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QUASI-CONFORMAL CURVATURE TENSOR OF GENERALIZED SASAKIAN-SPACE-FORMS

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Abstract. The present paper deals with the study of generalized Sasakian-space-forms with the conditions $C^q(\xi, X).S = 0$, $C^q(\xi, X).R = 0$ and $C^q(\xi, X).C^q = 0$, where R, S and C^q denote Riemannian curvature tensor, Ricci tensor and quasi-conformal curvature tensor of the space-form, respectively. In the end of the paper, we have given some examples to support our results.
Keywords: Quasi-conformal curvature tensor; generalized Sasakian-space-forms; Einstein manifold; Pseudosymmetric manifold.

1. Introduction

An almost contact metric manifold $M^{2n+1}(\phi, \xi, \eta, g)$ is said to be a generalized Sasakian-space-form if the curvature tensor of the manifold has the following form

$$\begin{aligned} R(X,Y)Z &= f_1\{g(Y,Z)X - g(X,Z)Y\} \\ &+ f_2\{(g(X,\phi Z)\phi Y - g(Y,\phi Z)\phi X + 2g(X,\phi Y)\phi Z)\} \\ &+ f_3\big((\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X \\ &+ g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi)\big), \end{aligned} \tag{1.1}$$

for any vector fields X, Y, Z on M^{2n+1}. By taking $f_1 = \frac{c+3}{4}$ and $f_2 = f_3 = \frac{c-1}{4}$, where c denotes constant ϕ-sectional curvature tensor, we get different kind of generalized Sasakian-space-forms. This idea was introduced by P. Alegre, D. Blair and A. Carriazo [13] in 2004. P. Alegre and Carriazo [15], A. Sarkar, S. K. Hui, etc. [19, 21, 22] studied generalized Sasakian-space-forms by considering the cosymplectic space of Kenmotsu space form as particular types of generalized Sasakian-space-forms. In 2006, U. Kim [22] studied conformally flat generalized Sasakian-space-form and locally symmetric generalized Sasakian-space-form. He proved some geometric properties of generalized Sasakian-space-forms which depends on the nature

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of the functions f_1, f_2 and f_3. Also, he proved that if a generalized Sasakian-space-form $M^{2n+1}(f_1, f_2, f_3)$ is locally symmetric then $(f_1 - f_3)$ is constant. In [21] De and Sarkar studied the projective curvature tensor of generalized Sasakian-space-forms and proved that generalized Sasakian-space-forms is projectively flat if and only if $f_3 = \frac{3f_2}{1-2n}$. D.G. Prakasha and H. G. Nagaraja [8] studied quasiconformally semi-symmetric generalized Sasakian-space-forms. They proved that a generalized Sasakian-space-forms is quasiconformally semi-symmetric if and only if either space form is quasiconformally flat or $f_1 = f_2$. Recently, Hui and Prakasha [17] have studied C-Bochner curvature tensor of generalized Sasakian-space-forms. S. K. Hui and D. G. Prakasha [17] studied the C-Bochner pseudosymmetric generalized Sasakian-space-forms. The generalized Sasakian-space-forms have also been studied in ([9], [18], [10], [11], [23]) and many others. Throughout their study, C-Bochner curvature tensor B satisfied the conditions $B(\xi, X).S = 0$, $B(\xi, X).R = 0$ and $B(\xi, X).B = 0$, where R and S denoted the Riemannian curvature tensor and Ricci curvature tensor of the space form respectively. After investigations of the above mentioned developments, we plan to study the quasi-conformal curvature tensor of generalized Sasakian-space-forms.

2. Preliminaries

A Riemannian manifold (M^{2n+1}, g) of dimension $(2n+1)$ is said to be an almost contact metric manifold [7] if there exists a tensior field ϕ of type (1, 1), a vector field ξ (called the structure vector field) and a 1-form η on M such that

$$\phi^2(X) = -X + \eta(X)\xi, \tag{2.1}$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y) \tag{2.2}$$

and

$$\eta(\xi) = 1, \tag{2.3}$$

for any vector fields X, Y on M . In an almost contact metric manifold, we have $\phi\xi = 0$ and $\eta o \phi = 0$. Then such type of manifold is called a contact metric manifold if $d\eta = \Phi$, where $\Phi(X, Y) = g(X, \phi Y)$ is called the fundamental 2-form of $M^{(2n+1)}$. A contact metric manifold is said to be K-contact manifold if and only if the covarient derivative of ξ satisfies

$$\nabla_X \xi = -\phi X, \tag{2.4}$$

for any vector field X on M.
The almost contact metric structure of M is said to be normal if

$$[\phi, \phi](X, Y) = -2d\eta(X, Y)\phi, \tag{2.5}$$

for any vector fields X and Y, where $[\phi, \phi]$ denotes the Nijenhuis torsion of ϕ.
A normal contact metric manifold is called Sasakian manifold. An almost contact metric manifold is Sasakian if and only if

$$(\nabla_X \phi)Y = g(X,Y)\xi - \eta(Y)X, \tag{2.6}$$

for any vector fields X, Y.
The generalized Sasakian-space-forms $M^{2n+1}(f_1, f_2, f_3)$ satisfies the following relations [13]

$$R(X,\, Y)\,\xi = (f_1 - f_3)\,\{\eta(Y)X - \eta(X)Y\}\,, \tag{2.7}$$

$$R(\xi,\, X)\,Y = (f_1 - f_3)\,\{g(X,Y)\xi - \eta(Y)X\}\,, \tag{2.8}$$

$$S(X,Y) = (2nf_1 + 3f_2 - f_3)g(X,Y) - \{3f_2 + (2n-1)f_3\}\,\eta(X)\eta(Y). \tag{2.9}$$

Replacing Y by ξ in the equation (2.9), we get

$$S(X,\xi) = 2n(f_1 - f_3)\eta(X). \tag{2.10}$$

Replacing X and Y by ξ in the equation (2.9), we get

$$S(\xi,\xi) = 2n(f_1 - f_3), \tag{2.11}$$

from the equation (2.9), we have

$$r = 2n(2n+1)f_1 + 6nf_2 - 4nf_3. \tag{2.12}$$

Again from (2.9), we have

$$QX = (2nf_1 + 3f_2 - f_3)X - (3f_2 + (2n-1)f_3)\eta(X)\xi. \tag{2.13}$$

Replacing X by ξ in the above equation, we get

$$Q\xi = 2n(f_1 - f_3)\xi. \tag{2.14}$$

In a Riemannian manifold of dimension $(2n+1)$ the quasi-conformal curvature tensor is defined by [12]

$$\begin{aligned} C^q(X,Y)Z &= aR(X,Y)Z + b\big(S(Y,Z)X - S(X,Z)Y \\ &+ g(Y,Z)QX - g(X,Z)QY\big) \\ &- \frac{r}{2n+1}\left[\frac{a}{2n} + 2b\right]\{g(Y,Z)X - g(X,Z)Y\}\,, \end{aligned} \tag{2.15}$$

where a and b are constants such that $a, b \neq 0$, Q is the Ricci operator, i.e., $g(QX,Y) = S(X,Y)$, for all X and Y and r is scalar curvature of the manifold.
Using the equations (2.7)-(2.15), we have

$$(2.16)\quad C^q(X,Y)\xi = \left[\frac{(a+(2n-1)b)((2n-1)f_3+3f_2)}{2n+1}\right]\{\eta(X)Y-\eta(Y)X\},$$

$$(2.17)\quad C^q(\xi,Y)Z = \left[\frac{(a+(2n-1)b)((2n-1)f_3+3f_2)}{2n+1}\right]\{\eta(Z)Y-g(Y,Z)\xi\},$$

and

$$\begin{aligned}\eta(C^q(X,Y)Z) &= \left[\frac{(a+(2n-1)b)((2n-1)f_3+3f_2)}{2n+1}\right]\big(g(Z,X)\eta(Y)\\ &- g(Y,Z)\eta(X)\big).\end{aligned} \tag{2.18}$$

This is required quasi-conformal curvature tensor in generalized Sasakian-space-forms.

3. Quasi-conformal Pseudosymmetric generalized Sasakian-Space-Forms

Let (M,g) be a Riemannian manifold and let ∇ be the Levi-Civita connection of (M,g). A Riemannian manifold is called locally symmetric if $\nabla R=0$, where R is the Riemannian curvature tensor of (M,g). The locally symmetric manifolds have been studied by different differential geometry through various aproaches and they extended semisymmetric manifolds by [2, 3, 4, 5, 6, 24], recurrent manifolds by Walker [1], conformally recurrent manifold by Adati and Miyazawa [20].
According to Z. I. $Szab^{'}o$ [24], if the manifold M satisfies the condition

$$(3.1)\qquad (R(X,Y).R)(U,V)W=0,\quad X,Y,U,V,W\in\chi(M)$$

for all vector fields X and Y, then the manifold is called semi-symmetric manifold. For a (0, k)- tensor field T on M, $k\geq 1$ and a symmetric (0, 2)-tensor field A on M the (0, k+2)-tensor fields R.T and Q(A, T) are defined by

$$\begin{aligned}(R.T)(X_1,.....X_k;X,Y) &= -T(R(X,Y)X_1,X_2,.......X_k)\\ &- .....-T(X_1,........X_{k-1},R(X,Y)X_k),\end{aligned} \tag{3.2}$$

and

$$\begin{aligned}Q(A,T)(X_1,.....X_k;X,Y) &= -T((X\wedge_A Y)X_1,X_2,.......X_k)\\ &- .....-T(X_1,.....X_{k-1},(X\wedge_A Y)X_k),\end{aligned} \tag{3.3}$$

where $X\wedge_A Y$ is the endomorphism given by

$$(3.4)\qquad (X\wedge_A Y)Z = A(Y,Z)X-A(X,Z)Y.$$

According to R. Deszcz [16], a Riemannian manifold is said to be pseudosymmetric if

$$R.R = L_R\, Q(g,R), \tag{3.5}$$

holds on $U_r = \left\{x \in M | R - \frac{r}{n(n-1)} G \neq 0 \ \ at\ x\right\}$, where G is (0, 4)-tensor defined by $G(X_1, X_2, X_3, X_4) = g((X_1 \wedge X_2)X_3, X_4)$ and L_R is some smooth function on U_R. A Riemannian manifold M is said to be quasi-conformal pseudosymmetric if

$$R.C^q = L_{C^q}\, Q(g, C^q), \tag{3.6}$$

holds on the set $U_{C^q} = \{x \in M : C^q \neq 0 \ \ at\ x\}$, where L_{C^q} is some function on U_{C^q} and C^q is the quasi-conformal curvature tensor.
Let $M^{2n+1}(f_1, f_2, f_3)$ be quasi-conformal pseudosymmetric generalized Sasakian-space-form then from the equation(3.6), we have

$$(R(X,\xi).C^q)(U,V)W = L_{C^q}[((X \wedge_g \xi).C^q)(U,V)W]. \tag{3.7}$$

Using the equations (3.2) and (3.3) in the equation (3.7), we get

$$\begin{aligned} R(X,\xi)C^q(U,V)W &- C^q(R(X,\xi)U,V)W - C^q(U,R(X,\xi)V)W \\ &- C^q(U,V)R(X,\xi)W \\ &= L_{C^q}\big((X \wedge_g \xi)C^q(U,V)W \\ &- C^q((X \wedge_g \xi)U,V)W \\ &- C^q(U,(X \wedge_g \xi)V)W - C^q(U,V)(X \wedge_g \xi)W\big). \end{aligned} \tag{3.8}$$

Again, using the equations (2.7) and (3.4) in (3.8), we conclude the following

$$\begin{aligned} (f_1 - f_3)\big(g(\xi, C^q(U,V)W)X &- g(X, C^q(U,V)W)\xi \\ &- \eta(U)C^q(X,V)W \\ &+ g(X,U)C^q(\xi,V)W - \eta(V)\ C^q(U,X)W \\ &+ g(X,V)C^q(U,\xi)W - \eta(W)C^q(U,V)X \\ &+ g(X,W)C^q(U,V)\xi\big) \\ &= L_{C^q}\big(g(\xi, C^q(U,V)W)X - g(X, C^q(U,V)W)\xi \\ &- \eta(U)C^q(X,V)W \\ &+ g(X,U)C^q(\xi,V)W - \eta(V)C^q(U,X)W \\ &+ g(X,V)C^q(U,\xi)W - \eta(W)C^q(U,V)X \\ &+ g(X,W)C^q(U,V)\xi\big). \end{aligned} \tag{3.9}$$

The above expression can be written as

$$\begin{aligned} (f_1 - f_3 - L_{C^q})\big(g(\xi, C^q(U,V)W)X &- g(X, C^q(U,V)W)\xi \\ &- \eta(U)C^q(X,V)W + g(X,U)C^q(\xi,V)W \\ &- \eta(V)C^q(U,X)W + g(X,V)C^q(U,\xi)W \\ &- \eta(W)C^q(U,V)X + g(X,W)C^q(U,V)\xi\big) = 0, \end{aligned} \tag{3.10}$$

which implies either $L_{C^q} = f_1 - f_3$ or

$$
\begin{aligned}
\big(g(\xi, C^q(U,V)W)X &- g(X, C^q(U,V)W)\xi - \eta(U)C^q(X,V)W \\
&+ g(X,U)C^q(\xi,V)W - \eta(V)C^q(U,X)W \\
&+ g(X,V)C^q(U,\xi)W - \eta(W)C^q(U,V)X \\
&+ g(X,W)C^q(U,V)\xi\big) = 0. \qquad (3.11)
\end{aligned}
$$

Putting $W = \xi$ in the equation (3.11) and using the equations (2.17) and (2.18), we have

$$
\begin{aligned}
C^q(U,V)X &= \left[\frac{(a+(2n-1)b)((2n-1)f_3+3f_2)}{2n+1}\right](g(X,U)V \\
&- g(X,V)U), \qquad (3.12)
\end{aligned}
$$

contracting V in the above equation, we have

$$\left[\frac{(a+(2n-1)b)((2n-1)f_3+3f_2)}{2n+1}\right]2ng(U,X) = 0, \qquad (3.13)$$

this implies that

$$\frac{(a+(2n-1)b)((2n-1)f_3+3f_2)}{2n+1} = 0, \qquad (3.14)$$

from the above equation two conditions arise, either

$$a = -(2n-1)b \qquad (3.15)$$

or

$$f_3 = \frac{3f_2}{(1-2n)}. \qquad (3.16)$$

Using the equations (3.15) or (3.16) in (2.16) and (2.17), we get

$$C^q(\xi, Y)Z = 0, \qquad (3.17)$$

and

$$C^q(X,Y)\xi = 0, \qquad (3.18)$$

this means $M^{2n+1}(f_1, f_2, f_3)$ is quasi-conformally flat.
Thus, we conclude:

Theorem 3.1. *Let $M^{2n+1}(f_1, f_2, f_3)$ be a (2n+1)-dimensional generalized Sasakian-space-form. If $M^{2n+1}(f_1, f_2, f_3)$ is quasi-conformal pseudosymmetric then $M^{2n+1}(f_1, f_2, f_3)$ is quasiconformally flat if at least one of the following conditions holds:*

$$(i) f_3 = \frac{3f_2}{(1-2n)} \quad (ii) a = -(2n-1)b, \quad (iii) L_{C^q} = f_1 - f_3.$$

Now we propose:

Theorem 3.2. *Let $M^{2n+1}(f_1, f_2, f_3)$ be a (2n+1)-dimensional generalized Sasakian-space-form. Then $M^{2n+1}(f_1, f_2, f_3)$ satisfies $C^q(\xi, X).S = 0$ if and only if at least one of the following conditions holds:*

$$(i)\, f_3 = \frac{3f_2}{(1-2n)}, \quad (ii)\, a = -(2n-1)b, \quad (iii)\, S(X,U) = 2n(f_1 - f_3)g(X,U).$$

Proof. If generalized Sasakian-space-form satisfies $C^q(\xi, X).S = 0$.
Then from the equation (3.2), we have

$$S(C^q(\xi, X)U, \xi) + S(U, C^q(\xi, X)\xi) = 0, \tag{3.19}$$

From the equation (2.10), we have

$$S(C^q(\xi, X)U, \xi) = 2n(f_1 - f_3)\eta(C^q(\xi, X)U). \tag{3.20}$$

Now with the help of equations (2.17) and (3.20), we can write

$$\begin{aligned} S(C^q(\xi, X)U, \xi) &= 2n(f_1 - f_3)\left[\frac{(a + (2n-1)b)((2n-1)f_3 + 3f_2)}{2n+1}\right](\eta(X)\eta(U) \\ &- g(X,U)). \end{aligned} \tag{3.21}$$

Again in view of the equation (2.17), we have

$$\begin{aligned} S(C^q(\xi, X)\xi, U) &= \left[\frac{(a + (2n-1)b)((2n-1)f_3 + 3f_2)}{2n+1}\right](S(X,U) \\ &- 2n(f_1 - f_3)\eta(X)\eta(U)). \end{aligned} \tag{3.22}$$

By using the expressions (3.21) and (3.22) in (3.19), we infer

$$\begin{aligned} &\left[\tfrac{(a+(2n-1)b)((2n-1)f_3+3f_2)}{2n+1}\right](S(X,U) \\ &-2n(f_1 - f_3)g(X,U)) = 0, \end{aligned} \tag{3.23}$$

which implies that if $C^q(\xi, X).S = 0$ then either $a = -(2n-1)b$ or $f_3 = \frac{3f_2}{(1-2n)}$ or $S(X,U) = 2n(f_1 - f_3)g(X,U)$.
Conversely, it is clear that if $a = -(2n-1)b$ or $f_3 = \frac{3f_2}{(1-2n)}$ or $S(X,U) = 2n(f_1 - f_3)g(X,U)$ then from (2.17), we have □

$$C^q(\xi, X).S = 0. \tag{3.24}$$

Now we take $C^q(\xi, U).R = 0$.
Then from the equation (3.2), we have

$$\begin{aligned} &C^q(\xi, U)R(X,Y)Z - R(C^q(\xi, U)X, Y)Z \\ &-R(X, C^q(\xi, U)Y)Z - R(X,Y)C^q(\xi, U)Z = 0, \end{aligned} \tag{3.25}$$

which in view of the equation (2.17), we have

$$\begin{aligned}&\left[\tfrac{(a+(2n-1)b)((2n-1)f_3+3f_2)}{2n+1}\right]\big(\eta(R(X,Y)Z)U\\&-g(U,R(X,Y)Z)\xi-\eta(X)R(U,Y)Z\\&+g(U,X)R(\xi,Y)Z-\eta(Y)R(X,U)Z+g(U,Y)R(X,\xi)Z\\&-\eta(Z)R(X,Y)U+g(U,Z)R(X,Y)\xi\big)=0,\end{aligned} \tag{3.26}$$

using $Z=\xi$ and (2.2) in the above equation, we infer

$$\begin{aligned}&\left[\tfrac{(a+(2n-1)b)((2n-1)f_3+3f_2)}{2n+1}\right]\\&\big((f_1-f_3)(g(U,Y)X-g(U,X)Y)-R(X,Y)U\big)=0,\end{aligned} \tag{3.27}$$

which implies that if $C^q(\xi,X).R=0$ then either $a=-(2n-1)b$ or $f_3=\frac{3f_2}{(1-2n)}$ or
$R(X,Y)U=(f_1-f_3)(g(U,Y)X-g(U,X)Y)$.
Thus, we conclude:

Theorem 3.3. *Let $M^{2n+1}(f_1,f_2,f_3)$ be a (2n+1)-dimensional generalized Sasakian-space-form. If $M^{2n+1}(f_1,f_2,f_3)$ satisfying $C^q(\xi,U).R=0$ then at least one of the following necessarily holds:*

$$(i)\ \ f_3=\tfrac{3f_2}{(1-2n)},\ (ii)\ \ a=-(2n-1)b,$$
$$(iii)\ R(X,Y)U=(f_1-f_3)(g(U,X)Y-g(U,Y)X).$$

Now we propose:

Theorem 3.4. *Let $M^{2n+1}(f_1,f_2,f_3)$ be a (2n+1)-dimensional generalized Sasakian-space-form. Then $M^{2n+1}(f_1,f_2,f_3)$ satisfies $C^q(\xi,X).C^q=0$ if and only if either $f_3=\frac{3f_2}{(1-2n)}$ or $a=-(2n-1)b$.*

Proof. If generalized Sasakian-space-form satisfies $C^q(\xi,X).C^q=0$. Then, from the equation (3.2) we have

$$\begin{aligned}&C^q(\xi,X)C^q(U,V)W-C^q(C^q(\xi,X)U,V)W\\&-C^q(U,C^q(\xi,X)V)W-C^q(U,V)C^q(\xi,X)W=0,\end{aligned} \tag{3.28}$$

by which in view of the equation (2.16) we get

$$\begin{aligned}&\left[\tfrac{(a+(2n-1)b)((2n-1)f_3+3f_2)}{2n+1}\right]\big(\eta(C^q(U,V)W)X\\&-g(X,C^q(U,V)W)\xi-\eta(U)C^q(X,V)W\\&+g(X,U)C^q(V,\xi)W-\eta(V)C^q(U,X)W+g(X,V)C^q(U,\xi)W\\&+g(W,X)C^q(U,V)\xi-\eta(W)C^q(U,V)X\big)=0.\end{aligned} \tag{3.29}$$

By using $V = \xi$ in the above equation, we infer

$$\Big[\tfrac{(a+(2n-1)b)((2n-1)f_3+3f_2)}{2n+1}\Big]\big((C^q(U,X)W + \Big[\tfrac{(a+(2n-1)b)((2n-1)f_3+3f_2)}{2n+1}\Big](g(X,W)U) -g(U,W)X)\big) = 0, \tag{3.30}$$

which implies that either $a = -(2n-1)b$ or $f_3 = \frac{3f_2}{(1-2n)}$ or

$$C^q(U,X)W = \big(\tfrac{(a+(2n-1)b)((2n-1)f_3+3f_2)}{2n+1}\big)(g(U,W)X - g(X,W)U), \tag{3.31}$$

contracting U in the above equation, we have

$$\left[\frac{(a+(2n-1)b)((2n-1)f_3+3f_2)}{2n+1}\right] 2ng(X,V) = 0, \tag{3.32}$$

this implies that either $a = -(2n-1)b$ or $f_3 = \frac{3f_2}{(1-2n)}$. Conversely, if $M^{2n+1}(f_1, f_2, f_3)$ satisfies $a = -(2n-1)b$ or $f_3 = \frac{3f_2}{(1-2n)}$, then in view of (2.17) we have $C^q(\xi, X).C^q = 0$. □

4. Examples

Example 4.1. [13] Let $N(\lambda_1, \lambda_2)$ be generalized Sasakian-space-forms of dimension 4, then by the warped product $M \times N$ endowed with the almost contact metric structure (ϕ, ξ, η, g_f), Sasakian space form $M(f_1, f_2, f_3)$ is generalized with

$$f_1 = \frac{\lambda_1 - (f^{'})^2}{f^2}, \quad f_2 = \frac{\lambda_2}{f^2}, \quad f_3 = \frac{\lambda_1 - (f^{'})^2}{f^2} + \frac{f^{''}}{f}, \tag{4.1}$$

where λ_1 and λ_2 are constants, $f = f(t)$, $t \in R$ and $f^{'}$ denotes the derivative of f with respect to t.
If we take $\lambda_1 = -\frac{3\lambda_2}{7}$ and $f(t) = e^{Kt}$, K is constant,
then $f_1 = -\frac{1}{e^{2Kt}}[\frac{3\lambda_2}{7} + K^2 e^{2Kt}]$, $f_2 = \frac{\lambda_2}{e^{2Kt}}$ and $f_3 = -\frac{1}{e^{2Kt}}[\frac{3\lambda_2}{7}]$. Hence $f_3 = \frac{3f_2}{(1-2n)}$, if $n = 4$.

Example 4.2. [14] Let $N(c)$ be a complex space form, and by the warped product $M = (-\frac{\pi}{2}, \frac{\pi}{2}) \times_f N$ endowed with the almost contact metric structure (ϕ, ξ, η, g_f), Sasakian space form $M(f_1, f_2, f_3)$ is generalized with functions

$$f_1 = \frac{c - 4(f^{'})^2}{4f^2}, \quad f_2 = \frac{c}{4f^2}, \quad f_3 = \frac{c - 4(f^{'})^2}{4f^2} + \frac{f^{''}}{f}, \tag{4.2}$$

where $f = f(t)$, $t \in R$ and $f^{'}$ denotes the derivative of f with respect to t.
If we take $c = 0$ and $f(t) = e^{Kt}$, K is constant,

then $f_1 = -K^2$, $f_2 = f_3 = 0$. Hence $f_3 = \frac{3f_2}{(1-2n)}$.

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