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ECONOMIC-EMISSION DISPATCH WITH SEMIDEFINITE PROGRAMMING AND RATIONAL FUNCTION APPROXIMATIONS

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Abstract. The emission function associated with the economic-emission dispatch problem contains exponential functions that model the emission pollutants. This paper presents a strategy of solving the economic-emission dispatch problem whereby the exponential function is approximated by a rational function that permits reduction to a standard polynomial optimization problem. This is reformulated as a hierarchy of semidefinite relaxation problems using the moment theory and the resulting SDP problem is solved. Different degrees of rational functional approximation were considered. The approach was tested on the IEEE 30-bus test systems to investigate its effectiveness. Solutions obtained were compared with those from some of the well known evolutionary methods. Results showed that SDP has inherently good convergence property and a lower but comparable diversity property.
Keywords: rational polynomial approximation, semidefinite program, multiobjective optimization, economic dispatch, emission dispatch

1. Introduction

Concern for the environmental impact of power plants and the high cost of retrofitting have made emission-economic dispatch (EED) a very promising option for optimizing their operation. In practice, the emission and fuel cost of generating stations are simultaneously minimized in a multi-objective optimization formulation [1]. Several methods have been proposed to solve the resulting multi-objective problem and notably most recent approaches revolve around the use of evolutionary algorithms [1, 2, 3, 4, 5, 6, 7, 8]. A drawback of evolutionary algorithms is the high computational burden which results in large time consumption and possible premature convergence [2]. There are few examples of the application of semidefinite programming (SDP) to emission dispatch (ED) problems where the objectives and constraints are either linear or quadratic [9], [10] and [11]. A straightforward application of the SDP to EED problem will necessitate dealing with an emission

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objective consisting of polynomial function (of at least second degree) and exponential functions when it is accurately modelled.

It is reasonable to express the exponential part of the objective as a power series and take advantage of algorithms that guarantee the infimum of the class of polynomial functions [12]. However, the exponential function is embedded in an infinite dimensional polynomial space. Using the results of Devolder et al. [13], the problem can be projected unto a finite dimensional space. Such projection reduces the infinite dimensional polynomial problem to the standard polynomial optimization problem (POP) form which can be efficiently solved via SDP [14]. Mostly, a good approximation can be achieved using a relatively high degree polynomial. However, the size of the matrix of the resulting SDP program grows prohibitively large with the degree of the polynomial. This also tends to increase the computational cost associated with solving the resulting semidefinite program. The problem can be mitigated by using an alternative found in rational function approximation. This achieves high accuracy and at the same time uses a rational function having lower degree of numerator and denominator.

A key motivation for the approach adopted in this paper is the ability of optimal rational approximating function to achieve higher accuracy than the optimal polynomial approximation with same number of coefficients [15]. Furthermore, recent advances in rational function optimization [12, 16] allow the reduction of the problem to a constrained polynomial optimization problem (POP) which can be solved using the semidefinite program. Although POPs are generally non-convex and difficult to solve, various hierarchy of convex relaxation of the problem have been proposed which monotonically converge to the exact global optimal solution [14, 17]. This simplifies and allows non-convex problems to be solved by convex optimization techniques. Furthermore, unlike most multiobjective evolutionary algorithms (MOEA) which are stochastic optimizers and which find it difficult or even impossible to attain the ideal Pareto surface, SDP provides a cheaply computable lower bound of the minimum value [18]. Therefore, it has a very good ability to converge to solution set that are close to the ideal Pareto surface.

It is noteworthy that the SDP applications considered in [9], [10] and [11] are limited to problems with quadratic constraint and objective functions. The contribution of this paper is the solution of EED problem that includes exponential function in the emission objective through the application of SDP. Specifically, the exponential function is projected unto a finite dimensional rational function space and the resulting problem is transformed into a POP.

The organization of the paper is as follows. In Section 2. of the paper, the formulation of the multiobjective dispatch problem is presented. A sketch of the solution of the problem is provided in Section 3.. The vector objective is convexified through semidefinite relaxations and then scalarized using the weighted sum method. This reduces the dispatch problem to a convex optimization problem. By employing the nonlinear weight selection method proposed earlier in [20], the SDP algorithm was guided to provide a better capture of the solution set. In order to make the paper self-contained, in Section 4. semindefinite programming is briefly reviewed along with the notations. In Section 5. rational function approximation and SDP relaxation methods are reviewed and key results presented. This paves the way for a formulation of the emission part of the EED taking into consideration the rational function approximation of the exponential function in Section 6.. In Section 7. performance of the method is evaluated by comparing its solution set with that generated by Non-dominated Sorting Genetic Algorithm -II (NSGA-II). The two methods are employed to solve the IEEE 30-bus 6-generator test system with the fuel cost and the transmission loss as the objectives to be minimized.

2. Problem Statement

The multiobjective economic-emission dispatch problem is formulated, for s generating plants, as follows:

$$\begin{array}{rll} \textit{minimize} & [C(x), E(x)] & \\ \textit{subject to:} & h(x) = 0 & x = [x_1, \ldots, x_s]^T \\ & g(x) \leq 0 & \end{array} \tag{2.1}$$

where x, the decision variable, is the vector of generated power, C is the fuel cost objective, E is the pollutant emission objective, g and h are the respective equality and the inequality constraints of the system. Further elaboration of the problem is now provided.

2.1. Problem Objectives

2.1.1. Total Fuel Cost, $C(x)$

The generator costs are represented by quadratic functions. and the total fuel cost, $C(x)$, can be expressed as

$$C(x) = \sum_{i=1}^{s} \alpha_i + \beta_i\, x_i + \gamma_i\, x_i^2, \tag{2.2}$$

where x_i is the real power output of the i^{th} plant, and α_i, β_i, and γ_i are the corresponding fuel cost coefficients of the plant.

2.1.2. Pollutant Emission

The total emission of atmospheric pollutants (e.g. SO_x, NO_x) in ton/h can be expressed as

$$E(x) = \sum_{i=1}^{s} 10^{-2}\left(a_i + b_i\, x_i + c_i\, x_i^2\right) + \zeta_i \exp(\lambda_i x_i), \tag{2.3}$$

where a_i, b_i, c_i, λ_i and ζ_i are the coefficients of emission characteristics of the i^{th} plant .

2.2. Problem Constraints

2.2.1. Generation capacity constraints

The real power output of each generating unit is constrained between an upper and lower limit as follows:

$$x_i^{min} \leq x_i \leq x_i{}^{max}, \quad i, \ldots, s. \tag{2.4}$$

This defines the inequality constraint $g(x)$.

2.2.2. Power balance constraint

The power balance constraint is given by

$$\sum_{i=1}^{s} x_i = P_D + P_L(x), \tag{2.5}$$

where P_D is the total load demand, and $P_L(x)$ is the transmission loss, which is given, by the Kron's loss formula, as

$$P_L(x) = \sum_{i=1}^{s}\sum_{j=1}^{s} x_i\, B_{ij}\, x_j + \sum_{i=1}^{s} B_{i01}\, x_i + B_{00}, \tag{2.6}$$

where B_{ij}, B_{i01} and B_{00} are the Kron's loss coefficients. Equation (2.6) defines the equality constraint $h(x)$.

3. Multiobjective Optimization

Scalarization is a class of methods of solving the multiobjective optimization problem (MOP). In one of the approaches, it reduces the vector objective function into a single objective (scalar) optimization problem by forming a weighted sum of objectives.

Consider the weight vector $w = (w_1, \ldots, w_s)^T \in \mathbb{R}^s$, the vector objective function $f(x) = (f_1(x), \ldots, f_s(x))^T \in \mathbb{R}^s$ and the map $\phi(x, w) : \mathbb{R}^s \times \mathbb{R}^s \mapsto \mathbb{R}$. The weighted sum method involves a convex combination of the objectives $f_i(x)$, $i = 1, \ldots, s$ to give the scalar objective $\phi(x, w)$:

$$\begin{aligned} \phi(x, \mathbf{w}) &= \sum_{i}^{s} w_i f_i(x) \\ &= w^T f(x) \end{aligned} \tag{3.1}$$

where

$$\sum_{i=1}^{s} w_i = 1,\ w_i \geqslant 0, i = 1, \ldots, s \tag{3.2}$$

This transforms the vector optimization to a scalar of the form:

$$\begin{array}{rll} \text{minimize} & \phi(x,w) & \\ \text{subject to} & x \in X; & X \subseteq \mathbb{R}^n \end{array} \tag{3.3}$$

This process maps the s-dimensional objective space onto the positive real line $\mathbb{R}$ and all the optimal (nondominated) points are mapped to the same point on the line.

For illustrative purpose, consider the bi-objective problem with $s = 2$, equations (3.1) and (3.2), respectively, reduce to

$$\phi(x,w) = w_1 f_1(x) + w_2 f_2(x) \tag{3.4}$$

and

$$w_1 + w_2 = 1,\ w_1, w_2 \geqslant 0 \tag{3.5}$$

The weighted sum method is the commonly used scalarization method because of its simplicity, ease of use, and direct translation of weights into the relative importance of the objectives [19]. However, it is known to miss solution points on the non-convex part of the Pareto surface, and even distribution of weights does not translate to uniform distribution of the solution points. If the weights in (3.4) are parameterized by λ, such that $w_1 = \lambda$ and $w_2 = 1 - \lambda$, a uniform spacing on λ does not produce a uniform spacing on the Pareto front. Furthermore, the distribution of solution points is highly dependent on the relative scaling of the objectives.

It was observed in [20], that the weight can be parameterized by λ and constrained on the surface of an ellipsoid, so that

$$\frac{\lambda_1^2}{k_1^2} + \frac{\lambda_2^2}{k_2^2} = 1. \tag{3.6}$$

where k_1 and k_2 are the axes of the ellipsoid. The parametrization has led to an improvement in distribution of the points on the Pareto front. The expression can be normalized by setting $k_2 = 1$. Variation in k_1 allows for the control of the curvature of the ellipsoidal surface. Thus, the non-linear weight selection gives a higher sensitivity and provides for further sensitivity improvement through the free parameter k_1. This parameter can be used to efficiently explore the Pareto surface. Further, it aids the control of the slope of the weight factor such that clustered points can be spread out, thereby improving computational efficiency of the weighted sum method.

4. Semidefinite programming

A semidefinite program (SDP) is a type of convex optimization that generalizes the linear program (LP) with the vector variables replaced by matrix variables and the element-wise nonnegativity replaced by positive semidefiniteness of the matrices. Of the various forms of SDP, this paper uses the primal form in its formulation. Thus, the optimization problem is defined as

$$\begin{array}{rl} \text{minimize} & \langle A_0, X\rangle \\ \text{subject to} & \langle A_i, X\rangle = b_i, \quad i = 1, \ldots, m \\ & X \geqslant 0 \end{array} \tag{4.1}$$

and the associated dual SDP is

$$\begin{array}{rl} \text{maximize} & \langle \mathbf{b}, \mathbf{y}\rangle \\ \text{subject to} & \sum_{i=1}^{m} y_i A_i \leqslant A_0, \quad \mathbf{y} \in \mathbb{R}^m \end{array} \tag{4.2}$$

where $X \in \mathcal{S}^n$ is the decision variable, $\mathbf{b} \in \mathbb{R}^n$ and $A_0, A_i \in \mathcal{S}^n$ (the set of all symmetric matrices in $\mathbb{R}^{n\times n}$). Let p^* and d^* be the optimal values of (4.1) and (4.2) respectively. Efficient interior point method has been developed for the primal/dual program [22, 23, 24]. More details on SDP can be found in [21].

5. SDP Relaxation for FPOP and Rational Functions

In this section, a brief overview of the rational function approximation and results of different convex SDP relaxations methods found in the literature are presented. We start with notations and objects from real algebra.

Let $\mathbb{R}[x]$ denotes the ring of all real polynomials in the variables $x_1, x_2, \ldots, x_n$ and $\mathcal{P}$ denote the $\mathbb{R}$-vector space spanned by the infinite monomial basis $v \in \mathcal{P}$, given by

$$v = \left[1, x_1, x_2, \ldots, x_n, x_1^2, x_1x_2, \ldots x_1x_n, x_2x_3, \ldots\right]^T \tag{5.1}$$

v_k is a finite monomial basis in v with $\deg(v_k) \leqslant k$ and defines a polynomial subspace $\mathcal{P}_k \subset \mathcal{P}$. The subset of $\mathbb{R}[x]$ consisting of the sums of squares of polynomials is denoted by $\sum^2[x]$. A quadratic module, $M(g_1, \ldots, g_m)$, generated by the polynomial $g_i(x) \in \mathbb{R}[x]$, $i = 1, 2, \ldots, m$ is defined as:

$$M(g_1, \ldots, g_m) := \left\{ \sigma_0 + \sum_{j=1}^{m} \sigma_j g_j \,\middle|\, \sigma_j \in \Sigma^2[x] \right\} \tag{5.2}$$

The truncated quadratic module of degree $2k$, $M_k(g_1, \ldots, g_m) \subset M(g_1, \ldots, g_m)$ have $\deg(\sigma_0) \leqslant 2k$, $\deg(\sigma_i g_i) \leqslant 2k$, $i = 1, \ldots, m$.

5.1. Rational Polynomial Function Approximation of e^x

The sum of an infinite geometric series (or rather an infinite degree polynomial), $s(x) = a + ax + ax^2 + \ldots ax^n + \ldots$ can be compactly written as a rational function

$s(x) = a/(1-x)$, $|x| \leqslant 1$. This is an example of low degree rational function representing very high degree (even infinite degree) polynomial accurately.

The Padé approximant is a rational function whose power series expansion agrees with a given power series to the highest possible order [15]. Given an arbitrary function $f(x)$ which can be described by an infinite series

$$f(x) = \sum_{i=1}^{\infty} c_i x^i \tag{5.3}$$

the Padé approximate rational function, $z(x)$, of degree $[m, n]$ to function $f(x)$ is given as

$$z(x) = \frac{\sum\limits_{k=0}^{m} a_k x^k}{1 + \sum\limits_{k=1}^{n} b_k x^k} = \frac{p(x)}{q(x)} \tag{5.4}$$

with

$$z(0) = f(0) \tag{5.5}$$

and

$$\left.\frac{d^k}{dx^k} z(x)\right|_{x=x_o} = \left.\frac{d^k}{dx^k} f(x)\right|_{x=x_o}, k = 1, 2, \ldots, m+n \tag{5.6}$$

The point $x = x_o$ is the point about which the series expansion is done. The unknown coefficient of both the numerator polynomial, $p(x)$, and the denominator polynomial, $q(x)$, are determined from equations (5.5) and (5.6).

Optimal determination of the coefficients of the rational function can be achieved by minimizing the ℓ_∞-norm of the residual $|f(x) - z(x)|$. Thus,

$$\min \; max_{\substack{q(x)>0 \\ (a,b)\in D}} |f(x) - z(x)| \tag{5.7}$$

with the constraint

$$D = \left\{(a, b) \in \mathbb{R}^{m+n} | q(x) > 0, \alpha \leqslant x \leqslant \beta\right\}$$

where $[\alpha, \beta]$ defines the interval over which $q(x)$ is positive. This improved approximation is called Chebychev-Padé approximation.

5.2. Rational Function Optimization Problem (RFOP)

The rational function optimization is of the form

$$z^* = \inf_{\substack{q(x)\neq 0\\ x\in\mathbf{K}}} \frac{p(x)}{q(x)} \tag{5.8}$$

where $p(x), q(x) \in \mathbb{R}[x]$ are relative primes, $\mathbf{K}$ is a basic closed semialgebraic set,

$$\mathbf{K} = \{x \in \mathbb{R}^n \,|g_i(x) \geqslant 0, i = 1, \ldots, s\} \tag{5.9}$$

defined by polynomial $g_i(x) \in \mathbb{R}[x]$, $i = 1, \ldots, s$.

The above rational function optimization reduces to a polynomial optimization problem if the denominator function, $q(x)$, is 1, i.e.

$$p^* = \inf \{p(x)\,|x \in \mathbf{K}\} \tag{5.10}$$

In order to optimize the rational function, one might turn to global optimization techniques. However, several of these techniques are inapplicable because the Lipschitz continuity requirement for global convergence does not hold in general for rational functions [25]. Recent techniques to mitigate this difficulty involve convex relaxation of the problem and aim to compute a tight lower bound on the objective function. Two of these convex relaxation approaches are usually considered and both proceed by reformulating the rational function objective as constrained polynomial objective, thus reducing the problem to a POP. A semidefinite program is then used to solve the resulting POP.

Jibetean [12], in a reformulation, considered the function $f(x) = \frac{p(x)}{q(x)} \geqslant \alpha$ and showed that if $q(x)$ changes sign in $\mathbf{K}$, then

$$\inf \frac{p(x)}{q(x)} = -\infty. \tag{5.11}$$

Otherwise, the problem reduces to

$$f^* = \sup \{\alpha\,|p(x) - \alpha\, q(x) \geqslant 0, \forall x \in \mathbf{K}\} \tag{5.12}$$

which is in the form (5.10). This was further reduced to a sum-of-squares (SOS) problem and solved through semidefinite program. The nonnegative polynomial $p(x) - \alpha\, q(x) \geqslant 0$ is then written as quadratic module

$$p^* = \sup \left\{ \alpha \,\middle|\, p(x) - \alpha\, q(x) = \sigma_0 + \sum_{j=1}^{r} \sigma_j g_j \in \Sigma^2[x] \right\} \tag{5.13}$$

where $\sigma_j \in \Sigma^2[x]$, $j = 0, \ldots, r$.This is transformed into an SDP form

$$\begin{array}{rl} \sup & \alpha \\ \text{subject to} & p(x) - \alpha q(x) = \langle Q, V\rangle \\ & V \geqslant 0 \end{array} \tag{5.14}$$

where Q is a positive semidefinite matrix and $V = v_t v_t^T$ is a positive semidefinite variable. Various hierarchy of SDP relaxation (approximation) is introduced by setting the polynomial to the truncated quadratic modules $M_t(g_1, \ldots, g_r)$ such that $\deg(\sigma_0) \leqslant 2t$, $\deg(\sigma_j g_j) \leqslant 2t$:

$$p_t^{sos} = \sup\left\{\alpha \,\middle|\, p(x) - \alpha\, q(x) = v_t^T Q v_t\right\} \tag{5.15}$$

It follows that p_t^{sos} can be computed through a semidefinite program. And as $t \to \infty$, $p_t^{sos} \to p^*$ provided that there exists a number $N \in \mathbf{N}$ such that $N - \|x\|^2 \in M(g_1, \ldots, g_r)$ [25].

In a second approached proposed by Bugarin et al.[26] the problem is reduced to a generalized moment problem:

$$p^* = \min\left\{\int p(x)d\mu \,|\mu \in \mathcal{M}(\mathbf{K})\,; \int q(x)d\mu = 1\right\} \tag{5.16}$$

This formulation defines a probability measure $\mathcal{M}(\mathbf{K})$ on $\mathbf{K}$, and replaces every point $x \in \mathbf{K}$ by its Dirac probability measure δ_x at x. The probability measure $\mu \in \mathcal{M}$ is equipped with the properties $\mu(\emptyset) = 0$ and $\mu(\mathbf{K}) = 1$. As a representing measure, μ defines the sequence $\mathbf{y} = \{\mathbf{y}_\alpha\}$, named the *moment* of order α, as

$$y_\alpha = \int_{\mathbf{K}} x^\alpha d\mu \;\; \forall \alpha \in \mathbf{N}^n \tag{5.17}$$

The sequences y is characterized by its *moment matrix*, $M(y)$, and the *localizing matrix*, $M(g_i y)$. Every polynomial $p(x) \in \mathcal{P}$ can be identified by its vector $\mathbf{p} = \{p_\alpha\}_{\alpha\in\mathbf{N}^n}$ of coefficients in the infinite basis v. To define the above two matrices, consider a linear mapping, $L_y : \mathcal{P} \mapsto \mathbb{R}$:

$$L_y(p) = \langle p, y\rangle = \sum_{\alpha\in\mathbf{N}^n} p_\alpha y_\alpha \tag{5.18}$$

and a bilinear mapping $\langle \cdot, \cdot \rangle_y : p_\alpha \times p_\alpha \mapsto \mathbb{R}$:

$$\begin{aligned} \langle p, q\rangle_y &= L_y(pq) = \int \langle p, v\rangle \langle v, q\rangle\, \mu(dx) && (5.19)\\ &= \int \left\langle p, vv^T q\right\rangle d\mu = \left\langle p, \int vv^T d\mu \;\; q\right\rangle && (5.20)\\ &= \langle p, M(y)\, q\rangle && (5.21) \end{aligned}$$

The moment matrix $M(y) = \int vv^T d\mu$ is indexed in the infinite basis v. Let v_k denote the finite basis of the subspaces $\mathcal{P}_k \subset \mathcal{P}$ of real polynomial with $\deg(\mathcal{P}_k) \leqslant$

k. Then, for all $p(x), q(x) \in \mathcal{P}_k$, $M_k(y) = \int v_k v_k^T d\mu$. It follows that if y has a representing measure, then $M_k(y) \geqslant 0, \quad k = 0, 1, \dots,$. Consider $g(x) \in \mathcal{P}$, $g(x) = \sum_\alpha g_\alpha x^\alpha$. The bilinear mapping associated with gy

$$\langle p, q \rangle_{gy} = L_y(gpq) = \langle p, M(gy)\, q \rangle \tag{5.22}$$

where $M(gy)$ is called the localizing matrix associated with y and g. For all polynomials in $\mathcal{P}_k$, $M_k(gy) \geqslant 0$ for all k.

A finite-dimensional relaxation of the problem can now be defined. For max (deg $p(x), \max g_i) \leqslant 2k$, a semidefinite program equivalent of (5.10) is

$$\begin{array}{lll} p_k^{mom} = & \inf y^T p \\ \text{s.t.} & y_0 = 1, M_k(y) \geqslant 0 \\ & M_{k-d_i}(g_i y) \geqslant 0, i = 1, \dots, r \end{array} \tag{5.23}$$

where $d_i = \deg(g_i)$. The problem in (5.23) can be clearly seen as an SDP relaxation of order k of the problem in (5.16) by writing $M_k(y) = \sum_\alpha B_\alpha y_\alpha$ and $M_{k-d_i}(g_i y) = \sum_\alpha C_\alpha^i y_\alpha,\ i = 1, \dots, r$ with appropriate symmetric matrices B_α, C_α^i. The SDP dual of (5.23) is the LMI problem [27, 28, 29]

$$\begin{array}{ll} & \max\limits_{\lambda, X, Z_i} \lambda \\ \text{s.t.} & \langle B_0, X \rangle + \sum\limits_{i=1}^r \left\langle C_0^i, Z_i \right\rangle = p(x) - \lambda \\ & \langle B_\alpha, X \rangle + \sum\limits_{i=1}^r \left\langle C_\alpha^i, Z_i \right\rangle = p_\alpha, i = 1, \dots, r; |\alpha| \leqslant 2k \\ & X \geqslant 0, Z_i \geqslant 0, i = 1, \dots, r \end{array} \tag{5.24}$$

and with $X, Z_i \in \Sigma^2[x]$, problem (5.24) can be written as problem (5.15). The two programs (5.15) and (5.16) give the dual formulation for the polynomial (5.10), while the programs (5.23) and (5.24) are SDP dual. By weak duality $p_k^{sos} \leqslant p_k^{mom} \leqslant p^*$, and equality $p_k^{sos} = p_k^{mom}$ when the set $\mathbf{K}$ is strictly feasible.

6. EED Problem Formulated as POP

In the EED problem of (2.1), the emission pollutant objective contain exponential terms that can be expressed as a power series using the Maclaurin series expansion:

$$e^x = \sum_{i=1}^{\infty} \frac{x^i}{i!} \tag{6.1}$$

The expansion indicates that the function resides in an infinite dimensional space spanned by the infinite monomial basis $\{1, x, x^2, \dots, x^k, \dots\}$.

Using (5.7), a Chebychev-Padé approximant of reasonable degree $[m, n]$ for exponential function can be determined such that

$$e^x \approx \frac{p(x)}{q(x)} \tag{6.2}$$

Fixing the degree $[m, n]$, Chebychev-Padé approximants, $p_i(x)/q_i(x)$, for each weighted exponential function $\zeta_i \exp(\lambda_i x_i)$ in $E_i(x)$, can be computed with Maple Chebyshev-Pade approximation function `chebpade()`.

Replacing the weighted exponential function in $E_i(x_i)$ by the rational function approximation, gives

$$E(x) = \sum_{i=1}^{N} E_i(x_i) \approx \sum_{i=1}^{N} 10^{-2}\left(a_i + b_i\, x_i + c_i\, x_i^2\right) + \frac{p_i(x_i)}{q_i(x_i)}, \tag{6.3}$$

Equation (6.3) is observed to have mixed parts, namely: polynomial and rational parts. This is different from most of the problems tackled in the literature which may have all-polynomial or all-rational functions. However, because of these fractional parts, the whole optimization problem can be reduced to an all-rational polynomial optimization problem by combining the polynomial and the rational function in each $E_i(x_i)$ into fractional polynomial function and then sum them to a single fractional function $E(x)$. This is observed to increase the degree of both the numerator and the denominator polynomials, and consequently the complexity of the problem.

Another approach, namely the epigraph approach [26], introduces additional lifting variables r_i, for each unit, with associated constraints

$$r_i \geqslant \frac{p_i(x_i)}{q_i(x_i)}, \text{ or } \; r_i q_i(x_i) - p_i(x_i) \geqslant 0;\; i = 1, \ldots, s \tag{6.4}$$

A minimum r_i is selected such that (6.4) is satisfied.

Using the epigraph approach, (6.3) becomes

$$E(x) \approx \sum_{i=1}^{s} 10^{-2}\left(a_i + b_i\, x_i + c_i\, x_i^2\right) + \min r_i, \tag{6.5}$$

with the new feasible set $\hat{\mathbf{K}}$,

$$\hat{\mathbf{K}} = \left\{(x, r) \in \mathbb{R}^{2s}\, |\mathbf{K};\; r_i q_i(x_i) - p_i(x_i) \geqslant 0; i = 1, \ldots, s\right\} \tag{6.6}$$

This approach is noted to preserve the pattern of the problem [26]. To ensure that r_is are minimized in the program, a regularization term $\lambda\|r\|^2$ is added to $E(x)$. Thus the EED problem reduces to a multiobjective polynomial optimization problem (MOPOP) as

$$\begin{array}{rl} \textit{minimize} & [C(x), E(x), \lambda\|r\|^2] \\ \textit{subject to:} & h(x) = 0 \\ & g(x) \leq 0 \\ & r_i q_i(x_i) - p_i(x_i) \geqslant 0 \quad i = 1, \ldots, p \end{array} \tag{6.7}$$

The MOPOP is further reduced to a scalar (or standard) POP by aggregating the objectives into one in $\phi(x) = w_1 C(x) + w_2 E_k^o(x) + \lambda\|r\|^2$ using the weighted sum method. The problem thus reduces to the standard POP form with a regularization term:

$$
\begin{array}{rl}
\textit{minimize} & \phi(x) = w_1 C(x) + w_2 E_k(x) + \lambda \|r\|^2 \\
\textit{subject to:} & h(x) \leq 0;\ h(x) \geq 0 \\
 & g(x) \leq 0 \\
 & r_i q_i(x_i) - p_i(x_i) \geqslant 0;\ i = 1, \ldots, p
\end{array} \tag{6.8}
$$

SDP relaxation of the resulting POP was performed using Gloptipoly; an efficient SDP parser for POP.

Gloptipoly is a freely available MATLAB software that implements POP solution algorithm based on the theory of moments [30]. It builds hierarchy of SDP relaxations of increasing dimension whose associated monotone sequence of optimal values converges to the global value and provides theoretical guarantee of the asymptotic convergence to the global optimum at low relaxation order. It also gives the global optimal value and can extract the global optimizer. It solves the resulting SDP using SeDuMi solver [31].

7. Simulations, results and discussion

The algorithm was tested on the standard IEEE 30-bus 6-generator test system to investigate the effectiveness of the approach. The total real load on the system is 283.4 MW. Details of the bus and line data of the test system, including the cost coefficients, emission coefficients and power generation limits, can be found in [32].

The rational polynomial approximation is compared with polynomial approximations. Two degrees of polynomial approximation (degree 4 and 6) and two rational polynomial approximations of degrees $[m, n] = [1, 1]$ and $[2, 2]$, were considered for the emission function using the Chebychev-Pade approximation in (5.7).

The resulting multiobjective POP was reduced to single objective POP using the weighted sum method. Gloptipoly was applied to solve each of the resulting POPs. Table 7.1 shows the simulation results for the extreme points of the Pareto front for the different approximations. It is interesting to note in Table 7.1, that although the polynomial approximations had greater number of coefficients to be determined, the computational time is less than that required by the problems with rational approximations. This is contrary to expectations. Analysis of the matrix of the semidefinite program generated by Sedumi [31], see Table 7.2, showed that, apart from the degree of the polynomials and the number of variables in the polynomial, the number of constraints which determines the sparsity of the resulting SDP matrix is a major factor determining the complexity of the resulting SDP problem. In Table 7.2, the sparsity of the matrix is measured by the number non-zero (nnz) elements in the matrix, while m and n specify the size of the matrix. For the polynomial approximations, while the size of the matrix did not change, nnz is observed to almost double by the inclusion of the transmission losses in the power balance equality constraint. This large change can also be observed for the rational approximations, too. Also to be noted in (6.6) is that for any number

Table 7.1: Best Solutions Comparison between Rational and Polynomial Approximations

Case		Rational	Rational	Polynomial	Polynomial
	$[m, n]$	[1,1]	[2,2]	[4,0]	[6,0]
Minimum Fuel Cost Case I					
	Cost ($/h)	**600.1114**	**600.1114**	**600.1114**	**600.1114**
	Emi	0.2219	0.2219	0.2222	0.22214
	Time (s)	37.33	32.17	0.45	19.75
Minimum Emission Case I					
	Cost ($/h)	638.721	638.3375	638.300	638.269
	Emi	**0.1944**	**0.1942**	**0.1942**	**0.1942**
	Time (s)	67.30	60.81	0.56	20.83
Minimum Fuel Cost Case II					
	Cost ($/h)	**606.2348**	**606.2348**	**606.2348**	**606.2348**
	Emi	0.2194	0.2196	0.2196	0.2196
	Time (s)	40.90	48.45	0.45	26.01
Minimum Emission Case II					
	Cost ($/h)	644.5435	644.1825	644.1087	644.1105
	Emi	**0.19438**	**0.19419**	**0.194192**	**0.194183**
	Time (s)	76.88	67.69	0.56	24.32

Table 7.2: The sparsity of the resulting SDP matrix

deg		m	n	nnz
4,0	w/out losses	209	127	1455
4,0	with losses	209	127	2631
6,0	w/out losses	923	477	18983
6,0	with losses	923	477	36035
1,1	w/out losses	1819	276	9807
1,1	with losses	1819	276	13629
2,2	w/out losses	1819	276	8091
2,2	with losses	1819	276	11913

Table 7.3: Best Solutions for Fuel Cost without Transmission Losses

	SDP	SDP	LP	NPGA	SPEA	NSGA-II
$[m, n]$	[1,1]	[2,2]				
P_{g_1}	0.1097	0.1097	0.1500	0.1080	0.1062	0.1050
P_{g_2}	0.2998	0.2998	0.3000	0.3284	0.2897	0.3177
P_{g_3}	0.5243	0.5243	0.5500	0.5386	0.5289	0.5216
P_{g_4}	1.0161	1.0161	1.0500	1.0067	1.0025	1.0146
P_{g_5}	0.5243	0.5243	0.4600	0.4949	0.5402	0.5159
P_{g_6}	0.3597	0.3597	0.3500	0.3574	0.3664	0.3583
Cost	**600.1114**	**600.1114**	**604.15**	**600.259**	**600.15**	**600.155**
Emi	0.2219	0.2219	0.2233	0.22116	0.2215	0.22188

Table 7.4: Best Solutions for Emission without Transmission Losses

	SDP	SDP	LP	NPGA	SPEA	NSGA-II
$[m, n]$	[1,1]	[2,2]				
P_{g_1}	0.4020	0.4058	0.4000	0.4002	0.4116	0.4077
P_{g_2}	0.4605	0.4587	0.4500	0.4474	0.4532	0.4577
P_{g_3}	0.5371	0.5387	0.5500	0.5166	0.5329	0.5389
P_{g_4}	0.3783	0.3818	0.4000	0.3688	0.3832	0.3837
P_{g_5}	0.5371	0.5387	0.5500	0.5751	0.5383	0.5352
P_{g_6}	0.5190	0.5103	0.5000	0.5259	0.5148	0.5110
Cost	638.721	638.338	639.600	639.182	638.51	638.269
Emi	**0.1944**	**0.1942**	**0.1942**	**0.1943**	**0.1942**	**0.1942**

of lifting variables introduced, there is the same number of inequality constraints added to the problem. This may be responsible for the exceptional increase in the computational time and complexity of problems with rational approximation.

The results for the best fuel cost and the best emission objectives against those reported using Linear Programming (LP) [33], Strength Pareto Evolutionary Algorithm (SPEA) [3], Non-dominated Sorting Genetic Algorithm-II (NSGA-II) [34] and Niched Pareto Genetic Algorithm (NPGA) [1], with and without the transmission losses, are shown in Tables 7.4 -7.6. Notice that the solutions with rational approximations were not dominated. It actually dominated most of the reported results. This is indicative of the effectiveness of the approximation.

In order to explore the Pareto front generated using the rational approximation considered, twenty one runs were carried out on the problem using the bi-quadratic rational approximation. The generated Pareto fronts, both with and without the transmission losses, using the weighted sum with nonlinear weight selection in [20], are as shown in Figure 7.1. In Figure 7.1, a typical value of 12.123 was used for the free parameter introduced in [20].

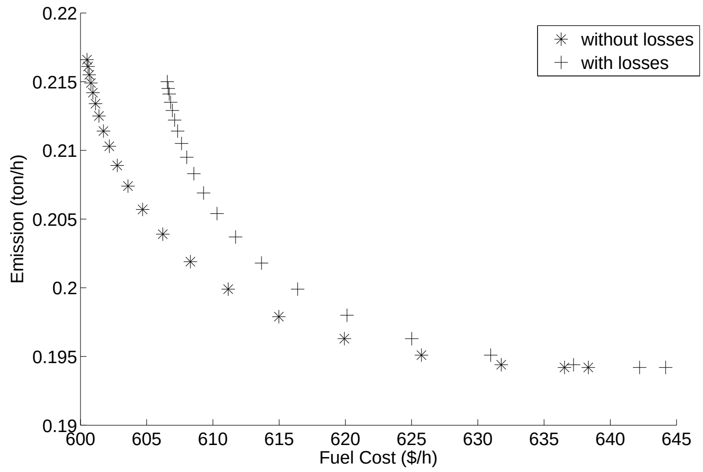

FIG. 7.1: Pareto fronts for the bi-quadratic rational approximation with and without Transmission Losses

Table 7.5: Best Solutions for Fuel Cost with Transmission Losses

	SDP	SDP	NPGA	SPEA	NSGA-II
$[m,n]$	[1,1]	[2,2]			
P_{g_1}	0.1134	0.1134	0.1245	0.1086	0.1182
P_{g_2}	0.2990	0.2990	0.2797	0.3056	0.3142
P_{g_3}	0.5977	0.5977	0.6284	0.5818	0.5910
P_{g_4}	0.9736	0.9737	1.0264	0.9846	0.9710
P_{g_5}	0.5218	0.5218	0.4693	0.5288	0.5172
P_{g_6}	0.3546	0.3546	0.3993	0.3584	0.3548
Cost	**606.2348**	**606.2348**	**608.147**	**607.807**	**608.147**
Emi	0.2194	0.2196	0.22364	0.22015	0.22364

Table 7.6: Best Solutions for Emission with Transmission Losses

	SDP	SDP	NPGA	SPEA	NSGA-II
$[m, n]$	[1,1]	[2,2]			
P_{g_1}	0.4055	0.4097	0.3923	0.4043	0.4141
P_{g_2}	0.4639	0.4624	0.4700	0.4525	0.4602
P_{g_3}	0.5419	0.5430	0.5565	0.5525	0.5429
P_{g_4}	0.3941	0.3876	0.3695	0.4079	0.4011
P_{g_5}	0.5420	0.5431	0.5599	0.5468	0.5422
P_{g_6}	0.5227	0.5143	0.5163	0.5005	0.5045
Cost	644.544	644.182	645.984	642.603	644.133
Emi	**0.19439**	**0.19419**	**0.19424**	**0.19422**	**0.19419**

8. Conclusion

In this paper, a multiobjective economic-emission dispatch problem with transmission losses is formulated as a convex optimization problem through SDP relaxation technique, and solved. Although the problem is an infinite-dimensional polynomial problem, a finite dimensional rational polynomial approximation was computed. Aggregation of the objectives using the non linear weight selection weighted sum reduced the multiobjective problem into a scalar form. A free MATLAB software, Gloptipoly, that efficiently solves POP was employed. The SDP-based weighted sum shows good convergence property and better exploration of the Pareto front was achieved through non linear weight selection.

A numerical example is considered which shows that the proposed formulation is efficient. And when compared with well known evolutionary algorithms, it was observed to have better convergence properties.

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