FACTA UNIVERSITATIS (NIŠ)
Ser. Math. Inform. Vol. 30, No 2 (2015), 209–216

INEXTENSIBLE FLOWS OF CURVES IN THE EQUIFORM GEOMETRY OF 4-DIMENSIONAL GALILEAN SPACE G_4

Hülya Gün Bozok and Handan Öztekin

Abstract. In this paper, inextensible flows of curves in the equiform geometry of Galilean space G_4 are investigated. Necessary and sufficient conditions for inextensible flows of curves are expressed as a partial differential equation involving the equiform curvature in a 4-dimensional Galilean space G_4.

1. Introduction

The flow of a curve is said to be inextensible if its arc length is preserved. Inextensible flow of curves has many applications such as computes vision or computer animation. Inextensible flows of curves and developable surfaces were first studied by Kwon et al. [6]. There have been a lot of studies in the literature on inextensible flows. Latifi et al. [5] studied inextensible flows of curves in Minkowski 3-space. Ögrenmis et al.[1] studied inextensible curves in Galilean space G_3 , Öztekin et al. [8] studied inextensible flows of curves in G_4, moreover, inelastic flows of curves according to equiform in a Galilean space given by D.Y. Woon [7].

A Galilean space may be considered as the limit case of a pseudo-Euclidean space in which the isotropic cone degenerates to a plane. Differential geometry of Galilean space was studied by Röschel [10]. The Frenet formulas of a curve in G_4 are given by S. Yilmaz [11]. Theory of curves and the curves of constant curvature in the equiform differential geometry of the isotropic space I_3^1 and I_3^2 and the Galilean space G_3 are described in [2] and [3], respectively. Also, Divjak et al.[4] studied the equiform differential geometry of curves in a pseudo-Galilean space.Then the equiform differential geometry of curves in a 4-dimensional Galilean space G_4 is studied in [9].

Received August 08, 2014.; Accepted December 13, 2014.
2010 *Mathematics Subject Classification.* Primary 53A35

In this paper we investigate inextensible flows of curves according to equiform geometry in a 4-dimensional Galilean space G_4 and then we obtain partial differential equations in terms of inextensible flows of curves in the equiform geometry of G_4.

2. Preliminaries

Let $\alpha : I \subset \mathbb{R} \to G_4$ be an arbitrary curve in a 4-dimensional Galilean space G_4 defined by

$$\alpha(t) = (x(t), y(t), z(t), w(t)) \tag{2.1}$$

where $x(t), y(t), z(t), w(t)$ are smooth functions.

For any vector $x = (x_1, y_1, z_1, w_1)$ and $y = (x_2, y_2, z_2, w_2)$in $G_4$the Galilean scalar product is defined by

$$\langle x, y\rangle = \begin{cases} x_1x_2, & if\, x_1 \neq 0 \;\; or\;\; x_2 \neq 0 \\ y_1y_2 + z_1z_2 + w_1w_2, & if\, x_1 = 0 \;\; and\;\; x_2 = 0\,. \end{cases} \tag{2.2}$$

For any vector $x = (x_1, y_1, z_1, w_1)$, $y = (x_2, y_2, z_2, w_2)$ and $z = (x_3, y_3, z_3, w_3)$ in G_4 the Galilean cross product is defined by

$$x \wedge y \wedge z = \begin{vmatrix} 0 & e_2 & e_3 & e_4 \\ x_1 & y_1 & z_1 & w_1 \\ x_2 & y_2 & z_2 & w_2 \\ x_3 & y_3 & z_3 & w_3 \end{vmatrix} \tag{2.3}$$

where e_i are standard basis vectors.

Let $\alpha : I \subset \mathbb{R} \to G_4$, $\alpha(s) = (s, y(s), z(s), w(s))$ be a curve parametrized by arc length s in G_4. Here we denote differentiation with respect to s by a dash. The first vector of the Frenet-Serret frame, that is, the tangent vector of α is defined by

$$t(s) = \alpha'(s) = (1, y'(s), z'(s), w'(s))$$

Similar to space G_3 we define the principal vector and binormal vector,

$$n(s) = \frac{1}{\kappa(s)}(0, y''(s), z''(s), w''(s)) \;\; ,$$

$$b(s) = \frac{1}{\tau(s)}\left(0, \left(\frac{y''(s)}{\kappa(s)}\right)', \left(\frac{z''(s)}{\kappa(s)}\right)', \left(\frac{w''(s)}{\kappa(s)}\right)'\right) \;\; ,$$

where $\kappa(s)$ is the first curvature and $\tau(s)$ is the second curvature of the curve α. The fourth unit vector is defined by

$$e(s) = \mu t(s) \wedge n(s) \wedge b(s) \;\; .$$

Here the coefficient μ is taken ± 1 to make $+1$ determinant of the matrix $[t, n, b, e]$. For the curve αin G_4 , we have the following Frenet-Serret equations

(2.4) $$t' = \kappa(s)\, n(s)$$

(2.5) $$b' = -\tau(s)\, n(s) + \sigma(s)\, e(s)$$

(2.6) $$n' = \tau(s)\, b(s)$$

(2.7) $$e' = -\sigma(s)\, b(s)$$

where $\sigma(s)$ is the third curvature of the curve α .

3. Frenet Formulas in Equiform Geometry in G_4

Let $\alpha : I \subset \mathbb{R} \to G_4$ be a curve parametrized by arc length s. We define the equiform parameter of α by

$$\sigma = \int \frac{ds}{\rho} = \int \kappa ds \ ,$$

where $\rho = \frac{1}{\kappa}$ is the radius of curvature of the curve α. It follows

$$\frac{d\sigma}{ds} = \frac{1}{\rho} \quad i.e. \quad \frac{ds}{d\sigma} = \rho$$

Let h be a homothety with the center in the origin and the coefficient λ. If we put $\tilde{\alpha} = h(\alpha)$, then it follows

$$\tilde{s} = \lambda s \quad and \quad \tilde{\rho} = \lambda \rho$$

where $\tilde{s}$ is the arc length parameter of $\tilde{\alpha}$ and $\tilde{\rho}$ the radius of curvature of this curve. Therefore, σ is an equiform invariant parameter of α[6]. From now on, we define the Frenet formula of the curve $\alpha$with respect to the equiform invariant parameter σ inG_4.

If we take

$$V_1 = \frac{d\alpha}{ds}$$

then using (2.2) we have

$$V_1 = \frac{d\alpha}{ds} . \frac{ds}{d\sigma} = \rho . \frac{d\alpha}{ds} = \rho t \quad .$$

Further, we define the the vectors V_2, V_3, V_4 by

$$V_2 = \rho n \quad , \quad V_3 = \rho b \ , \quad V_4 = \rho e \quad .$$

It is easy to check that the tetrahedron $\{V_1, V_2, V_3, V_4\}$ is an equiform invariant tetrahedron of the curve α.

Definition 3.1. The function $\mathbb{K}_i : I \to \mathbb{R}$defined by

$$\mathbb{K}_1 = \dot{\rho} \ , \ \mathbb{K}_2 = \frac{\tau}{\kappa} \ , \ \mathbb{K}_3 = \frac{\sigma}{\kappa}$$

is called the equiform curvature of the curve α.

Then the formulas analogous to the Frenet formulas in the equiform geometry of the 4-dimensional Galilean space G_4 have the following form

$$\begin{aligned} V_1' &= \mathbb{K}_1 V_1 + V_2\,, \\ V_2' &= \mathbb{K}_1 V_2 + \mathbb{K}_2 V_3\,, \\ V_3' &= -\mathbb{K}_2 V_2 + \mathbb{K}_1 V_3 + \mathbb{K}_3 V_4\,, \\ V_4' &= -\mathbb{K}_3 V_3 + \mathbb{K}_1 V_4\,. \end{aligned} \tag{3.1}$$

where the functions$\mathbb{K}_1, \mathbb{K}_2, \mathbb{K}_3$ are the equiform curvatures of this curve [9].

4. Inextensible Flows of Curves According to Equiform in G_4

Throughout this paper, we assume that $\alpha(u,t)$ is a one-parameter family of smooth curves in a 4-dimensional Galilean space G_4. The arc length of $\alpha$is given by

$$\sigma(u) = \int_0^u \left|\frac{\partial \alpha}{\partial u}\right| du \tag{4.1}$$

where

$$\left|\frac{\partial \alpha}{\partial u}\right| = \left|\left\langle \frac{\partial \alpha}{\partial u}, \frac{\partial \alpha}{\partial u}\right\rangle\right|^{\frac{1}{2}} \tag{4.2}$$

The operator $\frac{\partial}{\partial \sigma}$ is given in terms of u by

$$\frac{\partial}{\partial \sigma} = \frac{1}{v}\frac{\partial}{\partial u} \tag{4.3}$$

where $v = \left|\frac{\partial \alpha}{\partial u}\right|$ and the arc length parameter is $d\alpha = vdu$. Any flow of αcan be represented as

$$\frac{\partial \alpha}{\partial t} = f_1 V_1 + f_2 V_2 + f_3 V_3 + f_4 V_4 \tag{4.4}$$

we put

$$\sigma(u,t) = \int_0^u vdu\ ,$$

it is called the arc length variation of α.

In a 4-dimensional Galilean space G_4 the requirement that the curve not be subject to any elongation or compression can be expressed by the condition

$$\frac{\partial}{\partial t}\sigma(u,t) = \int_0^u \frac{\partial v}{\partial t} du = 0 \tag{4.5}$$

for all $u \in [0, l]$.

Definition 4.1. A curve evolution $\alpha(u,t)$and its flow $\frac{\partial \alpha}{\partial t}$ in a 4-dimensional Galilean space G_4 are said to be inextensible if

$$\frac{\partial}{\partial t}\left|\frac{\partial \alpha}{\partial u}\right| = 0.$$

Lemma 4.1. *Let $\frac{\partial\alpha}{\partial t} = f_1V_1 + f_2V_2 + f_3V_3 + f_4V_4$ be a smooth flow of the curve αin a 4-dimensional Galilean space G_4. The flow is inextensible if and only if*

$$\frac{\partial v}{\partial t} = \frac{\partial f_1}{\partial u} + f_1 v\mathbb{K}_1 \tag{4.6}$$

Proof. Suppose that $\frac{\partial\alpha}{\partial t}$ be a smooth flow of the curve α in a 4-dimensional Galilean space G_4. From the definition of v, we obtain

$$v\frac{\partial v}{\partial t} = \left\langle \frac{\partial\alpha}{\partial u}, \frac{\partial}{\partial u}\left(f_1V_1 + f_2V_2 + f_3V_3 + f_4V_4\right)\right\rangle. \tag{4.7}$$

By the formulas analogous to the Frenet formulas in the equiform geometry of the 4-dimensional Galilean space G_4, we have

$$\frac{\partial v}{\partial t} = \left\langle \begin{array}{l} V_1, \left(\frac{\partial f_1}{\partial u} + f_1 v\mathbb{K}_1\right)V_1 + \left(\frac{\partial f_2}{\partial u} + f_1 v + f_2 v\mathbb{K}_1 - f_3 v\mathbb{K}_2\right)V_2 \\ +\left(\frac{\partial f_3}{\partial u} + f_2 v\mathbb{K}_2 + f_3 v\mathbb{K}_1 - f_4 v\mathbb{K}_3\right)V_3 + \left(\frac{\partial f_4}{\partial u} + f_3 v\mathbb{K}_3 + f_4 v\mathbb{K}_1\right)V_4 \end{array}\right\rangle$$

Making necessary calculations from the above equation, we have (4.6), which proves the lemma. □

Theorem 4.1. *Let $\frac{\partial\alpha}{\partial t} = f_1V_1 + f_2V_2 + f_3V_3 + f_4V_4$ be a smooth flow of the curve α in a 4-dimensional Galilean space G_4. The flow is inextensible if and only if*

$$\frac{\partial f_1}{\partial u} = -f_1 v\mathbb{K}_1. \tag{4.8}$$

Proof. From (4.2) we have

$$\frac{\partial}{\partial t}\sigma(u,t) = \int_0^u \frac{\partial v}{\partial t}du = \int_0^u \left(\frac{\partial f_1}{\partial u} + f_1 v\mathbb{K}_1\right) = 0.$$

Substituting (4.6) in (4.5) completes the proof of the theorem. □

We now restrict ourselves to arc length parametrized curves. That is, $v = 1$ and the local coordinate $u$correspond to the curve arc length s. We require the following lemma.

Lemma 4.2. *Let $\frac{\partial\alpha}{\partial t} = f_1V_1 + f_2V_2 + f_3V_3 + f_4V_4$ be a smooth flow of the curve α in a 4-dimensional Galilean space G_4. By the formulas analogous to the Frenet formulas in the equiform geometry of the 4-dimensional Galilean space G_4, we have*

$$\frac{\partial V_1}{\partial t} = \left(\frac{\partial f_2}{\partial s} + f_1 + f_2\mathbb{K}_1 - f_3\mathbb{K}_2\right)V_2 \tag{4.9}$$

$$+ \left(\frac{\partial f_3}{\partial s} + f_2\mathbb{K}_2 + f_3\mathbb{K}_1 - f_4\mathbb{K}_3\right)V_3 + \left(\frac{\partial f_4}{\partial s} + f_3\mathbb{K}_3 + f_4\mathbb{K}_1\right)V_4 \tag{4.10}$$

$$\frac{\partial V_2}{\partial t} = -\left(\frac{\partial f_2}{\partial s} + f_1 + f_2\mathbb{K}_1 - f_3\mathbb{K}_2\right)V_1 + \Psi_1 V_3 + \Psi_2 V_4 \tag{4.11}$$

$$\frac{\partial V_3}{\partial t} = -\left(\frac{\partial f_3}{\partial s} + f_2\mathbb{K}_2 + f_3\mathbb{K}_1 - f_4\mathbb{K}_3\right)V_1 - \Psi_1 V_2 + \Psi_3 V_4 \tag{4.12}$$

$$\frac{\partial V_4}{\partial t} = -\left(\frac{\partial f_4}{\partial s} + f_3\mathbb{K}_3 + f_4\mathbb{K}_1\right)V_1 - \Psi_2 V_2 - \Psi_3 V_3 \tag{4.13}$$

where $\Psi_1 = \left\langle \frac{\partial V_2}{\partial t}, V_3 \right\rangle$, $\Psi_2 = \left\langle \frac{\partial V_2}{\partial t}, V_4 \right\rangle$, $\Psi_3 = \left\langle \frac{\partial V_3}{\partial t}, V_4 \right\rangle$ *provided that* $\left(\frac{\partial f_2}{\partial s} + f_1 + f_2\mathbb{K}_1 - f_3\mathbb{K}_2\right) = 0$ *and* $\left(\frac{\partial f_3}{\partial s} + f_2\mathbb{K}_2 + f_3\mathbb{K}_1 - f_4\mathbb{K}_3\right) = 0.$

Proof. Using the definition of α, we have

$$\frac{\partial V_1}{\partial t} = \frac{\partial}{\partial t}\frac{\partial \alpha}{\partial s} = \frac{\partial}{\partial s}\left(f_1 V_1 + f_2 V_2 + f_3 V_3 + f_4 V_4\right) \tag{4.14}$$

Using the formulas analogous to the Frenet formulas in the equiform geometry of the 4-dimensional Galilean space G_4, we have

$$\begin{aligned}\frac{\partial V_1}{\partial t} = &\left(\frac{\partial f_1}{\partial s} + f_1\mathbb{K}_1\right)V_1 + \left(\frac{\partial f_2}{\partial s} + f_1 + f_2\mathbb{K}_1 - f_3\mathbb{K}_2\right)V_2 \\ &+\left(\frac{\partial f_3}{\partial s} + f_2\mathbb{K}_2 + f_3\mathbb{K}_1 - f_4\mathbb{K}_3\right)V_3 + \left(\frac{\partial f_4}{\partial s} + f_3\mathbb{K}_3 + f_4\mathbb{K}_1\right)V_4 \ .\end{aligned}$$

On the other hand, using Theorem (4.1) in the above equation we get

$$\begin{aligned}\frac{\partial V_1}{\partial t} &= \left(\frac{\partial f_2}{\partial s} + f_1 + f_2\mathbb{K}_1 - f_3\mathbb{K}_2\right)V_2 + \left(\frac{\partial f_3}{\partial s} + f_2\mathbb{K}_2 + f_3\mathbb{K}_1 - f_4\mathbb{K}_3\right)V_3 \\ &+ \left(\frac{\partial f_4}{\partial s} + f_3\mathbb{K}_3 + f_4\mathbb{K}_1\right)V_4.\end{aligned}$$

Now we differentiate the Frenet frame by$t$:

$$\begin{aligned}0 &= \frac{\partial}{\partial t}\langle V_1, V_2\rangle = \left(\frac{\partial f_2}{\partial s} + f_1 + f_2\mathbb{K}_1 - f_3\mathbb{K}_2\right) + \left\langle V_1, \frac{\partial V_2}{\partial t}\right\rangle \ , \\ 0 &= \frac{\partial}{\partial t}\langle V_1, V_3\rangle = \left(\frac{\partial f_3}{\partial s} + f_2\mathbb{K}_2 + f_3\mathbb{K}_1 - f_4\mathbb{K}_3\right) + \left\langle V_1, \frac{\partial V_3}{\partial t}\right\rangle \ , \\ 0 &= \frac{\partial}{\partial t}\langle V_1, V_4\rangle = \left(\frac{\partial f_4}{\partial s} + f_3\mathbb{K}_3 + f_4\mathbb{K}_1\right) + \left\langle V_1, \frac{\partial V_4}{\partial t}\right\rangle \ , \\ 0 &= \frac{\partial}{\partial t}\langle V_2, V_3\rangle = \Psi_1 + \left\langle V_2, \frac{\partial V_3}{\partial t}\right\rangle \ , \\ 0 &= \frac{\partial}{\partial t}\langle V_2, V_4\rangle = \Psi_2 + \left\langle V_2, \frac{\partial V_4}{\partial t}\right\rangle \ , \\ 0 &= \frac{\partial}{\partial t}\langle V_3, V_4\rangle = \Psi_3 + \left\langle V_3, \frac{\partial V_4}{\partial t}\right\rangle \ .\end{aligned}$$

Then, a straightforward computation using the above system gives

$$\frac{\partial V_2}{\partial t} = -\left(\frac{\partial f_2}{\partial s} + f_1 + f_2\mathbb{K}_1 - f_3\mathbb{K}_2\right)V_1 + \Psi_1 V_3 + \Psi_2 V_4 \tag{4.15}$$

$$\frac{\partial V_3}{\partial t} = -\left(\frac{\partial f_3}{\partial s} + f_2\mathbb{K}_2 + f_3\mathbb{K}_1 - f_4\mathbb{K}_3\right)V_1 - \Psi_1 V_2 + \Psi_3 V_4 \tag{4.16}$$

$$\frac{\partial V_4}{\partial t} = -\left(\frac{\partial f_4}{\partial s} + f_3\mathbb{K}_3 + f_4\mathbb{K}_1\right)V_1 - \Psi_2 V_2 - \Psi_3 V_3 \tag{4.17}$$

where $\Psi_1 = \left\langle \frac{\partial V_2}{\partial t}, V_3 \right\rangle$, $\Psi_2 = \left\langle \frac{\partial V_2}{\partial t}, V_4 \right\rangle$, $\Psi_3 = \left\langle \frac{\partial V_3}{\partial t}, V_4 \right\rangle$ provided that $\left(\frac{\partial f_2}{\partial s} + f_1 + f_2 \mathbb{K}_1 - f_3 \mathbb{K}_2 \right) = 0$ and $\left(\frac{\partial f_3}{\partial s} + f_2 \mathbb{K}_2 + f_3 \mathbb{K}_1 - f_4 \mathbb{K}_3 \right) = 0.$ □

Theorem 4.2. *Let $\frac{\partial \alpha}{\partial t}$ be inextensible. Then, by the formulas analogous to the Frenet formulas in the equiform geometry of the 4-dimensional Galilean space G_4, the following system of partial differential equations holds:*

$$\frac{\partial \mathbb{K}_1}{\partial t} = 0 , \tag{4.18}$$

$$\frac{\partial \mathbb{K}_2}{\partial t} = \frac{\partial \Psi_1}{\partial s} - \mathbb{K}_3 \Psi_2 \tag{4.19}$$

where $\Psi_1 = \left\langle \frac{\partial V_2}{\partial t}, V_3 \right\rangle$, $\Psi_2 = \left\langle \frac{\partial V_2}{\partial t}, V_4 \right\rangle$, $\Psi_3 = \left\langle \frac{\partial V_3}{\partial t}, V_4 \right\rangle$ *provided that* $\left(\frac{\partial f_2}{\partial s} + f_1 + f_2 \mathbb{K}_1 - f_3 \mathbb{K}_2 \right) = 0$ *and* $\left(\frac{\partial f_3}{\partial s} + f_2 \mathbb{K}_2 + f_3 \mathbb{K}_1 - f_4 \mathbb{K}_3 \right) = 0.$

Proof. Using (4.9) we have

$$\frac{\partial}{\partial s}\frac{\partial V_1}{\partial t} = \frac{\partial}{\partial s}\left[\begin{array}{l} \left(\frac{\partial f_2}{\partial s} + f_1 + f_2 \mathbb{K}_1 - f_3 \mathbb{K}_2 \right) V_2 + \left(\frac{\partial f_3}{\partial s} + f_2 \mathbb{K}_2 + f_3 \mathbb{K}_1 - f_4 \mathbb{K}_3 \right) V_3 \\ + \left(\frac{\partial f_4}{\partial s} + f_3 \mathbb{K}_3 + f_4 \mathbb{K}_1 \right) V_4 \end{array} \right]$$

On the other hand, from the formulas analogous to the Frenet formulas in the equiform geometry of a 4-dimensional Galilean space G_4, we have

$$\frac{\partial}{\partial t}\frac{\partial V_1}{\partial s} = \frac{\partial}{\partial t}(\mathbb{K}_1 V_1 + V_2) = \frac{\partial \mathbb{K}_1}{\partial t} V_1 + \mathbb{K}_1 \frac{\partial V_1}{\partial t} + \frac{\partial V_2}{\partial t}$$

Hence we see that

$$\frac{\partial \mathbb{K}_1}{\partial t} = 0 .$$

Similarly, using (4.11) we have

$$\frac{\partial}{\partial s}\frac{\partial V_2}{\partial t} = \frac{\partial}{\partial s}\left[-\left(\frac{\partial f_2}{\partial s} + f_1 + f_2 \mathbb{K}_1 - f_3 \mathbb{K}_2 \right) V_1 + \Psi_1 V_3 + \Psi_2 V_4 \right]$$

On the other hand, from the formulas analogous to the Frenet formulas in the equiform geometry of a 4-dimensional Galilean space G_4, we have

$$\frac{\partial}{\partial t}\frac{\partial V_2}{\partial s} = \frac{\partial}{\partial t}(\mathbb{K}_1 V_2 + \mathbb{K}_2 V_3) = \frac{\partial \mathbb{K}_1}{\partial t} V_2 + \mathbb{K}_1 \frac{\partial V_2}{\partial t} + \frac{\partial \mathbb{K}_2}{\partial t} V_3 + \mathbb{K}_2 \frac{\partial V_3}{\partial t}$$

Hence we see that

$$\frac{\partial \mathbb{K}_2}{\partial t} = \frac{\partial \Psi_1}{\partial s} - \mathbb{K}_3 \Psi_2 ,$$

where $\Psi_1 = \left\langle \frac{\partial V_2}{\partial t}, V_3 \right\rangle$, $\Psi_2 = \left\langle \frac{\partial V_2}{\partial t}, V_4 \right\rangle$, $\Psi_3 = \left\langle \frac{\partial V_3}{\partial t}, V_4 \right\rangle$ provided that $\left(\frac{\partial f_2}{\partial s} + f_1 + f_2 \mathbb{K}_1 - f_3 \mathbb{K}_2 \right) = 0$ and $\left(\frac{\partial f_3}{\partial s} + f_2 \mathbb{K}_2 + f_3 \mathbb{K}_1 - f_4 \mathbb{K}_3 \right) = 0$. □

REFERENCES

1. A. O. Ogrenmis, M. Yeneroglu, *Inextensible curves in the Galilean space,* International Journal of the Physical Sciences, 5(9) (2010), 1424-1427.
2. B.J.Pavkovic, *Equiform geometry of curves in the isotropic space I_3^1 and I_3^2,* Rad JAZU., (1986), 39–44.
3. B.J.Pavkovic, I. Kamenarovic, *The equiform differential geometry of curves in the Galilean space G_3,* Glasnik mat., 22(42), (1987), 449-457.
4. B. Divjak and Z. Erjavec, *The equiform differential geometry of curves in the pseudo-Galilean space,* Mathematical Communitations., 13 (2008), 321-331.
5. D. Latifi, A. Razavi, *Inextensible flows of curves in Minkowskian Space*, Adv. Studies Theor. Phys., 2(16) (2008), 761-768.
6. D. Y. Kwon , F.C. Park, D.P. Chi, *Inextensible flows of curves and developable surfaces*, Applied Mathematics Letters, 18 (2005) 1156–1162.
7. D. W. Yoon, *Inelastic flows of curves according to equiform in Galilean space*, Journal of the Chungcheong Mathematical Society, 24(4),(2011).
8. H. Öztekin and H. Gün Bozok, *Inextensible flows of curves in 4-dimensional Galilean space G_4*, Mathematical Sciences and Applications E-Notes, Vol.1(2) (2013), 28-34.
9. M.E. Aydin and M. Ergüt, *The equiform differential geometry of curves in 4-dimensional Galilean space G_4*, Stud. Univ. Babes-Bolyai Math., 58(3) (2013), 393–400 .
10. O. Röschel, Die Geometrie Des Galileischen Raumes, Berichte der Math.-Stat. Sektion im Forschumgszentrum Graz, Ber., 256 (1986), 1-20.
11. S.Yilmaz, Consruction of the Frenet-Serret Frame of a Curve in 4D Galilean Space and Some Applications, Int. Jour. of the Phys. Sci. Vol. ,4 August (2010), 1284-1289.

Hülya Gün Bozok
Osmaniye Korkut Ata University
Faculty of Arts and Science
Department of Mathematics
80000 Osmaniye, Turkey
hulya-gun@hotmail.com

Handan Öztekin
Firat University
Faculty of Science
Department of Mathematics
23119 Elazig, Turkey
handanoztekin@gmail.com