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SOME COUPLED COINCIDENCE AND COMMON COUPLED FIXED POINT THEOREMS IN COMPLEX-VALUED METRIC SPACES

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Abstract. The aim of this paper is to obtain a coupled coincidence point theorem and a common coupled fixed point theorem of contractive type mappings involving rational expressions in the framework of a complex-valued metric spaces. The results of this paper generalize and extend the results of Bhaskar and Lakhmikantham [7], Azam et al. [3] and several known results in complex-valued metric spaces.

Keywords: Coupled coincidence point, common coupled fixed point, complex-valued metric space, w-compatible mappings.

1. Introduction

The fixed point theory has gained impetus, due to its wide range of applicability to resolve diverse problems emanating from the theory of non-linear differential equations, theory of non-linear integral equations, game theory, mathematical economics and so forth.

The first fixed point theorem was given by Brouwer [9] in 1912, but the credit of making concept useful and popular goes to the Polish mathematician Stephan Banach [5] who proved the famous contraction mapping theorem in 1922 which states that: Let (X, d) be a complete metric space and let $T : X \to X$ be a contraction on X, that is, there exists a constant $\lambda \in [0, 1)$ such that $d(Tx, Ty) \leqslant \lambda d(x, y)$ for all $x, y \in X$. Then T has a unique fixed point in X.

The Banach contraction principle [5] is one of the most important and useful results in the metric fixed point theory. It guarantees the existence and uniqueness of the fixed point of certain self-maps of metric spaces and provides a constructive method to find those fixed points. This principle includes different direction

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in different spaces adopted by mathematicians; for example, 2-metric spaces, G-metric spaces, partial metric spaces, cone metric spaces have already been obtained.

Recently, Azam et al [3] introduced a new space called complex-valued metric space which is more general than the well-known metric space, and obtained sufficient conditions for the existence of common fixed points of a pair of contractive type mappings involving rational expression. Subsequently, several authors have studied the existence and uniqueness of the fixed point and common fixed points of self-mappings in view of contrasting contractive conditions. Some of these investigations are noted in ([6], [10], [14],[17], [21]).

Though the complex-valued metric spaces form a special class of cone metric spaces, yet this idea is intended to define rational expressions which are not meaningful in cone metric spaces and thus many result of analysis cannot be generalized to cone metric spaces.

In [7], Bhaskar and Lakhmikantham introduced the concept of coupled fixed points for a given partially ordered set$X$. Samet et al ([19], [20]) proved that most of the coupled fixed point theorems (on ordered metric spaces) are infect immediate consequences of the well-known fixed point theorems in the literature. Very recently, Kutbi et al [14] proved the existence and uniqueness of the common coupled fixed point in complete complex-valued metric spaces in view of diverse contractive condition.

The aim of this paper is to establish a coupled coincidence point theorem for mappings on complex-valued metric spaces(in short CVMS) along with generalized contraction involving rational expression and a unique common coupled fixed point theorem using the notion of w-compatible mappings. Our results extend and improve several existing fixed point results in the literature.

2. Preliminaries

Let $\mathbb{C}$ be the set of complex numbers and $z_1, z_2 \in \mathbb{C}$, we define a partial order $\preceq$ on $\mathbb{C}$ as follows:

$$z_1 \preceq z_2 \text{ if and only if } Re(z_1) \leqslant Re(z_2) \text{ and } Im(z_1) \leqslant Im(z_2).$$

We write $z_1 \prec z_2$ if and only if $Re(z_1) < Re(z_2)$ and $Im(z_1) < Im(z_2)$.

Consistent with Azam et al. [3], we state some definitions and results about the complex-valued metric space to prove our main results.

Definition 2.1. [3] Let X be a nonempty set. Suppose that the mapping $d : X \times X \to \mathbb{C}$satisfies the following conditions:

(d1) $0 \precsim d(x, y)$ for all $x, y \in X$;

(d2) $d(x, y) = 0$ if and only if $x = y$ for all $x, y \in X$;

(d3) $d(x, y) = d(y, x)$ for all $x, y \in X$;

(d4) $d(x, y) \precsim d(x, z) + d(z, y)$ for all $x, y \in X$;

Then d is called a *complex-valued metric* on X, and (X, d) is called a *complex-valued metric space*.

Definition 2.2. [3] Let (X, d) be a complex-valued metric space.

I. A point $x \in X$ is called *interior point* of a set $B \subseteq X$ whenever there exists $0 \prec r \in \mathbb{C}$ such that $N(x, r) := \{y \in X : d(x, y) \prec r\} \subseteq B$.

II. A point $x \in X$ is called *limit point* of a set $B \subseteq X$ whenever there exists $0 \prec r \in \mathbb{C}$ such that $N(x, r) \bigcap (B - \{x\}) \neq \phi$.

III. A subset $B \subseteq X$ is called *open* whenever each element of B is an interior point of B.

IV. A subset $B \subseteq X$ is called *closed* whenever each limit point of B belongs to B.

V. The family $F = \{N(x, r) : x \in X, 0 \prec r\}$ is a sub-basis for a topology on X. We denote this complex topology by τ_c. Indeed, the topology τ_c is Hausdorff.

Definition 2.3. [3] Let (X, d) be a complex-valued metric space, and let $\{x_n\}$ be a sequence in X and $x \in X$.

I. If for every $c \in \mathbb{C}$ with $0 \prec c$ there is $N \in \mathbb{N}$ such that for all $n > N$, $d(x_n, x) \prec c$ then $\{x_n\}$ is said to be *convergent*, if $\{x_n\}$ converges to x and x is the limit point of $\{x_n\}$. We denote this by $x_n \to x$ as $n \to \infty$ or $\lim_{n\to\infty} x_n = x$.

II. If for every $c \in \mathbb{C}$ with $0 \prec c$ there is $N \in \mathbb{N}$ such that for all $n, m > N$, $d(x_n, x_m) \prec c$, then $\{x_n\}$ is said to be Cauchy sequence.

III. If every Cauchy sequence in X is convergent, then (X, d) is said to be a *complete complex-valued metric space*.

Lemma 2.1. *[3] Let (X, d) be a complex-valued metric space, and let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ converges to x if and only if $|d(x_n, x)| \to 0$ as $n \to \infty$.*

Lemma 2.2. *[3] Let (X, d) be a complex-valued metric space, and let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ is a Cauchy sequence if and only if $|d(x_n, x_{n+m})| \to 0$ as $n \to \infty$.*

Definition 2.4. [7] An element $(x, y) \in X \times X$ is said to be a *coupled fixed point* of the mapping $F : X \times X \to X$ if $F(x, y) = x$ and $F(y, x) = y$.

Definition 2.5. [15] An element $(x, y) \in X \times X$ is said to be

I. A *coupled coincidence point* of mappings $F : X \times X \to X$ and $g : X \to X$ if $gx = F(x, y)$ and $gy = F(y, x)$, and (gx, gy) is called a *coupled point of coincidence* if there exists $(u, v) \in X \times X$ such that $x = gu = F(u, v)$ and $y = gv = F(v, u)$.

II. A *common coupled fixed point* of mappings $F : X \times X \to X$ and $g : X \to X$ if $x = gx = F(x, y)$ and $y = gy = F(y, x)$.

Definition 2.6. [2] The mappings $F : X \times X \to X$ and $g : X \to X$ are called *w-compatible* if $g(F(x, y)) = F(gx, gy)$, whenever $gx = F(x, y)$ and $gy = F(y, x)$.

3. Main Results

3.1. Coupled Coincidence Point Result in Complex-Valued Metric Spaces

Theorem 3.1. *Let (X, d) be a complex-valued metric space. Let $F : X \times X \to X$ and $g : X \to X$ be two mappings. Suppose that there exist nonnegative constants $a_i \in [0, 1)$, $i = 1, 2, ..., 6$ such that $\sum_{i=1}^{6} a_i < 1$ and for all $x, y, u, v \in X$*

$$\begin{aligned} d(F(x,y),F(u,v)) &\preceq a_1 d(gx,gu) + a_2(gy,gv) + a_3\frac{d(gx,F(x,y))d(gu,F(u,v))}{d(gx,gu)} \\ &+ a_4\frac{d(gx,F(u,v))d(gu,F(x,y))}{d(gx,gu)} \\ &+ a_5\frac{d(gy,F(y,x))d(gv,F(v,u))}{d(gy.gv)} \\ &+ a_6\frac{d(gy,F(v,u))d(gv,F(y,x))}{d(gy,gv)}. \end{aligned}$$

Suppose $F(X \times X) \subseteq g(X)$ and $g(X)$ is a complete subspace of$X$. Then F and g have a coupled coincidence point $(x^*, y^*) \in X \times X$.

Proof. Let x_0 and y_0 are arbitrary elements of X. Set $gx_1 = F(x_0, y_0)$, $gy_1 = F(y_0, x_0)$, this can be done because $F(X \times X) \subseteq g(X)$. Continuing this process, we obtain two sequences $\{x_n\}$ and $\{y_n\}$ such that $gx_{n+1} = F(x_n, y_n)$ and $gy_{n+1} = F(y_n, x_n)$ for all $n \geqslant 0$. Then we have

$$\begin{aligned} d(gx_n, gx_{n+1}) &= d(F(x_{n-1}, y_{n-1}), F(x_n, y_n)) \\ &\preceq a_1 d(gx_{n-1}, gx_n) + a_2 d(gy_{n-1}, gy_n) \\ &+ a_3\frac{d(gx_{n-1}, F(x_{n-1}, y_{n-1}))d(gx_n, F(x_n, y_n))}{d(gx_{n-1}, gx_n)} \\ &+ a_4\frac{d(gx_{n-1}, F(x_n, y_n))d(gx_n, F(x_{n-1}, y_{n-1}))}{d(gx_{n-1}, gx_n)} \end{aligned}$$

$$\begin{aligned}
&+ \quad a_5 \frac{d(gy_{n-1}, F(y_{n-1}, x_{n-1}))d(gy_n, F(y_n, x_n))}{d(gy_{n-1}, gy_n)} \\
&+ \quad a_6 \frac{d(gy_{n-1}, F(y_n, x_n))d(gy_n, F(y_{n-1}, x_{n-1}))}{d(gy_{n-1}, gy_n)} \\
&\preceq \quad a_1 d(gx_{n-1}, gx_n) + a_2 d(gy_{n-1}, gy_n) \\
&+ \quad a_3 \frac{d(gx_{n-1}, gx_n)d(gx_n, gx_{n+1})}{d(gx_{n-1}, gx_n)} \\
&+ \quad a_4 \frac{d(gx_{n-1}, gx_{n+1})d(gx_n, gx_n)}{d(gx_{n-1}, gx_n)} \\
&+ \quad a_5 \frac{d(gy_{n-1}, gy_n)d(gy_n, gy_{n+1})}{d(gy_{n-1}, gy_n)} \\
&+ \quad a_6 \frac{d(gy_{n-1}, gy_{n+1})d(gy_n, gy_n)}{d(gy_{n-1}, gy_n)}.
\end{aligned}$$

Which implies that

$$\begin{aligned}
\left|d(gx_n, gx_{n+1})\right| &\leqslant a_1 \left|d(gx_{n-1}, gx_n)\right| + a_2 \left|d(gy_{n-1}, gy_n)\right| \\
&+ a_3 \left|d(gx_n, gx_{n+1})\right| + a_5 \left|d(gy_n, gy_{n+1})\right|. \qquad (3.1)
\end{aligned}$$

Similarly, we can prove that

$$\begin{aligned}
\left|d(gy_n, gy_{n+1})\right| &\leqslant a_1 \left|d(gy_{n-1}, gy_n)\right| + a_2 \left|d(gx_{n-1}, gx_n)\right| \\
&+ a_3 \left|d(gy_n, gy_{n+1})\right| + a_5 \left|d(gx_n, gx_{n+1})\right|. \qquad (3.2)
\end{aligned}$$

Put $d_n = \|d(gx_n, gx_{n+1})\| + \|d(gy_n, gy_{n+1})\|$.
Adding inequalities (3.1) and (3.2), one can assert that,

$$d_n \leqslant (a_1 + a_2)d_{n-1} + (a_3 + a_5)d_n, \qquad (3.3)$$

that is,

$$d_n \leqslant h d_{n-1} \quad \text{where} \quad h = \frac{a_1 + a_2}{1 - (a_3 + a_5)} < 1$$

Thus, we have

$$d_n \leqslant h d_{n-1} \leqslant h^2 d_{n-2} \leqslant h^3 d_{n-3} \leqslant \cdots \leqslant h^n d_0 \qquad (3.4)$$

We shall show that $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences. If $m > n$, then we have

$$\begin{aligned}
\left|d(gx_n, gx_m)\right| + \left|d(gy_n, gy_m)\right| &\leqslant \left|d(gx_n, gx_{n+1})\right| + \left|d(gy_n, gy_{n+1})\right| + \left|d(gx_{n+1}, gx_{n+2})\right| \\
&+ \left|d(gy_{n+1}, gy_{n+2})\right| + \cdots + \left|d(gx_{m-1}, gx_m)\right| + \left|d(gy_{m-1}, gy_m)\right| \\
&\leqslant d_n + d_{n+1} + \cdots + d_{m-1} \\
&\leqslant h^n d_0 + h^{n+1} d_0 + \cdots + h^{m-1} d_0 \\
&\leqslant \frac{h^n}{1-h} d_0 \to 0 \ \text{ as } \ n \to \infty.
\end{aligned}$$

Hence $\{gx_n\}$ and $\{gy_n\}$ are Cauchy sequences in $g(X)$. Since $g(X)$ is complete, there exists x^* and $y^* \in X$ such that $gx_n \to gx^*$ and $gy_n \to gy^*$ as $n \to \infty$.

On the other hand, we have

$$\begin{aligned}
d(F(x^*,y^*),gx^*) &\preceq d(F(x^*,y^*),gx_{n+1}) + d(gx_{n+1},gx^*) \\
&= d(F(x^*,y^*),F(x_n,y_n)) + d(gx_{n+1},gx^*) \\
&\preceq a_1 d(gx^*,gx_n) + a_2 d(gy^*,gy_n) \\
&+ a_3 \frac{d(gx^*,F(x^*,y^*))d(gx_n,F(x_n,y_n))}{d(gx^*,gx_n)} \\
&+ a_4 \frac{d(gx^*,F(x_n,y_n))d(gx_n,F(x^*,y^*))}{d(gx^*,gx_n)} \\
&+ a_5 \frac{d(gy^*,F(y^*,x^*))d(gy_n,F(y_n,x_n))}{d(gy^*,gy_n)} \\
&+ a_6 \frac{d(gy^*,F(y_n,x_n))d(gy_n,F(y^*,x^*))}{d(gy^*,gy_n)} \\
&+ d(gx_{n+1},gx^*) \\
&\preceq a_1 d(gx^*,gx_n) + a_2 d(gy^*,gy_n) \\
&+ a_3 \frac{d(gx^*,F(x^*,y^*))[d(gx_n,gx^*) + d(gx^*,gx_{n+1})]}{d(gx^*,gx_n)} \\
&+ a_4 \frac{d(gx^*,gx_{n+1})[d(gx_n,gx^*) + d(gx^*,F(x^*,y^*))]}{d(gx^*,gx_n)} \\
&+ a_5 \frac{d(gy^*,F(y^*,x^*))[d(gy_n,gy^*) + d(gy^*,gy_{n+1})]}{d(gy^*,gy_n)} \\
&+ a_6 \frac{d(gy*,gy_{n+1})[d(gy_n,gy*) + d(gy*,F(y*,x*))]}{d(gy*,gy_n)} \\
&+ d(gx_{n+1},gx*),
\end{aligned}$$

which implies that

$$\begin{aligned}
\left|d(F(x^*,y^*),gx^*)\right| &\leqslant a_1 \left|d(gx^*,gx_n)\right| + a_2 \left|d(gy^*,gy_n)\right| \\
&+ a_3 \frac{\left|d(gx^*,F(x^*,y^*))\right| \left\{\left|d(gx_n,gx^*)\right| + \left|d(gx^*,gx_{n+1})\right|\right\}}{\left|d(gx^*,gx_n)\right|} \\
&+ a_4 \frac{\left|d(gx^*,gx_{n+1})\right| \left\{\left|d(gx_n,gx^*)\right| + \left|d(gx^*,F(x^*,y^*))\right|\right\}}{\left|d(gx^*,gx_n)\right|} \\
&+ a_5 \frac{\left|d(gy^*,F(y^*,x^*))\right| \left\{\left|d(gy_n,gy^*)\right| + \left|d(gy^*,gy_{n+1})\right|\right\}}{\left|d(gy^*,gy_n)\right|} \\
&+ a_6 \frac{\left|d(gy^*,gy_{n+1})\right| \left\{\left|d(gy_n,gy^*)\right| + \left|d(gy^*,F(y^*,x^*))\right|\right\}}{\left|d(gy^*,gy_n)\right|}
\end{aligned}$$

$$+ \quad \left| d(gx_{n+1}, gx*) \right|$$

Since $gx_n \to gx^*$ and $gy_n \to gy^*$ as $n \to \infty$, we have $\left| d(F(x^*, y^*), gx^*) \right| \leqslant 0$.
That is,$F(x^*, y^*) = gx^*$.

Similarly one can show that $F(y^*, x^*) = gy^*$.

Hence (x^*, y^*) is a coupled coincidence point of F and g. □

3.2. Common Coupled Fixed Point Result in Complex-Valued Metric Spaces

The condition of Theorem 3.1 are not enough to prove the existence of a common coupled fixed point for the mappings F and g. By applying the condition of w-compatibility on F and g, we obtain the following common coupled fixed point theorem.

Theorem 3.2. *In addition to the hypotheses of Theorem 3.1 are not enough to prove the existence of a common coupled fixed point for the mappings F and g. By applying the condition of w-compatibility on F and g, we obtain the following common coupled fixed point theorem, if F and g are w-compatible, then F and g have a unique common coupled fixed point. Moreover, a common coupled fixed point of F and g is of the form* (u, v) *for some* $u, v \in X$.

Proof. The existence of coupled coincidence point (x^*, y^*) of F and g follows from Theorem 3.1. Then (gx^*, gy^*) is a coupled point of coincidence of F,g and so $gx^* = F(x^*, y^*)$ and $gy^* = F(y^*, x^*)$.

First we will show that this coupled point of coincidence is unique.

For this, suppose that F and g have another coupled point of coincidence (gx', gy'), that is, $gx' = F(x', y')$ and $gy' = F(y', x')$ where $(x', y') \in X \times X$. Then we have

$$\begin{aligned} d(gx^*, gx') &= d(F(x^*, y^*), F(x', y')) \\ &\preceq a_1 d(gx^*, gx') + a_2 d(gy^*, gy') \\ &+ a_3 \frac{d(gx^*, F(x^*, y^*)) d(gx', F(x', y'))}{d(gx^*, gx')} \\ &+ a_4 \frac{d(gx^*, F(x', y')) d(gx', F(x^*, y^*))}{d(gx^*, gx')} \\ &+ a_5 \frac{d(gy^*, F(y^*, x^*)) d(gy', F(y', x'))}{d(gy^*, gy')} \\ &+ a_6 \frac{d(gy^*, F(y', x')) d(gy', F(y^*, x^*))}{d(gy*, gy')}. \end{aligned}$$

This implies that

$$\begin{aligned}
\left|d(gx^*, gx')\right| &\leqslant a_1 \left|d(gx^*, gx')\right| + a_2 \left|d(gy^*, gy')\right| \\
&+ a_3 \frac{\left|d(gx^*, gx^*)\right| \left|d(gx', gx')\right|}{\left|d(x^*, gx')\right|} \\
&+ a_4 \frac{\left|d(gx^*, gx')\right| \left|d(gx', gx^*)\right|}{\left|d(gx^*, gx')\right|} \\
&+ a_5 \frac{\left|d(gy^*, gy^*)\right| \left|d(gy', gy')\right|}{\left|d(gy^*, gy')\right|} \\
&+ a_6 \frac{\left|d(gy^*, gy')\right| \left|d(gy', gy^*)\right|}{\left|d(gy^*, gy')\right|} \\
&\leqslant a_1 \left|d(gx^*, gx')\right| + a_2 \left|d(gy^*, gy')\right| \\
&+ a_4 \left|d(gx^*, gx')\right| + a_6 \left|d(gy^*, gy')\right|.
\end{aligned}$$

Hence

$$\left|d(gx^*, gx')\right| \leqslant (a_1 + a_4) \left|d(gx^*, gx')\right| + (a_2 + a_6) \left|d(gy^*, gy')\right|. \tag{3.5}$$

Similarly, we can show that

$$\left|d(gy^*, gy')\right| \leqslant (a_1 + a_4) \left|d(gy^*, gy')\right| + (a_2 + a_6) \left|d(gx^*, gx')\right|. \tag{3.6}$$

Adding inequalities (3.5) and (3.6), we get

$$\left|d(gx^*, gx')\right| + \left|d(gy^*, gy')\right| \leqslant (a_1 + a_2 + a_4 + a_6)\{\left|d(gx^*, gx')\right| + \left|d(gy^*, gy')\right|\}.$$

Since $(a_1 + a_2 + a_4 + a_6) < 1$. Therefore,

$$\left|d(gx^*, gx')\right| + \left|d(gy^*, gy')\right| \leqslant 0$$

.

Hence $d(gx^*, gx') = 0$ and $d(gy^*, gy') = 0$, i.e.,$gx^* = gx'$ and $gy^* = gy'$.

Thus, $(gx^*, gy^*) = (u, v)$ (say) is the unique coupled point of coincidence of F and g. Now if F and g are w-compatible, then $gu = g(F(x^*, y^*)) = F(gx^*, gy^*) = F(u, v) = w$(say). Similarly, we obtain $gv = g(F(y^*, x^*)) = F(gy^*, gx^*) = F(v, u) = z$ (say). So,(w, z) is another coupled point of coincidence of F and g. By uniqueness, we have $(u, v) = (w, z)$, that is, $gu = F(u, v) = u$ and $gv = F(v, u) = v$.Thus (u, v) is the unique common coupled fixed point of F and g. □

Next, we present an example to illustrate our results.

Example 3.1. Let $X = \{ix : x \in [0,1]\}$ and consider a complex-valued metric $d : X \times X \to X$ defined by

$$d(x,y) = i\left|x - y\right| \quad \text{for all} \quad x, y \in X$$

.

Then (X,d) is a complex-valued metric space.
Define the mappings $F : X \times X \to X$ and $g : X \to X$ by $F(x,y) = i\left(\frac{x}{9} + \frac{y}{7}\right)$ and $g(x) = i\frac{x}{2}$ for all $x, y \in [0,1]$.
Then we obtain,

$$\begin{aligned} d(F(x,y), F(u,v)) &= i\left|i\left(\frac{x}{9} + \frac{y}{7}\right) - i\left(\frac{u}{9} + \frac{v}{7}\right)\right| \\ &= i\left|\frac{2}{9}i\left(\frac{x}{2} - \frac{u}{2}\right) + \frac{2}{7}i\left(\frac{y}{2} - \frac{v}{2}\right)\right| \\ &\leqslant \frac{2}{9}i\left|i\frac{x}{2} - i\frac{u}{2}\right| + \frac{2}{7}i\left|i\frac{y}{2} - i\frac{v}{2}\right| \\ &\leqslant \frac{2}{9}d\left(gx, gu\right) + \frac{2}{7}d\left(gy, gv\right), \end{aligned}$$

where $a_1 = \frac{2}{9}$, $a_2 = \frac{2}{7}$, $a_i = 0$, $i = 3,4,5,6$. Note that $a_1 + a_2 = \frac{2}{9} + \frac{2}{7} < 1$, $F(X \times X) \subseteq g(X)$ and $g(X)$ is a complete subspace of X. Hence the condition of Theorem 3.1 are satisfied, that is, F and g have a coupled coincidence point (0,0). Furthermore, since F and g are w-compatible, hence, Theorem 3.2 shows that (0,0) is the unique common coupled fixed point of F and g.

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