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SOME SYMMETRIC PROPERTIES OF KENMOTSU MANIFOLDS ADMITTING SEMI-SYMMETRIC METRIC CONNECTION

Venkatesha Venkatesh,* Arasaiah Arasaiah, Vishnuvardhana Srivaishnava Vasudeva and Naveen Kumar Rahuthanahalli Thimmegowda

Abstract. The objective of the present paper is to study some symmetric properties of the Kenmotsu manifold endowed with a semi-symmetric metric connection. Here we consider pseudo-symmetric, Ricci pseudo-symmetric, projective pseudo-symmetric and ϕ-projective semi-symmetric Kenmotsu manifolds with respect to the semi-symmetric metric connection. Finally, we provide an example of the 3-dimensional Kenmotsu manifold admitting a semi-symmetric metric connection which verifies our result.
Keywords: Kenmotsu manifold; projective curvature tensor; semi-symmetric metric connection; η-Einstein manifold.

1. Introduction

In 1932, Hayden [12] introduced the idea of metric connection with a torsion on a Riemannian manifold. By considering the torsion tensor of a linear connection, Friedmann and Schouten [11] gave a new connection called semi-symmetric connection. The torsion tensor with respect to the semi-symmetric connection $\bar{\nabla}$ is given by

$$\bar{T}(X,Y) = \bar{\nabla}_X Y - \bar{\nabla}_Y X - [X,Y]. \tag{1.1}$$

The connection $\bar{\nabla}$ is called a semi-symmetric metric connection [12] if $\bar{\nabla} g = 0$, otherwise, non-metric connection. A relation between the semi-symmetric metric connection $\bar{\nabla}$ and the Levi-Civita connection ∇ on (M,g) established by Yano [18] is given by

$$\bar{\nabla}_X Y = \nabla_X Y + \eta(Y)X - g(X,Y)\xi. \tag{1.2}$$

Semi-symmetric manifolds form a subclass of the class of pseudo-symmetric manifolds. The concept of pseudo-symmetric manifold was introduced by Chaki and

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*corresponding author

Chaki [8] and Deszcz [10] in two different ways. Here we study the properties of pseudo-symmetric manifolds with a semi-symmetric metric connection in the Deszcz sense. An n-dimensional Riemannian manifold M is called pseudo-symmetric in the sense of Deszcz [10] if the Riemannian curvature tensor R satisfies the following relation

$$(R(X,Y)\cdot R)(U,V)W = L_R((X\wedge_g Y)\cdot R)(U,V)W, \tag{1.3}$$

for all the vector fields $X, Y, Z, U, V, W \in TM$. Where L_R is a smooth function on M and $X\wedge_g Y$ is an endomorphism defined by

$$(X\wedge_g Y)Z = g(Y,Z)X - g(X,Z)Y. \tag{1.4}$$

The notion of semi-symmetric metric connection has been weakened by many geometers such as [2, 3, 5, 9, 15, 17] etc., with different structures of manifolds and submanifolds. In particular, De [1] and Bagewadi et. al. [4] studied semi-symmetric metric connection on Kenmotsu manifolds with a projective curvature tensor. Also in [16], Singh et. al. studied the semi-symmetric metric connection in an ϵ-Kenmotsu manifold.

The projective curvature tensor $\bar{P}$ with respect to the semi-symmetric metric connection on a Kenmotsu manifold is defined by [1]

$$\bar{P}(X,Y)Z = \bar{R}(X,Y)Z - \frac{1}{n-1}[\bar{S}(Y,Z)X - \bar{S}(X,Y)Z], \tag{1.5}$$

for $X, Y, Z \in \chi(M)$. Here $\bar{S}$ is the Ricci tensor with respect to the semi-symmetric metric connection.

Further, a relation between the curvature tensor $\bar{R}$ of the semi-symmetric metric connection $\bar{\nabla}$ and the curvature tensor R of the Levi-Civita connection ∇ is given by [18]

$$\begin{aligned} \bar{R}(X,Y)Z &= R(X,Y)Z - \alpha(Y,Z)X + \alpha(X,Z)Y \\ &- g(Y,Z)LX + g(X,Z)LY, \end{aligned} \tag{1.6}$$

where α is a tensor field of type (0,2) and L is a tensor field of type (1,1) which is given by

$$\alpha(Y,Z) = g(LY,Z) = (\nabla_Y\eta)(Z) - \eta(Y)\eta(Z) + \frac{1}{2}\eta(\xi)g(Y,Z), \tag{1.7}$$

for any vector fields $X, Y, Z \in \chi(M)$. From (1.6), it follows that

$$\bar{S}(Y,Z) = S(Y,Z) - (n-2)\alpha(Y,Z) - ag(Y,Z), \tag{1.8}$$

where $\bar{S}$ denotes the Ricci tensor with respect to $\bar{\nabla}$ and a=trace of α.

Motivated by these studies, we investigate the semi-symmetric metric connection due to Yano [18] on Kenmotsu manifolds. The paper is organized as follows. After giving preliminaries and basic results of the Kenmotsu manifold in Section

2, in Section 3 we study pseudo-symmetric Kenmotsu manifolds with respect to the semi-symmetric metric connection, proving that either $L_{\bar{R}} = -2$ or the manifold is η-Einstein. In the next section we prove that in a Ricci pseudo-symmetric Kenmotsu manifold with respect to the semi-symmetric metric connection, either $L_{\bar{S}} = -2$ or the manifold is η-Einstein. Sections 5 and 6 are devoted to the study of projective pseudo-symmetric and ϕ-projective semi-symmetric Kenmotsu manifolds with respect to the semi-symmetric metric connection. Finally, we construct an example of a 3-dimensional Kenmotsu manifold admitting the semi-symmetric metric connection and verify the results.

2. Preliminaries

Let M be an n-dimensional almost contact Riemannian manifold equipped with the almost contact metric structure (ϕ, ξ, η, g), where ϕ is a (1,1) tensor field, ξ is a characteristic vector field, η is a 1-form and g is the Riemannian metric satisfying the following conditions [7];

(2.1) $\phi^2(X) = -X + \eta(X)\xi, \quad \eta(\xi) = 1, \quad \eta \circ \phi = 0, \quad \phi\xi = 0, \quad g(X, \xi) = \eta(X),$

(2.2) $g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y),$

for all vector fields X, Y on M. If an almost contact metric manifold satisfies

$$(\nabla_X \phi)(Y) = g(\phi X, Y)\xi - \eta(Y)\phi X, \tag{2.3}$$

then M is called a Kenmotsu manifold [14]. Here ∇ denotes the operator of covariant differentiation with respect to g. From (2.3), it follows that

$$\nabla_X \xi = X - \eta(X)\xi, \tag{2.4}$$

$$(\nabla_X \eta)(Y) = g(X, Y) - \eta(X)\eta(Y). \tag{2.5}$$

In a Kenmotsu manifold M, the following relations hold:

(2.6) $\eta(R(X, Y)Z) = [g(X, Z)\eta(Y) - g(Y, Z)\eta(X)],$

(2.7) $(a)\ R(\xi, X)Y = [\eta(Y)X - g(X, Y)\xi], \ (b)\ R(X, Y)\xi = [\eta(X)Y - \eta(Y)X],$

(2.8) $(a)\ S(X, Y) = -(n-1)g(X, Y), \ (b)\ QX = -(n-1)X,$

(2.9) $(a)\ S(X, \xi) = -(n-1)\eta(X), \ (b)\ S(\xi, \xi) = -(n-1), \ (c)\ Q\xi = -(n-1)\xi,$

(2.10) $(\nabla_W R)(X, Y)\xi = g(W, X)Y - g(W, Y)X - R(X, Y)W,$

(2.11) $S(\phi X, \phi Y) = S(X, Y) + (n-1)\eta(X)\eta(Y).$

Now by using (1.7), (2.1) and (2.5) in (1.6), we have the following relation

$$\begin{aligned} \bar{R}(X, Y)Z &= R(X, Y)Z - 3[g(Y, Z)X - g(X, Z)Y] + 2[\eta(Y)X \\ &- \eta(X)Y]\eta(Z) + 2[g(Y, Z)\eta(X) - g(X, Z)\eta(Y)]\xi. \end{aligned} \tag{2.12}$$

Contracting X in (2.12), we get

$$\bar{S}(Y, Z) = S(Y, Z) - (3n-5)g(Y, Z) + 2(n-2)\eta(Y)\eta(Z). \tag{2.13}$$

Again contracting Y and Z in (2.13), we get

$$\bar{r} = r - (n-1)(3n-4), \tag{2.14}$$

where $\bar{r}$ and r are the scalar curvatures with respect to the semi-symmetric metric connection and the Levi-Civita connection respectively.

3. Pseudo-symmetric Kenmotsu manifold with respect to the semi-symmetric metric connection

Definition: An n-dimensional Kenmotsu manifold M is said to be pseudosymmetric with respect to semi-symmetric metric connection if the curvature tensor $\bar{R}$ of $\bar{\nabla}$ satisfies the condition

$$(\bar{R}(X,Y)\cdot\bar{R})(U,V)W = L_{\bar{R}}((X\wedge_g Y)\cdot\bar{R})(U,V)W, \tag{3.1}$$

where $L_{\bar{R}}$ is a function on M. From (3.1), we have

$$\begin{aligned}&\bar{R}(X,Y)(\bar{R}(U,V)W) - \bar{R}(\bar{R}(X,Y)U,V)W - \bar{R}(U,\bar{R}(X,Y)V)W\\ &-\bar{R}(U,V)(\bar{R}(X,Y)W) = L_{\bar{R}}[(X\wedge_g Y)(\bar{R}(U,V)W) - \bar{R}((X\wedge_g Y)U,V)W\\ &-\bar{R}(U,(X\wedge_g Y)V)W - \bar{R}(U,V)(X\wedge_g Y)W].\end{aligned} \tag{3.2}$$

Replacing X by ξ in (3.2), we get

$$\begin{aligned}&\bar{R}(\xi,Y)(\bar{R}(U,V)W) - \bar{R}(\bar{R}(\xi,Y)U,V)W - \bar{R}(U,\bar{R}(\xi,Y)V)W\\ &-\bar{R}(U,V)(\bar{R}(\xi,Y)W) = L_{\bar{R}}[(\xi\wedge_g Y)(\bar{R}(U,V)W) - \bar{R}((\xi\wedge_g Y)U,V)W\\ &-\bar{R}(U,(\xi\wedge_g Y)V)W - \bar{R}(U,V)(\xi\wedge_g Y)W].\end{aligned} \tag{3.3}$$

Using (1.4) and (2.12) in (3.3) and then taking the inner product with ξ, we obtain

$$\begin{aligned}&(L_{\bar{R}}+2)[-\bar{R}(U,V,W,Y) + \eta(\bar{R}(U,V)W)\eta(Y) + 2g(Y,U)\eta(V)\eta(W)\\ &-2g(Y,U)g(V,W) - \eta(\bar{R}(Y,V)W)\eta(U) - 2g(Y,V)\eta(U)\eta(W)\\ &+2g(Y,V)g(U,W) - \eta(\bar{R}(U,Y)W)\eta(V) - \eta(\bar{R}(U,V)Y)\eta(W)] = 0.\end{aligned} \tag{3.4}$$

On plugging $U = Y = e_i$ in (3.4) and taking summation over i, we get

$$(L_{\bar{R}}+2)[S(V,W) - (n-5)g(V,W) + 2(n-1)\eta(V)\eta(W)] = 0. \tag{3.5}$$

This implies that either $L_{\bar{R}} = -2$ or

$$S(V,W) = (n-5)g(V,W) + 2(1-n)\eta(V)\eta(W). \tag{3.6}$$

On contracting (3.6), we get

$$r = n(n-7) + 2. \tag{3.7}$$

Hence we can state the following:

Theorem 3.1. Let M be an n-dimensional pseudo-symmetric Kenmotsu manifold with respect to semi-symmetric metric connection. Then either $L_{\bar{R}} = -2$ or the manifold is η-Einstein with constant scalar curvature $r = n(n-7)+2$ with respect to Levi-Civita connection.

4. Ricci pseudo-symmetric Kenmotsu manifold with respect to the semi-symmetric metric connection

Definition: An n-dimensional Kenmotsu manifold M is said to be Ricci pseudo-symmetric with respect to semi-symmetric metric connection, if

$$(\bar{R}(X,Y)\cdot\bar{S})(Z,U) = L_{\bar{S}}Q(g,\bar{S})(Z,U;X,Y), \tag{4.1}$$

holds true on M, where $L_{\bar{S}}$ is some function and $Q(g,S)$ is the Tachibana tensor on M. From (4.1), it follows that

$$\begin{aligned}&\bar{S}(\bar{R}(X,Y)Z,U) + \bar{S}(Z,\bar{R}(X,Y)U)\\ &= L_{\bar{S}}[\bar{S}((X\wedge_g Y)Z,U) + \bar{S}(Z,(X\wedge_g Y)U)].\end{aligned} \tag{4.2}$$

Putting $Y = U = \xi$ in (4.2), we have

$$\bar{S}(\bar{R}(X,\xi)Z,\xi) + \bar{S}(Z,\bar{R}(X,\xi)\xi) = L_{\bar{S}}[\bar{S}((X\wedge\xi)Z,\xi) + \bar{S}(Z,(X\wedge\xi)\xi)]. \tag{4.3}$$

Using (1.4), (2.12), (2.13) and (2.7) in (4.3), we can get

$$(L_{\bar{S}}+2)[S(X,Z) - (n-3)g(X,Z) + 2(n-2)\eta(X)\eta(Z)] = 0. \tag{4.4}$$

This implies that either $L_{\bar{S}} = -2$ or

$$S(X,Z) = (n-3)g(X,Z) + 2(2-n)\eta(X)\eta(Z). \tag{4.5}$$

On contracting (4.5) over X and Z, we get

$$r = (n-1)(n-4). \tag{4.6}$$

Thus we can state the following theorem:
Theorem 4.1. If a Kenmotsu manifold M is Ricci pseudo-symmetric with respect to semi-symmetric metric connection, then either $L_{\bar{S}} = -2$ or the manifold is η-Einstein with constant scalar curvature $r = (n-1)(n-4)$ with respect to Levi-Civita connection.

5. Projective pseudo-symmetric Kenmotsu manifold with respect to the semi-symmetric metric connection

Definition: An n-dimensional Kenmotsu manifold M is said to be projective pseudo-symmetric with respect to semi-symmetric metric connection if

$$(\bar{R}(X,Y)\cdot\bar{P})(U,V)W = L_{\bar{P}}((X\wedge_g Y)\cdot\bar{P})(U,V)W, \tag{5.1}$$

holds on M. Putting $Y = W = \xi$ in (5.1), we get

$$(\bar{R}(X,\xi)\cdot\bar{P})(U,V)\xi = L_{\bar{P}}((X\wedge_g \xi)\cdot\bar{P})(U,V)\xi. \tag{5.2}$$

Now right hand side of (5.2) can be written as

$$\begin{aligned} L_{\bar{P}}((X \wedge_g \xi)\cdot \bar{P})(U,V)\xi &= L_{\bar{P}}[((X \wedge_g \xi)\bar{P})(U,V)\xi - \bar{P}((X \wedge_g \xi)U,V)\xi \\ &- \bar{P}(U,(X \wedge_g \xi)V)\xi - \bar{P}(U,V)(X \wedge_g \xi)\xi]. \end{aligned} \tag{5.3}$$

By virtue of (1.4), (1.5), (2.12), (2.13) and (2.7) in (5.3), we obtain

$$L_{\bar{P}}((X \wedge_g \xi)\cdot \bar{P})(U,V)\xi = -L_{\bar{P}} \cdot \bar{P}(U,V)X. \tag{5.4}$$

Next by considering left hand side of (5.2), we have

$$\begin{aligned} (\bar{R}(X,\xi)\cdot \bar{P})(U,V)\xi &= \bar{R}(X,\xi)\bar{P}(U,V)\xi - \bar{P}(\bar{R}(X,\xi)U,V)\xi \\ &- \bar{P}(U,\bar{R}(X,\xi)V)\xi - \bar{P}(U,V)\bar{R}(X,\xi)\xi. \end{aligned} \tag{5.5}$$

Again using (1.5), (2.12), (2.13) and (2.7) in (5.5), we get

$$(\bar{R}(X,\xi)\cdot \bar{P})(U,V)\xi = 2\bar{P}(U,V)X. \tag{5.6}$$

Substituting (5.4) and (5.6) in (5.2), we obtain

$$(L_{\bar{P}} + 2)\bar{P}(U,V)X = 0. \tag{5.7}$$

This leads us to the following:

`Theorem 5.1.` If an n-dimensional Kenmotsu manifold is projective pseudo-symmetric with respect to the semi-symmetric metric connection, then either $L_{\bar{P}} = -2$ or the manifold is projectively flat.

Also, in a Kenmotsu manifold, Bagewadi, Prakasha and Venkatesha [4] proved the following:

`Lemma 5.1.`[4] If the projective curvature tensor of a Kenmotsu manifold M admitting the semi-symmetric metric connection vanishes, then M reduces to an Einstein manifold with the constant scalar curvature $-n(n-1)$.

Hence from `Theorem 5.1.` and `Lemma 5.1.`, we conclude that:

`Corollary 5.1.` A projective pseudo-symmetric Kenmotsu manifold admitting the semi-symmetric metric connection is an Einstein manifold with the constant scalar curvature with respect to the Levi-Civita connection provided $L_{\bar{P}} \neq -2$.

6. ϕ-projective semi-symmetric Kenmotsu manifold with respect to the semi-symmetric metric connection

`Definition:` An n-dimensional Kenmotsu manifold M is said to be ϕ-projectively semi-symmetric with respect to the semi-symmetric metric connection if $\bar{P}(X,Y)\cdot \phi = 0$.

Let us consider an n-dimensional Kenmotsu manifold M which is ϕ-projective semi-symmetric. Then we have

$$\bar{P}(X,Y)\phi Z - \phi\bar{P}(X,Y)Z = 0, \tag{6.1}$$

for any vector fields X, Y and Z on M.
By virtue of (1.5) in (6.1) gives

$$\bar{R}(X,Y)\phi Z - \phi\bar{R}(X,Y)Z + \frac{1}{n-1}[\bar{S}(Y,\phi Z)X - \bar{S}(X,\phi Z)Y + \bar{S}(Y,Z)\phi X - \bar{S}(X,Z)\phi Y] = 0. \tag{6.2}$$

On plugging $Y = \xi$ in (6.2) and then using (2.12), (2.13) and (2.7), we obtain

$$2g(X,\phi Z)\xi - \frac{1}{n-1}\bar{S}(X,\phi Z)\xi = 0. \tag{6.3}$$

Now taking the inner product of the above equation with ξ, we get

$$2g(X,\phi Z) - \frac{1}{n-1}\bar{S}(X,\phi Z) = 0. \tag{6.4}$$

Replacing Z by ϕZ in (6.4) and then by virtue of (2.1) and (2.13), we obtain

$$S(X,Z) = Ag(X,Z) + B\eta(X)\eta(Z), \tag{6.5}$$

where $A = 5n - 7$ and $B = -2(3n - 5)$.
Hence we can state the following:
Theorem 6.1. An n-dimensional ϕ-projective semi-symmetric Kenmotsu manifold with respect to the semi-symmetric metric connection is η-Einstein with respect to the Levi-Civita connection.

7. Example

Consider a $3-$dimensional manifold $M = \{(x,y,z) \in \mathbb{R}^3 : z \neq 0\}$, where (x,y,z) are the standard coordinates in $\mathbb{R}^3$. We choose the vector fields

$$E_1 = -e^{-z}\frac{\partial}{\partial x}, \qquad E_2 = e^{-z}\frac{\partial}{\partial y}, \qquad E_3 = \frac{\partial}{\partial z},$$

which are linearly independent at each point of M. Let g be the Riemannian metric defined by

$$g = e^{2z}(dx \otimes dx + dy \otimes dy) + \eta \otimes \eta, \tag{7.1}$$

where η is the $1-$form defined by $\eta(X) = g(X, E_3)$, for any vector field X on M. Then $\{E_1, E_2, E_3\}$ is an orthonormal basis of M. We define a $(1,1)$ tensor field ϕ as

$$\phi\left(X\frac{\partial}{\partial x} + Y\frac{\partial}{\partial y}\right) + Z\frac{\partial}{\partial z} = \left(Y\frac{\partial}{\partial x} - X\frac{\partial}{\partial y}\right). \tag{7.2}$$

Thus, we have

$$\phi(E_1) = E_2, \;\; \phi(E_2) = -E_1 \quad \text{and} \quad \phi(E_3) = 0. \tag{7.3}$$

The linearity property of ϕ and g yields that

$$\begin{aligned} \eta(E_3) &= 1, \quad \phi^2 X = -X + \eta(X) E_3, \\ g(\phi X, \phi Y) &= g(X,Y) - \eta(X)\eta(Y), \end{aligned}$$

for any vector fields X, Y on M.

Moreover, we get

$$[E_i, \xi] = E_i, \quad [E_i, E_j] = 0, \quad i,j = 1,2$$

Using Koszul's formula, we obtain

$$\nabla_{E_i} E_i = -\xi, \quad \nabla_{E_i}\xi = E_i, \quad i = 1,2.$$

and others are zero. Thus for $E_3 = \xi$, $M(\phi, \xi, \eta, g)$ is a Kenmotsu manifold. Now, the non-zero terms of the semi-symmetric metric connection on M become

$$\bar{\nabla}_{E_i} E_i = -2\xi, \quad \bar{\nabla}_{E_i}\xi = 2E_i \quad i = 1,2. \tag{7.4}$$

With the help of the above results it can be easily verified that

$$\begin{aligned}
&R(E_1,E_2)E_3 = 0, && R(E_2,E_3)E_3 = -E_2, && R(E_1,E_3)E_3 = -E_1,\\
&R(E_1,E_2)E_2 = -E_1, && R(E_2,E_3)E_2 = E_3, && R(E_1,E_3)E_2 = 0,\\
&R(E_1,E_2)E_1 = E_2, && R(E_2,E_3)E_1 = 0, && R(E_1,E_3)E_1 = E_3.\\
&\textit{and} && && \\
&\bar{R}(E_1,E_2)E_3 = 0, && \bar{R}(E_2,E_3)E_3 = -2E_2, && \bar{R}(E_1,E_3)E_3 = -2E_1,\\
&\bar{R}(E_1,E_2)E_2 = -4E_1, && \bar{R}(E_2,E_3)E_2 = 2E_3, && \bar{R}(E_1,E_3)E_2 = 0,\\
&\bar{R}(E_1,E_2)E_1 = 4E_2, && \bar{R}(E_2,E_3)E_1 = 0, && \bar{R}(E_1,E_3)E_1 = 2E_3. \quad (7.5)
\end{aligned}$$

In view of (1.1), one can obtain the torsion tensor $\bar{T}$ with respect to the semi-symmetric metric connection as

$$\begin{aligned} &\bar{T}(E_i,E_i) = 0 \quad for \quad i = 1,2,3; \\ \bar{T}(E_1,E_2) = 0, \ &\bar{T}(E_1,E_3) = E_1, \quad \bar{T}(E_2,E_3) = E_2. \end{aligned}$$

Since E_1, E_2, E_3 forms a basis, the vector fields $X, Y, Z \in \chi(M)$ can be written as

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix} \begin{pmatrix} E_1 \\ E_2 \\ E_3 \end{pmatrix}, \tag{7.6}$$

where $a_i, b_i, c_i \in \mathbb{R}^+$ (the set of all positive real numbers), $i = 1,2,3$. Using the expressions of the curvature tensors, we find values of the Riemannian curvature and Ricci curvature with respect to the semi-symmetric metric connection as;

$$\begin{aligned} \bar{R}(X,Y)Z &= [-4\{a_1b_2 - b_1a_2\}b_3 + 2\{c_1a_2 - a_1c_2\}c_3]E_1 \\ &+ [-4\{b_1a_2 - a_1b_2\}a_3 + 2\{c_1b_2 - b_1c_2\}c_3]E_2 \\ &+ [-2\{c_1a_2 - a_1c_2\}a_3 - 2\{c_1b_2 - b_1c_2\}b_3]E_3, \quad (7.7) \\ \bar{S}(E_1,E_1) &= \bar{S}(E_2,E_2) = -6, \ \bar{S}(E_3,E_3) = -4. \quad (7.8) \end{aligned}$$

In view of the expression of the endomorphism $(E_i \wedge_g E_j)E_w = g(E_j, E_w)E_i - g(E_i, E_w)E_j$ for $1 \leq i, j, w \leq 3$ and equations (7.5) and (7.8), one can easily verify that

$$\begin{aligned} \bar{S}(\bar{R}(E_i, E_3)E_j, E_3) + \bar{S}(E_j, \bar{R}(E_i, E_3)E_3) &= -2[\bar{S}((E_i \wedge_g E_3)E_j, E_3) \\ &+ \bar{S}(E_j, (E_i \wedge_g E_3)E_3)], \end{aligned} \tag{7.9}$$

in view of the above equation `Theorem 4.1.` is verified.

REFERENCES

1. Ajit Barman and U. C. De: *Projective curvature tensor of a semi-symmetric metric connection in a Kenmotsu manifold.* International Electronic Journal of Geometry. **6** (2013), 159-169.
2. K. Amur and S. S. Pujar: *On submanifolds of a Riemannian manifold admitting a metric semi-symmetric connection.* Tensor, N. S. **32** (1978), 35-38.
3. C. S. Bagewadi: *On totally real submanifolds of a Kahlerian manifold admitting Semi symmetric metric F-connection.* Indian. J. Pure. Appl. Math. **13** (1982), 528-536.
4. C. S. Bagewadi, D. G. Prakasha and Venkatesha: *Projective curvature tensor on a Kenmotsu manifold with respect to semi-symmetric metric connection.* Seria. Mathematica **17** (2007), 21-32.
5. A. Barman: *On para-Sasakian manifolds admitting semi-symmetric metric connection.* Publ. De L'Institut Math. **109** (2014), 239-247.
6. T. Q. Binh: *On semi-symmetric connection.* Periodica Math. Hungerica, **21** (1990), 101-107.
7. D. E. Blair: *Contact manifolds in Riemannian geometry.* Lecture Notes in Mathematics, Springer-Verlag, Berlin, 1976.
8. M. C. Chaki and B. Chaki: *On pseudosymmetric manifolds admitting a type of semisymmetric connection.* Soochow J. Math. **13** (1987), 1-7.
9. U. C. De: *On a type of semi-symmetric connection on a Riemannian manifold.* Indian J. Pure Appl. Math. **21** (1990), 334-338.
10. R. Deszcz: *On pseudosymmetric spaces.* Bull. Soc. Math. Belg., Ser. **44** (1992), 1-34.
11. A. Friedmann and J. A. Schouten: *Über die Geometric der halbsymmetrischen Übertragung.* Math. Zeitschr **21** (1924), 211-223.
12. H. A. Hayden: *Subspaces of a space with torsion.* Proc. London Math. Soc. **34** (1932), 27-50.
13. S. K. Hui and Richard S. Lemence: *Ricci pseudosymmetric generalized quasi-Einstein manifolds.* SUT J.Math. **51** (2015), 195-213.
14. K. Kenmotsu: *A class of almost contact Riemannian manifolds.* Tohoku Math. J. **24** (1972), 93-103.
15. D. G. Prakasha, Aysel Turgut Vanli and C. S. Bagewadi: *Some classes of Kenmotsu manifolds with respect to semi-symmetric metric connection.* Acta Mathematica Sinica. **29** (2013), 1311-1322.

16. R. N. Singh, S. K. Pandey, G. Pandey and K. Tiwari: *On a semi-symmetric metric connection in an ϵ-Kenmotsu manifold*. Commun. Korean Math. Soc., 29 (2014), No.2, 331-343.

17. Venkatesha, K. T. Pradeep Kumar and C. S. Bagewadi and Gurupadavva Ingalahalli: *On concircular ϕ-recurrent K-contact manifold admitting semisymmetric metric connection*. Hin. Publ. Corp. Intl. J. Math. Sci. **2012** (2012).

18. K. Yano: *On semi-symmetric metric connections*. Rev. Roumaine Math. Pures Appl. **15** (1970), 1579-1586.

Venkatesha Venkatesh
Associate Professor
Department of Mathematics, Kuvempu University,
Shankaraghatta-577 451, Karnataka, INDIA
vensmath@gmail.com

Arasaiah Arasaiah
Department of Mathematics, Kuvempu University
Shankaraghatta-577 451, Karnataka, INDIA
ars.gnr94@gmail.com

Vishnuvardhana Srivaishnava Vasudeva
Department of Mathematics, GITAM School of Technology
GITAM(Deemed to be university), Bangalore, Karnataka, INDIA
svvishnuvardhana@gmail.com

Naveen Kumar Rahuthanahalli Thimmegowda
Department of Mathematics, Siddaganga Institute of Technology
B H Road, Tumakuru-572 103, Karnataka, INDIA.
rtnaveenkumar@gmail.com