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# ON SOME NEW $\mathcal{P}_\delta$-TRANSFORMS OF KUMMER'S CONFLUENT HYPERGEOMETRIC FUNCTIONS

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**Abstract.** The aim of our paper is to present $\mathcal{P}_\delta$-transforms of the Kummer's confluent hypergeometric functions by employing the generalized Gauss's second summation theorem, Bailey's summation theorem and Kummer's summation theorem obtained earlier by Lavoie, Grondin and Rathie [9]. Relevant connections of certain special cases of the main results presented here are also pointed out.

**Keywords**. Hypergeometric functions; Gauss's second summation theorem; gamma functions; Summation theorems.

## 1. Introduction

The *generalized hypergeometric function* ${}_pF_q$ with $p$ numerator and $q$ denominator parameters is defined as follows:

$$ {}_pF_q\left[\begin{matrix} a_1, \cdots, a_p \\ b_1, \cdots, b_q \end{matrix} \middle| z\right] := \sum_{m=0}^{\infty} \frac{(a_1)_m \cdots (a_p)_m}{(b_1)_m \cdots (b_q)_m} \frac{z^m}{m!}, \tag{1.1}$$

where $b_j \in \mathbb{C} \setminus \mathbb{Z}_0^-, j \in \overline{1,q} := \{1, 2, \cdots, q\}$. Here and in the following text, let $\mathbb{C}$, $\mathbb{R}$ and $\mathbb{Z}_0^-$ be the sets of complex numbers, real numbers, and non-positive integers, respectively. The series converges for all $z \in \mathbb{C}$ if $p \leq q$. It is divergent for all $z \neq 0$ when $p > q + 1$, unless at least one numerator parameter is a negative integer in which case (1.1) is a polynomial. Finally, if $p = q+1$, the series converges on the unit circle $|z| = 1$ when $\mathrm{Re}\left(\sum b_j - \sum a_j\right) > 0$. The importance of the hypergeometric series lies in the fact that almost all elementary functions such as exponential, binomial, trigonometric, hyperbolic, logarithmic are its special cases. It should be remarked here that whenever generalized hypergeometric functions reduce to gamma functions, the results are important from the applications point of view. Thus, the well-known classical summation theorems such as those of the Gauss

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second summation theorem, Bailey summation theorem and Kummer's summation theorem for the series ${}_2F_1$ [11] given below play an important role in the theory of hypergeometric functions.

The Gauss's second summation theorem

$$ {}_2F_1\left[\begin{matrix} a, b \\ \frac{1}{2}(a+b+1) \end{matrix} \,\middle|\, \frac{1}{2}\right] = \frac{\Gamma(\frac{1}{2})\Gamma(\frac{1}{2}(a+b+1))}{\Gamma(\frac{1}{2}(a+1)\Gamma(\frac{1}{2}(b+1)}; \tag{1.2} $$

Bailey's summation theorem

$$ {}_2F_1\left[\begin{matrix} a, 1-a \\ b \end{matrix} \,\middle|\, \frac{1}{2}\right] = \frac{\Gamma(\frac{1}{2}b)\Gamma(\frac{1}{2}(b+1)}{\Gamma(\frac{1}{2}(b+a))\Gamma(\frac{1}{2}(b-a+1))}; \tag{1.3} $$

Kummer's summation theorem

$$ {}_2F_1\left[\begin{matrix} a, b \\ 1+a-b \end{matrix} \,\middle|\, -1\right] = \frac{\Gamma(1+\frac{1}{2}a)\Gamma(1+a-b)}{\Gamma(1+a)\Gamma(1+\frac{1}{2}a-b)}. \tag{1.4} $$

During 1992-96, in a series of three research papers, Lavoie *et al.* [7, 8, 9] have generalized various classical summation theorems such as the Gauss second, Bailey and Kummer ones for the ${}_2F_1$ series, as well as the Watson, Dixon and Whipple ones for the ${}_3F_2$ series. However, in our present investigation, we are interested in the following generalized Gauss's second summation theorem, Bailey's summation theorem and Kummer's summation theorem given in [9]

$$ \begin{aligned} {}_2F_1\left[\begin{matrix} a, b \\ \frac{1}{2}(a+b+i+1) \end{matrix} \,\middle|\, \frac{1}{2}\right] &= \frac{\Gamma(\frac{1}{2})\Gamma(\frac{1}{2}a+\frac{1}{2}b+\frac{1}{2}i+\frac{1}{2})\Gamma(\frac{1}{2}a-\frac{1}{2}b-\frac{1}{2}i+\frac{1}{2})}{\Gamma(\frac{1}{2}a-\frac{1}{2}b+\frac{1}{2}|i|+\frac{1}{2})} \\ &\times\left\{\frac{A_i(a,b)}{\Gamma(\frac{1}{2}a+\frac{1}{2})\Gamma(\frac{1}{2}b+\frac{1}{2}i+\frac{1}{2}-\lfloor\frac{1+i}{2}\rfloor)} + \frac{B_i(a,b)}{\Gamma(\frac{1}{2}a)\Gamma(\frac{1}{2}b+\frac{1}{2}i-\lfloor\frac{i}{2}\rfloor)}\right\}; \end{aligned} \tag{1.5} $$

$$ \begin{aligned} {}_2F_1\left[\begin{matrix} a, 1-a+i \\ b \end{matrix} \,\middle|\, \frac{1}{2}\right] &= \frac{\Gamma(\frac{1}{2})\Gamma(b)\Gamma(1-a)}{2^{b-i-1}\Gamma(1-a+\frac{1}{2}i+\frac{1}{2}|i|)} \\ &\times\left\{\frac{C_i(a,b)}{\Gamma(\frac{1}{2}b-\frac{1}{2}a+\frac{1}{2})+\Gamma(\frac{1}{2}b+\frac{1}{2}a-\lfloor\frac{1+i}{2}\rfloor)} + \frac{D_i(a,b)}{\Gamma(\frac{1}{2}b-\frac{1}{2}a)\Gamma(\frac{1}{2}b+\frac{1}{2}a-\frac{1}{2}-\lfloor\frac{i}{2}\rfloor)}\right\}; \end{aligned} \tag{1.6} $$

$$ \begin{aligned} {}_2F_1\left[\begin{matrix} a, b \\ 1+a-b+i \end{matrix} \,\middle|\, -1\right] &= \frac{2^{-a}\Gamma(\frac{1}{2})\Gamma(1-b)\Gamma(1+a-b+i)}{\Gamma(1-b+\frac{1}{2}i+\frac{1}{2}|i|)} \\ &\times\left\{\frac{E_i(a,b)}{\Gamma(\frac{1}{2}a-b+\frac{1}{2}i+1)\Gamma(\frac{1}{2}a+\frac{1}{2}i+\frac{1}{2}-\lfloor\frac{1+i}{2}\rfloor)} + \frac{F_i(a,b)}{\Gamma(\frac{1}{2}a-b+\frac{1}{2}i+\frac{1}{2})\Gamma(\frac{1}{2}a+\frac{1}{2}i-\lfloor\frac{i}{2}\rfloor)}\right\}, \end{aligned} \tag{1.7} $$

respectively. Here, and in what follows, $i = 0, \pm1, \pm2, \pm3, \pm4, \pm5$ . Also, throughout the paper, as usual, $[x]$ denotes the greatest integer less than or equal to the real number $x$ and its absolute value is denoted by $|x|$. The coefficients which appear in (1.5), (1.6) and (1.7) are listed in Tables 1-3 and that, for $i = 0$, these equations reduce, respectively to (1.2), (1.3) and (1.4).

**Table 1**

| $i$ | $A_i(a,b)$ | $B_i(a,b)$ |
|---|---|---|
| 5 | $-(a+b+6)^2+\frac{1}{2}(b-a+6)(b+a+6)+\frac{1}{4}(b-a+6)^2 11(b+a+6)-\frac{13}{2}(b-a+6)+20$ | $(a+b+6)^2+\frac{1}{2}(b-a+6)(b+a+6)-\frac{1}{4}(b-a+6)^2-17(b+a+6)-\frac{1}{2}(b-a+6)+62$ |
| 4 | $\frac{1}{2}(a+b+1)(a+b-3)-\frac{1}{4}(b-a+3)(b-a-3)$ | $-2(b+a-1)$ |
| 3 | $\frac{1}{2}(b-a+4)-(b+a+4)+3$ | $\frac{1}{2}(b-a+4)+(b+a+4)-7$ |
| 2 | $\frac{1}{2}(b+a+3)-2$ | -2 |
| 1 | -1 | 1 |
| 0 | 1 | 0 |
| -1 | 1 | 1 |
| -2 | $\frac{1}{2}(b+a-1)$ | 2 |
| -3 | $\frac{1}{2}(3a+b-2)$ | $\frac{1}{2}(3b+a-2)$ |
| -4 | $\frac{1}{2}(a+b-3)(a+b+1)-\frac{1}{4}(b-a-3)(b-a+3)$ | $2(b+a-1)$ |
| -5 | $(b+a-4)^2-\frac{1}{2}(b+a-4)(b-a-4)-\frac{1}{4}(b-a-4)^2+4(b+a-4)-\frac{7}{2}(b-a-4)$ | $(b+a-4)^2+\frac{1}{2}(b+a-4)(b-a-4)-\frac{1}{4}(b-a-4)^2+8(b+a-4)-\frac{1}{2}(b-a-4)+12$ |

**Table 2**

| $i$ | $C_i(a,b)$ | $D_i(a,b)$ |
|---|---|---|
| 5 | $-(4b^2-2ab-a^2-22b+13a+20)$ | $4b^2+2ab-a^2-34b-a+62$ |
| 4 | $2(b-2)(b-4)-(a-1)(a-4)$ | $-4(b-3)$ |
| 3 | $a-2b+3$ | $a+2b-7$ |
| 2 | $b-2$ | -2 |
| 1 | -1 | 1 |
| 0 | 1 | 0 |
| -1 | 1 | 1 |
| -2 | $b$ | 2 |
| -3 | $2b-a$ | $a+2b+2$ |
| -4 | $2b(b+2)-a(a+3)$ | $4(b+1)$ |
| -5 | $4b^2-2ab-a^2+8b-7a$ | $4b^2+2ab-a^2+16b-a+12$ |

**Table 3**

| $i$ | $E_i(a,b)$ | $F_i(a,b)$ |
|---|---|---|
| 5 | $-4(6+a-b)^2+2b(6+a-b)+b^2-22(6+a-b)-13b-20$ | $4(6+a-b)^2+2b(6+a-b)-b^2-34(6+a-b)-b+62$ |
| 4 | $2(a+b-3)(a-b+1)-(b-1)(b-4)$ | $-4(a-b+2)$ |
| 3 | $3b-2a-5$ | $2a-b+1$ |
| 2 | $1+a-b$ | -2 |
| 1 | -1 | 1 |
| 0 | 1 | 0 |
| -1 | 1 | 1 |
| -2 | $a-b-1$ | 2 |
| -3 | $2a-3b-4$ | $2a-b-2$ |
| -4 | $2(a-b-3)(a-b-1)-b(b+3)$ | $4(a-b-2)$ |
| -5 | $4(a-b-4)^2-2b(a-b-4)-b^2+8(a-b-4)-7b$ | $4(a-b-4)^2+2b(a-b-4)-b^2+16(a-b-4)-b+12$ |

The main objective of this paper is to derive three new interesting and general $\mathcal{P}_\delta$-transforms of the Kummer's confluent hypergeometric functions by employing the generalized Gauss's second summation theorem, Bailey's summation theorem and Kummer's summation theorem given in (1.5), (1.6) and (1.7), respectively. Relevant connections of certain special cases of the main results presented here with those earlier ones are also pointed out.

## 2. $\mathcal{P}_\delta$-transforms

The $\mathcal{P}_\delta$-transforms or pathway transforms of the function $f(t)\,(t\in\mathbb{R})$ is a function $F_{\mathcal{P}}(s)$ of a complex variable $s$ defined by (see, *e.g.*, [6])

$$\mathcal{P}_\delta\{f(t);s\} = F_{\mathcal{P}}(s) = \int_0^\infty [1+(\delta-1)s]^{-\frac{t}{\delta-1}}\, f(t)\,dt \quad (\delta>1), \tag{2.1}$$

For the sufficient condition for the existence of the $\mathcal{P}_\delta$-transform (2.1) to exist, we refer the reader to [6]. The $\mathcal{P}_\delta$-transform of the power function $t^{\mu-1}$ is given by [6, p. 7, Eq. (32)]

$$\mathcal{P}_\delta\{t^{\mu-1};s\} = \left(\frac{\delta-1}{\ln[1+(\delta-1)s]}\right)^\mu \Gamma(\mu) = \frac{\Gamma(\mu)}{[\Lambda(\delta;s)]^\mu} \quad (\mathrm{Re}(\mu)>0;\, \delta>1). \tag{2.2}$$

Furthermore, upon letting $\delta \mapsto 1$ in the definition (2.1), the $\mathcal{P}_\delta$-transform reduces to the classical Laplace transform (see, *e.g.*, [13]):

$$L\{f(t);s\} = \int_0^\infty e^{-st} f(t)\,dt \quad (\,\mathrm{Re}(s)>0\,). \tag{2.3}$$

In view of the power function formula (2.2), it is easy to derive the $\mathcal{P}_\delta$-transform of the generalized hypergeometric function to obtain the following formula (see, [6, p. 8, Eq. (42)]):

$$\int_0^\infty [1+(\delta-1)s]^{-\frac{t}{\delta-1}}\, t^{\mu-1}\, {}_pF_q\left[\begin{array}{c} a_1,\cdots,a_p \\ b_1,\cdots,b_q \end{array}\Big|\,\omega t\right] \mathrm{d}t = \frac{\Gamma(\mu)}{[\Lambda(\delta;s)]^\mu}\, {}_{p+1}F_q\left[\begin{array}{c} a_1,\cdots,a_p,\mu \\ b_1,\cdots,b_q \end{array}\Big|\,\frac{\omega}{\Lambda(\delta;s)}\right], \tag{2.4}$$

for $p < q$, $\mathrm{Re}(\mu) > 0$, $\mathrm{Re}\left(\frac{\ln[1+(\delta-1)s]}{\delta-1}\right) > 0$ and $\delta > 1$ or for $p = q$, $\mathrm{Re}(\mu) > 0$, $\mathrm{Re}\left(\frac{\ln[1+(\delta-1)s]}{\delta-1}\right) > \mathrm{Re}(\omega)$ and $\delta > 1$.

If $p = q = 1$, we get the following formula :

$$\int_0^\infty [1+(\delta-1)s]^{-\frac{t}{\delta-1}}\, t^{\mu-1}\, {}_1F_1\left[\begin{array}{c} a \\ c \end{array}\Big|\,\omega t\right] \mathrm{d}t = \frac{\Gamma(\mu)}{[\Lambda(\delta;s)]^\mu}\, {}_2F_1\left[\begin{array}{c} a,\mu \\ c \end{array}\Big|\,\frac{\omega}{\Lambda(\delta;s)}\right], \tag{2.5}$$

for $\mathrm{Re}(c) > 0$, $\mathrm{Re}(\mu) > 0$, $\mathrm{Re}\left(\frac{\ln[1+(\delta-1)s]}{\delta-1}\right) > \mathrm{Re}(\omega)$ and $\delta > 1$. In the next section, we shall demonstrate how one can obtain three rather general $\mathcal{P}_\delta$-transforms of the Kummer's confluent hypergeometric functions by employing the results (1.5), (1.6) and (1.7).

## 3. $\mathcal{P}_\delta$-transforms of ${}_1F_1(a;b;x)$

In this section, we establish the following integral formulas, asserted in Theorem(3.1), Theorem(3.2) and Theorem(3.3).

**Theorem 3.1.** *Let* $\mathrm{Re}(b) > 0$, $\mathrm{Re}\left(\frac{\ln[1+(\delta-1)s]}{\delta-1}\right) > 0$ *and* $\delta > 1$. *Then*

$$\int_0^\infty [1+(\delta-1)s]^{-\frac{t}{\delta-1}}\, t^{b-1}\, {}_1F_1\left[\begin{array}{c} a \\ \frac{1}{2}(a+b+i+1) \end{array}\Big|\,\frac{t\,\Lambda(\delta;s)}{2}\right] \mathrm{d}t = \frac{\Gamma(b)}{[\Lambda(\delta;s)]^b}\, \frac{\Gamma(\frac{1}{2})\Gamma(\frac{1}{2}a+\frac{1}{2}b+\frac{1}{2}i+\frac{1}{2})\Gamma(\frac{1}{2}a-\frac{1}{2}b-\frac{1}{2}i+\frac{1}{2})}{\Gamma(\frac{1}{2}a-\frac{1}{2}b+\frac{1}{2}|i|+\frac{1}{2})} \times\left\{\frac{A_i(a,b)}{\Gamma(\frac{1}{2}a+\frac{1}{2})\Gamma(\frac{1}{2}b+\frac{1}{2}i+\frac{1}{2}-\lfloor\frac{1+i}{2}\rfloor)} + \frac{B_i(a,b)}{\Gamma(\frac{1}{2}a)\Gamma(\frac{1}{2}b+\frac{1}{2}i-\lfloor\frac{i}{2}\rfloor)}\right\}. \tag{3.1}$$

**Theorem 3.2.** *Let* $\mathrm{Re}(1-a+i) > 0$ $(i = 0, \pm1, \pm2, \pm3, \pm4, \pm5)$, $\mathrm{Re}\left(\frac{\ln[1+(\delta-1)s]}{\delta-1}\right) > 0$ *and* $\delta > 1$. *Then*

$$\int_0^\infty [1+(\delta-1)s]^{-\frac{t}{\delta-1}}\, t^{-a+i}\, {}_1F_1\left[\begin{array}{c} a \\ b \end{array}\Big|\,\frac{t\,\Lambda(\delta;s)}{2}\right] \mathrm{d}t \tag{3.2}$$

$$= \frac{\Gamma(1-a+i)}{[\Lambda(\delta;s)]^{1-a+i}} \frac{\Gamma(\frac{1}{2})\Gamma(b)\Gamma(1-a)}{2^{b-i-1}\Gamma(1-a+\frac{1}{2}i+\frac{1}{2}|i|)}$$

$$\times\left\{\frac{C_i(a,b)}{\Gamma(\frac{1}{2}b-\frac{1}{2}a+\frac{1}{2})+\Gamma(\frac{1}{2}b+\frac{1}{2}a-\lfloor\frac{1+i}{2}\rfloor)} + \frac{D_i(a,b)}{\Gamma(\frac{1}{2}b-\frac{1}{2}a)\Gamma(\frac{1}{2}b+\frac{1}{2}a-\frac{1}{2}-\lfloor\frac{i}{2}\rfloor)}\right\}.$$

**Theorem 3.3.** *Let* $\mathrm{Re}(b) > 0$, $\mathrm{Re}\left(\frac{\ln[1+(\delta-1)s]}{\delta-1}\right) > 0$ *and* $\delta > 1$. *Then*

$$\int_0^\infty [1+(\delta-1)s]^{-\frac{t}{\delta-1}}\, t^{b-1}\, {}_1F_1\left[\begin{matrix} a \\ 1+a-b+i \end{matrix} \;\middle|\; -t\,\Lambda(\delta;s)\right] \mathrm{d}t = \tag{3.3}$$

$$= \frac{\Gamma(b)}{[\Lambda(\delta;s)]^{b}} \frac{\Gamma(\frac{1}{2})\Gamma(1-b)\Gamma(1+a-b+i)}{\Gamma(1-b+\frac{1}{2}i+\frac{1}{2}|i|)}$$

$$\times\left\{\frac{E_i(a,b)}{\Gamma(\frac{1}{2}a-b+\frac{1}{2}i+1)\Gamma(\frac{1}{2}a+\frac{1}{2}i+\frac{1}{2}-\lfloor\frac{1+i}{2}\rfloor)} + \frac{F_i(a,b)}{\Gamma(\frac{1}{2}a-b+\frac{1}{2}i+\frac{1}{2})\Gamma(\frac{1}{2}a+\frac{1}{2}i-\lfloor\frac{i}{2}\rfloor)}\right\}.$$

*Proof.* In order to prove Theorem (3.1), setting $\omega = \frac{\Lambda(\delta;s)}{2}$, $\mu = b$ and $c = \frac{1}{2}(a+b+i+1)$ for $i = 0, \pm1, \pm2, \pm3, \pm4, \pm5$ in (2.5), we have

$$\int_0^\infty [1+(\delta-1)s]^{-\frac{t}{\delta-1}}\, t^{b-1}\, {}_1F_1\left[\begin{matrix} a \\ \frac{1}{2}(a+b+i+1) \end{matrix} \;\middle|\; \frac{t\,\Lambda(\delta;s)}{2}\right] \mathrm{d}t \tag{3.4}$$

$$= \frac{\Gamma(b)}{[\Lambda(\delta;s)]^{b}}\; {}_2F_1\left[\begin{matrix} a, b \\ \frac{1}{2}(a+b+i+1) \end{matrix} \;\middle|\; \frac{1}{2}\right].$$

We observe that the ${}_2F_1$ appearing on the right-hand side of (3.4) can be evaluated with the help of generalized Gauss's second summation theorem (1.5). This yields the desired formula (3.1).

The results in Theorem (3.2) and Theorem (3.3) can also be proven in a similar way by applying summation theorems (1.6) and (1.7), respectively. □

## 4. Special Cases

The particular cases $i = 0$ of Theorem (3.1) to Theorem (3.3), reduce to the following interesting and presumably new results for classical ones.

**Corollary 4.1.** *Let* $\mathrm{Re}(b) > 0$, $\mathrm{Re}\left(\frac{\ln[1+(\delta-1)s]}{\delta-1}\right) > 0$ *and* $\delta > 1$. *Then*

$$\int_0^\infty [1+(\delta-1)s]^{-\frac{t}{\delta-1}}\, t^{b-1}\, {}_1F_1\left[\begin{matrix} a \\ \frac{1}{2}(a+b+1) \end{matrix} \;\middle|\; \frac{t\,\Lambda(\delta;s)}{2}\right] \mathrm{d}t \tag{4.1}$$

$$= \frac{\Gamma(b)}{[\Lambda(\delta;s)]^{b}} \frac{\Gamma(\frac{1}{2})\Gamma(\frac{1}{2}(a+b+1))}{\Gamma(\frac{1}{2}a+\frac{1}{2})\Gamma(\frac{1}{2}b+\frac{1}{2})}.$$

**Corollary 4.2.** *Let* $\mathrm{Re}(1-a) > 0, \mathrm{Re}\left(\frac{\ln[1+(\delta-1)s]}{\delta-1}\right) > 0$ *and* $\delta > 1$. *Then*

$$\int_0^\infty [1+(\delta-1)s]^{-\frac{t}{\delta-1}}\, t^{-a}\, {}_1F_1\left[\begin{matrix} a \\ b \end{matrix} \;\middle|\; \frac{t\,\Lambda(\delta;s)}{2}\right] \mathrm{d}t \tag{4.2}$$

$$= \frac{\Gamma(1-a)}{[\Lambda(\delta;s)]^{1-a}} \frac{\Gamma(\frac{1}{2}b)\Gamma(\frac{1}{2}b+\frac{1}{2})}{\Gamma(\frac{1}{2}b+\frac{1}{2}a)\Gamma(\frac{1}{2}b-\frac{1}{2}a+\frac{1}{2})}.$$

**Corollary 4.3.** *Let* $\mathrm{Re}(b) > 0, \mathrm{Re}\left(\frac{\ln[1+(\delta-1)s]}{\delta-1}\right) > 0$ *and* $\delta > 1$. *Then*

$$\int_0^\infty [1+(\delta-1)s]^{-\frac{t}{\delta-1}}\, t^{b-1}\, {}_1F_1\left[\begin{matrix} a \\ 1+a-b \end{matrix}\,\Big|\, -t\,\Lambda(\delta;s)\right] \mathrm{d}t = \frac{\Gamma(b)}{[\Lambda(\delta;s)]^b}\, \frac{\Gamma(1+\frac{1}{2}a)\Gamma(1+a-b)}{\Gamma(1+a)\Gamma(1+\frac{1}{2}a-b)}. \tag{4.3}$$

Similarly, for $i = \pm 1, \pm 2, \pm 3, \pm 4, \pm 5$, other results can also be obtained.

## 5. Concluding remarks

By letting $\delta \mapsto 1$ in the definition (2.1), the $\mathcal{P}_\delta$-transform is reduced to the classical Laplace transform. Hence, for $\delta \mapsto 1$, the results (3.1), (3.2) and (3.3) immediately reduce to the corresponding results due to Kim *et al.* [5].

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