

## CONNECTEDNESS IN SOFT m-STRUCTURE \*

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**Abstract.** In the present paper, we introduce the concept of soft connectedness in a soft m-structure and study some of its properties and characterizations.

**Keywords:** Soft m-structure, Soft m-connectedness and Soft m-connectedness between soft sets.

### 1. Introduction

The concept of soft set is fundamentally important in almost every scientific field. Soft set theory is a new mathematical tool dealing with uncertainty and has been applied in several directions since its introduction by Molodtsov [19] in 1999. The operations on soft sets and soft structures have been studied in [1, 16, 23]. Maji et. al [15] gave the first practical application of soft sets in decision theory. In 2011 Shabir and Naz [22] initiated a study of soft topological spaces. In recent years, many soft topological concepts such as soft connectedness and their strong forms [8, 11, 17, 20, 24], soft separation axioms [14, 20, 22], weak and strong forms of soft open sets and soft continuity [17, 2, 3, 4, 5, 6, 9, 10, 12, 13, 25] have been introduced and studied. Recently, the authors of this paper [21] initiated a study of soft m-structures. In the present paper we introduce the concept of soft connectedness in soft m-structures and we study some of its properties and characterizations.

### 2. Preliminaries

Let  $U$  be an initial universe set,  $E$  be a set of parameters,  $P(U)$  denote the power set of  $U$  and  $A \subseteq E$ .

**Definition 2.1.** [19] A pair  $(F, A)$  is called a soft set over  $U$ , where  $F$  is a mapping given by  $F: A \rightarrow P(U)$ . In other words, a soft set over  $U$  is a parameterized family of subsets of the universe  $U$ . For all  $e \in A$ ,  $F(e)$  may be considered a set of e-approximate elements of the soft set  $(F, A)$ .

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**Definition 2.2.** [16] For two soft sets  $(F, A)$  and  $(G, B)$  over a common universe  $U$ , we say that  $(F, A)$  is a soft subset of  $(G, B)$ , denoted by  $(F, A) \subseteq (G, B)$ , if

- (a)  $A \subseteq B$  and
- (b)  $F(e) \subseteq G(e)$  for all  $e \in E$ .

**Definition 2.3.** [16] Two soft sets  $(F, A)$  and  $(G, B)$  over a common universe  $U$  are said to be soft equal denoted by  $(F, A) = (G, B)$  if  $(F, A) \subseteq (G, B)$  and  $(G, B) \subseteq (F, A)$ .

**Definition 2.4.** [7] The complement of a soft set  $(F, A)$ , denoted by  $(F, A)^c$ , is defined by  $(F, A)^c = (F^c, A)$ , where  $F^c : A \rightarrow P(U)$  is a mapping given by  $F^c(e) = U - F(e)$ , for all  $e \in E$ .

**Definition 2.5.** [16] Let a soft set  $(F, A)$  over  $U$ .

- (a) A null soft set denoted by  $\phi$  if for all  $e \in A$ ,  $F(e) = \phi$ .
- (b) An absolute soft set denoted by  $\tilde{U}$ , if for each  $e \in A$ ,  $F(e) = U$ .

Clearly,  $\tilde{U}^c = \phi$  and  $\phi^c = \tilde{U}$ .

**Definition 2.6.** [7] The union of two sets  $(F, A)$  and  $(G, B)$  over a common universe  $U$  is a soft set  $(H, C)$ , where  $C = A \cup B$  and for all  $e \in C$ ,

$$H(e) = \begin{cases} F(e), & \text{if } e \in A - B \\ G(e), & \text{if } e \in B - A \\ F(e) \cup G(e), & \text{if } e \in A \cap B \end{cases}$$

**Definition 2.7.** [7] The intersection of two soft sets  $(F, A)$  and  $(G, B)$  over a common universe  $U$  is a soft set  $(H, C)$  where  $C = A \cap B$  and  $H(e) = F(e) \cap G(e)$  for each  $e \in E$ .

Let  $X$  and  $Y$  be initial universe sets and  $E$  and  $K$  be non-empty sets of the parameters,  $S(X, E)$  denotes the family of all soft sets over  $X$ , and  $S(Y, K)$  denotes the family of all soft sets over  $Y$ .

**Definition 2.8.** [12] Let  $S(X, E)$  and  $S(Y, K)$  be families of soft sets. Let  $u: X \rightarrow Y$  and  $p: E \rightarrow K$  be mappings. Then a mapping  $f_{pu}: S(X, E) \rightarrow S(Y, K)$  is defined as:

(i) Let  $(F, A)$  be a soft set in  $S(X, E)$ . The image of  $(F, A)$  under  $f_{pu}$ , written as  $f_{pu}(F, A) = (f_{pu}(F), p(A))$ , is a soft set in  $S(Y, K)$  such that

$$f_{pu}(F)(k) = \begin{cases} \bigcup_{e \in p^{-1}(k) \cap A} u(F(e)), & p^{-1}(k) \cap A \neq \phi \\ \phi, & p^{-1}(k) \cap A = \phi \end{cases}$$

For all  $k \in K$ .

(ii) Let  $(G, B)$  be a soft set in  $S(Y, K)$ . The inverse image of  $(G, B)$  under  $f_{pu}$ , written as  $f_{pu}^{-1}(G, B) = (f_{pu}^{-1}(G), p^{-1}(B))$ , is a soft set in  $S(X, E)$  such that

$$f_{pu}^{-1}(G)(e) = \begin{cases} u^{-1}G(p(e)), & p(e) \in B \\ \phi, & p(e) \notin B \end{cases}$$

For all  $e \in E$ .

**Definition 2.9.** [25] Let  $f_{pu} : S(X, E) \rightarrow S(Y, K)$  be a mapping and  $u : X \rightarrow Y$  and  $p : E \rightarrow K$  be mappings. Then  $f_{pu}$  is soft onto, if  $u : X \rightarrow Y$  and  $p : E \rightarrow K$  are onto and  $f_{pu}$  is soft one-one, if  $u : X \rightarrow Y$  and  $p : E \rightarrow K$  are one-one.

**Definition 2.10.** [22] A subfamily  $\tau$  of  $S(X, E)$  is called a soft topology over  $X$  if:

1.  $\tilde{\phi}, \tilde{X}$  belong to  $\tau$ .
2. The union of any number of soft sets in  $\tau$  belongs to  $\tau$ .
3. The intersection of any two soft sets in  $\tau$  belongs to  $\tau$ .

The triplet  $(X, \tau, E)$  is called a soft topological space over  $X$ . The members of  $\tau$  are called soft open sets in  $X$  and their complements are called soft closed sets in  $X$ .

**Definition 2.11.** If  $(X, \tau, E)$  is a soft topological space and a soft set  $(F, E)$  over  $X$ .

(a) The soft closure of  $(F, E)$  is denoted by  $Cl(F, E)$ , and defined as the intersection of all soft closed super sets of  $(F, E)$  [22].

(b) The soft interior of  $(F, E)$  is denoted by  $Int(F, E)$ , and defined as the soft union of all soft open subsets of  $(F, E)$  [25].

**Definition 2.12.** [25] The soft set  $(F, E) \in S(X, E)$  is called a soft point if there exist  $x \in X$  and  $e \in E$  such that  $F(e) = \{x\}$  and  $F(e') = \phi$  for each  $e' \in E - \{e\}$ , and the soft point  $(F, E)$  is denoted by  $x_e$ .

**Definition 2.13.** A soft set  $(A, E)$  of a soft topological space  $(X, \tau, E)$  is called :

- (a) Soft regular open  $(A, E) = Int(Cl(A, E))$  [6];
- (b) Soft  $\alpha$ -open if  $(A, E) \subset Int(Cl(Int(A, E)))$  [3];
- (c) Soft semi-open if  $(A, E) \subset Cl(Int(A, E))$  [17];
- (d) Soft preopen if  $(A, E) \subset Int(Cl(A, E))$  [2];
- (e) Soft b-open if  $(A, E) \subset Int(Cl(A, E)) \cup Cl(Int(A, E))$  [5].

(f) Soft  $\beta$ -open if  $(A, E) \subset \text{Cl}(\text{Int}(\text{Cl}(A, E)))$  [4]

The family of all soft regular open (resp. soft  $\alpha$ -open, soft semi-open, soft preopen, soft  $\beta$ -open, soft b-open) sets of  $X$  will be denoted by  $\text{SRO}(X, E)$  (resp.  $\text{S}\alpha\text{O}(X, E)$ ,  $\text{SSO}(X, E)$ ,  $\text{SPO}(X, E)$ ,  $\text{S}\beta\text{O}(X, E)$ ,  $\text{SbO}(X, E)$ ).

**Definition 2.14.** Let  $(A, E)$  be a soft subset of a soft topological space  $(X, \tau, E)$ . Then:

- (a) The intersection of all soft semi-open sets containing  $(A, E)$  is called semi-closure of  $(A, E)$ . It is denoted by  $\text{sCl}(A, E)$  [17].
- (b) The intersection of all soft preopen sets containing  $(A, E)$  is called preclosure of  $(A, E)$ . It is denoted by  $\text{pCl}(A, E)$ [2].
- (c) The intersection of all soft  $\alpha$  open sets containing  $(A, E)$  is called  $\alpha$ -closure of  $(A, E)$ . It is denoted by  $\alpha\text{Cl}(A, E)$  [3].
- (d) The intersection of all soft b-open sets containing  $(A, E)$  is called b-closure of  $(A, E)$ . It is denoted by  $\text{bCl}(A, E)$ [5].
- (e) The intersection of all soft  $\beta$ -open sets containing  $(A, E)$  is called  $\beta$ -closure of  $(A, E)$ . It is denoted by  $\beta\text{Cl}(A, E)$ [4].

**Definition 2.15.** A soft mapping  $f_{pu} : (X, \tau, E) \rightarrow (X, \sigma, K)$  is said to be :

- (a) Soft continuous if  $f_{pu}^{-1}(U, K) \in \tau$  for every soft set  $(U, K) \in \sigma$  [25].
- (b) Soft  $\alpha$ -continuous if  $f_{pu}^{-1}(U, K) \in \text{S}\alpha\text{O}(X, E)$  for every soft set  $(U, K) \in \sigma$  [3].
- (c) Soft semi-continuous if  $f_{pu}^{-1}(U, K) \in \text{SSO}(X, E)$  for every soft set  $(U, K) \in \sigma$  [17].
- (d) Soft precontinuous if  $f_{pu}^{-1}(U, K) \in \text{SPO}(X, E)$  for every soft set  $(U, K) \in \sigma$  [2].
- (e) Soft b-continuous if  $f_{pu}^{-1}(U, K) \in \text{SbO}(X, E)$  for every soft set  $(U, K) \in \sigma$  [5].
- (f) Soft  $\beta$ -continuous if  $f_{pu}^{-1}(U, K) \in \text{S}\beta\text{O}(X, E)$  for every soft set  $(U, K) \in \sigma$  [4].

**Definition 2.16.** A soft mapping  $f_{pu} : (X, \tau, E) \rightarrow (X, \sigma, K)$  is said to be :

- (a) Soft open if  $f_{pu}(U, E) \in \sigma$  for every soft set  $(U, E) \in \tau$  [26].
- (b) Soft  $\alpha$ -open if  $f_{pu}(U, E) \in \text{S}\alpha\text{O}(Y, K)$  for every soft set  $(U, E) \in \tau$  [3].
- (c) Soft semi-open if  $f_{pu}(U, E) \in \text{SSO}(Y, K)$  for every soft set  $(U, E) \in \tau$  [17].
- (d) Soft preopen if  $f_{pu}(U, E) \in \text{SPO}(Y, K)$  for every soft set  $(U, E) \in \tau$  [2].

(e) Soft b-open if  $f_{pu}(U, E) \in \text{SbO}(Y, K)$  for every soft set  $(U, E) \in \tau$  [5].

(f) Soft  $\beta$ -open if  $f_{pu}(U, E) \in \text{S}\beta\text{O}(Y, K)$  for every soft set  $(U, E) \in \tau$  [4].

**Definition 2.17.** [14] Let  $(X, \tau, E)$  be a soft topological space, and  $(A, E), (B, E)$  be two soft sets over  $X$ . The soft sets  $(A, E)$  and  $(B, E)$  are said to be soft-separated, if  $(A, E) \cap \text{Cl}(B, E) = \phi$  and  $\text{Cl}(A, E) \cap (B, E) = \phi$ .

**Definition 2.18.** [14] Let  $(X, \tau, E)$  be a soft topological space and if there exist two non-empty soft separated sets  $(A, E), (B, E)$  such that  $(A, E) \cup (B, E) = \tilde{X}$ , then  $(A, E)$  and  $(B, E)$  are said to be a soft disconnection for a soft topological space  $(X, \tau, E)$ .  $(X, \tau, E)$  is said to be soft-disconnected if  $(X, \tau, E)$  has a soft disconnection. Otherwise,  $(X, \tau, E)$  is said to be soft-connected.

**Definition 2.19.** [17] Let  $(X, \tau, E)$  be a soft topological space. The nonempty soft sets  $(F, A)$  and  $(F, B)$  in  $S(X, E)$  are called soft semi-separated iff  $\text{sCl}(F, A) \cap (F, B) = (F, A) \cap \text{sCl}(F, B) = \phi$ .

**Definition 2.20.** [17] Let  $(X, \tau, E)$  be a soft topological space. If there does not exist a soft semi-separation of  $X$ , then it is said to be soft s-connected.

**Definition 2.21.** [24] Let  $(X, \tau, E)$  be a soft topological space. The nonempty soft sets  $(F, A)$  and  $(F, B)$  in  $S(X, E)$  are called soft pre-separated iff  $\text{pCl}(F, A) \cap (F, B) = (F, A) \cap \text{pCl}(F, B) = \phi$ .

**Definition 2.22.** [24] Let  $(X, \tau, E)$  be a soft topological space. If there does not exist a soft pre-separation of  $X$ , then it is said to be soft P-connected.

**Definition 2.23.** [21] A subfamily  $m_{(X, E)}$  of  $S(X, E)$  is called a soft minimal structure (briefly soft m-structure) over  $X$  if  $\phi \in m_{(X, E)}$  and  $\tilde{X} \in m_{(X, E)}$ .

$(X, m_{(X, E)})$  is called a soft space with a soft minimal structure  $m_{(X, E)}$  or simply a soft m-space. Each member of  $m_{(X, E)}$  is called a soft m-open set and the complement of a soft m-open set is called a soft m-closed set.

**Remark 2.1.** [21] Let  $(X, \tau, E)$  be a soft topological space. Then the families  $\tau, \text{SSO}(X, E), \text{SPO}(X, E), \text{S}\alpha\text{O}(X, E), \text{S}\beta\text{O}(X, E), \text{SbO}(X, E), \text{SRO}(X, E)$  are all soft m-structures over  $X$ .

**Definition 2.24.** [21] Let  $X$  be a nonempty set,  $E$  be a set of parameters and  $m_{(X, E)}$  be a soft m-structure over  $X$ . The soft  $m_{(X, E)}$ -closure and the soft  $m_{(X, E)}$ -interior of the soft set  $(A, E)$  over  $X$  are defined as follows:

- (1)  $m_{(X, E)}\text{-Cl}(A, E) = \cap \{ (F, E) : (A, E) \subset (F, E), (F, E)^c \in m_{(X, E)} \}$ .
- (2)  $m_{(X, E)}\text{-Int}(A, E) = \cup \{ (F, E) : (F, E) \subset (A, E), (F, E) \in m_{(X, E)} \}$ .

**Remark 2.2.** [21] Let  $(X, \tau, E)$  be a soft topological space and  $(A, E)$  be a soft set over  $X$ . If  $m_{(X, E)} = \tau$  (respectively  $SO(X, E)$ ,  $SPO(X, E)$ ,  $S\alpha O(X, E)$ ,  $S\beta O(X, E)$ ,  $SbO(X, E)$ ), then we have:

- (1)  $m_{(X, E)}-Cl(A, E) = Cl(A, E)$  (resp.  $sCl(A, E)$ ,  $pCl(A, E)$ ,  $\alpha Cl(A, E)$ ,  $\beta Cl(A, E)$ ,  $bCl(A, E)$ ).  
 (2)  $m_{(X, E)}-Int(A, E) = Int(A, E)$  (resp.  $sInt(A, E)$ ,  $pInt(A, E)$ ,  $\alpha Int(A, E)$ ,  $\beta Int(A, E)$ ,  $bInt(A, E)$ ).

**Theorem 2.1.** [21] Let  $S(X, E)$  be a family of soft sets and  $m_{(X, E)}$  a soft minimal structure over  $X$ .

For soft sets  $(A, E)$  and  $(B, E)$  of  $X$ , the following holds:

- (a) (i):  $m_{(X, E)}-Int(A, E)^c = (m_{(X, E)} - Cl(A, E))^c$  and (ii) :  $m_{(X, E)}-Cl(A, E)^c = (m_{(X, E)} - Int(A, E))^c$ .  
 (b) If  $(A, E)^c \in m_{(X, E)}$ , then  $m_{(X, E)}-Cl(A, E) = (A, E)$  and if  $(A, E) \in m_{(X, E)}$ , then  $m_{(X, E)}-Int(A, E) = (A, E)$ .  
 (c)  $m_{(X, E)}-Cl(\phi) = \phi$ ,  $m_{(X, E)}-Cl(\tilde{X}) = \tilde{X}$ ,  $m_{(X, E)}-Int(\phi) = \phi$ ,  $m_{(X, E)}-Int(\tilde{X}) = \tilde{X}$ .  
 (d) If  $(A, E) \subset (B, E)$ , then  $m_{(X, E)}-Cl(A, E) \subset m_{(X, E)}-Cl(B, E)$ ,  $m_{(X, E)}-Int(A, E) \subset m_{(X, E)}-Int(B, E)$ .  
 (e)  $(A, E) \subset m_{(X, E)}-Cl(A, E)$  and  $m_{(X, E)}-Int(A, E) \subset (A, E)$ .  
 (f)  $m_{(X, E)}-Cl(m_{(X, E)}-Cl(A, E)) = m_{(X, E)}-Cl(A, E)$  and  $m_{(X, E)}-Int(m_{(X, E)}-Int(A, E)) = m_{(X, E)}-Int(A, E)$ .

**Definition 2.25.** [21] A soft mapping  $f_{pu} : (X, m_{(X, E)}) \rightarrow (Y, m_{(Y, K)})$ , where the minimal soft structure  $m_{(X, E)}$  and  $m_{(Y, K)}$  over  $X$  and  $Y$ , respectively, is said to be soft M-continuous if for each  $x_e \in S(X, E)$  and each  $(V, K) \in m_{(Y, K)}$  containing  $f_{pu}(x_e)$ , there exists  $(U, E) \in m_{(X, E)}$  containing  $x_e$  such that  $f_{pu}(U, E) \subset (V, K)$ .

Throughout this paper soft clopen means soft closed and open.

### 3. Connectedness in soft m-structure

**Definition 3.1.** [21] A soft minimal structure  $m_{(X, E)}$  over  $X$  is said to have the property **B** if the union of any family of subsets belongs to  $m_{(X, E)}$  belongs to  $m_{(X, E)}$ .

**Definition 3.2.** Let  $X$  be a nonempty set,  $E$  be a set of parameters and  $m_{(X, E)}$  be a soft m-structure over  $X$  with property **B**. In  $(X, m_{(X, E)})$  two nonempty soft sets  $(A, E)$  and  $(B, E)$  over  $X$  are called soft m-separated iff  $m_{(X, E)}-Cl(A, E) \cap (B, E) = (A, E) \cap m_{(X, E)}-Cl(B, E) = \phi$ .

**Remark 3.1.** Let  $(X, \tau, E)$  be a soft topological space over  $X$ . If  $m_{(X, E)} = \tau$  (resp.  $SSO(X, E), SPO(X, E), SbO(X, E)$ ) and  $m_{(X, E)}\text{-Cl}(A, E) = \text{Cl}(A, E)$  (resp.  $s\text{Cl}(A, E), p\text{Cl}(A, E), b\text{Cl}(A, E)$ ) we get definitions of soft separated (resp. soft semi-separated, soft pre-separated, soft b-separated) sets.

**Definition 3.3.** Let  $m_{(X, E)}$  be a soft m-structure over  $X$  with the property **B**. Then  $(X, m_{(X, E)})$  is said to be soft m-connected if there does not exist two nonempty soft m-separated sets  $(A, E)$  and  $(B, E)$  over  $X$ , such that  $(A, E) \cup (B, E) = \tilde{X}$ . Otherwise it is soft m-disconnected. In this case, the pair  $(A, E)$  and  $(B, E)$  is called soft m-disconnection over  $X$ .

**Remark 3.2.** Let  $(X, \tau, E)$  be a soft topological space over  $X$ . If we replace soft m-separated by soft separated (resp. soft semi-separated, soft pre-separated, soft b-separated) sets we get a definition for soft connectedness (resp. soft semi-connectedness, soft pre-connectedness, soft b-connectedness).

**Theorem 3.1.** Let  $(X, m_{(X, E)})$  be a soft m-space with the property **B**. Then the following conditions are equivalent:

- (1)  $(X, m_{(X, E)})$  has a soft m-disconnection.
- (2) There exist two disjoint soft m-closed sets  $(A, E), (B, E) \in m_{(X, E)}$  such that  $(A, E) \cup (B, E) = \tilde{X}$ .
- (3) There exist two disjoint soft m-open sets  $(A, E), (B, E) \in m_{(X, E)}$  such that  $(A, E) \cup (B, E) = \tilde{X}$ .
- (4)  $(X, m_{(X, E)})$  has a proper soft m-open and soft m-closed set over  $X$ .

Proof: (1)  $\rightarrow$  (2) : Let  $(X, m_{(X, E)})$  have a soft m-disconnection  $(A, E)$  and  $(B, E)$ . Then  $(A, E) \cap (B, E) = \phi$  and

$$m_{(X, E)}\text{-Cl}(A, E) = m_{(X, E)}\text{-Cl}(A, E) \cap ((A, E) \cup (B, E)) = (m_{(X, E)}\text{-Cl}(A, E) \cap (A, E)) \cup (m_{(X, E)}\text{-Cl}(A, E) \cap (B, E)) = (A, E).$$

Therefore,  $(A, E)$  is a soft m-closed set over  $X$ . Similarly, we can see that  $(B, E)$  is also a soft m-closed set over  $X$ .

(2)  $\rightarrow$  (3) : Let  $(X, m_{(X, E)})$  has a soft m-disconnection  $(A, E)$  and  $(B, E)$  such that  $(A, E)$  and  $(B, E)$  are soft m-closed. Then  $(A, E)^c$  and  $(B, E)^c$  are soft m-open sets in  $m_{(X, E)}$ . Then it is easy to see  $(A, E)^c \cap (B, E)^c = \phi$  and  $(A, E)^c \cup (B, E)^c = \tilde{X}$ .

(3)  $\rightarrow$  (4) : Let  $(X, m_{(X, E)})$  have a soft m-disconnection  $(A, E)$  and  $(B, E)$  such that  $(A, E)$  and  $(B, E)$  are soft m-open over  $X$ . Then  $(A, E)$  and  $(B, E)$  are also soft closed in  $(X, m_{(X, E)})$ .

(4)  $\rightarrow$  (1) : Let  $(X, m_{(X, E)})$  has a proper soft m-open and soft m-closed set  $(F, E)$  over  $X$ . Put  $(H, E) = (F, E)^c$ . Then  $(H, E)$  and  $(F, E)$  are non-empty soft m-closed sets in  $(X, m_{(X, E)})$ .  $(H, E) \cap (F, E) = \phi$  and  $(H, E) \cup (F, E) = \tilde{X}$ . Therefore,  $(H, E)$  and  $(F, E)$  is a soft m-disconnection of  $(X, m_{(X, E)})$ .

**Remark 3.3.** Let  $(X, \tau, E)$  be a soft topological space over  $X$ , if  $m_{(X, E)} = \tau$  (resp.  $\text{SSO}(X, E), \text{SPO}(X, E), \text{SbO}(X, E)$ ) Then the following conditions are equivalent:

- (1)  $(X, \tau, E)$  has a soft disconnection (resp. soft semi-disconnection, soft pre disconnection, soft b-disconnection).
- (2) There exist two disjoint soft closed (resp. soft semi-closed, soft pre-closed, soft b-closed) sets  $(A, E), (B, E)$  such that  $(A, E) \cup (B, E) = \tilde{X}$ .
- (3) There exist two disjoint soft open (resp. soft semi-open, soft pre-open, soft b-open) sets  $(A, E), (B, E)$  such that  $(A, E) \cup (B, E) = \tilde{X}$ .
- (4)  $(X, \tau, E)$  has a proper soft open (resp. soft semi-open, soft pre-open, soft b-open) and soft closed (resp. soft semi-closed, soft pre-closed, soft b-closed) set over  $X$ .

**Corollary 3.1.** Let  $(X, m_{(X, E)})$  be a soft  $m$ -space with the property **B**. Then the following conditions are equivalent: (1)  $(X, m_{(X, E)})$  is a soft  $m$ -connected.

- (2) There does not exist two disjoint soft  $m$ -closed sets  $(A, E), (B, E) \in m_{(X, E)}$  such that  $(A, E) \cup (B, E) = \tilde{X}$ .
- (3) There does not exist two disjoint soft  $m$ -open sets  $(A, E), (B, E) \in m_{(X, E)}$  such that  $(A, E) \cup (B, E) = \tilde{X}$ .
- (4)  $(X, m_{(X, E)})$  at most has two soft  $m$ -closed and soft  $m$ -open sets over  $X$ , that is,  $\phi$  and  $\tilde{X}$ .

**Remark 3.4.** Let  $(X, \tau, E)$  be a soft topological space over  $X$ , if  $m_{(X, E)} = \tau$  (resp.  $\text{SSO}(X, E), \text{SPO}(X, E), \text{SbO}(X, E)$ ). Then the following conditions are equivalent:

- (1)  $(X, \tau, E)$  is a soft connected (resp. soft semi-connected, soft preconnected, soft b-connected).
- (2) There does not exist two disjoint soft closed (resp. soft semi-closed, soft preclosed, soft b-closed) sets  $(A, E), (B, E)$  such that  $(A, E) \cup (B, E) = \tilde{X}$ .
- (3) There does not exist two disjoint soft open (resp. soft semi-open, soft pre-open, soft b-open) sets  $(A, E), (B, E)$  such that  $(A, E) \cup (B, E) = \tilde{X}$ .
- (4)  $(X, \tau, E)$  has a proper soft open (resp. soft semi-open, soft pre-open, soft b-open) and soft closed (resp. soft semi-closed, soft pre-closed, soft b-closed) set over  $X$ .

**Definition 3.4.** Let  $(X, m_{(X, E)})$  be a soft  $m$ -space with the property **B**,  $Y \subset X$  in  $(X, m_{(X, E)})$ . The soft space  $(Y, m_{(Y, E)})$  is called a soft  $m$ -subspace of  $(X, m_{(X, E)})$  if  $m_{(Y, E)} = \{(A, E) \cap \tilde{Y} : (A, E) \in m_{(X, E)}\}$ .

**Lemma 3.1.** Let  $(X, m_{(X, E)})$  be a soft  $m$ -space with the property **B**,  $(Y, m_{(Y, E)})$  be a soft  $m$ -subspace of  $(X, m_{(X, E)})$ . If  $(A, E) \subset \tilde{Y} \subset \tilde{X}$ . Then  $m_{(Y, E)}\text{-Cl}(A, E) = m_{(X, E)}\text{-Cl}(A, E) \cap \tilde{Y}$ .

Proof: We have  $m_{(Y, E)}\text{-Cl}(A, E) = \cap \{(F, E) : (A, E) \subset (F, E), \tilde{Y} - (F, E) \in m_{(Y, E)}\} = \cap \{(F, E) \cap \tilde{Y} : (A, E) \subset (F, E) \cap \tilde{Y}, \tilde{X} - (F, E) \in m_{(X, E)}\} = \cap \{(F, E) \cap \tilde{Y} : (A, E) \subset (F, E), \tilde{X} - (F, E) \in m_{(X, E)}\} = \cap \{(F, E) : (A, E) \subset (F, E), \tilde{X} - (F, E) \in m_{(X, E)}\} \cap \tilde{Y} = m_{(X, E)}\text{-Cl}(A, E) \cap \tilde{Y}$ .

Therefore, the lemma holds.



**Lemma 3.2.** *Let  $(X, m_{(X,E)})$  be a soft m-space with the property **B**,  $(Y, m_{(Y,E)})$  be a soft m-subspace of  $(X, m_{(X,E)})$ . If  $(A, E)$  and  $(B, E)$  are soft sets in  $(Y, m_{(Y,E)})$ , then  $(A, E)$  and  $(B, E)$  are soft m-separated in  $(Y, m_{(Y,E)})$  if and only if  $(A, E)$  and  $(B, E)$  are soft m-separated in  $(X, m_{(X,E)})$ .*

Proof: We have  $m_{(Y,E)}\text{-Cl}(A, E) \cap (B, E) = (m_{(X,E)}\text{-Cl}(A, E) \cap \tilde{Y}) \cap (B, E) = m_{(X,E)}\text{-Cl}(A, E) \cap (B, E)$  by lemma 3.1.

Similarly, we have

$$m_{(Y,E)}\text{-Cl}(B, E) \cap (A, E) = m_{(X,E)}\text{-Cl}(B, E) \cap (A, E).$$

Therefore, the lemma holds.

**Lemma 3.3.** *Let  $(X, m_{(X,E)})$  be a soft m-space with the property **B**,  $\tilde{Y} \subset \tilde{X}$ .  $(Y, m_{(Y,E)})$  be a soft m-subspace of  $(X, m_{(X,E)})$ .  $(Y, m_{(Y,E)})$  is soft m-connected. If  $(A, E)$  and  $(B, E)$  are soft m-separated in  $(X, m_{(X,E)})$ , such that  $\tilde{Y} \subset (A, E) \cup (B, E)$ , then  $\tilde{Y} \subset (A, E)$  or  $\tilde{Y} \subset (B, E)$ .*

Proof: We have  $\tilde{Y} \subset (A, E) \cup (B, E)$ , we have  $\tilde{Y} = (\tilde{Y} \cap (A, E)) \cup (\tilde{Y} \cap (B, E))$ . By lemma 3.2,  $\tilde{Y} \cap (A, E)$  and  $\tilde{Y} \cap (B, E)$  are soft m-separated in  $(Y, m_{(Y,E)})$ . Since  $(Y, m_{(Y,E)})$  is soft m-connected, we have  $\tilde{Y} \cap (A, E) = \phi$  or  $\tilde{Y} \cap (B, E) = \phi$ . Therefore,  $\tilde{Y} \subset (A, E)$  or  $\tilde{Y} \subset (B, E)$ .

**Lemma 3.4.** *Let  $\{(X_\alpha, m_{(X_\alpha,E)}): \alpha \in J\}$  be a soft family non-empty soft m-connected subspaces of  $(X, m_{(X,E)})$ . If  $\bigcap_{\alpha \in J} X_\alpha \neq \phi$ , then  $(\bigcup_{\alpha \in J} X_\alpha, \bigcup_{\alpha \in J} m_{(X_\alpha,E)})$  is a soft m-connected subspace of  $(X, m_{(X,E)})$ .*

Proof: Let  $Y = (\bigcup_{\alpha \in J} X_\alpha)$ . Choose a soft point  $x_e \in \tilde{Y}$ . Let  $(C, E)$  and  $(D, E)$  be a soft m-disconnection of  $(\bigcup_{\alpha \in J} X_\alpha, \bigcup_{\alpha \in J} m_{(X_\alpha,E)})$ . Then,  $x_e \in (C, E)$  and  $x_e \in (D, E)$ , we assume that  $x_e \in (C, E)$ . For each  $\alpha \in J$ . Since  $(X_\alpha, m_{(X_\alpha,E)})$  is soft m-connected, it follows from lemma 3.3 that  $\tilde{X}_\alpha \subset (C, E)$  or  $\tilde{X}_\alpha \subset (D, E)$ . Therefore, we have  $\tilde{Y} \subset (C, E)$  since  $x_e \in (C, E)$  and then  $(D, E) = \phi$ , which is a contradiction. Thus  $(\bigcup_{\alpha \in J} X_\alpha, \bigcup_{\alpha \in J} m_{(X_\alpha,E)})$  is a soft m-connected subspace of  $(X, m_{(X,E)})$ .

**Theorem 3.2.** *Let  $\{(X_\alpha, m_{(X_\alpha,E)}): \alpha \in J\}$  be a soft family non-empty soft m-connected subspaces of  $(X, m_{(X,E)})$ . If  $X_\alpha \cap X_\beta \neq \phi$  for  $\alpha, \beta \in J$ , then  $(\bigcup_{\alpha \in J} X_\alpha, m_{(\bigcup_{\alpha \in J} X_\alpha, E)})$  is a soft m-connected subspace of  $(X, m_{(X,E)})$ .*

Proof: Let  $\alpha_o \in J$ . For  $\beta \in J$ , Put  $A_\beta = X_{\alpha_o} \cup X_\beta$  By lemma 3.4,  $\{(A_\beta, m_{(X_\beta,E)}): \beta \in J\}$  is a family soft m-connected subspace of  $(X, m_{(X,E)})$  and  $\bigcap_{\beta \in J} A_\beta = X_{\alpha_o} \neq \phi$ . Obviously,  $(\bigcup_{\alpha \in J} X_\alpha, m_{(\bigcup_{\alpha \in J} X_\alpha, E)}) = (\bigcup_{\beta \in J} A_\beta, m_{(\bigcup_{\beta \in J} A_\beta, E)})$ . It follows from lemma 3.4 that  $(\bigcup_{\alpha \in J} X_\alpha, \bigcup_{\alpha \in J} m_{(X_\alpha,E)})$  is a soft m-connected subspace of  $(X, m_{(X,E)})$ .

**Theorem 3.3.** *Let  $(X, m_{(X,E)})$  be a soft m-space with the property **B**,  $\tilde{Y} \subset \tilde{X}$ .  $(Y, m_{(Y,E)})$  be a soft m-subspace of  $(X, m_{(X,E)})$ . If  $\tilde{Y} \subset \tilde{A} \subset m_{(X,E)}\text{-Cl}(F, E)$ , then*

$(A, m_{(A,E)})$  is a soft connected  $m$ -subspace of  $(X, m_{(X,E)})$ . In particular,  $m_{(X,E)}$ - $Cl(F, E)$  is a soft connected  $m$ -subspace of  $(X, m_{(X,E)})$ .

*Proof:* Let  $(C, E)$  and  $(D, E)$  be a soft  $m$ -disconnection of  $(A, m_{(A,E)})$ . By lemma 3.3, we have  $\tilde{A} \subset (C, E)$  or  $\tilde{A} \subset (D, E)$ . We assume that  $\tilde{A} \subset (C, E)$ . By lemma 3.2, we have  $m_{(X,E)}$ - $Cl(C, E) \cap (D, E) = \phi$  and, hence,  $\tilde{A} \cap (D, E) = \phi$ , which is a contradiction.

**Theorem 3.4.** Let  $f_{pu} : (X, m_{(X,E)}) \rightarrow (Y, m_{(Y,K)})$  be a soft  $M$ -continuous mapping, where  $m_{(X,E)}$  and  $m_{(Y,K)}$  are soft minimal structures over  $X$  and  $Y$ , respectively. If  $(X, m_{(X,E)})$  is soft  $m$ -connected, then the soft image of  $(X, m_{(X,E)})$  is also soft  $m$ -connected.

*Proof:* Let  $f_{pu} : (X, m_{(X,E)}) \rightarrow (Y, m_{(Y,K)})$  be a soft continuous mapping. Conversely, suppose that  $(Y, m_{(Y,K)})$  is soft  $m$ -disconnected and the pair  $(A, K)$  and  $(B, K)$  is a soft  $m$ -disconnection of  $(Y, m_{(Y,K)})$ . Since  $f_{pu} : (X, m_{(X,E)}) \rightarrow (Y, m_{(Y,K)})$  is soft continuous, then  $f_{pu}^{-1}(A, K) \in m_{(X,E)}$ ,  $f_{pu}^{-1}(B, K) \in m_{(X,E)}$ . Clearly, the pair  $f_{pu}^{-1}(A, K)$  and  $f_{pu}^{-1}(B, K)$  is a soft  $m$ -disconnection of  $(X, m_{(X,E)})$ , which is a contradiction. Hence,  $(Y, m_{(Y,K)})$  is soft  $m$ -connected. This completes the proof.

**Remark 3.5.** Let  $(X, \tau, E)$  and  $(Y, \vartheta, K)$  be two soft topological spaces over  $X$  and  $Y$ , respectively. If  $m_{(X,E)} = \tau$ ,  $m_{(Y,K)} = \vartheta$ .  $f_{pu} : (X, \tau, E) \rightarrow (Y, \vartheta, K)$  is a soft continuous mapping. If  $(X, \tau, E)$  is soft connected (resp. soft semi-connected, soft pre connected, soft  $b$ -connected) then the soft image of  $(X, \tau, E)$  is also soft connected (resp. soft semi-connected, soft preconnected, soft  $b$ -connected).

**Definition 3.5.** Let  $m_{(X,E)}$  be a soft  $m$ -structure over  $X$ . A soft set  $(F, E)$  in  $(X, m_{(X,E)})$  is soft  $m$ -connected if it is soft  $m$ -connected as a soft  $m$ -subspace.

**Remark 3.6.** Let  $(X, \tau, E)$  be a soft topological space over  $X$ . A soft set  $(F, E)$  in  $(X, \tau, E)$  is soft connected (resp. soft semi-connected, soft preconnected and soft  $b$ -connected) if it is soft connected (resp. soft semi-connected, soft preconnected and soft  $b$ -connected) as a soft subspace.

**Theorem 3.5.** Let  $m_{(X,E)}$  be a soft  $m$ -structure over  $X$ ,  $(G, E)$  be a soft  $m$ -connected set in  $(X, m_{(X,E)})$  and  $(F, E)$  be a soft set over  $X$  such that  $(G, E) \subset (F, E) \subset m_{(X,E)}$ - $Cl(G, E)$ . Then  $(F, E)$  is soft  $m$ -connected.

*Proof:* It is sufficient that  $m_{(X,E)}$ - $Cl(G, E)$  is soft  $m$ -connected. On the contrary, suppose that  $m_{(X,E)}$ - $Cl(G, E)$  is soft  $m$ -disconnected. Then there exists a soft  $m$ -disconnection  $((H, E), (K, E))$  of  $m_{(X,E)}$ - $Cl(G, E)$ . That is, there are  $((H, E) \cap (G, E), ((K, E) \cap (G, E)))$  soft sets in  $(G, E)$  such that  $((H, E) \cap (G, E)) \cap ((K, E) \cap (G, E)) = ((H, E) \cap (K, E)) \cap (G, E) = \phi$ , and  $((H, E) \cap (G, E)) \cup ((K, E) \cap (G, E)) = ((H, E) \cup (K, E)) \cap (G, E) = (G, E)$ . This yields that the pair  $((H, E) \cap (G, E))$  and  $((K, E) \cap (G, E))$  is a soft  $m$ -disconnection of  $(G, E)$ , which is a contradiction. This proves that  $m_{(X,E)}$ - $Cl(G, E)$  is soft  $m$ -connected. Hence, the proof is complete.

**Lemma 3.5.** *Let  $m_{(X,E)}$  be a soft m-structure over  $X$  with the property **B**, and let  $(A,E)$  and  $(B,E)$  be two soft sets over  $X$ . In  $(X, m_{(X,E)})$  the following statements are equivalent:*

- (1)  $\phi, \tilde{X}$  are only soft m-open and soft m-closed set in  $m_{(X,E)}$ .
- (2)  $(X, m_{(X,E)})$  is not a soft union of two disjoint soft sets  $(A,E)$  and  $(B,E) \in m_{(X,E)}$ .
- (3)  $(X, m_{(X,E)})$  is not a soft union of two disjoint soft sets  $(A, E)^c$  and  $(B, E)^c \in m_{(X,E)}$ .
- (4)  $(X, m_{(X,E)})$  is not a soft union of two nonempty soft m-separated sets.

**Remark 3.7.** Let  $(X, \tau, E)$  be a soft topological space over  $X$ , so we put  $m_{(X,E)} = \tau$  (resp.  $SSO(X,E), SPO(X,E), SbO(X,E)$ ). Also, let  $(A,E)$  and  $(B,E)$  be two soft sets over  $X$ . In  $(X, \tau, E)$  the following statements are equivalent:

- (1)  $\phi$  and  $\tilde{X}$  are only soft clopen (resp. soft semi-clopen, soft preclopen, soft b-clopen) sets in  $(X, \tau, E)$ .
- (2)  $(X, \tau, E)$  is not a soft union of two soft disjoint soft open (resp. soft semi-open, soft pre open, soft b-open) sets .
- (3)  $(X, \tau, E)$  is not a soft union of two soft disjoint soft closed (resp. soft semi-closed, soft preclosed, soft b-closed) sets.
- (4)  $(X, \tau, E)$  is not a soft union of two nonempty soft separated (soft semi separated, soft pre-separated, soft b-separated) sets.

**Theorem 3.6.** *Let  $m_{(X,E)}$  be a soft m-structure over  $X$  with the property **B**. In  $(X, m_{(X,E)})$  the following statements are equivalent:*

- (1)  $(X, m_{(X,E)})$  is a soft m-connected space.
- (2)  $(X, m_{(X,E)})$  is not a soft union of any two soft m-separated sets.

Proof : (1)  $\rightarrow$  (2) : Assume (1). Suppose (2) is false, then let  $(A,E)$  and  $(B,E)$  be two soft m-separated sets such that  $\tilde{X} = (A,E) \cup (B,E)$ . Since  $(X, m_{(X,E)})$  is soft m-connected  $m_{(X,E)}\text{-Cl}(A,E) \cap (B,E) = (A,E) \cap m_{(X,E)}\text{-Cl}(B,E) = \phi$ . Since  $(A,E) \subset m_{(X,E)}\text{-Cl}(A,E)$  and  $(B,E) \subset m_{(X,E)}\text{-Cl}(B,E)$ , then  $(A,E) \cup (B,E) = \phi$ . Now  $m_{(X,E)}\text{-Cl}(A,E) \subset (B, E)^c = (A,E)$ . Hence,  $m_{(X,E)}\text{-Cl}(A,E) = (A,E)$ . Therefore,  $(A, E)^c \in m_{(X,E)}$ . By the same way we show that  $(B, E)^c \in m_{(X,E)}$  which is a contradiction with remark 3.5. This shows that (2) is true. Therefore (1)  $\rightarrow$  (2).

(2)  $\rightarrow$  (1) : Assume that (2) is not true. Let  $(A, E)^c$  and  $(B, E)^c \in m_{(X,E)}$  such that  $\tilde{X} = (A, E)^c \cup (B, E)^c$ . Then,  $m_{(X,E)}\text{-Cl}(A, E)^c \cap (B, E) = (A, E) \cap m_{(X,E)}\text{-Cl}(B, E)^c = (A, E)^c \cap (B, E)^c = \phi$ . This contradicts the hypothesis in (2). This show that (1) is true. Therefore, (2)  $\rightarrow$  (1).

**Remark 3.8.** Let  $(X, \tau, E)$  be a soft topological space over  $X$ , so we put  $m_{(X,E)} = \tau$ . Then, the following statements are equivalent:

- (1)  $(X, \tau, E)$  is a soft connected (soft semi-connected, soft preconnected, soft b-connected) space.

(2)  $(X, \tau, E)$  is not the soft union of any two soft separated (soft semi separated, soft pre-separated, soft b-separated) sets.

**Remark 3.9.** (1) Let  $m_{(X,E)}$  be a soft m-structure over  $X$  with the property **B**, and let  $(A,E)$  be a soft set over  $X$ . If  $\phi \neq (A,E) \subset (X, m_{(X,E)})$  then  $(A,E)$  is a soft m-connected set in  $m_{(X,E)}$  whenever  $(X, m_{(X,E)})$  is a soft m-connected space.

(2) Let  $(X, \tau, E)$  be a soft topological space over  $X$ , so we put  $m_{(X,E)} = \tau$ . If  $\phi \neq (A,E) \subset (X, \tau, E)$  then  $(A,E)$  is a soft connected (soft semi-connected, soft pre-connected, soft b-connected) set over  $X$  whenever  $(X, \tau, E)$  is a soft connected (soft semi-connected, soft pre-connected, soft b-connected) space.

**Theorem 3.7.** Let  $m_{(X,E)}$  be a soft m-structure over  $X$  with the property **B**. In  $(X, m_{(X,E)})$ , let the soft set  $(A,E)$  be a soft m-connected set. Let  $(B,E)$  and  $(C,E)$  be soft m-separated sets. If  $(A,E) \subset (B,E) \cup (C,E)$ . Then, either  $(A,E) \subset (B,E)$  or  $(A,E) \subset (C,E)$ .

Proof: Suppose  $(A,E)$  is a soft m-connected set and  $(B,E), (C,E)$  are soft m-separated sets such that  $(A,E) \subset (B,E) \cup (C,E)$ . Let  $(A,E) \not\subset (B,E)$  and  $(A,E)$  is not a subset of  $(C,E)$ . Suppose  $(A_1,E) = (B,E) \cap (A,E) \neq \phi$  and  $(A_2,E) = (C,E) \cap (A,E) \neq \phi$ . Then,  $(A,E) = (A_1,E) \cup (A_2,E)$ . Since  $(A_1,E) \subset (B,E)$ . Hence,  $m_{(X,E)}\text{-Cl}(A_1,E) \subset m_{(X,E)}\text{-Cl}(B,E)$ . Since  $m_{(X,E)}\text{-Cl}(B,E) \cap (C,E) = \phi$  then  $m_{(X,E)}\text{-Cl}(A_1,E) \cap (A_2,E) = \phi$ . Since  $(A_2,E) \subset (C,E)$ . Hence,  $m_{(X,E)}\text{-Cl}(A_2,E) \subset m_{(X,E)}\text{-Cl}(C,E)$ . Since  $m_{(X,E)}\text{-Cl}(C,E) \cap (B,E) = \phi$ . Then  $m_{(X,E)}\text{-Cl}(A_2,E) \cap (A_1,E) = \phi$ . But  $(A,E) = (A_1,E) \cup (A_2,E)$ . Therefore,  $(A,E)$  is not a soft m-connected space. This is a contradiction. Then either  $(A,E) \subset (B,E)$  or  $(A,E) \subset (C,E)$ .

**Remark 3.10.** Let  $(X, \tau, E)$  be a soft topological space over  $X$ , so we put  $m_{(X,E)} = \tau$ . Also, let  $(A,E)$  be a soft connected (resp. soft semi-connected, soft pre-connected, soft b-connected) set. Let  $(B,E)$  and  $(C,E)$  be soft separated (resp. soft semi-separated, soft pre-separated, soft b-separated) sets. If  $(A,E) \subset (B,E) \cup (C,E)$  then either  $(A,E) \subset (B,E)$  or  $(A,E) \subset (C,E)$ .

Let  $m_{(X,E)}$  be a soft m-structure over  $X$  with the property **B**. In  $(X, m_{(X,E)})$ , let the soft set  $(A,E)$  be a soft m-connected set, then  $m_{(X,E)}\text{-Cl}(A,E)$  is soft m-connected.

Proof: Suppose the soft set  $(A,E)$  is a soft m-connected set and  $m_{(X,E)}\text{-Cl}(A,E)$  is not. Then there exist two soft m-separated sets  $(B,E)$  and  $(C,E)$  such that  $m_{(X,E)}\text{-Cl}(A,E) = (B,E) \cup (C,E)$ . But  $(A,E) \subset m_{(X,E)}\text{-Cl}(A,E)$ , then  $(A,E) = (B,E) \cup (C,E)$  and since  $(A,E)$  is a soft m-connected set, then by Theorem 3.7 either  $(A,E) \subset (B,E)$  or  $(A,E) \subset (C,E)$ .

(i) If  $(A,E) \subset (B,E)$  then  $m_{(X,E)}\text{-Cl}(A,E) \subset m_{(X,E)}\text{-Cl}(B,E)$ . But  $m_{(X,E)}\text{-Cl}(B,E) \cap (C,E) = \phi$ . Hence,  $m_{(X,E)}\text{-Cl}(A,E) \cap (C,E) = \phi$ . Since  $(C,E) \subset m_{(X,E)}\text{-Cl}(A,E)$ , then  $(C,E) = \phi$  this is a contradiction.

(ii) If  $(A,E) \subset (C,E)$  then in the same way we can prove that  $(B,E) = \phi$ , which is a contradiction. Therefore,  $m_{(X,E)}\text{-Cl}(A,E)$  is soft m-connected.

**Remark 3.11.** Let  $(X, \tau, E)$  be soft topological space over  $X$ , we put  $m_{(X,E)} = \tau$  let soft set  $(A,E)$  be a soft connected (resp. soft semi connected, soft pre connected, soft b-connected) set then  $m_{(X,E)}\text{-Cl}(A,E)$  is soft connected (resp. soft semi connected, soft pre connected, soft b-connected).

**Theorem 3.8.** Let  $m_{(X,E)}$  be a soft m-structure over  $X$  with the property **B**. In  $(X, m_{(X,E)})$ , let the soft set  $(A,E)$  be a soft m-connected set and  $(A,E) \subset (B,E) \subset m_{(X,E)}\text{-Cl}(A,E)$  then  $(B,E)$  is soft m-connected.

Proof: If  $(B,E)$  is not soft m-connected, then there exist two soft sets  $(C,E)$  and  $(D,E)$  such that  $m_{(X,E)}\text{-Cl}(C,E) \cap (D,E) = (C,E) \cap m_{(X,E)}\text{-Cl}(D,E) = \phi$  and  $(B,E) = (C,E) \cup (D,E)$ . Since  $(A,E) \subset (B,E)$ , thus either  $(A,E) \subset (C,E)$  or  $(A,E) \subset (D,E)$ . Suppose  $(A,E) \subset (C,E)$  then  $m_{(X,E)}\text{-Cl}(A,E) \subset m_{(X,E)}\text{-Cl}(C,E)$ , thus  $m_{(X,E)}\text{-Cl}(A,E) \subset (D,E) = m_{(X,E)}\text{-Cl}(C,E) \subset (D,E) = \phi$ . But  $(D,E) \subset (B,E) \subset m_{(X,E)}\text{-Cl}(A,E)$ , thus  $m_{(X,E)}\text{-Cl}(A,E) \cap (D,E) = (D,E)$ . Therefore,  $(D,E) = \phi$  which is a contradiction. Thus,  $(B,E)$  is a soft m-connected set.

If  $(A,E) \subset (B,E)$ , then we can prove that  $(C,E) = \phi$ . This is a contradiction. Then  $(B,E)$  is soft m-connected.

**Remark 3.12.** Let  $(X, \tau, E)$  be a soft topological space over  $X$ , so we put  $m_{(X,E)} = \tau$ . Also, let the soft set  $(A,E)$  be a soft connected (resp. soft semi-connected, soft preconnected, soft b-connected) set and  $(A,E) \subset (B,E) \subset m_{(X,E)}\text{-Cl}(A,E)$ , then  $(B,E)$  is soft connected (resp. soft semi-connected, soft preconnected, soft b-connected).

**Remark 3.13.** Let  $(X, \tau, E)$  be a soft topological space over  $X$ , and  $(F,E)$  be a soft set over  $X$ .  $(X, \tau, E)$  is soft connected (soft semi-connected, soft preconnected, soft b-connected) if and only if there does not exist nonempty soft set  $(F,E)$  over  $X$  which is both soft open (resp. soft semi-open, soft preopen, soft b-open) and soft closed (resp. soft semi-closed, soft pre-closed, soft b-closed) set over  $X$ .

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