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STUDY OF A HYPERBOLIC KAEHLERIAN MANIFOLDS EQUIPPED WITH A QUARTER-SYMMETRIC METRIC CONNECTION

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Abstract. This paper contains the study of a hyperbolic Kaehlerian manifold with different approaches. We consider a hyperbolic Kaehlerian manifold with a quarter-symmetric metric connection and obtained expressions for holomorphic conharmonic curvature tensor, conformal curvature tensor with respect to a quarter-symmetric metric connection. We have also studied holomorphic conharmonic recurrent, conformal recurrent and Weyl projective recurrent with respect to a quarter-symmetric metric connection.
Keywords: Kaehlerian manifold; curvature tensor; Riemannian connection; Ricci tensor.

1. Introduction

Let (M^n, g), $(n > 2)$, be an even dimensional differentiable manifold with the structure F_i^h. If F_i^h satisfies the relation

$$F_j^i F_i^h = \delta_j^h, \tag{1.1}$$

$$F_{ij} = -F_{ji}, \quad (F_{ij} = g_{jk} F_i^k), \tag{1.2}$$

and

$$F_{i,j}^h = 0, \tag{1.3}$$

then the manifold is called hyperbolic Kaehlerian (space) manifold i.e. in a hyperbolic Kaehlerian manifold, equations (1.1), (1.2) and (1.3) hold. There, F_i^h is a tensor field of type (1.1) and $F_{i,j}^h$ is a covariant derivative of F_i^h with respect to Riemannian connection. Yano and Imai [3] considered a quarter-symmetric metric connection ∇ and Riemannian connection D with coefficients Γ_{ij}^h and $\{{}^{\,h}_{i\,j}\}$, respectively. According to them, if the torsion tensor T of the connection ∇ on (M^n, g), $(n > 2)$, satisfies

$$T_{jk}^i = p_j A_k^i - p_k A_j^i, \tag{1.4}$$

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the relation between the coefficients of quarter-symmetric metric connection ∇ and Riemannian connection D is given by

$$\Gamma^i_{jk} = \{^{\,i}_{jk}\} - p_k U^i_j + p_j V^i_k - p^i V_{jk}, \tag{1.5}$$

where

$$U_{ij} = \frac{1}{2}(A_{ij} - A_{ji}), \quad V_{ij} = \frac{1}{2}(A_{ij} + A_{ji}), \tag{1.6}$$

$\nabla g = 0$ and p_i are the components of a 1-form . Also, A^i_j denotes the components of the tensor of the type (1,1). Equation (1.6) implies

$$A_{ij} = U_{ij} + V_{ij}. \tag{1.7}$$

In [4] a quarter-symmetric metric connection in a hyperbolic Kaehlerian manifold by taking $V_{ij} = g_{ij}$ and $U_{ij} = F_{ij}$ in (1.5) was constructed as the form

$$\Gamma^i_{jk} = \{^{\,i}_{jk}\} - p_k F^i_j + p_j \delta^i_k - p^i g_{jk}. \tag{1.8}$$

Also, it is shown [4] that the relation between the curvature tensor with respect to a quarter-symmetric metric connection and a Riemannian connection is given by

$$\begin{aligned} \overline{R}_{ijkh} &= R_{ijkh} - g_{ih}p_{kj} + g_{ik}\,p_{hj} - g_{jk}\,p_{hi} \\ &+ g_{hj}\,p_{ki} + p_j\,p_h F_{ik} + p_i\,p_k F_{jh} \\ &- p_j\,p_k F_{ih} - p_i\,p_h F_{jk}, \end{aligned} \tag{1.9}$$

where

$$p_{jk} = \nabla_j p_k - p_j\,p_k + p_k\,q_j + \frac{1}{2}p_s\,p^s\,g_{jk}. \tag{1.10}$$

Additionally, the Ricci tensor and the scalar curvature are found as, [4], respectively

$$\overline{R}_{jk} = R_{jk} - (n-2)p_{kj} - g_{jk}p^m_m - p_j\,q_k + p_k\,q_j - p^s\,p_s F_{kj}, \tag{1.11}$$

and

$$\overline{R} = R - 2(n-1)p^m_m. \tag{1.12}$$

We know that in a Kaehler manifold

$$(a) p_j = p^h g_{jh}, \quad (b) q_i = F_{ti}\,p^t, \quad (c)\ p^r = g^{ir}\,p_i. \tag{1.13}$$

2. Holomorphic conharmonic curvature tensor

We know that the holomorphic conharmonic curvature tensor in a Riemannian manifold is defined as

$$T_{ijkh} = R_{ijkh} + \frac{1}{n-2}(g_{ik}R_{jh} - g_{jk}R_{ih}). \tag{2.1}$$

Therefore the holomorphic conharmonic curvature tensor with respect to a quarter-symmetric metric connection is given by

$$\overline{T}_{ijkh} = \overline{R}_{ijkh} + \frac{1}{n-2}(g_{ik}\overline{R}_{jh} - g_{jk}\overline{R}_{ih}), \tag{2.2}$$

where $\overline{R}_{ijkh}$ and $\overline{R}_{ih}$ denote the curvature tensor and the Ricci tensor with respect to a quarter-symmetric metric connection, respectively.

Using (1.9), (1.11) in (2.2), we get

$$\begin{aligned} \overline{T}_{ijkh} &= R_{ijkh} - g_{ih}p_{kj} + g_{ik}\,p_{hj} - g_{jk}\,p_{hi} + g_{hj}\,p_{ki} \\ &+ p_j p_h F_{ik} + p_i p_k F_{jh} - p_j p_k F_{ih} - p_i p_h F_{jk} \\ &+ \frac{1}{n-2}\Big(g_{ik}(R_{jh} - (n-2)p_{hj} - g_{jh}\,p_m^m - p_j q_h \\ &+ p_h q_j - p_s p^s F_{hj}\Big) \\ &- g_{jk}\Big(R_{ih} - (n-2)p_{hi} - g_{ih}\,p_m^m - p_i q_h \\ &+ p_h q_i - p_s p^s F_{hi})\Big). \end{aligned} \tag{2.3}$$

Using [(1.13(*a*, *b*))] in (2.3), we find

$$\begin{aligned} \overline{T}_{ijkh} &= R_{ijkh} + \frac{1}{n-2}(g_{ik}R_{jh} - g_{jk}R_{ih}) \\ &- \frac{n}{n-2}(p_j p_k F_{ih} - p_i p_k F_{jh}) \\ &+ \frac{n-1}{n-2}(p_h p_j F_{ik} - p_i p_h F_{jk}) \\ &- \frac{p_m^m}{n-2}(g_{ki}\,g_{hj} - g_{jk}\,g_{hi}) + (g_{hj}\,p_{ki} - g_{ih}p_{kj}). \end{aligned} \tag{2.4}$$

From (2.1) and (2.4), we obtain

$$\begin{aligned} \overline{T}_{ijkh} &= T_{ijkh} - \frac{n}{n-2}(p_j p_k F_{ih} - p_i p_k F_{jh}) \\ &+ \frac{n-1}{n-2}(p_h p_j F_{ik} - p_i p_h F_{jk}) \\ &- \frac{p_m^m}{n-2}(g_{ki}\,g_{hj} - g_{jk}\,g_{hi}) + (g_{hj}\,p_{ki} - g_{ih}p_{kj}). \end{aligned} \tag{2.5}$$

In this case, if

$$p_j F_{ih} = p_i F_{jh}, \quad p_{ki}\,g_{hj} = p_{kj}\,g_{hi}, \quad \text{and} \quad g_{ki}\,g_{hj} = g_{jk}\,g_{hi}, \tag{2.6}$$

then from (2.5), we get

$$\overline{T}_{ijkh} = T_{ijkh}. \tag{2.7}$$

Thus, we conclude:

Theorem 2.1. *In a hyperbolic Kaehlerian manifold equipped with a quarter-symmetric metric connection, the holomorphic conharmonic curvature tensor with respect to a quarter-symmetric metric connection will be equal to the holomorphic conharmonic curvature tensor with respect to a Riemannian connection if the following conditions hold:*

(1) $p_j F_{ih} = p_i F_{jh}$, (2) $p_{ki} g_{hj} = p_{kj} g_{hi}$, (3) $g_{ki} g_{hj} = g_{jk} g_{hi}$.

Now, we propose:

Theorem 2.2. *In a hyperbolic Kaehlerian manifold equipped with a quarter-symmetric metric connection, the holomorphic conharmonic curvature tensor with respect to a quarter-symmetric metric connection satisfies the first Bianchi identity if*

$$p_j F_{ik} + p_i F_{kj} + p_k F_{ji} = 0, \text{ and } p_{ij} g_{kh} = p_{ik} g_{jh},$$

i.e.

$$\overline{T}_{ijkh} + \overline{T}_{jkih} + \overline{T}_{kijh} = 0,$$

if

$$p_j F_{ik} + p_i F_{kj} + p_k F_{ji} = 0, \text{ and } p_{ij} g_{kh} = p_{ik} g_{jh}.$$

Proof. Interchanging i, j and k in a cyclic order in (2.5), we get

$$\begin{aligned} \overline{T}_{ijkh} &= T_{ijkh} \\ &- \frac{n}{n-2}(p_j p_k F_{ih} - p_i p_k F_{jh}) \\ &+ \frac{n-1}{n-2}(p_h p_j F_{ik} - p_i p_h F_{jk}) \\ &- \frac{p_m^m}{n-2}(g_{ki} g_{hj} - g_{jk} g_{hi}) + (p_{ki} g_{hj} - p_{kj} g_{hi}), \end{aligned} \tag{2.8}$$

$$\begin{aligned} \overline{T}_{jkih} &= T_{jkih} \\ &- \frac{n}{n-2}(p_k p_i F_{jh} - p_j p_i F_{kh}) \\ &+ \frac{n-1}{n-2}(p_h p_k F_{ji} - p_j p_h F_{ki}) \\ &- \frac{p_m^m}{n-2}(g_{ij} g_{hk} - g_{ki} g_{hj}) + (p_{ij} g_{hk} - p_{ik} g_{hj}), \end{aligned} \tag{2.9}$$

and

$$\begin{aligned} \overline{T}_{kijh} &= T_{kijh} \\ &- \frac{n}{n-2}(p_i p_j F_{kh} - p_k p_j F_{ih}) \\ &+ \frac{n-1}{n-2}(p_h p_i F_{kj} - p_k p_h F_{ij}) \\ &- \frac{p_m^m}{n-2}(g_{jk} g_{hi} - g_{ij} g_{hk}) + (p_{jk} g_{hi} - p_{ji} g_{hk}). \end{aligned} \tag{2.10}$$

Adding equation (2.8), (2.9) and (2.10), we have

$$\begin{aligned}\overline{T}_{ijkh}+\overline{T}_{jkih}+\overline{T}_{kijh} &= T_{ijkh}+T_{jkih}+T_{kijh}\\ &+ \frac{2(n-1)}{n-2}p_h(p_jF_{ik}+p_iF_{kj}+p_kF_{ji})\\ &+ (p_{ki}g_{hj}-p_{kj}g_{hi})\\ &+ (p_{ij}g_{hk}-p_{ik}g_{hj})\\ &+ (p_{jk}g_{hi}-p_{ji}g_{kh}).\end{aligned} \tag{2.11}$$

Since in a Riemannian manifold the holomorphic conharmonic curvature tensor satisfies the first Bianchi identity, i.e.

$$T_{ijkh}+T_{jkih}+T_{kijh}=0. \tag{2.12}$$

Using (2.12) in (2.11), we have

$$\begin{aligned}\overline{T}_{ijkh}+\overline{T}_{jkih}+\overline{T}_{kijh} &= \tfrac{2(n-1)}{n-2}p_h(p_jF_{ik}+p_iF_{kj}+p_kF_{ji})\\ &\quad+(p_{ki}g_{hj}-p_{kj}g_{hi})\\ &\quad+(p_{ij}g_{hk}-p_{ik}g_{hj})\\ &\quad+(p_{jk}g_{hi}-p_{ji}g_{kh}).\end{aligned} \tag{2.13}$$

Now, assuming that

$$p_jF_{ik}+p_iF_{kj}+p_kF_{ji}=0, \quad and \quad p_{ij}g_{hk}=p_{ik}g_{hj}, \tag{2.14}$$

then from (2.13), we get

$$\overline{T}_{ijkh}+\overline{T}_{jkih}+\overline{T}_{kijh}=0. \tag{2.15}$$

This completes the proof. □

Now, we propose:

Theorem 2.3. *In a hyperbolic Kaehlerian manifold equipped with a quarter-symmetric metric connection, the holomorphic conharmonic curvature tensor with respect to a quarter-symmetric metric connection satisfies*

$$\overline{T}_{ijkh}=-\overline{T}_{jikh}.$$

Proof. Interchanging i and j in (2.5), we have

$$\begin{aligned}\overline{T}_{jikh} &= T_{jikh}\\ &- \frac{n}{n-2}(p_ip_kF_{jh}-p_jp_kF_{ih})\\ &+ \frac{n-1}{n-2}(p_hp_iF_{jk}-p_jp_hF_{ik})\\ &- \frac{p_m^m}{n-2}(g_{kj}g_{hi}-g_{ik}g_{hj})+(g_{hi}p_{kj}-g_{jh}p_{ki}).\end{aligned} \tag{2.16}$$

Adding (2.5) and (2.16), we have

$$\overline{T}_{ijkh} + \overline{T}_{jikh} = T_{ijkh} + T_{jikh}. \tag{2.17}$$

Since in a Riemannian manifold the holomorphic conharmonic curvature tensor satisfies

$$T_{ijkh} + T_{jikh} = 0. \tag{2.18}$$

Then by using (2.18) in (2.17) , we get

$$\overline{T}_{ijkh} = -\overline{T}_{jikh}. \tag{2.19}$$

□

3. Conformal curvature tensor

In the Riemannian manifold (M^n, g), $(n > 2)$, the conformal curvature tensor of the type (0,4) is defined as

$$\begin{aligned} C_{ijkh} &= R_{ijkh} \\ &- \frac{1}{n-2}(R_{jk}\, g_{ih} - R_{ik}\, g_{jh} + R_{ih}\, g_{jk} - R_{jh}\, g_{ki}) \\ &+ \frac{R}{(n-1)(n-2)}(g_{ih}\, g_{jk} - g_{jh}\, g_{ki}). \end{aligned} \tag{3.1}$$

The conformal curvature tensor with respect to a quarter-symmetric metric connection is given by

$$\begin{aligned} \overline{C}_{ijkh} &= \overline{R}_{ijkh} \\ &- \frac{1}{n-2}(\overline{R}_{jk}\, g_{ih} - \overline{R}_{ik}\, g_{jh} + \overline{R}_{ih}\, g_{jk} - \overline{R}_{jh}\, g_{ki}) \\ &+ \frac{\overline{R}}{(n-1)(n-2)}(g_{ih}\, g_{jk} - g_{jh}\, g_{ki}). \end{aligned} \tag{3.2}$$

Using (1.9), (1.11) and (1.12) in (3.2), we have

$$\begin{aligned} \overline{C}_{ijkh} = R_{ijkh} & \\ &- g_{ih}p_{kj} + g_{ik}\, p_{hj} - g_{jk}\, p_{hi} + g_{hj}\, p_{ki} \\ &+ p_j\, p_h\, F_{ik} + p_i\, p_k\, F_{jh} - p_j\, p_k\, F_{ih} - p_i\, p_h\, F_{jk} \\ &- \frac{1}{n-2}\Big(g_{ih}(R_{jk} - (n-2)p_{kj} - g_{jk}p^m_m - p_j\, q_k + p_k\, q_j - p^s\, p_s\, F_{kj}) \\ &- g_{jh}(R_{ik} - (n-2)p_{ki} - g_{ik}p^m_m - p_i\, q_k + p_k\, q_i - p^s\, p_s\, F_{ki}) \\ &+ g_{kj}(R_{ih} - (n-2)p_{hi} - g_{ih}p^m_m - p_i\, q_h + p_h\, q_i - p^s\, p_s\, F_{hi}) \\ &- g_{ik}(R_{jh} - (n-2)p_{hj} - g_{jh}p^m_m - p_j\, q_h + p_h\, q_j - p^s\, p_s\, F_{hj})\Big) \\ &+ \frac{R - 2(n-1)\, p^m_m}{(n-1)(n-2)}(g_{ih}\, g_{jk} - g_{jh}\, g_{ik}). \end{aligned} \tag{3.3}$$

Now using [(1.13(a, b))] in (3.3), we have

$$\overline{C}_{ijkh} = R_{ijkh} - \frac{1}{n-2}(R_{jk}\, g_{ih} - R_{ik}\, g_{jh} + R_{ih}\, g_{jk} - R_{jh}\, g_{ki}) + \frac{R}{(n-1)(n-2)}(g_{ih}\, g_{jk} - g_{jh}\, g_{ki}) + \frac{n+1}{n-2}(p_j p_h F_{ik} - p_j p_k F_{ih}) + \frac{n+1}{n-2}(p_i p_k F_{jh} - p_i p_h F_{jk}). \tag{3.4}$$

Using (3.1) in (3.4), we have

$$\overline{C}_{ijkh} = C_{ijkh} + \frac{n+1}{n-2}(p_j p_h F_{ik} - p_j p_k F_{ih}) + \frac{n+1}{n-2}(p_i p_k F_{jh} - p_i p_h F_{jk}). \tag{3.5}$$

If we take $p_h F_{ik} = p_k F_{ih}$ then (3.5) reduces to the form

$$\overline{C}_{ijkh} = C_{ijkh}. \tag{3.6}$$

Thus, we conclude:

Theorem 3.1. *In a hyperbolic Kaehlerian manifold equipped with a quarter-symmetric metric connection, the conformal curvature tensor with respect to a quarter-symmetric metric connection will be equal to the conformal curvature tensor with respect to a Riemannian connection if and only if*

$$p_h F_{ik} = p_k F_{ih}. \tag{3.7}$$

Now, we propose:

Theorem 3.2. *In a hyperbolic Kaehlerian manifold equipped with a quarter-symmetric metric connection, the conformal curvature tensor with respect to a quarter-symmetric metric connection satisfies the first Bianchi identity if*

$$p_j F_{ik} + p_i F_{kj} + p_k F_{ji} = 0. \tag{3.8}$$

Proof. Interchanging i, j and k in a cyclic order in (3.5), we find

$$\overline{C}_{ijkh} = C_{ijkh} + \frac{n+1}{n-2}(p_j p_h F_{ik} - p_j p_k F_{ih}) + \frac{n+1}{n-2}(p_i p_k F_{jh} - p_i p_h F_{jk}), \tag{3.9}$$

$$\begin{aligned}\overline{C}_{jkih} &= C_{jkih}\\ &+ \frac{n+1}{n-2}(p_k p_h F_{ji} - p_k p_i F_{jh})\\ &+ \frac{n+1}{n-2}(p_j p_i F_{kh} - p_j p_h F_{ki}),\end{aligned} \tag{3.10}$$

and

$$\begin{aligned}\overline{C}_{kijh} &= C_{kijh}\\ &+ \frac{n+1}{n-2}(p_i p_h F_{kj} - p_i p_j F_{kh})\\ &+ \frac{n+1}{n-2}(p_k p_j F_{ih} - p_k p_h F_{ij}).\end{aligned} \tag{3.11}$$

Adding (3.9), (3.10) and (3.11), we get

$$\begin{aligned}\overline{C}_{ijkh} + \overline{C}_{jkih} + \overline{C}_{kijh} &= C_{ijkh} + C_{jkih} + C_{kijh}\\ &+ 2\frac{n+1}{n-2}p_h(p_j F_{ik} + p_i F_{kj} + p_k F_{ji}).\end{aligned} \tag{3.12}$$

Since in a Riemannian manifold the conformal curvature tensor satisfies the condition

$$C_{ijkh} + C_{jkih} + C_{kijh} = 0, \tag{3.13}$$

by using (3.13) in (3.12), we find

$$\overline{C}_{ijkh} + \overline{C}_{jkih} + \overline{C}_{kijh} = 2\frac{n+1}{n-2}p_h(p_j F_{ik} + p_i F_{kj} + p_k F_{ji}). \tag{3.14}$$

If we take $p_j F_{ik} + p_i F_{kj} + p_k F_{ji} = 0$ then from (3.14), we get

$$\overline{C}_{ijkh} + \overline{C}_{jkih} + \overline{C}_{kijh} = 0. \tag{3.15}$$

□

Now, we propose:

Theorem 3.3. *In a hyperbolic Kaehlerian manifold equipped with a quarter-symmetric metric connection, the conformal curvature tensor with respect to a quarter-symmetric metric connection satisfies the following properties:*

(1) $\overline{C}_{ijkh} = -\overline{C}_{jikh}$,
(2) $\overline{C}_{ijkh} = -\overline{C}_{ijhk}$.

Proof. Interchanging i and j in (3.5), we find

$$\begin{aligned}\overline{C}_{jikh} &= C_{jikh}\\ &+ \frac{n+1}{n-2}(p_i p_h F_{jk} - p_i p_k F_{jh})\\ &+ \frac{n+1}{n-2}(p_j p_k F_{ih} - p_j p_h F_{ik}).\end{aligned} \tag{3.16}$$

Adding (3.5) and (3.16), we obtain

$$\overline{C}_{ijkh} + \overline{C}_{jikh} = C_{ijkh} + C_{jikh}. \tag{3.17}$$

Since in a Riemannian manifold the conformal curvature tensor satisfies

$$C_{ijkh} + C_{jikh} = 0, \tag{3.18}$$

then by using (3.18) in (3.17), we get the expression (1). Now interchanging k and h in (3.5), we have

$$\begin{aligned} \overline{C}_{ijhk} &= C_{ijhk} \\ &+ \frac{n+1}{n-2}(p_j p_k F_{ih} - p_j p_h F_{ik}) \\ &+ \frac{n+1}{n-2}(p_i p_h F_{jk} - p_i p_k F_{jh}). \end{aligned} \tag{3.19}$$

Adding (3.5) and (3.19), we have

$$\overline{C}_{ijkh} + \overline{C}_{ijhk} = C_{ijkh} + C_{ijhk}. \tag{3.20}$$

Since in a Riemannian manifold the conformal curvature tensor satisfies

$$C_{ijkh} + C_{ijhk} = 0, \tag{3.21}$$

by using (3.21) in (3.20), we get expression (2). □

Taking the covariant derivative of the conformal curvature tensor with respect to the Riemannian connection and quarter-symmetric metric connection, respectively, we get

$$\begin{aligned} D_m C_{ijkh} &= \partial_m C_{ijkh} - C_{rjkh} \left\{ {r \atop m\,i} \right\} - C_{irkh} \left\{ {r \atop m\,j} \right\} \\ &- C_{ijrh} \left\{ {r \atop m\,k} \right\} - C_{ijkr} \left\{ {r \atop m\,h} \right\}, \end{aligned} \tag{3.22}$$

and

$$\begin{aligned} \nabla_m C_{ijkh} &= \partial_m C_{ijkh} - C_{rjkh} \Gamma^r_{mi} - C_{irkh} \Gamma^r_{mj} \\ &- C_{ijrh} \Gamma^r_{mk} - C_{ijkr} \Gamma^r_{mh}. \end{aligned} \tag{3.23}$$

Subtracting (3.22) from (3.23), we get

$$\begin{aligned} \nabla_m C_{ijkh} - D_m C_{ijkh} &= C_{rjkh} \left(\left\{ {r \atop m\,i} \right\} - \Gamma^r_{mi} \right) \\ &+ C_{irkh} \left(\left\{ {r \atop m\,j} \right\} - \Gamma^r_{mj} \right) \\ &+ C_{ijrh} \left(\left\{ {r \atop m\,k} \right\} - \Gamma^r_{mk} \right) \\ &+ C_{ijkr} \left(\left\{ {r \atop m\,h} \right\} - \Gamma^r_{mh} \right). \end{aligned} \tag{3.24}$$

Now using (1.8) in (3.24), we find

$$\begin{aligned}\nabla_m C_{ijkh} - D_m C_{ijkh} = C_{rjkh}\,(p_i F^r_m - p_m \delta^r_i + p^r g_{mi}) \\ + C_{irkh}\,(p_j F^r_m - p_m \delta^r_j + p^r g_{mj}) \\ + C_{ijrh}(p_k F^r_m - p_m \delta^r_k + p^r g_{mk}) \\ + C_{ijkr}\,(p_h F^r_m - p_m \delta^r_h + p^r g_{mh}).\end{aligned} \tag{3.25}$$

Using [1.13(c)] in (3.25), we get

$$\begin{aligned}\nabla_m C_{ijkh} - D_m C_{ijkh} &= (C_{rjkh}\, p_i + C_{irkh}\, p_j \\ &+ C_{ijrh}\, p_k + C_{ijkr}\, p_h)F^r_m.\end{aligned} \tag{3.26}$$

Considering that the expression

$$C_{rjkh}\, p_i + C_{irkh}\, p_j + C_{ijrh}\, p_k + C_{ijkr}\, p_h = 0 \tag{3.27}$$

is satisfied, then it is finally obtained that

$$\nabla_m C_{ijkh} = D_m C_{ijkh}. \tag{3.28}$$

Thus, we conclude:

Theorem 3.4. *In a hyperbolic Kaehlerian manifold equipped with a quarter-symmetric metric connection, if the conformal curvature tensor with respect to a Riemannian connection is recurrent with respect to the Riemannian connection then it is also recurrent with respect to the quarter-symmetric metric connection if and only if*

$$C_{rjkh} p_i + C_{irkh} p_j + C_{ijrh} p_k + C_{ijkr} p_h = 0.$$

Taking the covariant derivative of the holomorphic conharmonic curvature tensor with respect to the Riemannian connection and quarter-symmetric metric connection, respectively, we have

$$\begin{aligned}D_m T_{ijkh} &= \partial_m T_{ijkh} - T_{rjkh} \left\{ {r \atop m\,i} \right\} \\ &- T_{irkh} \left\{ {r \atop m\,j} \right\} - T_{ijrh} \left\{ {r \atop m\,k} \right\} - T_{ijkr} \left\{ {r \atop m\,h} \right\},\end{aligned} \tag{3.29}$$

and

$$\begin{aligned}\nabla_m T_{ijkh} &= \partial_m T_{ijkh} - T_{rjkh}\, \Gamma^r_{mi} \\ &- T_{irkh}\, \Gamma^r_{mj} - T_{ijrh}\, \Gamma^r_{mk} - T_{ijkr}\, \Gamma^r_{mh}.\end{aligned} \tag{3.30}$$

Subtracting (3.29) from (3.30), we get

$$\begin{aligned}\nabla_m T_{ijkh} - D_m T_{ijkh} &= T_{rjkh} \left(\left\{ {r \atop m\,i} \right\} - \Gamma^r_{mi}\right) \\ &+ T_{irkh} \left(\left\{ {r \atop m\,j} \right\} - \Gamma^r_{mj}\right) \\ &+ T_{ijrh} \left(\left\{ {r \atop m\,k} \right\} - \Gamma^r_{mk}\right) \\ &+ T_{ijkr} \left(\left\{ {r \atop m\,h} \right\} - \Gamma^r_{mh}\right).\end{aligned} \tag{3.31}$$

Now, using (1.8) in (3.31), we find

$$
\begin{aligned}
\nabla_m T_{ijkh} - D_m T_{ijkh} &= T_{rjkh}\,(p_i F^r_m - p_m \delta^r_i + p^r g_{mi}) \\
&+ T_{irkh}\,(p_j F^r_m - p_m \delta^r_j + p^r g_{mj}) \\
&+ T_{ijrh}(p_k F^r_m - p_m \delta^r_k + p^r g_{mk}) \\
&+ T_{ijkr}\,(p_h F^r_m - p_m \delta^r_h + p^r g_{mh}).
\end{aligned} \tag{3.32}
$$

Using [1.13(c)] in (3.32), we obtain

$$
\begin{aligned}
\nabla_m T_{ijkh} - D_m T_{ijkh} &= (T_{rjkh}\, p_i + T_{irkh}\, p_j \\
&+ T_{ijrh}\, p_k + T_{ijkr}\, p_h)F^r_m.
\end{aligned} \tag{3.33}
$$

If we take the condition

$$T_{rjkh}p_i + T_{irkh}p_j + T_{ijrh}p_k + T_{ijkr}p_h = 0, \tag{3.34}$$

then

$$\nabla_m T_{ijkh} = D_m T_{ijkh}. \tag{3.35}$$

Thus, we conclude:

Theorem 3.5. *In a hyperbolic Kaehlerian manifold equipped with a quarter-symmetric metric connection, if the holomorphic conharmonic curvature tensore with respect to a Riemannian connection is recurrent with respect to the Riemannian connection then it is also recurrent with respect to the quarter-symmetric metric connection if and only if the following condition holds*

$$T_{rjkh}p_i + T_{irkh}p_j + T_{ijrh}p_k + T_{ijkr}p_h = 0.$$

Taking the covariant derivative of the Weyl projective curvature tensor with respect to a Riemannian connection and quarter-symmetric metric connection, respectively, we can write

$$
\begin{aligned}
D_m W_{ijkh} &= \partial_m W_{ijkh} - W_{rjkh}\left\{{r \atop mi}\right\} - W_{irkh}\left\{{r \atop mj}\right\} \\
&- W_{ijrh}\left\{{r \atop mk}\right\} - W_{ijkr}\left\{{r \atop mh}\right\},
\end{aligned} \tag{3.36}
$$

and

$$
\begin{aligned}
\nabla_m W_{ijkh} &= \partial_m W_{ijkh} - W_{rjkh}\,\Gamma^r_{mi} - W_{irkh}\,\Gamma^r_{mj} \\
&- W_{ijrh}\,\Gamma^r_{mk} - W_{ijkr}\,\Gamma^r_{mh}.
\end{aligned} \tag{3.37}
$$

Subtracting (3.36) from (3.37), we get

$$
\begin{aligned}
\nabla_m W_{ijkh} - D_m W_{ijkh} &= W_{rjkh}\left(\left\{{r \atop mi}\right\} - \Gamma^r_{mi}\right) + W_{irkh}\left(\left\{{r \atop mj}\right\}\right. \\
&- \left.\Gamma^r_{mj}\right) + W_{ijrh}\left(\left\{{r \atop mk}\right\} - \Gamma^r_{mk}\right) \\
&+ W_{ijkr}\left(\left\{{r \atop mh}\right\} - \Gamma^r_{mh}\right).
\end{aligned} \tag{3.38}
$$

Now, using (1.8) in (3.38), we find

$$\begin{aligned} \nabla_m W_{ijkh} - D_m W_{ijkh} &= W_{rjkh}(p_i F^r_m - p_m \delta^r_i + p^r g_{mi}) \\ &+ W_{irkh}(p_j F^r_m - p_m \delta^r_j + p^r g_{mj}) \\ &+ W_{ijrh}(p_k F^r_m - p_m \delta^r_k + p^r g_{mk}) \\ &+ W_{ijkr}(p_h F^r_m - p_m \delta^r_h + p^r g_{mh}). \end{aligned} \tag{3.39}$$

Using [1.13(*c*)] in (3.39), it is obtained that

$$\begin{aligned} \nabla_m W_{ijkh} - D_m W_{ijkh} &= (W_{rjkh} p_i + W_{irkh} p_j \\ &+ W_{ijrh} p_k + W_{ijkr} p_h) F^r_m. \end{aligned} \tag{3.40}$$

If we consider that the expression

$$W_{rjkh} p_i + W_{irkh} p_j + W_{ijrh} p_k + W_{ijkr} p_h = 0 \tag{3.41}$$

holds, then we find

$$\nabla_m W_{ijkh} = D_m W_{ijkh}. \tag{3.42}$$

Thus, we conclude:

Theorem 3.6. *In a hyperbolic Kaehlerian manifold equipped with a quarter-symmetric metric connection, if the Weyl projective curvature tensor with respect to a Riemannian connection is recurrent with respect to the Riemannian connection then it is also recurrent with respect to a quarter-symmetric metric connection if and only if*

$$W_{rjkh} p_i + W_{irkh} p_j + W_{ijrh} p_k + W_{ijkr} p_h = 0.$$

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