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ON A KÄHLER MANIFOLD EQUIPPED WITH LIFT OF QUARTER SYMMETRIC NON-METRIC CONNECTION

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Abstract. The aim of the present paper is to study Kähler manifolds equipped with the lift of a quarter-symmetric non-metric connection. In this paper, a condition on the manifold for being a Kähler manifold with respect to the lift of the quarter-symmetric non-metric connection is obtained. It is further shown that the Nijenhuis tensor with respect to the lift of the quarter-symmetric non-metric connection vanishes. Also, a necessary and sufficient condition for a contravariant almost analytic vector field in a Kähler manifold equipped with the lift of a quarter-symmetric non-metric connection has been found.
Keywords Kähler manifold; non-metric connection; differentiable manifolds.

1. Introduction

In 1975, a linear connection was introduced by S. Golab [5] called quarter-symmetric connection.

A linear connection $\overline{\nabla}$ is said to be a quarter-symmetric connection if the torsion tensor T of $\overline{\nabla}$ has the form

$$T(X,Y) = \omega(Y)\phi X - \omega(X)\phi Y,$$

where ϕ is the tensor field of type (1,1) and X, Y are arbitrary vector fields. A linear connection $\overline{\overline{\nabla}}$ is said to be a non-metric connection if the covariant derivative of the metric tensor g with respect to $\overline{\overline{\nabla}}$ does not vanish *i.e.* $\overline{\overline{\nabla}}g \neq 0$.

The lift function plays an important role in the study of differentiable manifolds. In the last few decades, the theory of lift has been studied by several authors. Furthermore, the study of tangent bundles has been continued by L. S. Das and M. N. I. Khan [3] (2005). They [3] considered a manifold with an almost r-contact structure and obtained an almost complex structure on the tangent bundle. Recently, M. Tekkoyun and S. Civelek [8] (2008) studied and extended the concept of lifts by

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considering the structures on complex manifolds. In 2014, the lift was studied with a quarter-symmetric semi-metric connection on tangent bundles by M. N. I. Khan [6]. The same author [7] (2015) also studied the lift equipped with a semi-symmetric non-metric connection on a Kähler manifold. The semi-symmetric non-metric connection has also been considered by B. B. Chaturvedi and P. N. Pandey [2] (2008) in a Kähler manifold. In [2] they showed that the Nijenhuis tensor vanishes in a Kähler manifold equipped with a semi-symmetric non-metric connection. In the same paper, they [2] also proved that if V is a contra-variant almost analytic vector field in a Kähler manifold then V is also a contra-variant almost analytic vector field in a Kähler manifold equipped with a semi-symmetric non-metric connection. Recently, B. B. Chaturvedi and P. Pandey [1] (2015) studied a new type of the metric connection in an almost Hermitian manifold. In that paper, they [1] obtained a condition for a vector field V to be a contravariant almost analytic vector field in an almost Hermitian manifold equipped with a new type of the metric connection.

1.1. Kähler manifold

Let M be an n-(even) dimensional differentiable manifold. If for a tensor field F of type (1,1) and a Riemannian metric g the conditions

$$F^2(X) + X = 0, \quad g(FX, FY) = g(X, Y), \quad (\nabla_X F)Y = 0,$$

hold then M is called Kähler manifold, X, Y are arbitrary vector fields.

1.2. Quarter-symmetric non-metric connection

Let F be a tensor field of type (1,1) then a linear connection $\overline{\nabla}$ defined by

$$\overline{\nabla}_X Y = \nabla_X Y + \omega(Y)FX, \tag{1.1}$$

is called quarter-symmetric non-metric connection, ∇ is the Riemannian connection, ω is 1-form defined by $g(X, \rho) = \omega(X)$ for the associated vector field ρ.
The torsion tensor T and the metric tensor g of $\overline{\nabla}$ are given respectively by

$$T(X, Y) = \omega(Y)FX - \omega(X)FY \quad and \quad (\overline{\nabla}_X g)(Y, Z) = \omega(Y)g(X, FZ) + \omega(Z)g(X, FY),$$

for arbitrary vector fields X and Y.

1.3. Tangent Bundle

let M be a differentiable manifold and T_pM denotes the tangent space of M at any point $p \in M$ then the collection of all tangent spaces at $p \in M$ is called the tangent bundle of M and denoted by $T(M) = \cup_{p \in M} T_pM$. Let $\tilde{p} \in T(M)$ then the projection $\pi : T(M) \to M$ defined by $\pi(\tilde{p}) = p$ is called the bundle projection of $T(M)$ over M and the set $\pi^{-1}(p)$ is called the fiber over $p \in M$ and M the base space.

Vertical lift: The composition of two maps $\pi : T(M) \to M$ and $f : M \to \mathbb{R}$ defined by $f^V = f o\pi$ is called the vertical lift of f, where f is a smooth function in M. For $\tilde{p} \in \pi^{-1}(U)$ with induced coordinates (x^h, y^h), the value of $f^V(\tilde{p})$ is constant along each fiber $T_p(M)$ and equal to $f(p)$ *i.e.* $f^V(\tilde{p}) = f^V(x, y) = f o\pi(\tilde{p}) = f(p) = f(x)$.
Complete lift: For a smooth function f in M, a function f^C defined by $f^C = i(df)$ on $T(M)$ is called the complete lift of f. If ∂f is denoted locally by $y^i \partial_i f$ then the complete lift of f is locally denoted by $f^C = y^i \partial_i f = \partial f$.
Let X be a vector field, then for a smooth function f on M, a vector field $X^C \in T(M)$ defined by $X^C f^C = (Xf)^C$ is called the complete lift of X in $T(M)$. If X has the component x^h in M then the component of the complete lift X^C in $T(M)$ is given by $X^C : (x^h, \partial x^h)$ with respect to the induced coordinates in $T(M)$.
For a 1-form ω in M and an arbitrary vector field X, the complete lift of ω is denoted by ω^C and defined by $\omega^C(X^C) = (\omega(X))^C$ [7].

1.4. Induced metric and connection

Let $\tau : S \to M$ be the immersion of an $(n-1)$-dimensional manifold S in M. If we denote the differentiable map $d\tau : T(S) \to T(M)$ of τ by B called the tangent map of τ, $T(S)$ and $T(M)$ being the tangent bundles of S and M, respectively, then the tangent map of B is denoted by $\tilde{B} : T(T(S)) \to T(T(M))$ [7].
Let g be a Riemannian metric in M and the complete lift of g is g^C in $T(M)$. If $\tilde{g}$ denotes the induced metric of g^C on $T(S)$ then we have $\tilde{g}(X, Y) = g^C(\tilde{B}X^C, \tilde{B}Y^C)$, where X, Y are vector fields in S. If ∇ denotes a Riemannian connection on M then ∇^C, the complete lift of ∇, is also a Riemannian connection satisfying $\nabla^C_{X^C} Y^C = (\nabla_X Y)^C$ and $\nabla^C_{X^C} Y^V = (\nabla_X Y)^V$, for the vector fields X, Y in M.
From [7], we know that the lift function has the following properties,

$$\begin{aligned} \omega^V(\tilde{B}X^C) &= \omega^V(\tilde{B}X)^{\overline{C}} = \#(\omega^V(X^C)) = \#((\omega(X))^V) \\ &= (\omega(BX))^{\overline{V}}, \omega^C(\tilde{B}X^C) = \omega^C(\tilde{B}X)^{\overline{C}} \\ &= \#(\omega^C(X^C)) = \#((\omega(X))^C) = (\omega(BX))^{\overline{C}}, [X^C, Y^C] \\ &= [X, Y]^C, \quad F^C(X^C) = (F(X))^C, \quad \omega^V(X^C) = (\omega(X))^V, \omega^C(X^C) \\ &= (\omega(X))^C, \quad g^C(X^V, Y^C) = g^C(X^C, Y^V) = (g(X, Y))^V, \end{aligned} \tag{1.2}$$

where X^C, ω^C, F^C, g^C and X^V, ω^V, F^V, g^V are the complete and vertical lifts of X, ω, F, g. $\#$, $\overline{V}$ and $\overline{C}$ denote the operation of restriction, vertical lift and complete lift on $\pi_M^{-1}(\tau(S))$ respectively, X, Y are vector fields in S.

2. Lift of quarter-symmetric non-metric connection on a Kähler manifold

Taking the complete lift of the equation (1.1), we get

$$(\overline{\nabla}_{BX} BY)^{\overline{C}} = (\nabla_{BX} BY)^{\overline{C}} + (\omega(BY)B(FX))^{\overline{C}}. \tag{2.1}$$

Simplifying (2.1), we have

$$\overline{\nabla}^C_{\tilde{B}X^C}\tilde{B}Y^C = \nabla^C_{\tilde{B}X^C}\tilde{B}Y^C + \omega^C(\tilde{B}Y^C)\tilde{B}(FX)^V + \omega^V(\tilde{B}Y^C)\tilde{B}(FX)^C. \tag{2.2}$$

A connection $\overline{\nabla}^C$ defined by (2.2) is called the lift of a quarter-symmetric non-metric connection $\overline{\nabla}$.
Replacing Y by FY, the equation (2.2) gives

$$\begin{aligned}\overline{\nabla}^C_{\tilde{B}X^C}\tilde{B}(FY)^C &= \nabla^C_{\tilde{B}X^C}\tilde{B}(FY)^C + \omega^C(\tilde{B}(FY)^C)\tilde{B}(FX)^V \\ &\quad +\omega^V(\tilde{B}(FY)^C)(\tilde{B}(FX))^C.\end{aligned} \tag{2.3}$$

Also, operating F^C on the equation (2.2), we get

$$F^C(\overline{\nabla}^C_{\tilde{B}X^C}\tilde{B}Y^C) = F^C(\nabla^C_{\tilde{B}X^C}\tilde{B}Y^C) - \omega^C(\tilde{B}Y^C)\tilde{B}X^V - \omega^V(\tilde{B}Y^C)\tilde{B}X^C. \tag{2.4}$$

Subtracting (2.4) from (2.3), we have

$$\begin{aligned}(\overline{\nabla}^C_{\tilde{B}X^C}\tilde{B}F^C)(\tilde{B}Y^C) &= \omega^C(\tilde{B}(FY)^C)\tilde{B}(FX)^V + \omega^V(\tilde{B}(FY)^C)\tilde{B}(FX)^C \\ &+ \omega^C(\tilde{B}Y^C)\tilde{B}X^V + \omega^V(\tilde{B}Y^C)\tilde{B}X^C.\end{aligned} \tag{2.5}$$

Thus, we can state

Theorem 2.1. *Let M be a Kähler manifold equipped with the lift of a quarter-symmetric non-metric connection $\overline{\nabla}^C$ then M is a Kähler manifold with respect to $\overline{\nabla}^C$ if and only if*

$$\begin{aligned}&\omega^C(\tilde{B}(FY)^C)\tilde{B}(FX)^V + \omega^V(\tilde{B}(FY)^C)\tilde{B}(FX)^C \\ &+\omega^C(\tilde{B}Y^C)\tilde{B}X^V + \omega^V(\tilde{B}Y^C)\tilde{B}X^C = 0.\end{aligned} \tag{2.6}$$

Now, if we denote

$$\overline{H}^C(\tilde{B}X^C, \tilde{B}Y^C) = \omega^C(\tilde{B}Y^C)\tilde{B}(FX)^V + \omega^V(\tilde{B}Y^C)\tilde{B}(FX)^C. \tag{2.7}$$

and define a tensor $'\overline{H}^C$ of type (0,3) by

$$'\overline{H}^C(\tilde{B}X^C, \tilde{B}Y^C, \tilde{B}Z^C) = g^C(\overline{H}^C(\tilde{B}X^C, \tilde{B}Y^C), \tilde{B}Z^C), \tag{2.8}$$

then, the equations (2.7) and (2.8) together give

$$\begin{aligned}'\overline{H}^C(\tilde{B}X^C, \tilde{B}Y^C, \tilde{B}Z^C) &= \omega^C(\tilde{B}Y^C)g^C(\tilde{B}(FX)^V, \tilde{B}Z^C) \\ &\quad +\omega^V(\tilde{B}Y^C)g^C(\tilde{B}(FX)^C, \tilde{B}Z^C)\end{aligned} \tag{2.9}$$

Replacing Y and Z by FY and FZ in (2.9), respectively, we get

$$\begin{aligned}'\overline{H}^C(\tilde{B}X^C, \tilde{B}(FY)^C, \tilde{B}(FZ)^C) &= \omega^C(\tilde{B}(FY)^C)g^C(\tilde{B}(FX)^V, \tilde{B}(FZ)^C) \\ &+ \omega^V(\tilde{B}(FY)^C)g^C(\tilde{B}(FX)^C, \tilde{B}(FZ)^C)\end{aligned} \tag{2.10}$$

Subtracting (2.9) from (2.10), we find

$$
\begin{aligned}
&{}'\overline{H}^C(\tilde{B}X^C, \tilde{B}(FY)^C, \tilde{B}(FZ)^C) -{}' \overline{H}^C(\tilde{B}X^C, \tilde{B}Y^C, \tilde{B}Z^C)\\
&= \omega^C(\tilde{B}(FY)^C)g^C(\tilde{B}(FX)^V, \tilde{B}(FZ)^C)\\
&+\omega^V(\tilde{B}(FY)^C)g^C(\tilde{B}(FX)^C, \tilde{B}(FZ)^C)\\
&-\omega^C(\tilde{B}Y^C)g^C(\tilde{B}(FX)^V, \tilde{B}Z^C) - \omega^V(\tilde{B}Y^C)g^C(\tilde{B}(FX)^C, \tilde{B}Z^C),
\end{aligned} \tag{2.11}
$$

which shows that ${}'\overline{H}^C$ is a hybrid in the last two slots if and only if the right hand side of (2.11) vanishes.
We also know that a necessary and sufficient condition to be a Kähler manifold with respect to the connection D defined by $D_XY = \nabla_XY + H(X, Y)$ is that ${}'H$ defined by ${}'H(X, Y, Z) = g(H(X, Y), Z)$ is a hybrid in the last two slots [4].
Hence from the above discussions, we conclude the following

Theorem 2.2. *Let M be a Kähler manifold equipped with the lift of a quarter-symmetric non-metric connection $\overline{\nabla}^C$ then a necessary and sufficient condition for M to be a Kähler manifold with respect to the connection $\overline{\nabla}^C$ is that*

$$
\begin{aligned}
&\omega^C(\tilde{B}(FY)^C)g^C(\tilde{B}(FX)^V, \tilde{B}(FZ)^C) + \omega^V(\tilde{B}(FY)^C)g^C(\tilde{B}(FX)^C, \tilde{B}(FZ)^C)\\
&-\omega^C(\tilde{B}Y^C)g^C(\tilde{B}(FX)^V, \tilde{B}Z^C) - \omega^V(\tilde{B}Y^C)g^C(\tilde{B}(FX)^C, \tilde{B}Z^C) = 0.
\end{aligned} \tag{2.12}
$$

Corollary 2.1. *Also, replacing X by FX in (2.12), we have*

$$
\begin{aligned}
&\omega^C(\tilde{B}(FY)^C)\tilde{B}(FX)^V + \omega^V(\tilde{B}(FY)^C)\tilde{B}(FX)^C\\
&+\omega^C(\tilde{B}Y^C)\tilde{B}X^V + \omega^V(\tilde{B}Y^C)\tilde{B}X^C = 0,
\end{aligned} \tag{2.13}
$$

which verifies the condition of the Kähler manifold obtained in (2.6). □

Let ${}'F$ denotes the 2-form of the Riemannian metric g defined by ${}'F(Y, Z) = g(FY, Z)$ then the complete lift of ${}'F$ is denoted and defined by

$$
{}'F^C(\tilde{B}Y^C, \tilde{B}Z^C) = g^C(\tilde{B}(FY)^C, \tilde{B}Z^C). \tag{2.14}
$$

Taking the covariant differentiation of (2.14), we get

Corollary 2.2.

$$
\begin{aligned}
&(\overline{\nabla}^C_{\tilde{B}X^C}{}'F^C)(\tilde{B}Y^C, \tilde{B}Z^C) = (\nabla^C_{\tilde{B}X^C}{}'F^C)(\tilde{B}Y^C, \tilde{B}Z^C)\\
&+\omega^C(\tilde{B}Y^C)g^C(\tilde{B}X^V, \tilde{B}Z^C) + \omega^V(\tilde{B}Y^C)g^C(\tilde{B}X^C, \tilde{B}Z^C)\\
&-\omega^C(\tilde{B}Z^C)g^C(\tilde{B}Y^C, \tilde{B}X^V) - \omega^V(\tilde{B}Z^C)g^C(\tilde{B}Y^C, \tilde{B}X^C).
\end{aligned}
$$

By taking the cyclic sum over X, Y, Z of the equation (2.15), we obtain

$$
(\overline{\nabla}^C_{\tilde{B}X^C}{}'F^C)(\tilde{B}Y^C, \tilde{B}Z^C) + (\overline{\nabla}^C_{\tilde{B}Y^C}{}'F^C)(\tilde{B}Z^C, \tilde{B}X^C)
$$

$$
\begin{aligned}
&+(\overline{\nabla}^{C}_{\tilde{B}Z^C}{}'F^C)(\tilde{B}X^C,\tilde{B}Y^C)\\
&=\omega^C(\tilde{B}Y^C)g^C(\tilde{B}X^V,\tilde{B}Z^C)+\omega^V(\tilde{B}Y^C)g^C(\tilde{B}X^C,\tilde{B}Z^C)\\
&+\omega^C(\tilde{B}Z^C)g^C(\tilde{B}Y^V,\tilde{B}X^C)+\omega^V(\tilde{B}Z^C)g^C(\tilde{B}Y^C,\tilde{B}X^C)\\
&+\omega^C(\tilde{B}X^C)g^C(\tilde{B}Z^V,\tilde{B}Y^C)+\omega^V(\tilde{B}X^C)g^C(\tilde{B}Z^C,\tilde{B}Y^C)\\
&-\omega^C(\tilde{B}Z^C)g^C(\tilde{B}Y^C,\tilde{B}X^V)-\omega^V(\tilde{B}Z^C)g^C(\tilde{B}Y^C,\tilde{B}X^C)\\
&-\omega^C(\tilde{B}X^C)g^C(\tilde{B}Z^C,\tilde{B}Y^V)-\omega^V(\tilde{B}X^C)g^C(\tilde{B}Z^C,\tilde{B}Y^C)\\
&-\omega^C(\tilde{B}Y^C)g^C(\tilde{B}X^C,\tilde{B}Z^V)-\omega^V(\tilde{B}Y^C)g^C(\tilde{B}X^C,\tilde{B}Z^C). \qquad (2.15)
\end{aligned}
$$

Thus, we can state the following

Theorem 2.3. *Let M be a Kähler manifold equipped with the lift of a quarter-symmetric non-metric connection $\overline{\nabla}^C$ then the relation (2.16) holds.*

Also, it is well known that the Nijenhuis tensor N with respect to the Riemannian connection ∇ is given by

$$N(X,Y) = [FX,FY]-[X,Y]-F[FX,Y]-F[X,FY] \qquad (2.16)$$

$$= \nabla_{FX}FY-\nabla_{FY}FX-\nabla_XY+\nabla_YX \qquad (2.17)$$

$$-F\nabla_{FX}Y+F\nabla_YFX-F\nabla_XFY+F\nabla_{FY}X. \qquad (2.18)$$

If N^C denotes the complete lift of the Nijenhuis tensor N then the equation (2.17) gives the Nijenhuis tensor $\overline{N}^C$ with respect to the connection $\overline{\nabla}^C$ as follows

$$
\begin{aligned}
\overline{N}^C(\tilde{B}X^C,\tilde{B}Y^C) &= \overline{\nabla}^C_{\tilde{B}(FX)^C}\tilde{B}(FY)^C-\overline{\nabla}^C_{\tilde{B}(FY)^C}\tilde{B}(FX)^C\\
&-\overline{\nabla}^C_{\tilde{B}X^C}\tilde{B}Y^C+\overline{\nabla}^C_{\tilde{B}Y^C}\tilde{B}X^C-F^C(\overline{\nabla}^C_{\tilde{B}(FX)^C}\tilde{B}Y^C)\\
&+F^C(\overline{\nabla}^C_{\tilde{B}Y^C}\tilde{B}(FX)^C)-F^C(\overline{\nabla}^C_{\tilde{B}X^C}\tilde{B}(FY)^C) \qquad (2.19)\\
&+F^C(\overline{\nabla}^C_{\tilde{B}(FY)^C}\tilde{B}X^C). \qquad (2.20)
\end{aligned}
$$

By help of (2.2), the equation (2.18) reduces to

$$\overline{N}^C(\tilde{B}X^C,\tilde{B}Y^C)=0. \qquad (2.21)$$

Hence, we have

Theorem 2.4. *Let M be a Kähler manifold equipped with the lift of a quarter-symmetric non-metric connection $\overline{\nabla}^C$ then the Nijenhuis tensor $\overline{N}^C$ with respect to the connection $\overline{\nabla}^C$ vanishes.*

3. Contravariant almost analytic vector field on a Kähler manifold

We know that in an almost Hermitian manifold, a necessary and sufficient condition for a vector field W to be a contravariant almost analytic vector field is that

$$\nabla_{FX}W=(\nabla_WF)X+F(\nabla_XW). \qquad (3.1)$$

For a Kähler manifold the equation (3.1) reduces to

$$\nabla_{FX}W - F(\nabla_X W) = 0. \tag{3.2}$$

Now, replacing X by FX and Y by W in (2.2) we have

$$\overline{\nabla}^C_{\tilde{B}(FX)^C}\tilde{B}W^C = \nabla^C_{\tilde{B}(FX)^C}\tilde{B}W^C - \omega^C(\tilde{B}W^C)\tilde{B}X^V - \omega^V(\tilde{B}W^C)\tilde{B}X^C. \tag{3.3}$$

Again, replacing Y by W and then taking F^C in (2.2), we get

$$F^C(\overline{\nabla}^C_{\tilde{B}X^C}\tilde{B}W^C) = F^C(\nabla^C_{\tilde{B}X^C}\tilde{B}W^C) - \omega^C(\tilde{B}W^C)\tilde{B}X^V - \omega^V(\tilde{B}W^C)\tilde{B}X^C. \tag{3.4}$$

Subtracting (3.4) from (3.3), we obtain

$$\overline{\nabla}^C_{\tilde{B}(FX)^C}\tilde{B}W^C - F^C(\overline{\nabla}^C_{\tilde{B}X^C}\tilde{B}W^C) = \nabla^C_{\tilde{B}(FX)^C}\tilde{B}W^C - F^C(\nabla^C_{\tilde{B}X^C}\tilde{B}W^C). \tag{3.5}$$

Thus, we have the following theorem

Theorem 3.1. *Let M be a Kähler manifold equipped with the lift of a quarter-symmetric non-metric connection $\overline{\nabla}^C$ then a necessary and sufficient condition for a vector field W to be a contravariant almost analytic vector field with respect to the connection $\overline{\nabla}^C$ is that it is a contravariant almost analytic vector field with respect to the connection ∇^C.* □

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