FACTA UNIVERSITATIS (NIŠ)
Ser. Math. Inform. Vol. 32, No 4 (2017), 503–514
https://doi.org/10.22190/FUMI1704503P

CONFORMAL CURVATURE TENSOR ON K-CONTACT MANIFOLDS WITH RESPECT TO THE QUARTER-SYMMETRIC METRIC CONNECTION

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Abstract. In the present paper, we study certain curvature conditions on the conformal curvature tensor in K-contact manifolds with respect to the quarter-symmetric metric connection.
Keywords. K-contact manifold, conformal curvature tensor, η-Einstein manifold, quarter-symmetric metric connection

1. Introduction

An emerging branch of modern mathematics is the geometry of contact manifolds. The notion of contact geometry has evolved from the mathematical formalism of classical mechanics [8]. Two important classes of contact manifolds are K-contact manifolds and Sasakian manifolds ([7], [17]). K-contact manifolds have been studied by several authors ([4], [10], [13], [14], [20], [22]) and many others.

Let $\bar{\nabla}$ be a linear connection in a Riemannian manifold M. The torsion tensor T is given by

$$T(X,Y) = \bar{\nabla}_X Y - \bar{\nabla}_Y X - [X,Y].$$

The connection $\bar{\nabla}$ is symmetric if its torsion tensor vanishes, otherwise it is non-symmetric. The connection $\bar{\nabla}$ is a metric connection if there is a Riemannian metric g in M such that $\bar{\nabla}_X g = 0$, otherwise it is non-metric. It is well known that a linear connection is symmetric and metric if and only if it is the Levi-Civita connection.

A. Friedmann and J.A. Schouten introduced the idea of a semi-symmetric linear connection [2]. A linear connection $\bar{\nabla}$ is said to be a semi-symmetric connection if

Received March 04, 2017; accepted May 04, 2017
2010 *Mathematics Subject Classification.* Primary 53C05; Secondary 53D10, 53C25

its torsion tensor T is of the form

$$T(X,Y) = \eta(Y)X - \eta(X)Y$$

for all $X, Y \in \chi(M)$, where $\chi(M)$ is the Lie algebra of vector fields on the manifold M.

S. Golab introduced the idea of a quarter symmetric linear connection in a differentiable manifold [16]. A linear connection is said to be a quarter-symmetric connection if its torsion tensor T is of the form

$$T(X,Y) = \eta(Y)\phi X - \eta(X)\phi Y,$$

where η is a 1-form and ϕ is a $(1,1)$ tensor field. If we put $\phi X = X$ and $\phi Y = Y$, then the quarter-symmetric metric connection reduces to the semi-symmetric metric connection [2]. Thus the notion of the quarter-symmetric connection generalizes the notion of the semi-symmetric connection. A quarter-symmetric metric connection have been studied by various authors ([1], [3], [12], [15], [18], [19], [21]).

A relation between the quarter-symmetric metric connection $\bar{\nabla}$ and the Levi-Civita connection ∇ in an n-dimensional K-contact manifold M is given by [6]

$$\bar{\nabla}_X Y = \nabla_X Y - \eta(X)\phi Y. \tag{1.1}$$

Motivated by the above studies, in this paper we study certain curvature conditions on the conformal curvature tensor in K-contact manifolds with respect to the quarter-symmetric metric connection. The paper is organized as follows: In Section 2, we give a brief introduction of K-contact manifolds. In Section 3, we deduce the relation between the curvature tensor of K-contact manifolds with respect to the quarter-symmetric metric connection and the Levi-Civita connection. In Section 4, we consider conformal curvature tensor with respect to the quarter-symmetric metric connection and discuss its characteristic properties. Section 5 is devoted to study flatness conditions on K-contact manifolds with respect to the quarter-symmetric metric connection.

2. Preliminaries

Let M be an almost contact metric manifold of dimension n equipped with an almost contact metric structure (ϕ, ξ, η, g) admitting a $(1,1)$ tensor field ϕ, a vector field ξ, a 1-form η and a Riemannian metric g. Then

$$\phi^2 X = -X + \eta(X)\xi, \quad \eta(\xi) = 1, \quad \phi\xi = 0, \quad \eta(\phi X) = 0, \tag{2.1}$$

$$g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y), \quad g(X,\xi) = \eta(X), \tag{2.2}$$

$$g(X, \phi Y) = -g(\phi X, Y) \tag{2.3}$$

for all vector fields X, Y on M.

An almost contact metric manifold is
(i) a contact manifold if $g(X, \phi Y) = d\eta(X, Y)$; and
(ii) a Sasakian manifold if $(\nabla_X \phi)Y = g(X, Y)\xi - \eta(Y)X$
for every X, Y on M.

A contact metric manifold is K-contact if and only if the $(1, 1)$ type tensor field h defined by $h = \frac{1}{2}\pounds_\xi \phi$ is equal to zero, where $\pounds$ denotes Lie differentiation. Every Sasakian manifold is K-contact but the converse is not true, in general. However a three-dimensional K-contact manifold is a Sasakian manifold [9]. It is well known that (M, g) is Sasakian if and only if

$$R(X, Y)\xi = \eta(Y)X - \eta(X)Y \tag{2.4}$$

for all vector fields X, Y on M.

In a K-contact manifold M, the following relations hold:

$$\nabla_X \xi = -\phi X, \tag{2.5}$$

$$g(R(X, Y)Z, \xi) = g(Y, Z)\eta(X) - g(X, Z)\eta(Y), \tag{2.6}$$

$$R(\xi, X)Y = g(X, Y)\xi - \eta(Y)X, \quad R(\xi, X)\xi = -X + \eta(X)\xi, \tag{2.7}$$

$$S(X, \xi) = (n - 1)\eta(X), \quad Q\xi = (n - 1)\xi \tag{2.8}$$

for any vector fields X, Y and Z, where R and S are the Riemannian curvature tensor and the Ricci tensor of M, respectively.

Definition 2.1. A K-contact manifold M is said to be an η-Einstein manifold if its Ricci tensor S is of the form [11]

$$S(X, Y) = ag(X, Y) + b\eta(X)\eta(Y), \tag{2.9}$$

where a and b are smooth functions on M.

Definition 2.2. The conformal curvature tensor C on a K-contact manifold M is defined by [11]

$$C(X, Y)Z = R(X, Y)Z - \frac{1}{(n-2)}[S(Y, Z)X - S(X, Z)Y \tag{2.10}$$

$$+g(Y, Z)QX - g(X, Z)QY] + \frac{r}{(n-1)(n-2)}[g(Y, Z)X - g(X, Z)Y],$$

where R, S, Q and r are the Riemannian curvature tensor, the Ricci tensor, the Ricci operator and the scalar curvature, respectively.

3. Curvature tensor on K-contact manifolds with respect to the quarter-symmetric metric connection

If R and $\bar{R}$, respectively, are the curvature tensors of the Levi-Civita connection ∇ and the quarter-symmetric metric connection $\bar{\nabla}$ on a K-contact manifold M. Then we have [6]

$$\bar{R}(X,Y)Z = R(X,Y)Z + 2g(\phi X,Y)\phi Z + [\eta(X)g(Y,Z) - \eta(Y)g(X,Z)]\xi + \eta(Z)[\eta(Y)X - \eta(X)Y], \tag{3.1}$$

$$\bar{R}(X,Y)\xi = 2[\eta(Y)X - \eta(X)Y], \tag{3.2}$$

$$\bar{R}(\xi,X)Y = 2[g(X,Y)\xi - \eta(Y)X], \tag{3.3}$$

$$\bar{S}(Y,Z) = S(Y,Z) - g(Y,Z) + n\eta(Y)\eta(Z), \tag{3.4}$$

$$\bar{S}(X,\xi) = 2(n-1)\eta(X), \quad \bar{S}(\xi,\xi) = 2(n-1), \tag{3.5}$$

$$\bar{Q}Y = QY - Y + n\eta(Y)\xi, \quad \bar{Q}\xi = 2(n-1)\xi, \tag{3.6}$$

$$\bar{r} = r \tag{3.7}$$

for all vector fields $X, Y, Z \in \chi(M)$.

4. Conformal curvature tensor on K-contact manifolds with respect to the quarter-symmetric metric connection

Analogous to the Definition 2.2, the conformal curvature tensor $\bar{C}$ on a K-contact manifold M with respect to the quarter-symmetric metric connection $\bar{\nabla}$ is given by

$$\bar{C}(X,Y)Z = \bar{R}(X,Y)Z - \frac{1}{(n-2)}[\bar{S}(Y,Z)X - \bar{S}(X,Z)Y + g(Y,Z)\bar{Q}X - g(X,Z)\bar{Q}Y] + \frac{\bar{r}}{(n-1)(n-2)}[g(Y,Z)X - g(X,Z)Y], \tag{4.1}$$

where $\bar{R}$, $\bar{S}$, $\bar{Q}$ and $\bar{r}$ are the Riemannian curvature tensor, the Ricci tensor, the Ricci operator and the scalar curvature with respect to the connection $\bar{\nabla}$, respectively on M.

Using (3.1), (3.4), (3.6) and (3.7) in (4.1), we get

$$\bar{C}(X,Y)Z = C(X,Y)Z + 2g(\phi X,Y)\phi Z + \frac{2}{(n-2)}[g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi + \eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X + g(Y,Z)X - g(X,Z)Y], \tag{4.2}$$

where

$$C(X,Y)Z = R(X,Y)Z - \frac{1}{(n-2)}[S(Y,Z)X - S(X,Z)Y$$

$$+g(Y,Z)QX - g(X,Z)QY] + \frac{r}{(n-1)(n-2)}[g(Y,Z)X - g(X,Z)Y]$$

is the conformal curvature tensor with respect to the Levi-Civita connection ∇. Putting $Z = \xi$ in (4.2) and using (2.1) and (2.2), we get

$$\bar{C}(X,Y)\xi = C(X,Y)\xi. \tag{4.3}$$

Hence we can state the following theorem:

Theorem 4.1. *An n-dimensional K-contact manifold is ξ-conformally flat with respect to the quarter-symmetric metric connection if and only if the manifold is also ξ-conformally flat with respect to the Levi-Civita connection.*

Taking the inner product of (4.2) with U, we have

$$\bar{C}(X,Y,Z,U) = C(X,Y,Z,U) + 2g(\phi X,Y)g(\phi Z,U) \tag{4.4}$$

$$+\frac{2}{(n-2)}[g(X,Z)\eta(Y)\eta(U) - g(Y,Z)\eta(X)\eta(U) + g(Y,U)\eta(X)\eta(Z)$$

$$-g(X,U)\eta(Y)\eta(Z) + g(Y,Z)g(X,U) - g(X,Z)g(Y,U)],$$

where $g(C(X,Y)Z,U) = C(X,Y,Z,U)$ and $g(\bar{C}(X,Y)Z,U) = \bar{C}(X,Y,Z,U)$ are the conformal curvature tensors with respect to the connections ∇ and $\bar{\nabla}$, respectively on M.
Interchanging X and Y in (4.4), we have

$$\bar{C}(Y,X,Z,U) = C(Y,X,Z,U) + 2g(\phi Y,X)g(\phi Z,U) \tag{4.5}$$

$$+\frac{2}{(n-2)}[g(Y,Z)\eta(X)\eta(U) - g(X,Z)\eta(Y)\eta(U) + g(X,U)\eta(Y)\eta(Z)$$

$$-g(Y,U)\eta(X)\eta(Z) + g(X,Z)g(Y,U) - g(Y,Z)g(X,U)].$$

On adding (4.4) and (4.5), we get

$$\bar{C}(X,Y,Z,U) + \bar{C}(Y,X,Z,U) = 0. \tag{4.6}$$

Interchanging Z and U in (4.4), we have

$$\bar{C}(X,Y,U,Z) = C(X,Y,U,Z) + 2g(\phi X,Y)g(\phi U,Z) \tag{4.7}$$

$$+\frac{2}{(n-2)}[g(X,U)\eta(Y)\eta(Z)-g(Y,U)\eta(X)\eta(Z)+g(Y,Z)\eta(X)\eta(U)$$

$$-g(X,Z)\eta(Y)\eta(U)+g(Y,U)g(X,Z)-g(X,U)g(Y,Z)].$$

Adding (4.4) and (4.7), we get

$$\bar{C}(X,Y,Z,U)+\bar{C}(X,Y,U,Z)=0. \tag{4.8}$$

Again interchanging pair of slots in (4.4), we have

$$\bar{C}(Z,U,X,Y)=C(Z,U,X,Y)+2g(\phi Z,U)g(\phi X,Y) \tag{4.9}$$

$$+\frac{2}{(n-2)}[g(Z,X)\eta(U)\eta(Y)-g(U,X)\eta(Z)\eta(Y)+g(U,Y)\eta(Z)\eta(X)$$

$$-g(Z,Y)\eta(U)\eta(X)+g(U,X)g(Z,Y)-g(Y,U)g(X,Z)].$$

Now, subtracting (4.9) from (4.4), we get

$$\bar{C}(X,Y,Z,U)-\bar{C}(Z,U,X,Y)=0. \tag{4.10}$$

Thus in view of (4.6), (4.8) and (4.10), we can state the following theorem:

Theorem 4.2. *In an n-dimensional K-contact manifold with respect to the quarter-symmetric metric connection, we have*
(i) $\bar{C}(X,Y,Z,U)+\bar{C}(Y,X,Z,U)=0$;
(ii) $\bar{C}(X,Y,Z,U)+\bar{C}(X,Y,U,Z)=0$;
(iii) $\bar{C}(X,Y,Z,U)-\bar{C}(Z,U,X,Y)=0$
for any vector fields $X,Y,Z,U\in\chi(M)$.

Now, let $\bar{R}(X,Y)Z=0$, then from (3.2), we have

$$R(X,Y)Z=2g(X,\phi Y)\phi Z+[\eta(Y)g(X,Z)-\eta(X)g(Y,Z)]\xi \tag{4.11}$$

$$+[\eta(X)Y-\eta(Y)X]\eta(Z).$$

Taking the inner product of (4.11) with ξ and using (2.1) and (2.2), we have

$$g(R(X,Y)Z,\xi)=-[g(Y,Z)\eta(X)-g(X,Z)\eta(Y)] \tag{4.12}$$

which can be written as

$$g(R(X,Y)Z,U)=-[g(Y,Z)g(X,U)-g(X,Z)g(Y,U)]. \tag{4.13}$$

Thus we can state the following theorem:

Theorem 4.3. *If the curvature tensor of a quarter-symmetric metric connection in a K-contact manifold M vanishes, then the manifold is of constant curvature tensor -1 and consequently it is locally isometric to the hyperbolic space $H^n(-1)$.*

5. Flatness conditions on K-contact manifolds with respect to the quarter-symmetric metric connection

Definition 5.1. A K-contact manifold is said to be
(i) conformally flat with respect to the quarter-symmetric metric connection, if

$$\bar{C}(X,Y)Z = 0 \qquad for\ all\ \ X,Y,Z \in \chi(M); \tag{5.1}$$

(ii) ξ-conformally flat with respect to the quarter-symmetric metric connection, if

$$\bar{C}(X,Y)\xi = 0 \qquad for\ all\ \ X,Y \in \chi(M); \tag{5.2}$$

(iii) quasi-conformally flat with respect to the quarter-symmetric metric connection, if

$$g(\bar{C}(X,Y)Z,\phi W) = 0 \qquad for\ all\ \ X,Y,Z,W \in \chi(M); \quad and \tag{5.3}$$

(iv) ϕ-conformally flat with respect to the quarter-symmetric metric connection, if

$$\phi^2\bar{C}(\phi X,\phi Y)\phi Z = 0 \qquad for\ all\ \ X,Y,Z \in \chi(M). \tag{5.4}$$

Firstly, we consider that the manifold M with respect to the quarter-symmetric metric connection is conformally flat. Then from (4.1) and (5.1) it follows that

$$\bar{R}(X,Y)Z = \frac{1}{(n-2)}[\bar{S}(Y,Z)X - \bar{S}(X,Z)Y + g(Y,Z)\bar{Q}X - g(X,Z)\bar{Q}Y]$$
$$-\frac{\bar{r}}{(n-1)(n-2)}[g(Y,Z)X - g(X,Z)Y]. \tag{5.5}$$

Taking the inner product of (5.5) with ξ and using then (2.2), we have

$$g(\bar{R}(X,Y)Z,\xi) = \frac{1}{(n-2)}[\bar{S}(Y,Z)\eta(X) - \bar{S}(X,Z)\eta(Y)$$
$$+g(Y,Z)\bar{S}(X,\xi) - g(X,Z)\bar{S}(Y,\xi)] - \frac{\bar{r}}{(n-1)(n-2)}[g(Y,Z)\eta(X) - g(X,Z)\eta(Y)]. \tag{5.6}$$

Putting $X=\xi$ in (5.6) and using (2.1), (3.3) and (3.5), we have

$$\bar{S}(Y,Z) = (\frac{\bar{r}}{n-1} - 2)g(Y,Z) - (\frac{\bar{r}}{n-1} - 2n)\eta(Y)\eta(Z). \tag{5.7}$$

In view of (3.4) and (3.7), (5.7) takes the form

$$S(Y,Z) = (\frac{r}{n-1} - 1)g(Y,Z) + (n - \frac{r}{n-1})\eta(Y)\eta(Z). \tag{5.8}$$

Secondly, we consider that the manifold M with respect to the quarter-symmetric metric connection is ξ-conformally flat. Then from (4.1) and (5.2) it follows that

$$g[\bar{R}(X,Y)\xi - \frac{1}{(n-2)}(\bar{S}(Y,\xi)X - \bar{S}(X,\xi)Y + g(Y,\xi)\bar{Q}X - g(X,\xi)\bar{Q}Y) + \frac{\bar{r}}{(n-1)(n-2)}(g(Y,\xi)X - g(X,\xi)Y), W] = 0. \tag{5.9}$$

Using (2.2), (3.2) and (3.5) in (5.9), we have

$$(\frac{\bar{r}}{n-1} - 2)(\eta(Y)g(X,W) - \eta(X)g(Y,W)) - \eta(Y)\bar{S}(X,W) + \eta(X)\bar{S}(Y,W) = 0. \tag{5.10}$$

Taking $Y = \xi$ in (5.10) and using (2.1), (2.2), (3.5) and (3.7), we have

$$\bar{S}(X,W) = (\frac{r}{n-1} - 2)g(X,W) + (2n - \frac{r}{n-1})\eta(X)\eta(W)$$

which in view of (3.4), takes the form

$$S(X,W) = (\frac{r}{n-1} - 1)g(X,W) + (n - \frac{r}{n-1})\eta(X)\eta(W). \tag{5.11}$$

Thirdly, we consider that the manifold M with respect to the quarter-symmetric metric connection is quasi-conformally flat. Then from (4.1) and (5.3) it follows that

$$g[\bar{R}(X,Y)Z,\phi W] = \frac{1}{(n-2)}[\bar{S}(Y,Z)g(X,\phi W) - \bar{S}(X,Z)g(Y,\phi W) + g(Y,Z)\bar{S}(X,\phi W) - g(X,Z)\bar{S}(Y,\phi W)] - \frac{\bar{r}}{(n-1)(n-2)}[g(Y,Z)g(X,\phi W) - g(X,Z)g(Y,\phi W)]. \tag{5.12}$$

By considering $Y = Z = \xi$ and using (2.1), (2.2), (3.2) and (3.5), (5.12) reduces to

$$\bar{S}(X,\phi W) = (\frac{\bar{r}}{n-1} - 2)g(X,\phi W). \tag{5.13}$$

Replacing W by ϕW in (5.13) and using (2.1), (3.5) and (3.7), we have

$$\bar{S}(X,W) = (\frac{r}{n-1} - 2)g(X,W) + (2n - \frac{r}{n-1})\eta(X)\eta(W)$$

which in view of (3.4), takes the form

$$S(X,W) = (\frac{r}{n-1} - 1)g(X,W) + (n - \frac{r}{n-1})\eta(X)\eta(W). \tag{5.14}$$

Finally, we consider that the manifold is ϕ-conformally flat with respect to the quarter-symmetric metric connection. Then from (5.4), we have

$$g(\bar{C}(\phi X, \phi Y)\phi Z, \phi W) = 0 \tag{5.15}$$

for any $X, Y, Z, W \in \chi(M)$. In view of (4.1), (5.15) takes the form

$$g(\bar{R}(\phi X, \phi Y)\phi Z, \phi W) = \frac{1}{(n-2)}[g(\phi Y, \phi Z)\bar{S}(\phi X, \phi W) \tag{5.16}$$

$$-g(\phi X, \phi Z)\bar{S}(\phi Y, \phi W) + g(\phi X, \phi W)\bar{S}(\phi Y, \phi Z) - g(\phi Y, \phi W)\bar{S}(\phi X, \phi Z)]$$

$$-\frac{\bar{r}}{(n-1)(n-2)}[g(\phi Y, \phi Z)g(\phi X, \phi W) - g(\phi X, \phi Z)g(\phi Y, \phi W)].$$

Now using (2.1), (3.1), (3.4) and (3.7) in (5.16), we have

$$g(R(\phi X, \phi Y)\phi Z, \phi W) = 2g(Y, \phi X)g(Z, \phi W) \tag{5.17}$$

$$+\frac{1}{(n-2)}[S(\phi Y, \phi Z)g(\phi X, \phi W) - g(\phi Y, \phi Z)g(\phi X, \phi W) - S(\phi X, \phi Z)g(\phi Y, \phi W)$$

$$+g(\phi X, \phi Z)g(\phi Y, \phi W) + S(\phi X, \phi W)g(\phi Y, \phi Z) - g(\phi X, \phi W)g(\phi Y, \phi Z)$$

$$-S(\phi Y, \phi W)g(\phi X, \phi Z) + g(\phi Y, \phi W)g(\phi X, \phi Z)]$$

$$-\frac{r}{(n-1)(n-2)}[g(\phi Y, \phi Z)g(\phi X, \phi W) - g(\phi X, \phi Z)g(\phi Y, \phi W)].$$

Let $\{e_1, e_2, ..., e_{n-1}, \xi\}$ be a local orthonormal basis of vector fields in M. Using that $\{\phi e_1, \phi e_2, ..., \phi e_{n-1}, \xi\}$ is also a local orthonormal basis, if we put $X = W = e_i$ in (5.17) and sum up with respect to i, then

$$\sum_{i=1}^{n-1} g[R(\phi e_i, \phi Y)\phi Z, \phi e_i] = 2\sum_{i=1}^{n-1} g(Y, \phi e_i)g(Z, \phi e_i) + \frac{1}{(n-2)}\sum_{i=1}^{n-1}[S(\phi Y, \phi Z)g(\phi e_i, \phi e_i) \tag{5.18}$$

$$-g(\phi Y, \phi Z)g(\phi e_i, \phi e_i) - S(\phi e_i, \phi Z)g(\phi Y, \phi e_i) + g(\phi e_i, \phi Z)g(\phi Y, \phi e_i)$$

$$+S(\phi e_i, \phi e_i)g(\phi Y, \phi Z) - g(\phi e_i, \phi e_i)g(\phi Y, \phi Z) - S(\phi Y, \phi e_i)g(\phi e_i, \phi Z)$$

$$+g(\phi Y, \phi e_i)g(\phi e_i, \phi Z)] + \frac{r}{(n-1)(n-2)}\sum_{i=1}^{n-1}[g(\phi e_i, \phi Z)g(\phi Y, \phi e_i) - g(\phi Y, \phi Z)g(\phi e_i, \phi e_i)].$$

It can be verified easily that [5]

$$\sum_{i=1}^{n-1} g(R(\phi e_i, \phi Y)\phi Z, \phi e_i) = S(\phi Y, \phi Z) - g(\phi Y, \phi Z), \tag{5.19}$$

$$\sum_{i=1}^{n-1} S(\phi e_i, \phi e_i) = r - (n-1), \tag{5.20}$$

$$\sum_{i=1}^{n-1} g(\phi e_i, \phi Z) S(\phi Y, \phi e_i) = S(\phi Y, \phi Z), \tag{5.21}$$

$$\sum_{i=1}^{n-1} g(\phi e_i, \phi e_i) = n - 1, \tag{5.22}$$

$$\sum_{i=1}^{n-1} g(\phi e_i, \phi Z) g(\phi Y, \phi e_i) = g(\phi Y, \phi Z), \tag{5.23}$$

$$\sum_{i=1}^{n-1} g(\phi e_i, Y) g(\phi e_i, Z) = g(Y, Z) - \eta(Y)\eta(Z). \tag{5.24}$$

By virtue of (5.19)-(5.24), the equation (5.18) can be written as

$$S(\phi Y, \phi Z) - g(\phi Y, \phi Z) = +2g(Y, Z) - 2\eta(Y)\eta(Z) \tag{5.25}$$

$$+\frac{1}{(n-2)}[(r - 3n + 5)g(\phi Y, \phi Z) + (n-3)S(\phi Y, \phi Z)] - \frac{r}{(n-1)} g(\phi Y, \phi Z)$$

from which it follows that

$$S(\phi Y, \phi Z) = 2(n-2)g(Y, Z) - 2(n-2)\eta(Y)\eta(Z) \tag{5.26}$$

$$+\left(\frac{r}{n-1} - 2n + 3\right) g(\phi Y, \phi Z).$$

By replacing Y by ϕY and Z by ϕZ and using (2.1), (2.2) and (2.8), (5.26) becomes

$$S(Y, Z) = \left(\frac{r}{n-1} - 1\right) g(Y, Z) - \left(\frac{r}{n-1} - n\right) \eta(Y)\eta(Z). \tag{5.27}$$

Equations (5.8), (5.11), (5.14) and (5.27) are of the form $S(X, Y) = ag(X, Y) + b\eta(X)\eta(Y)$, where $a = (\frac{r}{n-1} - 1)$ and $b = (n - \frac{r}{n-1})$. Thus, we can state the following theorem:

Theorem 5.1. *Conformally flat, ξ-conformally flat, quasi-conformally flat and ϕ-conformally flat K-contact manifolds of dimensional n $(n > 3)$ with respect to the quarter-symmetric metric connection are an η-Einstein manifold.*

Acknowledgement. The authors are thankful to the editor and anonymous referees for their valuable suggestions in the improvement of the paper.

REFERENCES

1. A. Barman: *Weakly symmetric and weakly Ricci-symmetric LP-Sasakian manifolds admitting a quarter-symmetric metric connection*, Novi Sad J. Math. **45**(2015), 143-153.
2. A. Friedmann and J. A. Schouten: *Uber die Geometric der halbsymmetrischen Ubertragung*, Math. Zeitschr. **21**(1924), 211-223.
3. A. Haseeb: *Some new results on para-Sasakian manifolds with a quarter-symmetric metric connection*, Facta Universitatis (NIŠ), Ser. Math. Inform. **30 (5)** (2015), 765-776.
4. A. Yildiz and E. Ata: *On a type of K-conatct manifolds*, Hacettepe J. Math. Stat. **41(4)**(2012), 567-571.
5. C. Özgür: *ϕ-conformally flat Lorentzian para-Sasakian manifolds*, Radovi Matematički **12**(2003), 99-106.
6. C. S. Bagewadi and G. Ingalahalli : *A study on the ϕ-symmetric K-contact manifold admitting quarter-symmetric metric connection*, Journal of Mathematical Physics, Analysis, Geometry **10(4)** (2014), 399-411.
7. D. E. Blair: *Contact manifolds in Riemannian geometry*, Lecture Notes in Math. **509**, Springer, Berlin-New York, 1976.
8. H. Geiges: *A brief history of contact geometry and topology*, Expo. Math. **19(1)** (2001), 25-53.
9. J. B. Jun and U. K. Kim: *On 3-dimensional almost contact metric manifolds*, Kyungpook Math. J. **34**(1994), 293-301.
10. J. L. Cabrerizo, L. M. Fernandes, M. Fernandes and G. Zhen: *The structure of a class of K-contact manifolds*, Acta Math. Hungarica **82(4)** (1999), 331-340.
11. K. Yano: *On semi-symmetric metric connections*, Rev. Roumaine Math. Pures Appl. **15**(1970), 1579-1586.
12. M. Ahmad, J. B. Jun and A. Haseeb : *Hypersurfaces of an almost r-paracontact Riemannian manifold endowed with a quarter-symmetric metric connection*, Bull. Korean Math. Soc. **46(3)**(2009), 477-487.
13. N. Guha and U. C. De: *On K-contact manifolds*, Serdica **19(4)** (1993), 267-272.
14. P. Majhi and U. C. De : *Concircular curvature tensor on K-contact manifolds*, Acta Math. Acad. Paeda. Nyiregyhaziensis **29**(2013), 89-99.
15. R. Prasad and A. Haseeb: *On a Lorentzian para-Sasakian manifold with respect to the quarter-symmetric metric connection*, Novi Sad J. Math. **46(2)** (2016), 103-116.
16. S. Golab: *On semi-symmetric and quarter-symmetric linear connections*, Tensor (N.S.) **29**(1975), 293-301.
17. S. Sasaki: *"Lecture Notes on Almost Contact Manifolds, Part I"*, Tohoku Univ., 1965.
18. S. Sular, C. Özgür and U. C. De: *Quarter-symmetric metric connection in a Kenmotsu manifold*, SUT J. Math., **44**(2008), 297-306.
19. S. K. Hui: *On ϕ-pseudo symmetric Kenmotsu manifolds with respect to quarter-symmetric metric connection*, Applied Sci. **15** (2013), 71-84.

20. U. C. De and A. De: *On some curvature properties of K-contact manifolds*, Extracta Math. **27** (2012), 125-134.

21. U. C. De and J. Sengupta: *Quarter-symmetric metric connection on a Sasakian manifold*, Commun. Fac. Sci. Univ. Ank. Series **49** (2000), 7-13.

22. U. C. De and K. Mandal: *On K-contact Einstein manifolds*, Novi Sad J. Math. **46(1)** (2016), 105-114.

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