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RADICAL TRANSVERSAL SCR-LIGHTLIKE SUBMANIFOLDS OF INDEFINITE KAEHLER MANIFOLDS

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Abstract. In this paper, we introduce the notion of radical transversal screen Cauchy-Riemann (SCR)-lightlike submanifolds of indefinite Kaehler manifolds giving a characterization theorem with some non-trivial examples of such submanifolds. Integrability conditions of distributions D_1, D_2, D and $D^{\perp}$ on radical transversal SCR-lightlike submanifolds of an indefinite Kaehler manifold have been obtained. Further, we obtain necessary and sufficient conditions for foliations determined by the above distributions to be totally geodesic.

Keywords. Semi-Riemannian manifold, degenerate metric, radical distribution, screen distribution, screen transversal vector bundle, lightlike transversal vector bundle, Gauss and Weingarten formulae.

1. Introduction

The theory of lightlike submanifolds of a semi-Riemannian manifold was introduced by Duggal and Bejancu ([7]). Various classes of lightlike submanifolds of indefinite Kaehler manifolds are defined according to the behaviour of distributions on these submanifolds with respect to the action of (1, 1) tensor field $\overline{J}$ in Kaehler structure of the ambient manifolds. Such submanifolds have been studied by Duggal and Sahin in ([8], [10]). In [9], Duggal and Sahin introduced the notion of generalized CR-lightlike submanifolds of an indefinite Kaehler manifold which contains CR-lightlike and SCR-lightlike submanifolds as its sub-cases. In [3], Sahin and Gunes studied geodesic CR-lightlike submanifolds and found some geometric properties of CR-lightlike submanifolds of an indefinite Kaehler manifold.

However, all these submanifolds of an indefinite Kaehler manifold mentioned above have invariant radical distribution on their tangent bundles i.e $\overline{J}(RadTM) \subset TM$, where $RadTM$ is the radical distribution and TM is the tangent bundle.

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In [2], Sahin introduced radical transversal and transversal lightlike submanifolds of an indefinite Kaehler manifold for which the action of (1, 1) tensor field $\overline{J}$ on radical distribution of such submanifolds does not belong to the tangent bundle, more precisely, $\overline{J}(Rad(TM)) = ltr(TM)$, where $ltr(TM)$ is the lightlike transversal bundle of lightlike submanifolds.

Thus motivated sufficiently, we introduce the notion of radical transversal screen Cauchy-Riemann (SCR)-lightlike submanifolds of an indefinite Kaehler manifold. This new class of lightlike submanifolds of an indefinite Kaehler manifold includes invariant, screen real, screen Cauchy-Riemann, radical transversal, totally real and generalized transversal lightlike submanifolds as its sub-cases. The paper is arranged as follows. There are some basic results in Section 2. In Section 3, we study radical transversal screen Cauchy-Riemann (SCR)-lightlike submanifolds of an indefinite Kaehler manifold, giving some examples. Section 4 is devoted to the study of foliations determined by distributions D_1, D_2, D and $D^\perp$ involved in the definition of the above submanifolds of an indefinite Kaehler manifold.

2. Preliminaries

A submanifold (M^m, g) immersed in a semi-Riemannian manifold $(\overline{M}^{m+n}, \overline{g})$ is called a lightlike submanifold [7] if the metric g induced from $\overline{g}$ is degenerate and the radical distribution $RadTM$ is of rank r, where $1 \leq r \leq m$. Let $S(TM)$ be a screen distribution which is a semi-Riemannian complementary distribution of $RadTM$ in TM, that is

$$TM = RadTM \oplus_{orth} S(TM). \tag{2.1}$$

Now consider a screen transversal vector bundle $S(TM^\perp)$, which is a semi-Riemannian complementary vector bundle of $RadTM$ in $TM^\perp$. Since for any local basis $\{\xi_i\}$ of $RadTM$, there exists a local null frame $\{N_i\}$ of sections with values in the orthogonal complement of $S(TM^\perp)$ in $[S(TM)]^\perp$ such that $\overline{g}(\xi_i, N_j) = \delta_{ij}$ and $\overline{g}(N_i, N_j) = 0$, it follows that there exists a lightlike transversal vector bundle $ltr(TM)$ locally spanned by $\{N_i\}$. Let $tr(TM)$ be a complementary (but not orthogonal) vector bundle to TM in $T\overline{M}|_M$. Then

$$tr(TM) = ltr(TM) \oplus_{orth} S(TM^\perp), \tag{2.2}$$

$$T\overline{M}|_M = TM \oplus tr(TM), \tag{2.3}$$

$$T\overline{M}|_M = S(TM) \oplus_{orth} [RadTM \oplus ltr(TM)] \oplus_{orth} S(TM^\perp), \tag{2.4}$$

where $\oplus$ denotes the direct sum and $\oplus_{orth}$ denotes the orthogonal direct sum. Following are four cases of a lightlike submanifold $\big(M, g, S(TM), S(TM^\perp)\big)$:

Case.1 r-lightlike if $r < min\,(m, n)$, Case.2 co-isotropic if $r = n < m$, $S\left(TM^{\perp}\right) = \{0\}$,

Case.3 isotropic if $r = m < n$, $S\left(TM\right) = \{0\}$,

Case.4 totally lightlike if $r = m = n$, $S(TM) = S(TM^{\perp}) = \{0\}$.

The Gauss and Weingarten formulae are given as

$$\overline{\nabla}_X Y = \nabla_X Y + h(X,Y), \quad \forall X,Y \in \Gamma(TM), \tag{2.5}$$

$$\overline{\nabla}_X V = -A_V X + \nabla^t_X V, \quad \forall V \in \Gamma(tr(TM)), \tag{2.6}$$

where $\{\nabla_X Y, A_V X\}$ and $\{h(X,Y), \nabla^t_X V\}$ belong to $\Gamma(TM)$ and $\Gamma(tr(TM))$, respectively. ∇ and ∇^t are linear connections on M and the vector bundle $tr(TM)$, respectively. The second fundamental form h is a symmetric $F(M)$-bilinear form on $\Gamma(TM)$ with values in $\Gamma(tr(TM))$ and the shape operator A_V is a linear endomorphism of $\Gamma(TM)$. From (2.5) and (2.6), we have

$$\overline{\nabla}_X Y = \nabla_X Y + h^l\left(X,Y\right) + h^s\left(X,Y\right), \quad \forall X,Y \in \Gamma(TM), \tag{2.7}$$

$$\overline{\nabla}_X N = -A_N X + \nabla^l_X\left(N\right) + D^s\left(X,N\right), \quad \forall N \in \Gamma(ltr(TM)), \tag{2.8}$$

$$\overline{\nabla}_X W = -A_W X + \nabla^s_X\left(W\right) + D^l\left(X,W\right), \quad \forall W \in \Gamma(S(TM^{\perp})), \tag{2.9}$$

where $h^l(X,Y) = L\left(h(X,Y)\right)$, $h^s(X,Y) = S\left(h(X,Y)\right)$, $D^l(X,W) = L(\nabla^t_X W)$, $D^s(X,N) = S(\nabla^t_X N)$. L and S are the projection morphisms of $tr(TM)$ on $ltr(TM)$ and $S(TM^{\perp})$, respectively. $\nabla^l$and ∇^s are linear connections on $ltr(TM)$ and $S(TM^{\perp})$ called the lightlike connection and screen transversal connection on M, respectively. For any vector field X tangent to M, we put

$$\overline{J}X = PX + FX, \tag{2.10}$$

where PX and FX are tangential and transversal parts of $\overline{J}X$, respectively.

Now by using (2.5), (2.7)-(2.9) and metric connection $\overline{\nabla}$, we obtain

$$\overline{g}(h^s(X,Y),W) + \overline{g}(Y, D^l(X,W)) = g(A_W X, Y), \tag{2.11}$$

$$\overline{g}(D^s(X,N),W) = \overline{g}(N, A_W X). \tag{2.12}$$

Denote the projection of TM on $S(TM)$ by $\overline{P}$. Then from the decomposition of the tangent bundle of a lightlike submanifold, we have

$$\nabla_X \overline{P}Y = \nabla^*_X \overline{P}Y + h^*(X, \overline{P}Y), \quad \forall X,Y \in \Gamma(TM), \tag{2.13}$$

$$\nabla_X \xi = -A^*_\xi X + \nabla^{*t}_X \xi, \quad \xi \in \Gamma(RadTM). \tag{2.14}$$

By using the above equations, we obtain

$$\overline{g}(h^l(X,\overline{P}Y),\xi) = g(A^*_\xi X,\overline{P}Y), \tag{2.15}$$

$$\overline{g}(h^*(X,\overline{P}Y),N) = g(A_N X,\overline{P}Y), \tag{2.16}$$

$$\overline{g}(h^l(X,\xi),\xi) = 0, \qquad A^*_\xi \xi = 0. \tag{2.17}$$

It is important to note that in general ∇ is not a metric connection. Since $\overline{\nabla}$ is a metric connection, by using (2.7), we get

$$(\nabla_X g)(Y,Z) = \overline{g}(h^l(X,Y),Z) + \overline{g}(h^l(X,Z),Y). \tag{2.18}$$

An indefinite almost Hermitian manifold $(\overline{M},\overline{g},\overline{J})$ is a 2m-dimensional semi-Riemannian manifold $\overline{M}$ with a semi-Riemannian metric $\overline{g}$ of the constant index q, $0 < q < 2m$ and a (1, 1) tensor field $\overline{J}$ on $\overline{M}$ such that the following conditions are satisfied:

$$\overline{J}^2 X = -X, \qquad \forall X \in \Gamma(T\overline{M}), \tag{2.19}$$

$$\overline{g}(\overline{J}X,\overline{J}Y) = \overline{g}(X,Y), \tag{2.20}$$

for all $X, Y \in \Gamma(T\overline{M})$.

An indefinite almost Hermitian manifold $(\overline{M},\overline{g},\overline{J})$ is called an indefinite Kaehler manifold if $\overline{J}$ is parallel with respect to $\overline{\nabla}$, i.e.,

$$(\overline{\nabla}_X \overline{J})Y = 0, \tag{2.21}$$

for all $X, Y \in \Gamma(T\overline{M})$, where $\overline{\nabla}$ is the Levi-Civita connection with respect to $\overline{g}$.

A plane section S in tangent space $T_x\overline{M}$ at a point x of a Kaehler manifold $\overline{M}$ is called a holomorphic section if it is spanned by a unit vector X and $\overline{J}X$, where X is a non-zero vector field on $\overline{M}$. The sectional curvature $K(X,\overline{J}X)$ of a holomorphic section is called a holomorphic sectional curvature. A simply connected complete Kaehler manifold $\overline{M}$ of the constant sectional curvature c is called a complex space-form and denoted by $\overline{M}(c)$. The curvature tensor of the complex space-form $\overline{M}(c)$ is given by ([12])

$$\begin{aligned}\overline{R}(X,Y)Z =& \frac{c}{4}[\overline{g}(Y,Z)X - \overline{g}(X,Z)Y + \overline{g}(\overline{J}Y,Z)\overline{J}X \\ &- \overline{g}(\overline{J}X,Z)\overline{J}Y + 2\overline{g}(X,\overline{J}Y)\overline{J}Z],\end{aligned} \tag{2.22}$$

for any smooth vector fields X, Y and Z on $\overline{M}$. This result is also true for an indefinite Kaehler manifold $\overline{M}$.

3. Radical Transversal SCR-Lightlike Submanifolds

In this section, we introduce the notion of radical transversal SCR-lightlike submanifolds of an indefinite Kaehler manifold.

Definition 3.1. Let $(M, g, S(TM), S(TM^{\perp}))$ be a lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then we say that M is the radical transversal SCR-lightlike submanifold of $\overline{M}$ if the following conditions are satisfied:

(i) there exist orthogonal distributions D_1, D_2, D and $D^{\perp}$ on M such that $RadTM = D_1 \oplus_{orth} D_2$ and $S(TM) = D \oplus_{orth} D^{\perp}$,

(ii) the distributions D_1 and D are invariant distributions with respect to $\overline{J}$, i.e. $\overline{J}D_1 = D_1$ and $\overline{J}D = D$,

(iii) the distributions D_2 and $D^{\perp}$ are transversal distributions with respect to $\overline{J}$, i.e. $\overline{J}D_2 \subset \Gamma(ltr(TM))$ and $\overline{J}D^{\perp} \subset \Gamma S(TM^{\perp})$.

From the above definition, we have the following decomposition

$$TM = D_1 \oplus_{orth} D_2 \oplus_{orth} D \oplus_{orth} D^{\perp}. \tag{3.1}$$

In particular, we have

(i) if $D_1 = 0$, then M is a generalized transversal lightlike submanifold,

(ii) if $D_1 = 0$ and $D = 0$, then M is a transversal lightlike submanifold,

(iii) if $D_1 = 0$ and $D^{\perp} = 0$, then M is a radical transversal lightlike submanifold,

(iv) if $D_2 = 0$, then M is a screen CR-lightlike submanifold,

(v) if $D_2 = 0$ and $D = 0$, then M is a screen real lightlike submanifold,

(vi) if $D_2 = 0$ and $D^{\perp} = 0$, then M is an invariant lightlike submanifold.

Thus this new class of lightlike submanifolds of an indefinite Kaehler manifold includes radical transversal, transversal, generalized transversal, invariant, screen real, screen Cauchy-Riemann lightlike submanifolds which have been studied in ([2], [8], [10], [15]) as its sub-cases.

Let $(\mathbb{R}^{2m}_{2q}, \overline{g}, \overline{J})$ denote the manifold $\mathbb{R}^{2m}_{2q}$ with its usual Kaehler structure given by

$\overline{g} = \frac{1}{4}(-\sum_{i=1}^{q} dx^i \otimes dx^i + dy^i \otimes dy^i + \sum_{i=q+1}^{m} dx^i \otimes dx^i + dy^i \otimes dy^i)$,

$\overline{J}(\sum_{i=1}^{m}(X_i\partial x_i + Y_i\partial y_i)) = \sum_{i=1}^{m}(Y_i\partial x_i - X_i\partial y_i)$,

where (x^i, y^i) are the cartesian coordinates on $\mathbb{R}^{2m}_{2q}$.

Now, we construct some examples of radical transversal SCR-lightlike submanifolds of an indefinite Kaehler manifold.

Example 1. Let $(\mathbb{R}^{16}_4, \overline{g}, \overline{J})$ be an indefinite Kaehler manifold, where $\overline{g}$ is of signature $(-,-,+,+,+,+,+,+,-,-,+,+,+,+,+,+)$ with respect to $\{\partial x_1, \partial x_2, \partial x_3, \partial x_4, \partial x_5, \partial x_6, \partial x_7, \partial x_8, \partial y_1, \partial y_2, \partial y_3, \partial y_4, \partial y_5, \partial y_6, \partial y_7, \partial y_8\}$.

Suppose M is a submanifold of $\mathbb{R}^{16}_4$ given by $x^1 = -y^3 = u_1$, $x^3 = y^1 = u_2$, $x^1 = -y^4 = u_3$, $x^4 = -y^1 = u_4$, $x^5 = -y^6 = u_5$, $x^6 = y^5 = u_6$, $x^7 = y^8 = u_7$, $x^8 = y^7 = u_8$.

The local frame of TM is given by $\{Z_1, Z_2, Z_3, Z_4, Z_5, Z_6, Z_7, Z_8\}$, where

$Z_1 = 2(\partial x_1 - \partial y_3), \quad Z_2 = 2(\partial x_3 + \partial y_1),$

$Z_3 = 2(\partial x_1 - \partial y_4), \quad Z_4 = 2(\partial x_4 - \partial y_1),$

$Z_5 = 2(\partial x_5 - \partial y_6), \quad Z_6 = 2(\partial x_6 + \partial y_5),$

$Z_7 = 2(\partial x_7 + \partial y_8), \quad Z_8 = 2(\partial x_8 + \partial y_7).$

Hence $RadTM = span\{Z_1, Z_2, Z_3, Z_4\}$ and $S(TM) = span\{Z_5, Z_6, Z_7, Z_8\}$.

Now $ltr(TM)$ is spanned by $N_1 = \partial x_1 + \partial y_3$, $N_2 = \partial x_3 - \partial y_1$, $N_3 = 2(\partial x_1 + \partial y_4), N_4 = 2(\partial x_4 + \partial y_1)$ and $S(TM^{\perp})$ is spanned by

$W_1 = 2(\partial x_5 + \partial y_6), \quad W_2 = 2(\partial x_6 - \partial y_5),$

$W_3 = 2(\partial x_7 - \partial y_8), \quad W_4 = 2(\partial x_8 - \partial y_7).$

It follows that $D_1 = span\{Z_1, Z_2\}$ such that $\overline{J}Z_1 = -Z_2$, $\overline{J}Z_2 = Z_1$, which implies that D_1 is invariant with respect to $\overline{J}$ and $D_2 = span\{Z_3, Z_4\}$ such that $\overline{J}Z_3 = -N_4$, $\overline{J}Z_4 = -N_3$, which implies that $\overline{J}D_2 \subset ltr(TM)$. On the other hand, we can see that $D = span\{Z_5, Z_6\}$ such that $\overline{J}Z_5 = -Z_6$, $\overline{J}Z_6 = Z_5$, which implies that D is invariant with respect to $\overline{J}$ and $D^{\perp} = span\{Z_7, Z_8\}$ such that $\overline{J}Z_7 = W_4$, $\overline{J}Z_8 = W_3$, which implies that $D^{\perp}$ is anti-invariant with respect to $\overline{J}$. Hence M is a radical transversal SCR-lightlike submanifold of $\mathbb{R}^{16}_4$.

Example 2. Let $(\mathbb{R}^{16}_4, \overline{g}, \overline{J})$ be an indefinite Kaehler manifold, where $\overline{g}$ is of signature $(-,-,+,+,+,+,+,+,+,-,-,+,+,+,+,+,+)$ with respect to $\{\partial x_1, \partial x_2, \partial x_3, \partial x_4, \partial x_5, \partial x_6, \partial x_7, \partial x_8, \partial y_1, \partial y_2, \partial y_3, \partial y_4, \partial y_5, \partial y_6, \partial y_7, \partial y_8\}$.

Suppose M is a submanifold of $\mathbb{R}^{16}_4$ given by $x^3 = u_1$, $y^3 = u_2$, $x^2 = u_1 \cos\alpha - u_2 \sin\alpha$, $y^2 = u_1 \sin\alpha + u_2 \cos\alpha$, $x^2 = u_3$, $y^2 = -u_4$, $x^4 = u_3 \cos\beta - u_4 \sin\beta$, $y^4 = u_3 \sin\beta + u_4 \cos\beta$, $x^5 = u_5 \cos\gamma$, $y^6 = u_5 \sin\gamma$, $x^6 = u_6 \sin\gamma$, $y^5 = -u_6 \cos\gamma$, $x^7 = u_8 \cos\delta$, $y^8 = u_8 \sin\delta$, $x^8 = u_7 \cos\delta$, $y^7 = u_7 \sin\delta$.

The local frame of TM is given by $\{Z_1, Z_2, Z_3, Z_4, Z_5, Z_6, Z_7, Z_8\}$, where

$Z_1 = 2(\partial x_3 + \cos\alpha\partial x_2 + \sin\alpha\partial y_2), \quad Z_2 = 2(\partial y_3 - \sin\alpha\partial x_2 + \cos\alpha\partial y_2),$

$Z_3 = 2(\partial x_2 + \cos\beta\partial x_4 + \sin\beta\partial y_4), \quad Z_4 = 2(-\partial y_2 - \sin\beta\partial x_4 + \cos\beta\partial y_4),$

$Z_5 = 2(\cos\gamma\partial x_5 + \sin\gamma\partial y_6), \quad Z_6 = 2(\sin\gamma\partial x_6 - \cos\gamma\partial y_5),$

$Z_7 = 2(\cos\delta\partial x_8 + \sin\delta\partial y_7), \quad Z_8 = 2(\cos\delta\partial x_7 + \sin\delta\partial y_8).$

Hence $RadTM = span\{Z_1, Z_2, Z_3, Z_4\}$ and $S(TM) = span\{Z_5, Z_6, Z_7, Z_8\}$.

Now $ltr(TM)$ is spanned by $N_1 = -\partial x_3 + \cos\alpha\partial x_2 + \sin\alpha\partial y_2$, $N_2 = -\partial y_3 - \sin\alpha\partial x_2 + \cos\alpha\partial y_2$, $N_3 = 2(-\partial x_2 + \cos\beta\partial x_4 + \sin\beta\partial y_4)$, $N_4 = 2(-\partial y_2 + \sin\beta\partial x_4 - \cos\beta\partial y_4)$ and $S(TM^{\perp})$ is spanned by

$W_1 = 2(\sin\gamma\partial x_5 - \cos\gamma\partial y_6), \quad W_2 = 2(\cos\gamma\partial x_6 + \sin\gamma\partial y_5),$

$W_3 = 2(\sin\delta\partial x_8 - \cos\delta\partial y_7), \quad W_4 = 2(\sin\delta\partial x_7 - \cos\delta\partial y_8).$

It follows that $D_1 = span\{Z_1, Z_2\}$ such that $\overline{J}Z_1 = -Z_2$, $\overline{J}Z_2 = Z_1$, which implies that D_1 is invariant with respect to $\overline{J}$ and $D_2 = span\{Z_3, Z_4\}$ such that $\overline{J}Z_3 = N_4$, $\overline{J}Z_4 = N_3$, which implies that $\overline{J}D_2 \subset ltr(TM)$. On the other hand, we can see that $D = span\{Z_5, Z_6\}$ such that $\overline{J}Z_5 = Z_6$, $\overline{J}Z_6 = -Z_5$, which implies that D is invariant with respect to $\overline{J}$ and $D^{\perp} = span\{Z_7, Z_8\}$ such that $\overline{J}Z_7 = W_4$,

$\overline{J}Z_8 = W_3$, which implies that $D^{\perp}$ is anti-invariant with respect to $\overline{J}$. Hence M is a radical transversal SCR-lightlike submanifold of $\mathbb{R}^{16}_4$.

Now, we denote the projection morphisms on D_1, D_2, D and $D^{\perp}$ in TM by P_1, P_2, P_3 and P_4 respectively. Similarly, we denote the projection morphisms of $tr(TM)$ on ν, $\overline{J}D_2$, μ and $\overline{J}D^{\perp}$ by Q_1, Q_2, Q_3 and Q_4 respectively, where ν and μ are orthogonal complementry distributions of $\overline{J}D_2$ and $\overline{J}D^{\perp}$ in $ltr(TM)$ and $S(TM^{\perp})$ respectively. Then, we get

$$X = P_1X + P_2X + P_3X + P_4X, \quad \forall X \in \Gamma(TM). \tag{3.2}$$

Now applying $\overline{J}$ to (3.2), we have

$$\overline{J}X = \overline{J}P_1X + \overline{J}P_2X + \overline{J}P_3X + \overline{J}P_4X. \tag{3.3}$$

Thus we get $\overline{J}P_1X \in D_1 \subset RadTM$, $\overline{J}P_2X \in \overline{J}D_2 \subset ltr(TM)$, $\overline{J}P_3X \in D \subset S(TM)$, $\overline{J}P_4X \in \overline{J}D^{\perp} \subset S(TM^{\perp})$. Also, we have

$$W = Q_1W + Q_2W + Q_3W + Q_4W, \quad \forall W \in \Gamma(tr(TM)). \tag{3.4}$$

Applying $\overline{J}$ to (3.4), we obtain

$$\overline{J}W = \overline{J}Q_1W + \overline{J}Q_2W + \overline{J}Q_3W + \overline{J}Q_4W. \tag{3.5}$$

Thus we get $\overline{J}Q_1W \in \nu \subset ltr(TM), \overline{J}Q_2W \in D_2 \subset RadTM$, $\overline{J}Q_3W \in \mu \subset S(TM^{\perp})$ and $\overline{J}Q_4W \in D^{\perp} \subset S(TM)$.

Now, by using (2.21), (3.3), (3.5) and (2.7)-(2.9) and identifying the components on D_1, D_2, D, $D^{\perp}$, ν, $\overline{J}D_2$, μ and $\overline{J}D^{\perp}$, we obtain

$$P_1(\nabla_X\overline{J}P_1Y) + P_1(\nabla_X\overline{J}P_3Y) - P_1(A_{\overline{J}P_2Y}X) - P_1(A_{\overline{J}P_4Y}X) = \overline{J}P_1\nabla_XY, \tag{3.6}$$

$$P_2(\nabla_X\overline{J}P_1Y) + P_2(\nabla_X\overline{J}P_3Y) - P_2(A_{\overline{J}P_2Y}X) - P_2(A_{\overline{J}P_4Y}X) = \overline{J}Q_2h^l(X,Y), \tag{3.7}$$

$$P_3(\nabla_X\overline{J}P_1Y) + P_3(\nabla_X\overline{J}P_3Y) - P_3(A_{\overline{J}P_2Y}X) - P_3(A_{\overline{J}P_4Y}X) = \overline{J}P_3\nabla_XY, \tag{3.8}$$

$$P_4(\nabla_X\overline{J}P_1Y) + P_4(\nabla_X\overline{J}P_3Y) - P_4(A_{\overline{J}P_2Y}X) - P_4(A_{\overline{J}P_4Y}X) = \overline{J}Q_4h^s(X,Y), \tag{3.9}$$

$$Q_1h^l(X,\overline{J}P_1Y) + Q_1h^l(X,\overline{J}P_3Y) + Q_1\nabla^l_X\overline{J}P_2Y + Q_1D^l(X,\overline{J}P_4Y) = \overline{J}Q_1h^l(X,Y), \tag{3.10}$$

$$Q_2h^l(X,\overline{J}P_1Y) + Q_2h^l(X,\overline{J}P_3Y) + Q_2\nabla^l_X\overline{J}P_2Y + Q_2D^l(X,\overline{J}P_4Y) = \overline{J}P_2\nabla_XY, \tag{3.11}$$

$$Q_3h^s(X,\overline{J}P_1Y)+Q_3h^s(X,\overline{J}P_3Y)+Q_3\nabla^s_X\overline{J}P_4Y+Q_3D^s(X,\overline{J}P_2Y) = \overline{J}Q_3h^s(X,Y), \tag{3.12}$$

$$Q_4h^s(X,\overline{J}P_1Y)+Q_4h^s(X,\overline{J}P_3Y)+Q_4\nabla^s_X\overline{J}P_4Y+Q_4D^s(X,\overline{J}P_2Y) = \overline{J}P_4\nabla_XY. \tag{3.13}$$

Theorem 3.1. *Let M be a radical transversal SCR-lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then μ is an invariant distribution with respect to $\overline{J}$.*

Proof. Let M be a radical transversal SCR-lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. For any $X\in\Gamma(\mu)$, $\xi\in\Gamma(RadTM)$ and $N\in\Gamma(ltr(TM))$, we have $\overline{g}(\overline{J}X,\xi)=-\overline{g}(X,\overline{J}\xi)=0$ and $\overline{g}(\overline{J}X,N)=-\overline{g}(X,\overline{J}N)=0$. Thus $\overline{J}X$ has no components in $RadTM$ and $ltr(TM)$.

Now, for $X\in\Gamma(\mu)$ and $Y\in\Gamma(D^\perp)$, we have $\overline{g}(\overline{J}X,Y)=-\overline{g}(X,\overline{J}Y)=0$, as $\overline{J}Y\in\Gamma(\overline{J}D^\perp)$, which implies that $\overline{J}X$ has no components in $D^\perp$. Hence $\overline{J}(\mu)\subset\Gamma(\mu)$, which complete the proof.

Now we give a characterization theorem for radical transversal SCR-lightlike submanifold.

Theorem 3.2. *Let M be a lightlike submanifold of an indefinite complex space-form $(\overline{M}(c),\overline{g})$, $c\neq 0$. Then M is a radical transversal SCR-lightlike submanifold if and only if*

(i) the maximal invariant subspace of T_pM, $p\in M$ defines a distribution $\overline{D}=D_1\oplus D$, where $RadTM=D_1\oplus D_2$ and D is a non-degenerate invariant distribution on M,

(ii) $\overline{g}(\overline{R}(\xi,N)\xi_1,\xi_2)\neq 0$, for all $\xi\in\Gamma(D_1)$, $N\in\Gamma(ltr(TM))$ and $\xi_1,\xi_2\in\Gamma(D_2)$,

(iii) $\overline{g}(\overline{R}(X,Y)Z,W)=0$, for all $X,Y\in\Gamma(D)$ and $Z,W\in\Gamma(D^\perp)$, where $D^\perp$ is the complementry distribution of D in $S(TM)$.

Proof. Let M be a radical transversal SCR-lightlike submanifold of an indefinite complex space-form $(\overline{M}(c),\overline{g})$, $c\neq 0$. Then proof of (i) follows from the definition of radical transversal SCR-lightlike submanifold. For $\xi\in\Gamma(D_1), N\in\Gamma ltr(TM)$ and $\xi_1,\xi_2\in\Gamma(D_2)$, from (2.22), we have

$$\overline{g}(\overline{R}(\xi,N)\xi_1,\xi_2)=\frac{c}{2}\overline{g}(\overline{J}\xi,N)\overline{g}(\xi_1,\overline{J}\xi_2). \tag{3.14}$$

Since D_1 is invariant distribution, we obtain $\overline{g}(\overline{J}\xi,N)\neq 0$, $\forall\xi\in\Gamma(D_1), N\in\Gamma ltr(TM)$. Also $\overline{J}D_2\subset ltr(TM)$, so we get $\overline{g}(\xi_1,\overline{J}\xi_2)\neq 0$, $\forall\xi_1,\xi_2\in\Gamma(D_2)$. Hence $\overline{g}(\overline{R}(\xi,N)\xi_1,\xi_2)\neq 0$ for all $\xi\in\Gamma(D_1)$, $N\in\Gamma(ltr(TM))$ and $\xi_1,\xi_2\in\Gamma(D_2)$, which proves (ii). For $X,Y\in\Gamma(D)$ and $Z,W\in\Gamma(D^\perp)$, from (2.22), we have

$$\overline{g}(\overline{R}(X,Y)Z,W)=\frac{c}{2}g(\overline{J}X,Y)\overline{g}(Z,\overline{J}W). \tag{3.15}$$

In view of $\overline{J}W \in S(TM^{\perp})$, we have $\overline{g}(Z, \overline{J}W) = 0$, $\forall Z, W \in \Gamma(D^{\perp})$. Hence $\overline{g}(\overline{R}(X,Y)Z,W) = 0$, which proves (iii).

Now, conversely suppose that the conditions (i), (ii), (iii) are satisfied. Since D_1 is invariant distribution, we have $\overline{g}(\overline{J}\xi, N) \neq 0$ $\forall \xi \in \Gamma(D_1)$, $N \in \Gamma(ltr(TM))$. Thus from (ii) and (3.14), we have $\overline{g}(\xi_1, \overline{J}\xi_2) \neq 0$ $\forall \xi_1, \xi_2 \in \Gamma(D_2)$, which implies $\overline{J}D_2 \subset ltr(TM)$.

Further, since D is non-degenerate invariant distribution, we may choose $X, Y \in \Gamma(D)$ such that $g(\overline{J}X, Y) \neq 0$. Thus from (iii) and (3.15), we have $\overline{g}(Z, \overline{J}W) = 0$, $\forall Z, W \in \Gamma(D^{\perp})$, which implies that $\overline{J}W$ have no components in $(D^{\perp})$. For any $X \in \Gamma(D)$, we have $\overline{g}(\overline{J}W, X) = -\overline{g}(W, \overline{J}X) = 0$, which implies that $\overline{J}W$ have no components in D.

Now, form (i) and (ii), we also have $\overline{g}(\overline{J}W, \xi) = -\overline{g}(W, \overline{J}\xi) = 0$ and $\overline{g}(\overline{J}W, N) = -\overline{g}(W, \overline{J}N) = 0$, $\forall \xi \in \Gamma(RadTM)$ and $N \in \Gamma(ltr(TM))$, which implies that $\overline{J}W$ have no components in $RadTM$ and $ltr(TM)$. Thus, we get $\overline{J}D^{\perp} \subseteq S(TM^{\perp})$, which completes the proof.

Theorem 3.3. *Let M be a radical transversal SCR-lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then the induced connection ∇ is a metric connection if and only if $P_3\nabla_X\overline{J}P_1\xi = P_3A_{\overline{J}P_2\xi}X$, $Q_4h^s(X, \overline{J}P_1\xi) = 0$ and $Q_4D^s(X, \overline{J}P_2\xi) = 0$, $\forall X \in \Gamma(TM)$ and $\xi \in \Gamma(RadTM)$.*

Proof. Let M be a radical transversal SCR-lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then the induced connection ∇ on M is a metric connection if and only if $RadTM$ is parallel distribution with respect to ∇ ([10]), i.e. $\nabla_X\xi \in \Gamma(RadTM)$, $\forall X \in \Gamma(TM), \forall \xi \in \Gamma(RadTM)$. From (2.21), we have

$$\overline{\nabla}_X\overline{J}\xi = \overline{J}\overline{\nabla}_X\xi \qquad \forall X \in \Gamma(TM), \forall \xi \in \Gamma(RadTM). \tag{3.16}$$

From (2.7), (2.8), (2.19) and (3.16), we obtain

$$\begin{aligned} &\overline{J}\nabla_X\overline{J}P_1\xi + \overline{J}h^l(X, \overline{J}P_1\xi) + \overline{J}h^s(X, \overline{J}P_1\xi) - \overline{J}A_{\overline{J}P_2\xi}X + \\ &\overline{J}\nabla^l_X\overline{J}P_2\xi + \overline{J}D^s(X, \overline{J}P_2\xi) + \nabla_X\xi + h^l(X,\xi) + h^s(X,\xi) = 0. \end{aligned} \tag{3.17}$$

Now, taking tangential components in (3.17), we get

$$\begin{aligned} &\overline{J}P_1\nabla_X\overline{J}P_1\xi + \overline{J}P_3\nabla_X\overline{J}P_1\xi + \overline{J}Q_2h^l(X, \overline{J}P_1\xi) + \overline{J}Q_4h^s(X, \overline{J}P_1\xi) - \\ &\overline{J}P_1A_{\overline{J}P_2\xi}X - \overline{J}P_3A_{\overline{J}P_2\xi}X + \overline{J}Q_2\nabla^l_X\overline{J}P_2\xi + \overline{J}Q_4D^s(X, \overline{J}P_2\xi) + \nabla_X\xi = 0. \end{aligned} \tag{3.18}$$

Thus $\nabla_X\xi = \overline{J}P_1A_{\overline{J}P_2\xi}X - \overline{J}P_1\nabla_X\overline{J}P_1\xi - \overline{J}Q_2h^l(X, \overline{J}P_1\xi) - \overline{J}Q_2\nabla^l_X\overline{J}P_2\xi \in \Gamma(RadTM)$ if and only if $P_3\nabla_X\overline{J}P_1\xi = P_3A_{\overline{J}P_2\xi}X$, $Q_4h^s(X, \overline{J}P_1\xi) = 0$ and $Q_4D^s(X, \overline{J}P_2\xi) = 0$, $\forall X \in \Gamma(TM)$ and $\xi \in \Gamma(RadTM)$, which completes the proof.

Theorem 3.4. *Let M be the radical transversal SCR-lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then D_1 is integrable if and only if*

(i) $Q_2h^l(Y,\overline{J}P_1X) = Q_2h^l(X,\overline{J}P_1Y)$ *and* $Q_4h^s(Y,\overline{J}P_1X) = Q_4h^s(X,\overline{J}P_1Y)$,

(ii) $P_3(\nabla_X\overline{J}P_1Y) = P_3(\nabla_Y\overline{J}P_1X)$, $\quad \forall X,Y \in \Gamma(D_1)$.

Proof. Let M be the radical transversal SCR-lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Let $X,Y \in \Gamma(D_1)$. From (3.8), we get $P_3(\nabla_X\overline{J}P_1Y) = \overline{J}P_3\nabla_XY$, which gives $P_3(\nabla_X\overline{J}P_1Y) - P_3(\nabla_Y\overline{J}P_1X) = \overline{J}P_3[X,Y]$. In view of (3.11), we have $Q_2h^l(X,\overline{J}P_1Y) = \overline{J}P_2\nabla_XY$. Thus $Q_2h^l(X,\overline{J}P_1Y)-Q_2h^l(Y,\overline{J}P_1X) = \overline{J}P_2[X,Y]$. Also from (3.13), we obtain $Q_4h^s(X,\overline{J}P_1Y) = \overline{J}P_4\nabla_XY$, which gives $Q_4h^s(X,\overline{J}P_1Y) - Q_4h^s(Y,\overline{J}P_1X) = \overline{J}P_4[X,Y]$. This concludes the theorem.

Theorem 3.5. *Let M be the radical transversal SCR-lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then D_2 is integrable if and only if*

(i) $P_1(A_{\overline{J}P_2Y}X) = P_1(A_{Y\overline{J}P_2X}Y)$ *and* $P_3(A_{\overline{J}P_2Y}X) = P_3(A_{\overline{J}P_2X}Y)$,

(ii) $Q_4D^s(Y,\overline{J}P_2X) = Q_4D^s(X,\overline{J}P_2Y)$, $\quad \forall X,Y \in \Gamma(D_2)$.

Proof. Let M be the radical transversal SCR-lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Let $X,Y \in \Gamma(D_2)$. From (3.6), we get $P_1(A_{\overline{J}P_2Y}X) = -\overline{J}P_1\nabla_XY$, which gives $P_1(A_{\overline{J}P_2X}Y)-P_1(A_{\overline{J}P_2Y}X) = \overline{J}P_1[X,Y]$. In view of (3.8), we obtain $P_3(A_{\overline{J}P_2Y}X) = -\overline{J}P_3\nabla_XY$, which implies $P_3(A_{\overline{J}P_2X}Y)-P_3(A_{\overline{J}P_2Y}X) = \overline{J}P_3[X,Y]$. Also from (3.13), we have $Q_4D^s(X,\overline{J}P_2Y) = \overline{J}P_4\nabla_XY$, which gives $Q_4D^s(X,\overline{J}P_2Y) - Q_4D^s(Y,\overline{J}P_2X) = \overline{J}P_4[X,Y]$. This completes the proof.

Theorem 3.6. *Let M be the radical transversal SCR-lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then D is integrable if and only if*

(i) $Q_2h^l(Y,\overline{J}P_3X) = Q_2h^l(X,\overline{J}P_3Y)$ *and* $Q_4h^s(Y,\overline{J}P_3X) = Q_4h^s(X,\overline{J}P_3Y)$,

(ii) $P_1(\nabla_X\overline{J}P_3Y) = P_1(\nabla_Y\overline{J}P_3X)$, $\quad \forall X,Y \in \Gamma(D)$.

Proof. Let M be the radical transversal SCR-lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Let $X,Y \in \Gamma(D)$. From (3.6), we get $P_1(\nabla_X\overline{J}P_3Y) = \overline{J}P_1\nabla_XY$, which gives $P_1(\nabla_X\overline{J}P_3Y) - P_1(\nabla_Y\overline{J}P_3X) = \overline{J}P_1[X,Y]$. In view of (3.11), we have $Q_2h^l(X,\overline{J}P_3Y) = \overline{J}P_2\nabla_XY$. Thus $Q_2h^l(X,\overline{J}P_3Y)-Q_2h^l(Y,\overline{J}P_3X) = \overline{J}P_2[X,Y]$. Also from (3.13), we obtain $Q_4h^s(X,\overline{J}P_3Y) = \overline{J}P_4\nabla_XY$, which gives $Q_4h^s(X,\overline{J}P_3Y)-Q_4h^s(Y,\overline{J}P_3X) = \overline{J}P_4[X,Y]$. Thus, we obtain the required results.

Theorem 3.7. *Let M be the radical transversal SCR-lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then $D^\perp$ is integrable if and only if*

(i) $P_1(A_{\overline{J}P_4Y}X) = P_1(A_{Y\overline{J}P_4X}Y)$ *and* $P_3(A_{\overline{J}P_4Y}X) = P_3(A_{\overline{J}P_4X}Y)$,

(ii) $Q_2D^l(Y,\overline{J}P_4X) = Q_2D^l(X,\overline{J}P_4Y)$, $\quad \forall X,Y \in \Gamma(D^\perp)$.

Proof. Let M be the radical transversal SCR-lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Let $X, Y \in \Gamma(D^{\perp})$. From (3.6), we get $P_1(A_{\overline{J}P_4Y}X) = -\overline{J}P_1\nabla_X Y$, which gives $P_1(A_{\overline{J}P_4X}Y) - P_1(A_{\overline{J}P_4Y}X) = \overline{J}P_1[X,Y]$. In view of (3.8), we have $P_3(A_{\overline{J}P_4Y}X) = -\overline{J}P_3\nabla_X Y$, which implies $P_3(A_{\overline{J}P_4X}Y) - P_3(A_{\overline{J}P_4Y}X) = \overline{J}P_3[X,Y]$. Also from (3.11), we obtain $Q_2D^l(X, \overline{J}P_4Y) = \overline{J}P_2\nabla_X Y$, which gives $Q_2D^l(X, \overline{J}P_4Y) - Q_2D^l(Y, \overline{J}P_4X) = \overline{J}P_2[X,Y]$. This proves the theorem.

4. Foliations Determined by Distributions

In this section, we obtain necessary and sufficient conditions for foliations determined by distributions to be totally geodesic on the radical transversal SCR-lightlike submanifold of an indefinite Kaehler manifold.

Theorem 4.1. *Let M be the radical transversal SCR-lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then $RadTM$ defines a totally geodesic foliation in M if and only if*

(i) $h^l(X, \overline{J}Z) = 0$ and $D^l(X, \overline{J}W) = 0$,

(ii) $\nabla_X \overline{J}Z$ and $A_{\overline{J}W}X$ have no components in $Rad\,TM$,

$\forall X \in \Gamma(RadTM)$, $Z \in \Gamma(D)$ and $W \in \Gamma(D^{\perp})$.

Proof. Let M be the radical transversal SCR-lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. The distribution RadTM defines a totally geodesic foliation if and only if $\nabla_X Y \in \mathrm{Rad}TM$, $\forall X, Y \in \Gamma(RadTM)$. Since $\overline{\nabla}$ is metric a connection, from (2.7), (2.20) and (2.21), for any $X, Y \in \Gamma(RadTM)$ and $Z \in \Gamma(D)$, we have $\overline{g}(\nabla_X Y, Z) = \overline{g}(\overline{\nabla}_X \overline{J}Y, \overline{J}Z)$, which gives $\overline{g}(\nabla_X Y, Z) = -\overline{g}(\overline{\nabla}_X \overline{J}Z, \overline{J}Y) = -\overline{g}(\nabla_X \overline{J}Z, \overline{J}P_2Y) - \overline{g}(h^l(X, \overline{J}Z), \overline{J}P_1Y)$. In view of (2.7), (2.20) and (2.21), for any $X, Y \in \Gamma(RadTM)$ and $W \in \Gamma(D^{\perp})$, we obtain $\overline{g}(\nabla_X Y, W) = \overline{g}(\overline{\nabla}_X \overline{J}Y, \overline{J}W)$, which implies $\overline{g}(\nabla_X Y, W) = \overline{g}(A_{\overline{J}W}X, \overline{J}P_2Y) - \overline{g}(D^l(X, \overline{J}W), \overline{J}P_1Y)$. This completes the proof.

Theorem 4.2. *Let M be the radical transversal SCR-lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then D defines a totally geodesic foliation in M if and only if $A_{\overline{J}W}X$, $A_{\overline{J}Q_1N}X$ and $A^*_{\overline{J}Q_2N}X$ have no components in D, $\forall X \in \Gamma(D)$, $\forall N \in \Gamma(ltr(TM))$ and $\forall W \in \Gamma(D^{\perp})$.*

Proof. Let M be the radical transversal SCR-lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. The distribution D defines a totally geodesic foliation if and only if $\nabla_X Y \in D$, $\forall X, Y \in \Gamma(D)$. Since $\overline{\nabla}$ is metric a connection, from (2.7), (2.20) and (2.21), for any $X, Y \in \Gamma(D)$ and $W \in \Gamma(D^{\perp})$, we have $\overline{g}(\nabla_X Y, W) = \overline{g}(\overline{\nabla}_X \overline{J}Y, \overline{J}W)$, which gives $\overline{g}(\nabla_X Y, W) = -\overline{g}(\overline{\nabla}_X \overline{J}W, \overline{J}Y) = \overline{g}(A_{\overline{J}W}X, \overline{J}Y)$. In view of (2.7), (2.20) and (2.21), for any $X, Y \in \Gamma(D)$ and $N \in \Gamma(ltr(TM))$, we obtain $\overline{g}(\nabla_X Y, N) = \overline{g}(\overline{\nabla}_X \overline{J}Y, \overline{J}N)$, which implies $\overline{g}(\nabla_X Y, N) = -\overline{g}(\overline{J}Y, \overline{\nabla}_X(\overline{J}Q_1N + \overline{J}Q_2N))$. This concludes the theorem.

Theorem 4.3. *Let M be the radical transversal SCR-lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. Then $D^{\perp}$ defines a totally geodesic foliation in M if and only if*

(i) $D^s(X, \overline{J}Q_1N) = 0$ *and* $h^s(X, \overline{J}Q_2N) = 0$, $\forall N \in \Gamma(ltr(TM))$,

(ii)$h^s(X, \overline{J}Z) = 0$, $\forall X \in \Gamma(D^{\perp})$ *and* $\forall Z \in \Gamma(D)$.

Proof. Let M be the radical transversal SCR-lightlike submanifold of an indefinite Kaehler manifold $\overline{M}$. The distribution $D^{\perp}$ defines a totally geodesic foliation if and only if $\nabla_X Y \in D^{\perp}$, $\forall X, Y \in \Gamma(D^{\perp})$. Since $\overline{\nabla}$ is metric a connection, in view of (2.7), (2.20) and (2.21), for any $X, Y \in \Gamma(D^{\perp})$ and $Z \in \Gamma(D)$, we have $\overline{g}(\nabla_X Y, Z) = \overline{g}(\overline{\nabla}_X \overline{J}Y, \overline{J}Z)$, which gives $\overline{g}(\nabla_X Y, Z) = -\overline{g}(\overline{\nabla}_X \overline{J}Z, \overline{J}Y) = \overline{g}(h^s(X, \overline{J}Z), \overline{J}Y)$. From (2.7), (2.20) and (2.21), for any $X, Y \in \Gamma(D^{\perp})$ and $N \in \Gamma(ltr(TM))$, we obtain $\overline{g}(\nabla_X Y, N) = \overline{g}(\overline{\nabla}_X \overline{J}Y, \overline{J}N)$, which implies $\overline{g}(\nabla_X Y, N) = -\overline{g}(\overline{\nabla}_X(\overline{J}Q_1N + \overline{J}Q_2N), \overline{J}Y) = -\overline{g}(h^s(X, \overline{J}Q_2N) + D^s(X, \overline{J}Q_1N), \overline{J}Y)$. Thus, we obtain the required results.

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