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ON ISOTROPIC RIEMANNIAN MANIFOLDS WITH A SEMI-SYMMETRIC NON-METRIC CONNECTION

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Abstract. In this paper, we investigate some geometrical properties of Riemannian manifolds equipped with a semi-symmetric non-metric connection. First, it is proved that an isotropic Riemannian manifold with a semi-symmetric non-metric connection is Einstein. Then, it is shown that an isotropic Riemannian manifold admitting a proper concircular vector field with the above mentioned connection is a warped product. Moreover, the physical properties of a spacetime with a semi-symmetric non-metric connection are also investigated.

Keywords: semi-symmetric non-metric connection, sectional curvature, subprojective manifold, perfect fluid spacetime, energy momentum tensor, Einstein's field equation

1. Introduction

Let ∇^* be a linear connection in an n-dimensional differentiable manifold M. The torsion tensor T and the curvature tensor R^* of ∇^* are given respectively by

$$T(X,Y) = \nabla^*_X Y - \nabla^*_Y X - [X,Y]$$

$$R^*(X,Y)Z = \nabla^*_X \nabla^*_Y Z - \nabla^*_Y \nabla^*_X Z - \nabla^*_{[X,Y]} Z$$

By a triple (M,g,T), we mean (M,g) is a Riemannian manifold with a torsion tensor T defined on M which is a smooth section of the tensor bundle (TM). Along with the Levi-Civita connection ∇, we introduce the linear connection $\nabla^* = \nabla + T$ on the manifold (M,g,T) for the torsion tensor T. Here and below, unless otherwise stated, the symbols X, Y and Z stand for arbitrary smooth vector fields on M.

H.A. Hayden [16] introduced a metric connection with a non-zero torsion on a Riemannian manifold. Such a connection is called Hayden connection, [16]. In [14] Friedmann and Schouten introduced the notion of a semi-symmetric linear connection on a differentiable manifold. The connection ∇^* is symmetric if its torsion

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tensor vanishes, otherwise it is not symmetric. The connection ∇^* is a metric connection if there is a Riemannian metric g in M such that $\nabla^* g = 0$, otherwise it is non-metric. K. Yano and M. Kon found a relation between a semi-symmetric metric connection and the Levi-Civita connection, [27]. Also, K.Yano studied some properties of the Riemannian manifold endowed with a semi-symmetric metric connection in [28].

The semi-symmetric metric connection plays an important role in the study of Riemannian manifolds. There are various physical problems involving the semi-symmetric metric connection. For example, if a man is moving on the surface of the earth always facing one defined point, the north pole, then this displacement is semi-symmetric and metric [20, 23].

In [2], Agashe and Chafle introduced the idea of a semi-symmetric non-metric connection on a Riemannian manifold and this was further developed by Agashe and Chafle [3], De and Kamilya [11], De, Sengupta, Binh [21], S.C. Biswas and U.C. De [5], [12] and others.

Let M be an m dimensional Riemannian manifold with a Riemannian metric g. A linear connection ∇^* on a Riemannian manifold M is called a semi-symmetric connection if the torsion tensor T of the connection

$$T(X,Y) = \nabla^*_X Y - \nabla^*_Y X - [X,Y]$$

satisfies

$$T(X,Y) = \omega(Y)X - \omega(X)Y \tag{1.1}$$

for any vector fields X and Y on M, where ω is a 1-form associated with the vector field U on M defined by

$$\omega(X) = g(X,U) \tag{1.2}$$

Let ∇ be the Levi-Civita connection of a Riemannian manifold M. The semi-symmetric non-metric connection ∇^* is given by

$$\nabla^*_X Y = \nabla_X Y + \omega(Y)X \tag{1.3}$$

where $\omega(X) = g(X,U)$ and X, Y, U are vector fields on M [8]. Using (1.3) we have,

$$(\nabla^*_X g)(Y,Z) = -\omega(Y)g(X,Z) - \omega(Z)g(X,Y) \tag{1.4}$$

In [2], N. S. Agashe and M. R. Chafle found a relation between the curvature tensor with respect to the semi-symmetric non-metric connection ∇^* and the Levi-Civita connection ∇

$$R^*(X,Y,Z,W) = R(X,Y,Z,W) - \alpha(Y,Z)g(X,W) + \alpha(X,Z)g(Y,W) \tag{1.5}$$

where R^* and R denote curvature tensors with respect to the connections ∇^* and ∇, respectively, and α is a tensor field of the type $(0,2)$ defined by

$$\alpha(X,Y) = g(\mathcal{A}X,Y) = (\nabla_X \omega)(Y) - \omega(X)\omega(Y) = (\nabla^*_X \omega)(Y) \tag{1.6}$$

where $\mathcal{A}$ is a $(1,1)$-tensor field which is metrically equivalent to α.

The concept of semi-symmetric non-metric connection has been applied to the hypersurface of the Riemannian manifold [13], lightlike submanifold [31], K-contact Riemannian manifold, [17]. In [22] J. Sengupta and U.C.De defined another type of semi-symmetric non-metric connection. They also considered the hypersurface of the Riemannian manifold with a semi-symmeric non-metric connection in their sense.

From (1.5), we get

$$S^*(Y,Z) = S(Y,Z) - (n-1)\alpha(X,Y) \tag{1.7}$$

where S^* and S denote respectively the Ricci tensor with respect to ∇^* and ∇. The tensor α of type $(0,2)$ given in (1.7) is not symmetric in general and hence from (1.7) it follows that the Ricci tensor S^* is not symmetric. But if 1-form ω associated with the torsion tensor T is closed then it can be shown that the relation $(\nabla_X\omega)(Y) = (\nabla_Y\omega)(X)$ holds for all vector fields X, Y. r^* and r denote the scalar curvatures with respect to the linear connection ∇^* and Levi-Civita connection ∇, respectively. Then, they are related by the following form:

$$r^* = r - (n-1)trace(\alpha) \tag{1.8}$$

Manifolds with a semi-symmetric non-metric connection have been studied by U. C. De and many authors, for example, De, Yıldız, Turan and Acet introduced 3-dimensional quasi-Sasakian manifolds with a semi-symmetric non-metric connection, [12].

2. Sectional Curvatures of a Riemannian Manifold having Semi-symmetric Non-metric Connection.

Let Π be a tangent plane to Riemannian manifold with semi-symmetric non-metric connection M_n at $P \in M_n$ given by $X, Y \in X(M_n)$. The sectional curvature $K^*(\pi)$ of Π defined by

$$K^*(X,Y)(g(X,X)g(Y,Y) - g^2(X,Y)) = g(R^*(X,Y)Y,X) \tag{2.1}$$

which is independent of the choice of the basis X, Y, i.e. assume that at any point $P \in M_n(\nabla^*, g)$, the sectional curvature is the same for all planes in $T_P(M)$. The case of a 2-dimensional Riemannian manifold having a semi-symmetric non-metric connection needs to be considered, since it has only one plane at each point. If at each point the sectional curvature $K^*(\pi)$ of the Riemannian manifold with a semi-symmetric non-metric connection is independent of the vector field X, Y, then this manifold is called an isotropic manifold [18]. Thus we have

$$R^*(X,Y,Z,W) = K^*(\pi)(g(X,W)g(Y,Z) - g(X,Z)g(Y,W)) \tag{2.2}$$

From (2.2) we have

$$S^*(X,Y) = K^*(\pi)(n-1)g(X,Y) \tag{2.3}$$

and

$$r^* = K^*(\pi)n(n-1) \tag{2.4}$$

Using (1.6) – (1.7) and (2.3) we get $d\omega(X,Y) = 0$. Thus 1-form ω is closed. N. S. Agashe and M. R. Chafle defined the projective curvature tensor of the Riemannian manifold with respect to a semi-symmetric non-metric connection by, [2]

$$P^*(X,Y)Z = R^*(X,Y)Z - \frac{1}{n-1}(S^*(Y,Z)X - S^*(X,Z)Y) \tag{2.5}$$

and the authors found

$$P^*(X,Y)Z = P(X,Y)Z \tag{2.6}$$

From (2.2), (2.3) and (2.5) we say that if a Riemannian manifold with a semi-symmetric non-metric connection is isotropic then we have

$$P^*(X,Y)Z = 0 \tag{2.7}$$

Therefore, from (2.6) and (2.7) we get

$$P(X,Y)Z = 0 \tag{2.8}$$

on M_n.

A necessary and sufficient condition for a manifold with a symmetric linear connection to be projectively flat is that the projective curvature tensor with respect to vanishes identically on a manifold. It is well known that a Riemannian manifold is of constant curvature if and only if it is projectively flat and a Riemannian manifold of constant curvature is conformally flat, [29]. Thus we have

$$R(X,Y)Z = \frac{r}{n(n-1)}[g(Y,Z)X - g(X,Z)Y] \tag{2.9}$$

and

$$C(X,Y)Z = 0 \tag{2.10}$$

on M_n. Hence, we have the following theorem:

Theorem 2.1. *If (M,g) is an isotropic Riemannian manifold admitting a semi-symmetric non-metric connection, then it is an Einstein manifold.*

By using (1.7), (2.3) and (2.4) we can express $\alpha(X,Y)$ as

$$\alpha(X,Y)=\frac{r-r^*}{n(n-1)}g(X,Y) \tag{2.11}$$

In view of the relation (1.6), it follows that $\omega(X)$ is a proper concircular vector field, that is

$$(\nabla_X\omega)(Y)=\frac{r-r^*}{n(n-1)}g(X,Y)+\omega(X)\omega(Y) \tag{2.12}$$

Thus, from (2.12) we can state the following:

Theorem 2.2. *An isotropic Riemannian manifold admitting a semi-symmetric non-metric connection has a proper concircular vector field.*

It is known that, [1], if a conformally flat manifold admits a proper concircular vector field, then this manifold is a subprojective manifold in the sense of Kagan.

Hence, we can state the following theorem:

Theorem 2.3. *An isotropic Riemannian manifold admitting a semi-symmetric non-metric connection is a subprojective manifold, provided that $r^*\neq r$.*

In [30] K.Yano proved that in a Riemannian manifold M which admits a concircular vector field, it is necessary and sufficient that there exists a coordinate system with respect to M so the fundamental quadratic differential form may be written in the form

$$ds^2=(dx^1)^2+e^q g^*_{\alpha\beta}dx^\alpha dx^\beta \tag{2.13}$$

where $g^*_{\alpha\beta}=g^*_{\alpha\beta}(x^\nu)$ are the function of x^ν; $(\alpha,\beta,\nu=2,3,...,n)$ and $q=q(x^1)\neq$ *const.* is a differentiable function on I only. Since an isotropic Riemannian manifold admitting a semi-symmetric non-metric connection has a proper concircular vector field, the manifold under this consideration is a warped product $I\times_{e^q}M^*$ where (M^*,g^*) is an $(n-1)$-dimensional Riemannian manifold. Since this manifold is conformally flat, we have

$$(\nabla_X S)(Y,Z)-(\nabla_Z S)(Y,X)=\frac{1}{2(n-1)}(g(Y,Z)dr(X)-g(Y,X)dr(Y)) \tag{2.14}$$

Gebarowski, [15], proved that the warped product $I\times_{e^q}M^*$ satisfies (2.14) if and only if M^* is an Einstein manifold.

Thus we can state the following theorem:

Theorem 2.4. *An isotropic Riemannian manifold admitting a semi-symmetric non-metric connection is a warped product $I\times_{e^q}M^*$ where M^* is an Einstein manifold.*

It is known from the theorem in [9] that every simply connected subprojective manifold can be isometrically immersed in a Euclidean space as a hypersurface.

This leads to the following result:

Theorem 2.5. *A simply connected isotropic Riemannian manifold admitting a semi-symmetric non-metric connection can be isometrically immersed in a Euclidean space as a hypersurface.*

3. Spacetimes admitting a type of semi-symmetric non- metric connection

General relativity explains gravity as a curvature of spacetime. It is all about geometry. The basic equation of general relativity is called Einstein's equation. If we assume $c = 8\pi G = 1$, then $G_{\alpha\beta} = T_{\alpha\beta}$. It looks simple, but what does it mean? Unfortunately, the beautiful geometrical meaning of this equation is a bit hard to find in most treatments of relativity.

Einstein manifolds play an important role in Riemannian Geometry as well as in general relativity. Also, Einstein manifolds form a natural subclass of various classes of Riemannian manifolds by a curvature condition imposed on their Ricci tensor. For example, every Einstein manifold belongs to the class of Riemannian manifolds realizing the following

$$S(X,Y) = ag(X,Y) + bA(X)A(Y)$$

where $a, b \in R$ and A is a non-zero 1-form such that $g(X,U) = A(X)$ for all vector fields X. Quasi Einstein manifolds arose during the study of exact solutions of the Einstein field equations as well as during considerations of quasi-umbilical hypersurfaces of semi-Euclidean spaces. For example, the Robertson-Walker spacetimes are quasi Einstein manifolds.

As already mentioned in the second section, an isotropic spacetime admitting a semi-symmetric non-metric connection is an Einstein spacetime, that is, this spacetime is of constant curvature.

Spaces with a constant curvature play a significant role in cosmology. The assumption that the universe is isotropic and homogeneous is given in the simplest cosmological model. This principle is known as the cosmological principle and, when translated into the language of Riemannian geometry, asserts that the three dimensional position space is a space of maximal symmetry, that is, space-like surfaces of a constant curvature are the Robertson-Walker metrics, while a four-dimensional space of a constant curvature is the de-Sitter model of universe [26, 19].

The equations of the gravitational field theory are higher order non-linear partial differential equations so finding exact solutions of these equations is very difficult, [26, 6]. If some geometric symmetry properties are assumed to be possessed by the metric tensor, the solutions to these equations can be obtained easily. These geometric symmetry properties are described by Killing vector fields and lead to conservation laws in the form of the first integrals of a dynamical system, [8]. Geometrical symmetries of spacetime are expressed through the equation

$$\mathcal{L}_\xi A = 2\Omega A \tag{3.1}$$

where A represents a geometrical/physical quantity $\mathcal{L}_\xi$ denotes the Lie derivative with respect to the vector field ξ and Ω is a scalar.

One of the most simple and widely used example is the metric inheritance symmetry for which $A = g$ in (3.1); and for this case, ξ is a Killing vector field if Ω is zero.

In this section, we denote an isotropic spacetime admitting a semi-symmetric non-metric connection by (M_4, ST). Since (M_4, ST) is an Einstein spacetime, we have

$$S(X,Y) = \frac{r}{4} g(X,Y) \tag{3.2}$$

Let us consider a spacetime satisfying the Einstein field equation with the cosmological constant given by the following equation:

$$S(X,Y) - \frac{r}{2} g(X,Y) + \lambda g(X,Y) = kT(X,Y) \tag{3.3}$$

for all vector fields X and Y where S and r denote the Ricci tensor and the scalar curvature, T is the energy momentum tensor, λ is the cosmological constant and k is the non-zero gravitational constant.

By virtue of the equations (3.2), (3.3) the energy momentum tensor reduces to the form

$$T(X,Y) = \frac{(4\lambda - r)}{4k} g(X,Y) \tag{3.4}$$

Taking the covariant derivative of (3.4), we get the following result:

Theorem 3.1. *In a (M_4, ST) satisfying the Einstein field equation with the cosmological constant, the energy momentum tensor is of the form*

$$T(X,Y) = \frac{(4\lambda - r)}{4k} g(X,Y) \tag{3.5}$$

which is covariantly constant.

Now, we assume that ξ is a conformal Killing vector in (M_4, ST). Taking the Lie derivative of both sides of (3.4)

$$(\mathcal{L}_\xi T)(X,Y) = \frac{(4\lambda - r)}{4k} (\mathcal{L}_\xi g)(X,Y) \tag{3.6}$$

By using (3.1) and (3.6), we find

$$(\mathcal{L}_\xi T)(X,Y) = 2\Omega T(X,Y) \tag{3.7}$$

In this case, it can be said that the energy momentum tensor has the symmetry inheritance property. Conversely, if the condition (3.7) holds, it follows from (3.6) that ξ is a conformal Killing vector field. Thus, we can state the following:

Theorem 3.2. *If (M_4, ST) obeys the Einstein field equations with the cosmological constant, then a vector field ξ is a conformal Killing vector field on this spacetime if and only if the energy-momentum tensor has the symmetry inheritance property along ξ.*

The well-known symmetry of the energy-momentum tensor T is the matter collineation defined by $(\mathcal{L}_\xi T)(X,Y) = 0$. Let ξ be a Killing vector field on the spacetime under consideration. Then $(\mathcal{L}_\xi g)(X,Y) = 0$. By taking the Lie derivative of both sides of (3.4) with respect to ξ, we get $(\mathcal{L}_\xi T)(X,Y) = 0$. The converse is trivial.

Hence, we have the following theorem:

Theorem 3.3. *If (M_4, ST) obeys the Einstein field equations with a cosmological constant, then the spacetime admits matter collineation with respect to the vector field ξ if and only if ξ is a Killing vector field.*

Let us consider the existence of a perfect fluid of (M_4, ST) obeying Einstein field equations without the cosmological constant. In a perfect fluid spacetime, the energy momentum tensor is of the form

$$T(X,Y) = (\sigma + p)A(X)A(Y) + pg(X,Y) \tag{3.8}$$

where σ is the energy density, p is the isotropic pressure and A is the associated 1-form of the spacetime defined as $g(X,U) = A(X)$ and U is the unit timelike velocity vector field of the perfect fluid, i.e $g(U,U) = A(U) = -1$. In (M_4, ST), from (3.2), (3.3), (3.8) and the Einstein field equations, we get

$$-(kp + \frac{r}{4})g(X,Y) = k(\sigma + p)A(X)A(Y) \tag{3.9}$$

By contracting (3.9) over X and Y, we have

$$r = k\sigma - 3kp \tag{3.10}$$

Putting $X = Y = U$ in (3.9) leads to

$$r = 4k\sigma \tag{3.11}$$

Since the scalar curvature r of this spacetime is constant, it follows from (3.11) that the energy density is constant. Combining the equations (3.10) and (3.11), we get $\sigma + p = 0$. Hence, we say that the isotropic pressure is constant. On the other hand, as $\sigma + p = 0$, the fluid behaves as a cosmological constant, [10]. This is also termed Phantom Barier, [24]. In cosmology, we know such a choice $\sigma = -p$ leads to rapid expansion of spacetime, [4]. Consequently, we can state that:

Theorem 3.4. *The spacetime (M_4, ST) obeying the Einstein field equations without the cosmological constant has constant energy density and isotropic pressure and it represents inflation. Also, the fluid behaves as a constant cosmological constant.*

In a dust or pressureless fluid spacetime, the energy momentum tensor is of the form

$$T(X, Y) = \sigma A(X)A(Y) \tag{3.12}$$

where σ is the energy-density of the dust-like matter and A is a non-zero 1-form such that $A(X) = g(X, U)$, for all X and U is a timelike vector field of the flow, [25]. Now, using (3.12) in (3.3), contracting the resulting equation over X and Y and then putting $X = Y = U$, we get $\sigma = 0$ and so $T(X, Y) = 0$. This means that the spacetime is devoid of the matter. Thus, we have:

Theorem 3.5. *A dust fluid isotropic spacetime equipped with a semi-symmetric non-metric connection satisfying Einstein's field equations with the cosmological constant is vacuum.*

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