\tau_2 \subseteq \tau_1$. If (X, τ_1, E) is soft b-connected, then (X, τ_2, E) is soft b-connected.*

Proof. It is Obvious.

Definition 4.4. Let (X, τ, E) be a soft topological space and $Z_E \tilde{\subseteq} \tilde{X}$ with $x \in Z_E$. Then, the soft b-component of Z_E w.r.t. x_α is the maximal of all soft b-connected subspaces of (Z, τ_Z, E) containing x_α and denoted by $\tilde{SC}_b[Z_E, x_\alpha]$ or $\tilde{SC}_b(Z_E, x_\alpha)$ *for short*, i.e

$$\tilde{SC}_b(Z_E, x_\alpha) = \tilde{\cup}\{Y_E \tilde{\subseteq} Z_E : x_\alpha \in Y_E,\ Y_E \textit{ is soft } \text{b-}\textit{connected}\}.$$

Theorem 4.2. *Every soft b-component of a soft topological space (X, τ, E) is a maximal soft b-connected subset of $\tilde{X}$.*

Proof. It is obvious from Definition 4.4.

Corollary 4.3. *The soft topological space (X, τ, E) is soft b-connected if and only if it is a soft b-component on $\tilde{X}$.*

Proof. It is clear.

Theorem 4.3. *If the non-null soft sets G_E and H_E of a soft topological space (X, τ, E) are soft b-separated, then ($G_E \tilde{\cup} H_E$) is soft b-disconnected.*

Proof. Let G_E and H_E be non-null soft b-separated sets, then there exist b-open soft sets U_E and V_E such that $G_E \tilde{\subseteq} U_E$, $H_E \tilde{\subseteq} V_E$ and $G_E \tilde{\cap} V_E = \tilde{\phi}$, $H_E \tilde{\cap} U_E = \tilde{\phi}$ follows immediately from Theorem 3.2. Hence, $(G_E \tilde{\cup} H_E) \tilde{\cap} U_E = G_E$ and $(G_E \tilde{\cup} H_E) \tilde{\cap} V_E = H_E$. Consequently, $G_E \tilde{\cup} H_E$ is soft b-disconnected.

Next, we characterized soft b-connectedness in terms of soft boundaries as follows:

Theorem 4.4. *A soft topological space (X, τ, E) is soft b-connected if and only if every non-null proper subset has a non-null soft b-boundary.*

Proof. Let (X, τ, E) be soft b-disconnected, then (X, τ, E) has a proper b-clopen soft set F_E. Then, $bScl(F_E) = F_E = bSint(F_E) = \tilde{X} - bScl(\tilde{X} - F_E)$. Therefore, b-$Sbd(F_E) = bScl(F_E) \tilde{\cap} bScl(\tilde{X} - F_E) = \tilde{\phi}$. Therefore, F_E has an empty soft b-boundary.
Conversely, suppose that a non-null proper soft subset F_E has an empty soft b-boundary. Then, b-$Sbd(F_E) = bScl(F_E) \tilde{\cap} bScl(\tilde{X} - F_E) = \tilde{\phi}$. Consequently, $bScl(F_E) \tilde{\subseteq} [\tilde{X} - bScl(\tilde{X} - F_E)] = bSint(F_E)$, and thus $F_E \tilde{\subseteq} bScl(F_E) \tilde{\subseteq} bSint(F_E) \tilde{\subseteq} F_E$. Thus, F_E is a proper b-clopen soft set and consequently, (X, τ, E) is soft b-disconnected.

Theorem 4.5. *Let (Z, τ_Z, E) be a soft subspace of a soft topological space(X, τ, E) and $F_{1E}, F_{2E} \tilde{\subseteq} Z_E \tilde{\subseteq} \tilde{X}$. Then, F_{1E}, F_{2E} are soft b-separated on τ_Z if and only if F_{1E}, F_{2E} are soft b-separated on τ.*

Proof. Suppose that F_{1E}, F_{2E} are soft b-separated on $\tau_Z \Leftrightarrow bScl_{\tau_Z} F_{1E} \tilde{\cap} F_{2E} = \tilde{\phi}$ and $F_{1E} \tilde{\cap} bScl_{\tau_Z} F_{2E} = \tilde{\phi} \Leftrightarrow [bScl_\tau F_{1E} \tilde{\cap} Z_E] \tilde{\cap} F_{2E} = bScl_\tau F_{1E} \tilde{\cap} F_{2E} = \tilde{\phi}$ and $[bScl_\tau F_{2E} \tilde{\cap} Z_E] \tilde{\cap} F_{1E} = bScl_\tau F_{2E} \tilde{\cap} F_{1E} = \tilde{\phi} \Leftrightarrow F_{1E}, F_{2E}$ are soft b-separated on τ.

Theorem 4.6. *Let Z_E be a soft subset of a soft topological space (X, τ, E). Then, Z_E is soft b-connected relative to (X, τ, E) if and only if it is soft b-connected relative to (Z, τ_Z, E).*

Proof. Suppose that Z_E is soft b-disconnected relative to $(Z, \tau_Z, E) \Leftrightarrow Z_E = F_{1E}\tilde{\cup}F_{2E}$, where F_{1E} and F_{2E} are soft b-separated on $\tau_Z \Leftrightarrow Z_E = F_{1E}\tilde{\cup}F_{2E}$, where F_{1E} and F_{2E} are soft b-separated on τ_Z from Theorem 4.5 $\Leftrightarrow Z_E$ is soft b-disconnected relative to (X, τ, E).

Theorem 4.7. *Let (Z, τ_Z, E) be a soft b-connected subspace of a soft topological space (X, τ, E) and F_E, G_E are soft b-separated sets of $\tilde{X}$ with $Z_E\tilde{\subseteq}F_E\tilde{\cup}G_E$, then either $Z_E\tilde{\subseteq}F_E$ or $Z_E\tilde{\subseteq}G_E$.*

Proof. Let $Z_E\tilde{\subseteq}F_E\tilde{\cup}G_E$ for some soft b-separated sets F_E, G_E on τ. Since $Z_E = (Z_E\tilde{\cap}F_E)\tilde{\cup}(Z_E\tilde{\cap}G_E)$. Then, $(Z_E\tilde{\cap}F_E)\tilde{\cup}bScl_\tau(Z_E\tilde{\cap}G_E)\tilde{\subseteq}F_E\tilde{\cap}bScl_\tau(G_E) = \tilde{\phi}$. Also, $bScl_\tau(Z_E\tilde{\cap}F_E)\tilde{\cup}(Z_E\tilde{\cap}G_E)\tilde{\subseteq}cl_\tau(F_E)\tilde{\cap}G_E = \tilde{\phi}$. If $Z_E\tilde{\cap}F_E$ and $Z_E\tilde{\cap}G_E$ are non-null soft sets. Then, Z_E is soft b-disconnected, which is a contradiction with the hypothesis. Thus, either $Z_E\tilde{\cap}F_E = \tilde{\phi}$ or $Z_E\tilde{\cap}G_E = \tilde{\phi}$. It follows that, $Z_E = Z_E\tilde{\cap}F_E$ or $Z_E = Z_E\tilde{\cap}G_E$. Therefore, $Z_E\tilde{\subseteq}F_E$ or $Z_E\tilde{\subseteq}G_E$.

Theorem 4.8. *The soft b-closure of a soft b-connected set is soft b-connected.*

Proof. Assume that, $bScl(F_E)$ is soft b-disconnected. Then, there are two non-null soft b-separated sets G_E and H_E such that $bScl(F_E) = G_E\tilde{\cup}H_E$. Since F_E is soft b-connected. By using Theorem 4.7, either $F_E\tilde{\subseteq}G_E$ or $F_E\tilde{\subseteq}H_E$. If $F_E\tilde{\subseteq}G_E$, then $bScl(F_E\tilde{\subseteq}bScl(G_E)$ and so $bScl(F_E\tilde{\cap}H_E = \tilde{\phi}$. But, $H_E\tilde{\subseteq}bScl(F_E)$. Therefore, $H_E = \tilde{\phi}$ which is a contradiction. Similarly, if $F_E\tilde{\subseteq}H_E$, we can get $G_E = \tilde{\phi}$, which is also a contradiction. Consequently, $bScl(F_E)$ is soft b-connected.

Theorem 4.9. *Every soft b-component of a soft topological space (X, τ, E) is b-closed soft set.*

Proof. It is obvious from Definition 4.4 and from the fact that the soft b-closure of a soft b-connected set is a soft b-connected.

Theorem 4.10. *Let (X, τ, E) be a soft topological space. Then:*

(1) *Each soft point $x_\alpha\tilde{\in}\tilde{X}$ is contained in exactly one soft b-component of $\tilde{X}$.*

(2) *Any two soft b-components w.r.t. two different soft points are either disjoint or identical.*

Proof.

(1) Let $x_\alpha\tilde{\in}\tilde{X}$ and consider the collection $\tilde{SC_b} = \{Z_E\tilde{\subseteq}\tilde{X} : x_\alpha\tilde{\in}Z_E,\ Z_E\ is\ soft$ b-*connected*$\}$. Then, we have:

- **(a)** $\tilde{SC_b} \neq \tilde{\phi}$, for the soft point x_α is a soft b-connected subset of $\tilde{X}$ containing x_α. Then, $x_\alpha \in \tilde{SC_b}$.

(b) $\tilde{\cap}\{Z_E\tilde{\subseteq}\tilde{X} : x_\alpha\tilde{\in}Z_E : Z_E\ is\ soft$ b-*connected*$\} \neq \tilde{\phi}$. Since $x_\alpha\tilde{\in}Z_E\ \forall Z_E \in \tilde{SC_b}$.

(c) The soft set $\tilde{\cup}\{Z_E\tilde{\subseteq}\tilde{X} : x_\alpha\tilde{\in}Z_E,\ Z_E\ is\ soft$ b-*connected*$\}$, having a non-null soft intersection, is soft b-connected subset of $\tilde{X}$ containing x_α.

(d) $\tilde{\cup}\{Z_E\tilde{\subseteq}\tilde{X} : x_\alpha\tilde{\in}Z_E,\ Z_E\ is\ soft$ b-*connected*$\}$ is the largest soft b-connected subset of $\tilde{X}$ containing x_α, which is the soft b-component $\tilde{SC_b}(\tilde{X}, x_\alpha)$ of $\tilde{X}$ w.r.t x_α and containing x_α from Definition 4.4.

Now, suppose $\tilde{SC}^*_b(\tilde{X}, x_\alpha)$ be another soft b-component containing x_α, then $\tilde{SC}^*_b(\tilde{X}, x_\alpha)$ is a soft b-connected subset of $\tilde{X}$ containing x_α, but $\tilde{SC_b}(\tilde{X}, x_\alpha)$ is a soft b-component, then $\tilde{SC_b}(\tilde{X}, x_\alpha)$ is the largest soft b-connected subset of $\tilde{X}$ containing x_α, consequently, $\tilde{SC}^*_b(\tilde{X}, x_\alpha)\tilde{\subseteq}\tilde{SC_b}(\tilde{X}, x_\alpha)$. Similarly, $\tilde{SC_b}(\tilde{X}, x_\alpha)\tilde{\subseteq}\tilde{SC}^*_b(\tilde{X}, x_\alpha)$, and hence x_α is contained in exactly one soft b-component of $\tilde{X}$.

(2) Let $\tilde{SC_b}(\tilde{X}, x_{1\alpha})$, $\tilde{SC_b}(\tilde{X}, x_{2\alpha})$ be the soft b-components of x_α w.r.t two different soft points $x_{1\alpha}$, $x_{2\alpha}$ of $\tilde{X}$ with $x_{1\alpha} \neq x_{2\alpha}$ respectively.
If $\tilde{SC_b}(\tilde{X}, x_{1\alpha})\tilde{\cap}\tilde{SC_b}(\tilde{X}, x_{2\alpha}) = \tilde{\phi}$, then we get the proof.
So, let $\tilde{SC_b}(\tilde{X}, x_{1\alpha})\tilde{\cap}\tilde{SC_b}(\tilde{X}, x_{2\alpha}) \neq \phi$. We may choose a $x_\beta \in \tilde{SC_b}(\tilde{X}, x_{1\alpha})\tilde{\cap}\tilde{SC_b}(\tilde{X}, x_{2\alpha})$. Clearly, $x_\beta \in \tilde{SC_b}(\tilde{X}, x_{1\alpha})$ and $x_\beta \in \tilde{SC_b}(\tilde{X}, x_{2\alpha})$, which mean that $\tilde{SC_b}(\tilde{X}, x_{1\alpha})$ is the largest soft b-connected subset of $\tilde{X}$ containing x_β , $\tilde{SC_b}(\tilde{X}, x_{2\alpha})$ is the largest soft b-connected subset of $\tilde{X}$ containing x_β. Therefore, $\tilde{SC_b}(\tilde{X}, x_{1\alpha}) = \tilde{SC_b}(\tilde{X}, x_{2\alpha})$, and hence $\tilde{SC_b}(\tilde{X}, x_{1\alpha})$ and $\tilde{SC_b}(\tilde{X}, x_{2\alpha})$ are identical. This completes the proof.

Theorem 4.11. *Let F_E be soft b-connected subsets of a soft topological space (X, τ, E) and G_E be a soft set such that $F_E\tilde{\subseteq}G_E\tilde{\subseteq}bScl(F_E)$, then G_E is soft b-connected.*

Proof. Let G_E be a soft b-disconnected, then there exist two non-null b-open soft sets U_E and V_E such that $G_E = U_E\tilde{\cup}V_E$. Since $F_E\tilde{\subseteq}G_E$ and F_E be soft b-connected, then by using Lemma 4.10 either $F_E\tilde{\subseteq}U_E$ or $F_E\tilde{\subseteq}V_E$. If $F_E\tilde{\subseteq}U_E$, then $bScl(F_E)\tilde{\subseteq}bScl(U_E)$ and so $bScl(F_E)\tilde{\cap}V_E = \tilde{\phi}$ i.e, $V_E\tilde{\subseteq}\tilde{X} - bScl(F_E)$, but $V_E\tilde{\subseteq}G_E\tilde{\subseteq}bScl(F_E)$. Thus, $V_E = \tilde{\phi}$, which is a contradiction, and so G_E is soft b-connected. Similarly, if $F_E\tilde{\subseteq}V_E$, thus $U_E = \tilde{\phi}$ this is a contradiction. Consequently, G_E is soft b-connected.

Theorem 4.12. *Let (Z, τ_Z, E) be a soft b-connected subspace of a soft b-connected topological space (X, τ, E) such that Z^c_E is the soft union of two soft b-separated sets F_E, G_E of $\tilde{X}$, then $Z_E\tilde{\cup}F_E$ and $Z_E\tilde{\cup}G_E$ are soft b-connected.*

Proof. Suppose that $Z_E\tilde{\cup}F_E$ is soft b-disconnected relative to (X, τ, E). Then, there exist two non-null soft b-separated sets K_E and H_E of $\tilde{X}$ such that $Z_E\tilde{\cup}F_E = K_E\tilde{\cup}H_E$. Since Z_E is soft b-connected and $Z_E\tilde{\subseteq}Z_E\tilde{\cup}F_E = K_E\tilde{\cup}H_E$. It follows that, either $Z_E\tilde{\subseteq}K_E$ or $Z_E\tilde{\subseteq}H_E$ from Theorem 4.7. Suppose $Z_E\tilde{\subseteq}K_E$. Since $Z_E\tilde{\cup}F_E = K_E\tilde{\cup}H_E$ and $Z_E\tilde{\subseteq}K_E$, implies that $Z_E\cup F_E\tilde{\subseteq}K_E\cup F_E$. So, $K_E\tilde{\cup}H_E\tilde{\subseteq}K_E\tilde{\cup}F_E$. Hence,

$H_E\tilde{\subseteq}F_E$, $G_E\tilde{\subseteq}K_E$. By Theorem 3.1 (1), H_E, G_E are soft b-separated. Now,

$$bScl_\tau H_E\tilde{\cap}[K_E\tilde{\cup}G_E] = [bScl_\tau H_E\tilde{\cap}K_E]\tilde{\cup}[bScl_\tau H_E\tilde{\cap}G_E] = \tilde{\phi}$$

and $H_E\tilde{\cap}cl_\tau\ [K_E\tilde{\cup}G_E] = H_E\tilde{\cap}[cl_\tau K_E\tilde{\cup}cl_\tau G_E] = [H_E\tilde{\cap}cl_\tau K_E]\tilde{\cup}[H_E\tilde{\cap}cl_\tau G_E] = \tilde{\phi}$. It follows that, H_E and $[K_E\tilde{\cup}G_E]$ are soft b-separated on τ. Since $Z_E^c = F_E\tilde{\cup}G_E$. Then,

$$\tilde{X} = Z_E\tilde{\cup}Z_E^c = Z_E\tilde{\cup}[F_E\tilde{\cup}G_E] = [Z_E\tilde{\cup}F_E]\tilde{\cup}G_E = [K_E\tilde{\cup}H_E]\tilde{\cup}G_E = H_E\tilde{\cup}[K_E\tilde{\cup}G_E].$$

Thus, $\tilde{X}$ is the soft union of two non null soft b-separated sets H_E and $K_E\tilde{\cup}G_E$, which is a contradiction. A similar contradiction will arise if $Z_E\tilde{\subseteq}H_E$. Hence, $Z_E\tilde{\cup}F_E$ is soft b-connected. Similarly, we can prove that$Z_E\tilde{\cup}G_E$ is soft b-connected.

Theorem 4.13. *If Z_E, Y_E are soft b-connected sets such that none of them is soft b-separated sets, then $Z_E\tilde{\cup}Y_E$ is soft b-connected set.*

Proof. Let (Z, τ_Z, E) and (Y, τ_Y, E) be soft b-connected subspaces of $\tilde{X}$ such that $Z_E\tilde{\cup}Y_E$ is not soft b-connected relative to (X, τ, E). Then, there exist two non-null soft b-separated sets K_E and H_E of $\tilde{X}$ such that $Z_E\tilde{\cup}Y_E = K_E\tilde{\cup}H_E$. Since Z_E, Y_E are soft b-connected and $Z_E, Y_E\tilde{\subseteq}Z_E\tilde{\cup}Y_E = K_E\tilde{\cup}H_E$. Then, either $Z_E\tilde{\subseteq}K_E$ or $Z_E\tilde{\subseteq}H_E$, also, either $Y_E\tilde{\subseteq}K_E$ or $Y_E\tilde{\subseteq}H_E$ from Theorem 4.7. If $Z_E\tilde{\subseteq}K_E$ or $Z_E\tilde{\subseteq}H_E$. Then, $Z_E\tilde{\cap}H_E\tilde{\subseteq}K_E\tilde{\cap}H_E = \tilde{\phi}$ or $Z_E\tilde{\cap}K_E\tilde{\subseteq}H_E\tilde{\cap}K_E = \tilde{\phi}$. Therefore, $[Z_E\tilde{\cup}Y_E]\tilde{\cap}K_E = [Z_E\tilde{\cap}K_E]\tilde{\cup}[Y_E\tilde{\cap}K_E] = \tilde{\phi}\tilde{\cup}[Y_E\tilde{\cap}K_E] = Y_E\tilde{\cap}K_E = Y_E$, since $Y_E\tilde{\subseteq}K_E$. Similarly, if $Y_E\tilde{\subseteq}K_E$ or$Y_E\tilde{\subseteq}H_E$. We can get, $[Z_E\tilde{\cup}Y_E]\tilde{\cap}H_E = Z_E$. Now,

$$\begin{aligned}\tilde{\cap}bScl_\tau[(Z_E\tilde{\cup}Y_E)\tilde{\cap}K_E]&\tilde{\subseteq}[(Z_E\tilde{\cup}Y_E)\tilde{\cap}H_E]\tilde{\cap}[bScl_\tau(Z_E\tilde{\cup}Y_E)\tilde{\cap}bScl_\tau K_E]\\&= [(Z_E\tilde{\cup}Y_E)]\tilde{\cap}[bScl_\tau K_E\tilde{\cap}H_E] = \tilde{\phi}\end{aligned}$$

and

$$\begin{aligned}bScl_\tau[(Z_E\tilde{\cup}Y_E)\tilde{\cap}H_E]\tilde{\cap}[(Z_E\tilde{\cup}Y_E)\tilde{\cap}K_E]&\tilde{\subseteq}[bScl_\tau(Z_E\tilde{\cup}Y_E)\tilde{\cap}bScl_\tau H_E]\tilde{\cap}[(Z_E\tilde{\cup}Y_E)\tilde{\cap}K_E]\\&= [(Z_E\tilde{\cup}Y_E)]\tilde{\cap}[bScl_\tau H_E\tilde{\cap}K_E] = \tilde{\phi}.\end{aligned}$$

It follows that $[Z_E\tilde{\cup}Y_E]\tilde{\cap}K_E = Z_E$ and $[Z_E\tilde{\cup}Y_E]\tilde{\cap}H_E = Y_E$ are soft b-separated, which is a contradiction. Hence, $Z_E\tilde{\cup}Y_E$ is soft b-connected relative to (X, τ, E).

Theorem 4.14. *If for all pair of soft points x_α, $y_\beta\tilde{\in}\tilde{X}$ with $x_\alpha \neq y_\beta$ there exists a soft b-connected set $Z_E\tilde{\subseteq}\tilde{X}$ with x_α, $y_\beta\tilde{\in}Z_E$, then $\tilde{X}$ is soft b-connected.*

Proof. Suppose that $\tilde{X}$ is soft b-disconnected. Then, $\tilde{X} = F_E\tilde{\cup}G_E$, for some F_E, G_E soft b-separated sets. It follows that, $F_E\tilde{\cap}G_E = \tilde{\phi}$. So, $\exists\ x_\alpha\tilde{\in}F_E$ and $y_\beta\tilde{\in}G_E$. Since $F_E\tilde{\cap}G_E = \tilde{\phi}$. Then, x_α, $y_\beta\tilde{\in}\tilde{X}$ with $x_\alpha \neq y_\beta$. By hypothesis, there exists a soft b-connected set $Z_E \subseteq \tilde{X}$ with x_α, $y_\beta\tilde{\in}Z_E$. Moreover, we have Z_E is soft b-connected subset of a soft b-disconnected space. By Theorem 4.7, either $Z_E\tilde{\subseteq}F_E$ or $Z_E\tilde{\subseteq}G_E$, and both cases is a contradiction with the hypothesis. This implies that, $\tilde{X}$ is soft b-connected.

Definition 4.5. A soft set N_E is said to be a soft b-neighborhood (briefly, soft b-nbd.) of a soft point $x_e\tilde{\in}(X,E)$ if there exists a b-open soft set $U_E\tilde{\subseteq}N_E$ such that $x_e\tilde{\in}U_E\tilde{\subseteq}N_E$.

Definition 4.6. A soft point $x_e\tilde{\in}\tilde{X}$is called a soft b-limit point of a soft set F_E if every soft b-nbd U_E of x_e contains a point of F_E other than x_e.

Theorem 4.15. *Let F_E and G_E be non-null disjoint soft sets of a soft topological space (X,τ,E) and $Y_E = F_E\tilde{\cup}G_E$. Then, F_E and G_E are soft b-separated if and only if each of F_E and G_E is b-closed soft (b-open soft) with respect to Y_E.*

Proof. Let F_E and G_E be soft b-separated sets. By Definition 3.1, F_E contains no soft b-limit points of G_E. Then, G_E contains all soft b-limit points of G_E. It follows that, $F_E\tilde{\cup}G_E$ also contains all soft b-limit points of G_E and G_E is b-closed soft in Y_E. Therefore, G_E is b-closed soft with respect to Y_E. Similarly, F_E is b-closed soft with respect to Y_E. The converse is clear from Theorem 4.14.

Theorem 4.16. *The soft union of any family of soft b-connected sets having a non-null soft intersection is soft b-connected set.*

Proof. Assume that $(Z,\tau_Z,E) = (\tilde{\cup}_{j\in J}Z_j, \tau_{\tilde{\cup}_{j\in J}Z_j}, E)$ is soft b-disconnected. Then, $Z_E = F_E\tilde{\cup}G_E$, where F_E, G_E are soft b-separated sets on τ_Z. Since $\tilde{\cap}_{j\in J}Z_{jE} \neq \tilde{\phi}$. Then, $\exists\ x_\alpha\tilde{\in}\tilde{\cap}_{j\in J}Z_{jE}$. It follows that, $x_\alpha\tilde{\in}Z_E$. So, either $x_\alpha\tilde{\in}F_E$ or $x_\alpha\tilde{\in}G_E$. Suppose that $x_\alpha\tilde{\in}F_E$. Since $x_\alpha\tilde{\in}Z_{jE}$ $\forall j \in J$ and $Z_{jE}\tilde{\subseteq}Z_E$. Thus, we have Z_{jE} is soft b-connected subset of a soft b-disconnected space. By Theorem 4.7, either $Z_{jE}\tilde{\subseteq}F_E$ or $Z_{jE}\tilde{\subseteq}G_E$ $\forall j \in J$. If $Z_{jE}\tilde{\subseteq}F_E$ $\forall j \in J$. Then, $Z_E\tilde{\subseteq}F_E$. This implies that, $G_E = \tilde{\phi}$, which is a contradiction. Also, if $Z_{jE}\tilde{\subseteq}G_E$ $\forall j \in J$. By a similar way, we can get $F_E = \tilde{\phi}$, which is a contradiction. Thus, $(Z,\tau_Z,E) = (\tilde{\cup}_{j\in J}Z_j, \tau_{\tilde{\cup}_{j\in J}Z_j}, E)$ is soft b-connected soft subspace of (X,τ,E).

Proposition 4.1. *Let $\{(Z_j,\tau_{Z_j},E)$ be a family of soft b-connected subspaces of soft topological space (X,τ,E) such that one of the members of the family intersects every other members, then $(\tilde{\cup}_{j\in J}Z_j, \tau_{\tilde{\cup}_{j\in J}Z_j}, E)$ is soft b-connected.*

Proof. Let $(Z,\tau_Z,E) = (\tilde{\cup}_{j\in J}Z_j, \tau_{\tilde{\cup}_{j\in J}Z_j}, E)$ and $Z_{j_oE} \in \{Z_{jE} : j \in J\}$ such that $Z_{j_oE}\tilde{\cap}Z_{jE} \neq \tilde{\phi}$ $\forall j \in J$. Then, $Z_{j_oE}\tilde{\cup}Z_{jE}$ is soft b-connected $\forall j \in J$ from Theorem 4.16. Therefore, the collection $\{Z_{j_oE}\tilde{\cup}Z_{jE} : j \in J\}$ is a collection of a soft b-connected subsets of $\tilde{X}$, which having a non-null soft intersection. Thus,$(\tilde{\cup}_{j\in J}Z_j, \tau_{\tilde{\cup}_{j\in J}Z_j}, E)$ is soft b-connected from Theorem 4.16.

Remark 4.2. *For a soft topological space (X,τ,E), the following statements hold:*

(1) *Every soft β-connected set is soft b-connected.*

(2) *Every soft b-connected is soft pre-connected.*

(3) *Every b-connected is soft semi-connected.*

Definition 4.7. A soft topological space (X, τ, E) is said to be soft locally b-connected at a soft point x_α if every soft b-nbd of the soft point x_α contains a soft b-connected open nbd of x_α. $\tilde{X}$ is said to be soft locally b-connected if it is soft locally b-connected at each of its soft points.

Theorem 4.17. *Every soft b-connected space is a soft locally b-connected space, but the converse is not true in general.*

Proof. Suppose that (X, τ, E) be a soft b-connected. Then, There is no proper b-clopen soft set in (X, τ, E) from Theorem 4.1. Hence, $\forall\ x_\alpha \tilde{\in} \tilde{X}\ \exists\ \tilde{X} \in \tau$ which is soft b-connected set such that $x_\alpha \tilde{\in} \tilde{X} \tilde{\subseteq} \tilde{X}$. Therefore, $\tilde{X}$ is soft locally b-connected. On the other hand, the indiscrete soft topological space, is soft locally b-connected but not soft b-connected.

Theorem 4.18. *The soft b-component of a soft locally b-connected soft topological space is b-open soft set.*

Proof. Let (X, τ, E) be a soft locally b-connected , $x_\alpha \tilde{\in} \tilde{X}$ and $\tilde{SC_b}$ be a soft b-component of $\tilde{X}$ w.r.t x_α. Since (X, τ, E) is a soft locally b-connected space. Therefore, every b-open soft set containing x_α contains a soft b-connected open set G_E containing x_α. But, $\tilde{SC_b}$ is the largest soft b-connected set containing x_α. Hence,$x_\alpha \tilde{\in} G_E \tilde{\subseteq} \tilde{SC_b}$, i.e $\tilde{SC_b}$ is a soft b-nbd of x_α. Thus, $\tilde{SC_b}$ is a soft b-nbd of each of its points. This means that, $\tilde{SC_b}$ is a b-open soft set.

Theorem 4.19. *The property of soft locally b-connectedness is hereditary on b-open soft subspaces.*

Proof. Suppose that (Z, τ_Z, E) be a b-open soft subspace of a soft locally b-connected topological space (X, τ, E) and let $x_\alpha \tilde{\in} \tilde{Z}$. Since $\tilde{X}$ is soft locally b-connected. Then, $\exists\ G_E \in \tau$ such that G_E is τ-soft b-connected subset of $\tilde{X}$ and $x_\alpha \tilde{\in} G_E \tilde{\subseteq} \tilde{Z}$. Since $G_E \in \tau$ and G_E is τ-soft b-connected set. Then, $G_E = G_E \cap \tilde{\tilde{Z}} \in \tau_Z$ and $G_E \cap \tilde{Z}$ is τ_Z-soft b-connected subset of $\tilde{Z}$ by Theorem 4.6. So, $\tilde{Z}$ is soft locally b-connected for each $x_\alpha \tilde{\in} \tilde{X}$. Hence, $\tilde{Z}$ is soft locally b-connected.

Theorem 4.20. *The soft b-components of every b-open soft subspace of a soft locally b-connected soft topological space are b-open soft.*

Proof. Suppose that (Y, τ_Y, E) be a b-open soft subspace of a soft locally b-connected soft topological space (X, τ, E), let $\tilde{SC_b}$ be a soft b-component of $\tilde{Y}$. Since $\tilde{X}$ is soft locally b-connected, then $\tilde{Y}$ is soft locally b-connected by Theorem 4.19. By Theorem 4.18 we get the proof.

Definition 4.8. A soft topological space (X, τ, E) is said to be soft b-hyperconnected if and only if every pair of non-null proper b-open soft sets F_E, G_E, has a non-null soft intersection, i.e (X, τ, E) is said to be soft b-hyperconnected if and only if for each $F_E, G_E \in BOS(X)$, we have $F_E \tilde{\cap} G_E \neq \tilde{\phi}$.

Theorem 4.21. *Every soft b-hyperconnected soft topological space is soft b-connected.*

Proof. Suppose that (X,τ,E) be a soft b-disconnected soft topological space. Then, there exists a proper b-clopen soft set F_E. Then, $F_E, F_E^c \in BOS(X)$ such that $F_E \tilde{\cap} F_E^c = \tilde{\phi}$. Hence, $\tilde{X}$ is not soft b-hyperconnected, which is a contradiction. Thus, (X,τ,E) is soft b-connected.

Remark 4.3. *The converse of Theorem 4.21 is not true in general, as shown in the following example.*

Example 4.2. Let $X = \{h_1, h_2, h_3, h_4\}$, $E = \{e_1, e_2\}$ and $\tau = \{\tilde{X}, \tilde{\phi}, F_{1E}, F_{2E}, F_{3E}\}$ where F_{1E}, F_{2E}, F_{3E} are soft sets over X defined as follows:
$F_1(e_1) = \{h_1\}$, $F_1(e_2) = \{h_2\}$,
$F_2(e_1) = \{h_2, h_3\}$, $F_2(e_2) = \{h_1, h_3\}$,
$F_3(e_1) = \{h_1, h_2, h_3\}$, $F_3(e_2) = \{h_1, h_2, h_3\}$.
Then, τ defines a soft topology on X. Hence, the space (X,τ,E) is soft b-connected but not soft b-hyperconnected.

Definition 4.9. A soft subset F_E of a soft topological space (X,τ,E) is said to be soft b-dense if $bScl(F_E) = \tilde{X}$.

Corollary 4.4. *A soft topological space (X,τ,E) is said to be a soft b-hyperconnected if every non-null open soft set F_E is soft b-dense.*

Theorem 4.22. *If (X,τ,E) is soft b-hyperconnected soft topological space, then (X,τ,E) is soft hyperconnected.*

Proof. Immediate.

Remark 4.4. *The converse of Theorem 4.22 is not true in general, as shown in the following example.*

Example 4.3. Let $X = \{a, b\}$, $E = \{e_1, e_2\}$ and $\tau = \{\tilde{X}, \tilde{\phi}, F_{1E}, F_{2E}, F_{3E}, F_{4E}, F_{5E}\}$ where $F_{1E}, F_{2E}, F_{3E}, F_{4E}, F_{5E}$ are soft sets over X defined as follows:
$F_1(e_1) = X$, $F_1(e_2) = \{b\}$,
$F_2(e_1) = \{a\}$, $F_2(e_2) = X$,
$F_3(e_1) = \{a\}$, $F_3(e_2) = \{b\}$,
$F_4(e_1) = \{a\}$, $F_4(e_2) = \{a\}$,
$F_5(e_1) = \{a\}$, $F_5(e_2) = \phi$.
Then, τ defines a soft topology on X. Hence, the space (X,τ,E) is soft hyperconnected, but not soft b-hyperconnected.

Theorem 4.23. . *If (X,τ,E) is soft b-hyperconnected soft topological space, then (X,τ,E) is connected.*

Proof. Immediate by Remark 4.1 and Theorem 4.21.

Corollary 4.5. *The following implications hold from Theorems 4.21, 4.22, 4.23 and [Corollary 3.3, [16]] for a soft topological space (X, τ, E).*

$$\begin{array}{ccc} (X,\tau,E) \text{ is soft b-hyperconnected} & \Rightarrow & (X,\tau,E) \text{ is soft hyperconnected} \\ \Downarrow & & \Downarrow \\ (X,\tau,E) \text{ is soft b-connected} & \Rightarrow & (X,\tau,E) \text{ is soft connected} \end{array}$$

5. Irresolute soft functions via b-open soft sets

In this section, we introduce a new type of soft functions called b-irresolute soft function as a generalization to the b-continuous soft functions and obtain some of their properties and characterizations.

Definition 5.1. Let (X, τ_1, A) and (Y, τ_2, B) be soft topological spaces and $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. Then, The function f_{pu} is called b-irresolute soft if $f_{pu}^{-1}(G_B) \in BOS(X) \forall\, G_B \in BOS(Y)$.

Theorem 5.1. *Every b-irresolute soft function is b-continuous soft.*

Proof. Straightforward.

Remark 5.1. *The converse of Theorem 5.1 is not true in general, as shown in the following example.*

Example 5.1. Let $X = \{a, b, c\}$, $Y = \{x, y, z\}$, $A = \{e_1, e_2\}$ and $B = \{k_1, k_2\}$. Define $u : X \to Y$ and $p : A \to B$ as follows:
$u(a) = x$, $u(b) = z$, $u(c) = y$,
$p(e_1) = k_2$, $p(e_2) = k_1$. Let (X, τ_1, A) be a soft topological space over X where,
$\tau_1 = \{\tilde{X}, \tilde{\phi}, F_A\}$, where F_A is a soft set over X defined as follows:
$F(e_1) = \{b\}$, $\quad F(e_2) = \{a\}$.
Let (Y, τ_2, B) be a soft topological space over Y where,
$\tau_2 = \{\tilde{Y}, \tilde{\phi}, G_B\}$, where G_B is a soft set over Y defined by:
$G(k_1) = \{x\}$, $\quad G(k_2) = \{x, z\}$.
Let $f_{pu} : (X, \tau_1, A) \to (Y, \tau_2, B)$ be a soft function. Then, $f_{pu}^{-1}(G_B) = \{(e_1, \{a, b\}), (e_2, \{a\})\}$ is a b-open soft set over X. Hence, f_{pu} is a b-continuous soft function. On the other hand, the soft set H_B, where H_B is a soft set over Y defined by:
$H(k_1) = \{y, z\}$, $\quad G(k_2) = \{x, y\}$.
Then, H_B is b-open soft set in Y. Also, $f_{pu}^{-1}(H_B) = \{(e_1, \{a, c\}), (e_2, \{b, c\})\}$ is not b-open soft set over X. Therefore, f_{pu} is not b-irresolute soft function.

Theorem 5.2. *Let (X, τ_1, A) and (Y, τ_2, B) be soft topological spaces. Let $u : X \to Y$ and $p : A \to B$ be a mappings. Let $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. Then, the following are equivalent:*

(1) *f_{pu} is b-irresolute soft function.*

(2) $f_{pu}^{-1}(H_B) \in BCS(X)\ \forall\ H_B \in \tau_2^c$.

(3) $f_{pu}(bScl_{\tau_1}(G_A)) \tilde{\subseteq} bScl_{\tau_2}(f_{pu}(G_A))\ \forall\ (G_A) \in SS(X)_A$.

(4) $bScl_{\tau_1}(f_{pu}^{-1}(H_B)) \tilde{\subseteq} f_{pu}^{-1}(bScl_{\tau_2}(H_B))\ \forall\ H_B \in SS(Y)_B$.

(5) $f_{pu}^{-1}(bSint_{\tau_2}(H_B)) \tilde{\subseteq} bSint_{\tau_1}(f_{pu}^{-1}(H_B))\ \forall\ H_B \in SS(Y)_B$.

Proof.

(1) $\Rightarrow$ (2) Let $H_B \in \tau_2^c$. Then, $H_B^c \in \tau_2$ and $f_{pu}^{-1}(H_B^c) \in BOS(X)$ from Definition 5.1. Since $f_{pu}^{-1}(H_B^c) = (f_{pu}^{-1}(H_B))^c$ from [[30], Theorem 3.14]. Thus, $f_{pu}^{-1}(H_B) \in BCS(X)$.

(2) $\Rightarrow$ (3) Let $G_A \in SS(X)_A$. Since $G_A \tilde{\subseteq} f_{pu}^{-1}(f_{pu}(G_A)) \tilde{\subseteq} f_{pu}^{-1}\ (bScl_{\tau_2}(f_{pu}(G_A))) \in BCS(Y)$ from (2) and [[30], Theorem 3.14]. Then, $G_A \tilde{\subseteq} bScl_{\tau_1}(G_A) \tilde{\subseteq} f_{pu}^{-1}(bScl_{\tau_2}$ $(f_{pu}(G_A)))$. Hence,

$$f_{pu}(bScl(G_A)) \tilde{\subseteq} f_{pu}(f_{pu}^{-1}(bScl_{\tau_2}(f_{pu}(G_A)))) \tilde{\subseteq} bScl_{\tau_2}(f_{pu}(G_A)))$$

from [[30], Theorem 3.14]. Thus, $f_{pu}(bScl_{\tau_1}(G_A)) \tilde{\subseteq} bScl_{\tau_2}(f_{pu}(G_A))$.

(3) $\Rightarrow$ (4) Let $H_B \in SS(Y)_B$ and $G_A = f_{pu}^{-1}(H_B)$. Then, $f_{pu}(bScl_{\tau_1}(f_{pu}^{-1}(H_B))) \tilde{\subseteq} bScl_{\tau_2}(f_{pu}(f_{pu}^{-1}(H_B)))$ From (3). Hence,

$$\begin{aligned} bScl_{\tau_1}(f_{pu}^{-1}(H_B)) &\tilde{\subseteq} f_{pu}^{-1}(f_{pu}(bScl_{\tau_1}(f_{pu}^{-1}(H_B)))) \\ &\tilde{\subseteq} f_{pu}^{-1}(bScl_{\tau_2}(f_{pu}(f_{pu}^{-1}(H_B)))) \tilde{\subseteq} f_{pu}^{-1}(bScl_{\tau_2}(H_B)) \end{aligned}$$

from [[30], Theorem 3.14]. Thus, $bScl_{\tau_1}(f_{pu}^{-1}(H_B)) \tilde{\subseteq}\ f_{pu}^{-1}(bScl_{\tau_2}(H_B))$.

(4) $\Rightarrow$ (2) Let H_B be a closed soft set over Y. Then,

$$\begin{aligned} bScl_{\tau_1}(f_{pu}^{-1}(H_B)) &\tilde{\subseteq} f_{pu}^{-1}(bScl_{\tau_2}(H_B)) \\ &= f_{pu}^{-1}(H_B) = f_{pu}^{-1}(H_B)\ \forall\ H_B \in SS(Y)_B \end{aligned}$$

from (4), but clearly $f_{pu}^{-1}(H_B) \tilde{\subseteq} bScl_{\tau_1}(f_{pu}^{-1}(H_B))$. This means that

$$f_{pu}^{-1}(H_B) = bScl_{\tau_1}(f_{pu}^{-1}(H_B)) \in BCS(X).$$

(1) $\Rightarrow$ (5) Let $H_B \in BOS(Y)$. Then, $f_{pu}^{-1}(bSint_{\tau_2}(H_B)) \in BOS(X)$ from (1). Hence, $f_{pu}^{-1}(bSint_{\tau_2}(H_B)) = bSint_{\tau_1}(f_{pu}^{-1} bSint_{\tau_2}(H_B)) \tilde{\subseteq} bSint_{\tau_1}\ (f_{pu}^{-1}(H_B))$. Thus, $f_{pu}^{-1}(bSint_{\tau_2}(H_B)) \tilde{\subseteq} bSint_{\tau_1}(f_{pu}^{-1}(H_B))$.

(5) $\Rightarrow$ (1) Let $H_B \in BOS(Y)$. Then, $bSint_{\tau_2}(H_B) = H_B$ and

$$f_{pu}^{-1}(bSint_{\tau_2}(H_B)) = f_{pu}^{-1}((H_B)) \tilde{\subseteq} bSint_{\tau_1}(f_{pu}^{-1}(H_B))$$

from (5). But, we have
$bSint_{\tau_1}(f_{pu}^{-1}(H_B))\tilde{\subseteq} f_{pu}^{-1}(H_B)$. This means that,

$$bSint_{\tau_1}(f_{pu}^{-1}(H_B)) = f_{pu}^{-1}(H_B) \in BOS(X).$$

Thus, f_{pu} is b-irresolute soft function.

Theorem 5.3. *Let (X_1, τ_1, A) and (X_2, τ_1, B) be soft topological spaces, $f_{pu} : (X_1, \tau_1, A) \rightarrow (X_2, \tau_2, B,)$ be a surjective b-irresolute soft function and F_B, G_B are soft b-separated sets in X_2, then $f_{pu}^{-1}(F_B), f_{pu}^{-1}(G_B)$ are soft b-separated sets in X_1.*

Proof. Let F_B, G_B be soft b-separated sets in X_2. Then, $F_B \tilde{\cap} bScl(G_B) = \tilde{\phi}_B$ and $bScl(F_B)\tilde{\cap} G_B = \tilde{\phi}_B$. Since $bScl_{\tau_1}(f_{pu}^{-1}(H_B))\tilde{\subseteq} f_{pu}^{-1}(bScl_{\tau_2}(H_B))\ \forall\ H_B \in SS(X_2)_B$ from Theorem 5.2 (4). It follows that

$$\begin{aligned} f_{pu}^{-1}(G_B)\tilde{\cap} bScl_{\tau_1}(f_{pu}^{-1}(H_B)) &\tilde{\subseteq} f_{pu}^{-1}(G_B)\tilde{\cap} f_{pu}^{-1}(bScl_{\tau_2}(H_B)) \\ &= f_{pu}^{-1}[G_B \tilde{\cap} bScl_{\tau_2}(H_B)] = f_{pu}^{-1}(\tilde{\phi}_B) = \tilde{\phi}_A. \end{aligned}$$

By a similar way, we have $f_{pu}^{-1}(H_B)\tilde{\cap} bScl_{\tau_1}(f_{pu}^{-1}(G_B)) = \tilde{\phi}_A$. Therefore, $f_{pu}^{-1}(F_B), f_{pu}^{-1}(G_B)$ are soft b-separated sets in X_1.

Theorem 5.4. *Let (X_1, τ_1, A) and (X_2, τ_1, B) be soft topological spaces and $f_{pu} : (X_1, \tau_1, A) \rightarrow (X_2, \tau_2, B,)$ be a bijective b-irresolute soft function. If G_A is soft b-connected in X_1, then $f_{pu}(G_A)$ is soft b-connected in X_2.*

Proof. Suppose that $f_{pu}(G_A)$ is not soft b-connected in X_2. Then, $f_{pu}(G_A) = M_B \tilde{\cup} N_B$ for some soft b-separated sets M_B, N_B of $f_{pu}(G_A)$ in X_2 from Theorem 4.1. By Theorem 5.3, $f_{pu}^{-1}(M_B)$ and $f_{pu}^{-1}(N_B)$ are soft b-separated in X_1. Since f_{pu} is bijective soft function. So, $G_A = f_{pu}^{-1}(f_{pu}(G_A)) = f_{pu}^{-1}(M_B)\tilde{\cup} f_{pu}^{-1}(N_B)$. It follows that, G_A is not soft b-connected in X_1, which is a contradiction. Thus, $f_{pu}(G_A)$ is soft b-connected in X_2.

Corollary 5.1. *Let (X_1, τ_1, A) and (X_2, τ_1, B) be soft topological spaces and $f_{pu} : (X_1, \tau_1, A) \rightarrow (X_2, \tau_2, B,)$ be a surjective b-irresolute soft function. If X_1 is soft b-connected space, then so X_2.*

Proof. Follows from Theorem 5.4.

Theorem 5.5. *Let (X_1, τ_1, A) and (X_2, τ_1, B) be soft topological spaces and $f_{pu} : (X_1, \tau_1, A) \rightarrow (X_2, \tau_2, B,)$ be a surjective b-continuous soft function. If G_A is soft b-connected in X_1, then $f_{pu}(G_A)$ is soft connected in X_2.*

Proof. The proof is similar to the proof of Theorem 5.4.

Corollary 5.2. *Let (X_1, τ_1, A) and (X_2, τ_1, B) be soft topological spaces and $f_{pu} : (X_1, \tau_1, A) \rightarrow (X_2, \tau_2, B,)$ be a surjective b-continuous soft function. If X_1 is soft b-connected space, then X_2 is soft connected.*

Proof. Follows from Theorem 5.5.

6. Conclusion

In the present paper, we introduce and study the notion of connectedness to soft topological spaces by using the notion of b-open soft sets. We study the notions of soft b-connected sets, soft b-separated sets in soft topological spaces and have established several interesting properties. Further, we introduce the notion of b-irresolute soft functions as a generalization to the b-continuous soft function and study their properties in detail. Finally, we show that the surjective b-irresolute soft image of soft b-connected space is also soft b-connected. Our next step is to generalize these notions by using the soft ideal notions [17].

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