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ON THE CHARACTERIZABILITY OF SOME FAMILIES OF FINITE GROUP OF LIE TYPE BY ORDERS AND VANISHING ELEMENT ORDERS

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Abstract. In this paper, we show that the following simple groups are uniquely determined by their orders and vanishing element orders: $A_{p-1}(2)$, where $p \neq 3$, ${}^2D_{p+1}(2)$, where $p \geq 5$, $p \neq 2^m - 1$, $A_p(2)$, $C_p(2)$, $D_p(2)$, $D_{p+1}(2)$ which for all of them p is an odd prime and $2^p - 1$ is a Mersenne prime. Also, ${}^2D_n(2)$ where $2^{n-1} + 1$ is a Fermat prime and $n > 3$, ${}^2D_n(2)$ and $C_n(2)$ where $2^n + 1$ is a Fermat prime. Then we give an almost general result to recognize the non-solvability of finite group H by an analogy between orders and vanishing element orders of H and a finite simple group of Lie type.
Keywords: simple groups; Mersenne prime; Fermat prime; Lie group.

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1. Introduction

Throughout this paper G and H are two finite groups. Let X be a finite set of positive integers. The prime graph $\Pi(X)$ is a graph whose vertices are the prime divisors of elements of X, and two distinct vertices p and q are adjacent if there exists an element of X divisible by pq. For a finite group G, we denote by $\omega(G)$, the set of element orders of G. The prime graph $\Pi(\omega(G))$ is denoted by $GK(G)$ and is called the Gruenberg-Kegel graph of G. Here, $s(G)$ denotes the number of connected components of $GK(G)$. For the group G, we denote by $\rho(G)$ some independence sets in $GK(G)$ with maximal number of vertices and put $t(G) = |\rho(G)|$, independence number of $GK(G)$. $g \in G$ is called a vanishing element of G if $\chi(g) = 0$ for some $\chi \in \mathrm{Irr}(G)$. Let us denote by $\mathrm{Van}(G)$ and $\mathrm{vo}(G)$ the set of all vanishing elements

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and the set of vanishing element orders of G, respectively. Also the prime graph $\Pi(\mathrm{vo}(G))$ is denoted by $\Gamma(G)$ and is called the vanishing prime graph of G.

If n is a natural number and π is a set of primes, then we denote the set of all prime divisors of n by $\pi(n)$, and the maximal divisor t of n such that $\pi(t) \subseteq \pi$ by n_π. If $\pi(G)$ is the set of prime divisors of $|G|$, then $\pi_i(G) = \pi(m_i)$ for some positive integers m_i, $1 \leq i \leq t$, such that $|G| = m_1 m_2 \cdots m_t$ and $t = s(G)$. Also for any group with even order, $2 \in \pi_1(G)$. We set $OC(G) = \{m_1, \cdots, m_t\}$ and call the set of order components of G. A finite simple group G is said characterizable by its order components, if $G \cong H$ for each finite group H such that $OC(G) = OC(H)$. Some authors have proved that some non-abelian simple groups are recognizable by their order components. We refer the reader to [23] to find a list of papers with the OC-characterizability criterion for some finite simple groups.

It was shown in [38] that if G is a finite group such that $\mathrm{vo}(G) = \mathrm{vo}(A_5)$ then $G \cong A_5$. According to this result, M. Foroudi, A. Iranmanesh and F. Mavadatpour in [12] stated the conjecture as follows:

Conjecture 1.1. *Let G and H be two groups with the same order. If G is a non-abelian group and $\mathrm{vo}(G) = \mathrm{vo}(H)$, then $G \cong H$.*

First, this conjecture was proved for $L_2(q)$, where $q \in \{5, 7, 8, 9, 17\}$, $L_3(4)$, A_7, $Sz(8)$ and $Sz(32)$ in [12]. Then they proved this conjecture in [13] for finite simple K_n-groups with $n \in \{3, 4\}$, sporadics, alternatatings and $L_2(p)$ where p is an odd prime. In [24] it has been verified that the groups $Sz(q)$ satisfy this conjecture, where $q = 2^{2n+1}$ and either $q-1$, $q-\sqrt{2q}+1$ or $q+\sqrt{2q}+1$ is a prime, and $F_4(q)$, where $q = 2^n$ and either q^4+1 or q^4-q^2+1 is a prime. In this paper, we show that the above conjecture is valid for some families of simple groups of Lie type. Then we prove another result about non-solvability of some finite group using vanishing element orders. In fact, we prove the following theorems:

Theorem 1.1. *Let G and H be two groups with the same order and G be a simple group of Lie type $A_{p-1}(2)$ where $p \neq 3$, ${}^2D_{p+1}(2)$, where $p \geq 5$, $p \neq 2^m - 1$, $A_p(2)$, $C_p(2)$, $D_p(2)$, $D_{p+1}(2)$, which for all of them p is an odd prime and $2^p - 1$ is a Mersenne prime, ${}^2D_n(2)$ where $2^{n-1} + 1$ is a Fermat prime, ${}^2D_n(2)$ and $C_n(2)$ where for the last two groups $2^n + 1$ is a Fermat prime. If $\mathrm{vo}(G) = \mathrm{vo}(H)$, then $G \cong H$.*

Theorem 1.2. *Let G and H be two groups with the same order. Suppose G is a simple group of Lie type with $s(G) \geq 2$ except $A_2(q)$, where $(q-1)_3 \neq 3$, q is a Mersenne prime, ${}^2A_2(q)$, where $(q+1)_3 \neq 3$, q is a Fermat prime, $C_2(q)$ where $q > 2$. If $\mathrm{vo}(G) = \mathrm{vo}(H)$, then H is non-solvable.*

2. Preliminaries

In this section, we state some results which will be of use to the proof of the main theorems.

Definition 2.1. A group G is said to be a 2-Frobenius group if there exist two normal subgroups F and L of G with the following properties: L is a Frobenius group with kernel F, and G/F is a Frobenius group with kernel L/F.

Recall that a Frobenius group with kernel N and complement H is a semidirect product $G = H \ltimes N$ such that N is a normal subgroup in G, and $C_N(h) = 1$ for every non-identity element h of H. As is well-known, N is the Fitting subgroup of G.

Definition 2.2. G is a nearly 2-Frobenius group if there exists two normal subgroups F and L of G with the following properties: $F = F_1 \times F_2$ is nilpotent, where F_1 and F_2 are normal subgroups of G, furthermore G/F is a Frobenius group with kernel L/F, G/F_1 is a Frobenius group with kernel L/F_1, and G/F_2 is a 2-Frobenius group.

Lemma 2.1. *[11]*

(a) Let G be a solvable Frobenius group with kernel F and complement H. The graph $GK(G)$ has two connected components, whose vertex sets are $\pi_1 = \pi(F)$ and $\pi_2 = \pi(H)$, and which are both complete graphs.

(b) Let G be a finite solvable group. Then $\Gamma(G)$ has at most two connected components. Moreover, if $\Gamma(G)$ is disconnected, then G is either a Frobenius group or a nearly 2-Frobenius group.

(c) Let G be a nearly 2-Frobenius group. If $\Gamma(G)$ is disconnected, then each connected component is a complete graph.

(d) Let G be a solvable Frobenius group with kernel F and complement H. If $F \cap Van(G) \neq \emptyset$, then $\Gamma(G) = GK(G)$, and the otherwise $\Gamma(G)$ coincides with the connected component of $GK(G)$ with vertex set $\pi(H)$.

Lemma 2.2. *[10] If G is a finite non-abelian simple group, then $GK(G) = \Gamma(G)$, unless $G \cong A_7$.*

Theorem 2.1. *[13] Let G be a finite group and let M be a simple K_3-group or a K_4-group. If $|G| = |M|$ and $\mathrm{vo}(G) = \mathrm{vo}(M)$, then $G \cong M$.*

Recall that a finite simple group G is called a K_n-group if its order has exactly n distinct prime divisors, where n is a natural number.

Theorem 2.2. *[36] Let G be a finite simple group. Then all the connected components of $GK(G)$ are cliques if and only if G is one of the following: A_5, A_6, A_7, A_9, A_{12}, A_{13}, M_{11}, M_{22}, J_1, J_2, J_3, HS, $A_1(q)$, with $q > 2$, $\mathrm{Sz}(q)$ with $q = 2^{2m+1}$, $C_2(q)$, $G_2(3^k)$, $A_2(q)$ where q is a Mersenne prime, ${}^2A_2(q)$ where q is a Fermat prime, $A_2(4)$, ${}^2A_2(9)$, ${}^2A_3(3)$, ${}^2A_5(2)$, $C_3(2)$,$D_4(2)$, ${}^3D_4(2)$.*

3. Main results

To prove Theorem 1.2, we adopt Table I by [14] of components of prime graphs of simple groups of Lie type over a field of even characteristics which in this table p is an odd prime. In Table 1, m_2 coincides with the factor for primes in the second connected component. Table 2 shows OC-characterizable groups of Lie type with their prime graph having two connected components. We also use Tables 3 and 4 for the proof of Theorem 1.3. These tables were adopted from [37] and they show the independence number of prime graphs of finite simple groups of Lie type and. In Tables 3 and 4, n and k are natural numbers. $[x]$ denotes the integral part of x. We assume that G is a finite non-abelian simple group of Lie type over a field of characteristic p and order q. We define the primitive prime divisor of $q^m - 1$ by r_m. If p is odd then we say that 2 is a primitive prime divisor of $q-1$ if $q \equiv 1 \pmod 4$ and that 2 is a primitive prime divisor of q^2-1 if $q \equiv -1 \pmod 4$.

The following lemma is a conclusion from some noteworthy properties of a simple group G with $s(G) = 2$ and the conditions of Conjecture 1.1.

Lemma 3.1. *Let G and H be two groups with the same order. Suppose that G is a non-abelian simple group with $s(G) = 2$ and $GK(H)$ is disconnected. If* $\mathrm{vo}(G) = \mathrm{vo}(H)$*, then $OC(G) = OC(H)$.*

Proof. The assumption $\mathrm{vo}(G) = \mathrm{vo}(H)$ and Lemma 2.2 imply $GK(G) = \Gamma(G) = \Gamma(H)$. So the set of vertices of the vanishing prime graph of H is equal to $\pi(H)$. Since $\Gamma(H) \leq GK(H)$, the prime graph of H has two connected components. Let $OC(G) = \{m_1, m_2\}$ and $OC(H) = \{n_1, n_2\}$. It is sufficient to prove $m_1 = n_1$. Assume $m_1 \neq n_1$. Therefore, $\pi_1(G) \neq \pi_1(H)$. Without loss of generality, we suppose there is a prime p in $\pi_1(G)$ such that $p \notin \pi_1(H)$. So $p \in \pi_2(H)$. The connectedness of components implies $\pi_1(G) \subseteq \pi_2(H)$, that is, $2 \in \pi_2(H)$, a contradiction. If p is an isolated vertex, then $p = 2$ because the order of G is even. Therefore $2 \in \pi_2(H)$ which is impossible. □

Before bringing forward the proof of Theorem 1.2, we recall that an irreducible character χ of group G is called p-defect zero if $p \nmid |G|/\chi(1)$ where p is a prime.

3.1. Proof of Theorem 1.2

First we show that $GK(H)$ is disconnected. According to Table 1, $s(G) = 2$ and the second order component of G are prime. From $\mathrm{vo}(G) = \mathrm{vo}(H)$ and Lemma 2.2, we deduce $GK(G) = \Gamma(G) = \Gamma(H)$. The last equalities imply that $\Gamma(H)$ has a connected component with a single vertex p. On the other hand, H has a vanishing p-element. Since characters of degree not divisible by some prime number p never vanish on p-elements, it is then clear that H has a p-defect zero character, namely χ. We claim that $GK(H)$ is disconnected. We assume the assertion is false. Then there exists a non-vanishing element x of order pq in H where $q \in \pi_1(G)$. Since any p-defect zero characters vanish on elements of order divisible by p, we observe

$\chi(x) = 0$. It means that $\Gamma(H)$ is connected. This is a contradiction and hence $GK(G)$ is disconnected. Then by Lemma 3.1, $OC(G) = OC(H)$. According to Table 2, G is an OC-characterizable group with $s(G) = 2$ and therefore $G \cong H$. □

Lemma 3.1 will be of use to show the validity of Conjecture 1.1 for more OC-characterizable simple groups of Lie type that we state as a general result.

Theorem 3.1. *Let G and H be two groups with the same order. Suppose G is an OC-characterizable simple group of Lie type with $s(G) = 2$ and $GK(H)$ is disconnected. If $\mathrm{vo}(G) = \mathrm{vo}(H)$, then $G \cong H$.*

In particular, the Conjecture 1.1 is valid for any group of Table 2 with a prime m_2.

Table 1: The prime graph components of the simple groups of Lie type over the field of even characteristic.

Type	Factors for primes in π_1	m_2
$A_{p-1}(q), (p,q) \neq (3,2), (3,4)$	$q, q^i - 1, 1 \leq i \leq p-1$	$\frac{q^p-1}{(q-1)(q-1,p)}$
$A_p(q), q-1 \mid p+1$	$q, q^{p+1}-1, q^i - 1, 1 \leq i \leq p-1$	$\frac{q^p-1}{q-1}$
$C_k(q), k = 2^n$	$q, q^k - 1, q^{2i} - 1, 1 \leq i \leq k-1$	$q^k + 1$
$C_p(q), (q-1,p) = 1$	$q, q^p + 1, q^{2i} - 1, 1 \leq i \leq p-1$	$\frac{q^p-1}{q-1}$
$D_p(q), (q-1,p) = 1$	$q, q^{2i} - 1, 1 \leq i \leq p-1$	$\frac{q^p-1}{q-1}$
$D_{p+1}(2)$	$2, 2^{2i} - 1, 1 \leq i \leq p-1,$ $2^p + 1, 2^{p+1} - 1$	$2^p - 1$
${}^2A_3(2^2)$	2,3	5
${}^2A_{p-1}(q^2)$	$q, q^i - (-1)^i, 1 \leq i \leq p-1$	$\frac{q^p+1}{(q+1)(q+1,p)}$
${}^2A_p(q^2), q+1 \mid p+1$	$q, q^{p+1} - 1, q^i - (-1)^i,$ $1 \leq i \leq p-1$	$\frac{q^p+1}{q+1}$
${}^2D_k(q), k = 2^n, n \geq 2$	$q, q^{2i} - 1, 1 \leq i \leq k-1$	$q^k + 1$
${}^2D_{k+1}(2), k = 2^n, n \geq 2$	$2, 2^{2i} - 1, 1 \leq i \leq k-1,$ $2^k - 1, 2^{k+1} + 1$	$2^k + 1$
$G_2(q), q \equiv 1 \pmod 3$	$q, q^2 - 1, q^3 - 1$	$q^2 - q + 1$
$G_2(q), q \equiv -1 \pmod 3$	$q, q^2 - 1, q^3 + 1$	$q^2 + q + 1$
${}^3D_4(q^3)$	$q, q^6 - 1$	$q^4 - q^2 + 1$
${}^2F_4(2)'$	2,3,5	13
$E_6(q), q \equiv 1 \pmod 3$	$q, q^5 - 1, q^8 - 1, q^{12} - 1$	$\frac{q^6+q^3+1}{3}$
$E_6(q), q \equiv 1 \pmod 3$	$q, q^5 - 1, q^8 - 1, q^{12} - 1$	$q^6 + q^3 + 1$
${}^2E_6(q^2), q \equiv -1 \pmod 3$	$q, q^5 + 1, q^8 - 1, q^{12} - 1$	$\frac{q^6-q^3+1}{3}$
${}^2E_6(q^2), q \equiv 1 \pmod 3$	$q, q^5 + 1, q^8 - 1, q^{12} - 1$	$q^6 - q^3 + 1$

Table 2: OC-characterizable simple groups of Lie type with their prime graphs having two connected components.

G	Restriction on G	Reference
$A_{p-1}(q)$	$p \neq 3, q \neq 2, 4$	[16, 15, 26]
$A_p(q)$	$(q-1)\|(p+1)$	[8, 34]
${}^2A_p(q)$	$(q+1)\|(p+1)$, $p \neq 3, 5$, $q \neq 2, 3$	[29]
${}^2A_{p-1}(q)$		[18, 19, 20, 30]
$B_n(q)$	$n = 2^m \geq 2$,	[22, 39, 25, 28]
$B_p(3)$		[7]
$C_n(q)$	$n = 2^m \geq 2$	[22, 39, 25, 28]
$C_p(q)$	$q = 2, 3$	[7] and Table 4 of [23]
$D_p(5)$	$p \geq 5, q = 2, 3, 5$	Table 4 of [23]
$D_{p+1}(q)$	$q = 2, 3$	[6]
${}^2D_n(q)$	$n = 2^m$	[27, 31]
${}^2D_n(2)$	$n = 2^m + 1$, $m \geq 2$	[9]
${}^2D_p(3)$	$5 \leq p \neq 2^m + 1$	[35, 5]
${}^2D_n(3)$	$n = 2^m + 1 \neq p$, $m \geq 2$	[4]
${}^3D_4(q)$		[3]
$E_6(q)$		[33]
${}^2E_6(q)$	$q > 2$	[32]
$F_4(q)$	q odd	[21, 17]
$G_2(q)$	$2 < q \equiv \varepsilon \pmod 3, \varepsilon = \pm 1$	[1, 2]

3.2. Proof of Theorem 1.3

From $\mathrm{vo}(G) = \mathrm{vo}(H)$ and Lemma 2.2, we deduce that $GK(G) = \Gamma(G) = \Gamma(H)$. Since for a simple group G with $s(G) > 2$, non-solvability of H is concluded from Lemma 2.1 (b), it is sufficient that we investigate the case $s(G) = 2$. Let H be a solvable group and G be a simple group of Lie type with $s(G) = 2$. Since $\Gamma(H)$ has two connected components, Lemma 2.1 (b) implies that H is either a Frobenius group or a nearly 2-Frobenius group. For both cases, using Lemma 2.1 (a), (b) and (c), $GK(G)$ has two clique connected components. So G is the above mentioned simple group of Theorem 2.2. According to Tables 3 and 4 for simple groups of Lie type with $s(G) = 2$ except $A_2(q)$, where $(q-1)_3 \neq 3$ and q is a Mersenne prime, ${}^2A_2(q)$, where $(q+1)_3 \neq 3$ and q is a Fermat prime, $C_2(q)$ where $q > 2$, ${}^2A_2(9)$, $C_3(2)$, $D_4(2)$ and ${}^3D_4(2)$, we have $t(G) \geq 3$. Thus, if $p, q, r \in \rho(G)$, then at least two of them lie in a component such that they are non-adjacent, which is impossible. Now, if G is one of the following groups: ${}^2A_2(9)$, $C_3(2)$, $D_4(2)$ or ${}^3D_4(2)$, then G is a K_4-group and Theorem 2.1 implies $H \cong G$. Hence the desired conclusion holds. □

Table 3: Independence number and set of finite simple classical groups of Lie type.

G	Condition	$t(G)$	$\rho(G)$
$A_{n-1}(q)$	$n=2, q>3$	3	$\{p, r_1, r_2\}$
	$n=3, (q-1)_3=3$ and $q+1\neq 2^k$	4	$\{p, 3, r_2, r_3\}$
	$n=3, (q-1)_3\neq 3$ and $q+1\neq 2^k$	3	$\{p, r_2, r_3\}$
	$n=3, (q-1)_3=3$ and $q+1=2^k$	3	$\{p, 3, r_3\}$
	$n=3, (q-1)_3\neq 3$ and $q+1=2^k$	2	$\{p, r_3\}$
	$n=4$	3	$\{p, r_{n-1}, r_n\}$
	$n=5,6, q=2$	3	$\{5, 7, 31\}$
	$7\leq n\leq 11, q=2$	$\left[\frac{n-1}{2}\right]$	$\{r_i \mid i\neq 6, \left[\frac{n}{2}\right]<i\leq n\}$
	$n\geq 5$ and $q>2$ or $n\geq 12$ and $q=2$	$\left[\frac{n+1}{2}\right]$	$\{r_i \mid \left[\frac{n}{2}\right]<i\leq n\}$
${}^2A_{n-1}(q)$	$n=3, q\neq 2, (q+1)_3=3$, and $q-1\neq 2^k$	4	$\{p, 3, r_1, r_6\}$
	$n=3, (q+1)_3\neq 3$ and $q-1\neq 2^k$	3	$\{p, r_1, r_6\}$
	$n=3, (q+1)_3=3$ and $q-1=2^k$	3	$\{p, 3, r_6\}$
	$n=3, (q+1)_3\neq 3$ and $q-1=2^k$	2	$\{p, r_6\}$
	$n=4, q=2$	2	$\{2, 5\}$
	$n=4, q>2$	3	$\{p, r_4, r_6\}$
	$n=5, q=2$	3	$\{2, 5, 11\}$
	$n\geq 5$ and $(n,q)\neq(5,2)$	$\left[\frac{n+1}{2}\right]$	$\{r_{i/2} \mid \left[\frac{n}{2}\right]<i\leq n,$
			$i\equiv 2(\text{mod } 4)\}\cup$
			$\{r_{2i} \mid \left[\frac{n}{2}\right]<i\leq n,$
			$i\equiv 1(\text{mod } 2)\}\cup$
			$\{r_i \mid \left[\frac{n}{2}\right]<i\leq n,$
			$i\equiv 0(\text{mod } 4)\}$
$B_n(q)$ or	$n=2, q>2$	2	$\{p, r_4\}$
$C_n(q)$	$n=3, q=2$	2	$\{5, 7\}$
	$n=4, q=2$	3	$\{5, 7, 17\}$
	$n=5, q=2$	4	$\{7, 11, 17, 31\}$
	$n=6, q=2$	5	$\{7, 11, 13, 17, 31\}$
	$n>2, (n,q)\neq(3,2),(4,2),(5,2),(6,2)$	$\left[\frac{3n+5}{4}\right]$	$\{r_{2i} \mid \left[\frac{n+1}{2}\right]\leq i\leq n\}\cup$
			$\{r_i \mid \left[\frac{n}{2}\right]<i\leq n,$
			$i\equiv 1(\text{mod } 2)\}$
$D_n(q)$	$n=4$ and $q=2$	2	$\{5, 7\}$
	$n=5$ and $q=2$	4	$\{5, 7, 17, 31\}$
	$n=6$ and $q=2$	4	$\{7, 11, 17, 31\}$
	$n\geq 4,$	$\left[\frac{3n+1}{4}\right]$	$\{r_{2i} \mid \left[\frac{n+1}{2}\right]\leq i<n\}\cup$
	$(n,q)\neq(4,2),(5,2),(6,2)$		$\{r_i \mid \left[\frac{n}{2}\right]<i\leq n,$
			$i\equiv 1(\text{mod } 2)\}$
			$\{r_{2i} \mid \left[\frac{n+1}{2}\right]\leq i<n\}\cup$
			$\{r_i \mid \left[\frac{n}{2}\right]\leq i\leq n\}$
${}^2D_n(q)$	$n=4$ and $q=2$	3	$\{5, 7, 17\}$
	$n=5$ and $q=2$	3	$\{7, 11, 17\}$
	$n=6$ and $q=2$	5	$\{7, 11, 13, 17, 31\}$
	$n=7$ and $q=2$	5	$\{11, 13, 17, 31, 43\}$
	$n\geq 4, n\not\equiv 1(\text{mod } 4),$	$\left[\frac{3n+4}{4}\right]$	$\{r_{2i} \mid \left[\frac{n}{2}\right]\leq i\leq n\}\cup$
	$(n,q)\neq(4,2),(6,2),(7,2),$		$\{r_i \mid \left[\frac{n}{2}\right]<i<n\}$
			$i\equiv 1(\text{mod } 2)\}$
	$n>4, n\equiv 1(\text{mod } 4), (n,q)\neq(5,2)$	$\left[\frac{3n+4}{4}\right]$	$\{r_{2i} \mid \left[\frac{n}{2}\right]<i\leq n\}\cup$
			$\{r_i \mid \left[\frac{n}{2}\right]<i\leq n,$
			$i\equiv 1(\text{mod } 2)\}$

Table 4: Independence number and set of finite simple exceptional Lie-type groups.

G	Conditions	$t(G)$	$\rho(G)$
$G_2(q)$	$q > 2$	3	$\{p, r_3, r_6\}$
$F_4(q)$	$q = 2$ $q > 2$	4 5	$\{5, 7, 13, 17\}$ $\{r_3, r_4, r_6, r_8, r_{12}\}$
$E_6(q)$	$q = 2$ $q > 2$	5 6	$\{5, 13, 17, 19, 31\}$ $\{r_4, r_5, r_6, r_8, r_9, r_{12}\}$
${}^2E_6(q)$		5	$\{r_4, r_8, r_{10}, r_{12}, r_{18}\}$
$E_7(q)$		7	$\{r_7, r_8, r_9, r_{10}, r_{12}, r_{14}, r_{18}\}$
$E_8(q)$		11	$\{r_7, r_8, r_9, r_{10}, r_{12}, r_{14}, r_{15}, r_{18}, r_{20}, r_{24}, r_{30}\}$
${}^3D_4(q)$	$q = 2$ $q > 2$	2 3	$\{2, 13\}$ $\{r_3, r_6, r_{12}\}$
${}^2B_2(2^{2n+1})$	$n \geq 1$	4	$\{2, s_1, s_2, s_3\}$ where $s_1 \mid 2^{2n+1} - 1$ $s_2 \mid 2^{2n+1} - 2^{n+1} + 1$ $s_3 \mid 2^{2n+1} + 2^{n+1} + 1$
${}^2G_2(3^{2n+1})$	$n \geq 1$	5	$\{3, s_1, s_2, s_3, s_4\}$, where $s_1 \neq 2, s_1 \mid 3^{2n+1} - 1$ $s_2 \neq 2, s_2 \mid 3^{2n+1} + 1$ $s_3 \mid 3^{2n+1} - 3^{n+1} + 1$ $s_4 \mid 3^{2n+1} + 3^{n+1} + 1$
${}^2F_4(2^{2n+1})$	$n \geq 2$	5	$\{s_1, s_2, s_3, s_4, s_5\}$, where $s_1 \neq 3, s_1 \mid 2^{2n+1} + 1$ $s_2 \mid 2^{4n+2} + 1$ $s_3 \neq 3, s_3 \mid 2^{4n+2} - 2^{2n+1} + 1$ $s_4 \mid 2^{4n+2} - 2^{3n+2} + 2^{2n+1} - 2^{n+1} + 1$ $s_5 \mid 2^{4n+2} + 2^{3n+2} + 2^{2n+1} + 2^{n+1} + 1$
${}^2F_4(2)'$	none	3	$\{3, 5, 13\}$
${}^2F_4(8)$	none	4	$\{7, 19, 37, 109\}$

REFERENCES

1. G. Y. Chen: *A new characterization of $G_2(q)$,* J. Southwest China Normal Univ., **21** (1996), 47-51.
2. G. Y. Chen: *Characterization of Lie type group $G_2(q)$ by its order components,* J. Southwest China Normal Univ., (Natural Science), **26** (5) (2001), 503-509.
3. G. Y. Chen: *Characterization of ${}^3D_4(q)$,* Southeast Asian Bulletin of Math., **25**, (2001), 389-401.
4. G. Y. Chen and H. Shi: *${}^2D_n(3)$ ($9 \leq n = 2^m + 1$ not a prime) can be characterized by its order components,* J. Appl. Math. Comput., **19**, (2005), 355-362.
5. M. R. Darafsheh: *Characterizability of group ${}^2D_p(3)$ by its order components, where $p \geq 5$ is a prime number not of the form $2^m + 1$,* Acta Mathematica Sinica, English Series, **24** (7), (2008), 1117-1126.

6. M. R. DARAFSHEH: *Characterization of groups* $D_{p+1}(2)$ *and* $D_{p+1}(3)$ *using order components,* J. Korean Math. Soc., **47** (2), (2010), 311-329.

7. M. R. DARAFSHEH: *On non-isomorphic groups with the same set of order components,* J. Korean Math. Soc., **45** (1), (2008), 137-150.

8. M. R. DARAFSHEH and A. MAHMIANI: *A quantitative characterization of the linear group* $L_{p+1}(2)$ *where* p *is a prime number,* Kumamoto J. Math., **20**, (2007), 33-50.

9. M. R. DARAFSHEH and A. MAHMIANI: *A characterization of the group* ${}^2D_n(2)$ *where* $n = 2^m + 1 \geq 5$, J. Appl. Math. Comput., **31**, (2009), 447-457.

10. S. DOLFI, E. PACIFICI, L. SANUS and P. SPIGA: *On the vanishing prime graph of finite groups,* J. London Math. Soc.,**82** (1) (2010), 167-183.

11. S. DOLFI, E.PACIFICI, L. SANUS and P. SPIGA: *On the vanishing prime graph of solvable groups,* J. Group Theory, **13** (2010), 189-206.

12. M. FOROUDI GHASEMABADI, A. IRANMANESH and F. MAVADATPOUR: *A new characterization of some finite simple groups,* Siberian Math. J., **56**, (2015), 78-82.

13. M. FOROUDI GHASEMABADI, A. IRANMANESH and M. AHANJIDEH: *A new characterization of some families of finite simple groups,* Rend. Sem. Mat. Univ. Padova, European Mathematical Society, **137** (2017), 57-74.

14. N. IIYORI and H. YAMAKI: *Prime graph components of the simple groups of Lie type over the field of even characteristic,* Proc. Japan Acad., **67**, Ser. A, 1991.

15. A. IRANMANESH, S. H. ALAVI and B. KHOSRAVI: *A characterization of* $PSL(3,q)$ *where* q *is an odd prime power,* J. Pure Appl. Algebra, **170**, (2002), 243-254.

16. A. IRANMANESH, S. H. ALAVI and B. KHOSRAVI: *A characterization of* $PSL(3,q)$ *where* $q = 2^n$, Acta Math. App. Sinica (English Ser.), **18**, (2002), 463-472.

17. A. IRANMANESH and B. KHOSRAVI: *A characterization of* $F_4(q)$ *where* q *is an odd prime power,* Lecture Notes London Math. Soc., **304**, (2003), 277-283.

18. A. IRANMANESH and B. KHOSRAVI: *A characterization of* $PSU(5,q)$, Int. Math. J., **3**, (2003), 129-141.

19. A. IRANMANESH and B. KHOSRAVI: *A characterization of* $PSU(7,q)$, Int. J. Appl. Math., **15**, (2004), 329-340.

20. A. IRANMANESH and B. KHOSRAVI: *A characterization of* $PSU(11,q)$, Canad. Math. Bull., **47**, (2004), 530-539.

21. A. IRANMANESH and B. KHOSRAVI: *A characterization of* $F_4(q)$ *where* $q = 2^n$, $(n > 1)$, Far East J. Math. Sci., (2000), 853-859.

22. A. IRANMANESH and B. KHOSRAVI: *A characterization of* $C_2(q)$ *where* $q > 5$, Comment. Math. Univ. Carolin., **43**, (2002), 9-21.

23. M. KHADEMI, M. R. DARAFSHEH: *Characterization of the Group* $D_p(5)$ *by order components, where* $p \geq 5$ *is a prime number,* Southeast Asian Bulletin of Mathematics, **37** (2013), 867-885.

24. M. KHATAMI and A. BABAI: *Recognition of some families of finite simple groups by order and set of orders of vanishing elements,* Czechoslovak Math. J., **68**, (2018), 121-130.

25. B. Khosravi, B. Khosravi and B. Khosravi: *The number of isomorphism classes of finite groups with the set of order components of $C_4(q)$,* Applicable Algebra in Engineering Comunication and Computing, **15**, (2005), 349-359.

26. A. Khosravi and B. Khosravi: *A new characterization of $PSL(p, q)$,* Comm. Algebra, **32** (6), (2004), 2325-2339.

27. A. Khosravi and B. Khosravi: *A characterization of ${}^2D_n(q)$, where $n = 2^m$,* Int. J. Math. Game Theory Algebra, **13**, (2003), 253-266.

28. A. Khosravi and B. Khosravi: *r-Recognizability of $B_n(q)$ and $C_n(q)$ where $n = 2^m$,* J. Pure App. Algebra, (2005), 149-165.

29. A. Khosravi and B. Khosravi: *A characterizability of $PSU(p + 1, q)$ by its order components,* Rock Mountain J. Math., **36** (5), (2006), 1555-1575.

30. B. Khosravi, B. Khosravi and B. Khosravi: *A new characterization of $PSU(p, q)$,* Acta Math. Hungar., **107** (3), (2005), 235-252.

31. B. Khosravi: *A characterization of ${}^2D_4(q)$,* Pure Math. Appl., 12, (2001), 415-424.

32. B. Khosravi and B. Khosravi: *A characterization of ${}^2E_6(q)$,* Kumamoto J. Math., **16**, (2003), 1-11.

33. B. Khosravi and B. Khosravi: *A characterization of $E_6(q)$,* Algebras Groups Geom., **19**, (2002), 225-243.

34. B. Khosravi and B. Khosravi: *Characterizability of $PSL(p + 1, q)$ by its order components,* Houston Journal of Mathematics, **32** (3), (2006), 683-700.

35. B. Khosravi and A. Iranmanesh: *A characterization of ${}^2D_p(3)$, where $p = 2^n + 1 (n \geq 2)$,* Hadronic J. Suppl., **18** (4), (2003), 465-477.

36. M. Silvia Lucido and Ali Reza Moghaddamfar: *Groups with complete prime graph connected components,* J. Group Theory , (2004), 373-384.

37. A. V. Vasiliev and E. P. Vdovin: *An adjacency criterion for the prime graph of a finite simple group,* Algebra and Logic, **44** (6), (2005).

38. J. Zhang, Z. Li and C. Shao: *Finite groups whose irreducible characters vanish only on elements of prime power order,* International Electronic J. Algebra, **9** (2011), 114-123.

39. L. Zhang and W. Shi: *New characterization of $S_4(q)$ by its noncommuting graph,* Siberian Mathematical Journal, **50** (3), (2009), 533-540.

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