FACTA UNIVERSITATIS (NIŠ)
Ser. Math. Inform. Vol. 34, No 4 (2019), 651–658
https://doi.org/10.22190/FUMI1904651G

ON THE PARTIAL DIFFERENCE SETS IN CAYLEY DERANGEMENT GRAPHS

Modjtaba Ghorbani* **and Mina Rajabi-Parsa**

Abstract. Let G be a finite group. The set $D \subseteq G$ with $|D| = k$ is called a (n, k, λ, μ)-partial difference set (PDS) in G if the differences $d_1 d_2^{-1}, d_2, d_2 \in D, d_1 \neq d_2$, represent each non-identity element in D exactly λ times and each non-identity element in $G - \{D\}$ exactly μ times. In the present paper, we determine for which group $G \in \{D_{2n}, T_{4n}, U_{6n}, V_{8n}\}$ the derangement set is a PDS. We also prove that the derangement set of a Frobenius group is a PDS.
Keywords. Finite group; Frobenius group; derangement set.

1. Introduction

Let G be a finite group. A symmetric subset of group G is a subset $S \subseteq G$, where $1 \notin S$ and $S = S^{-1}$. The Cayley graph $\Gamma = Cay(G, S)$ with respect to S is a graph whose vertex set is $V(\Gamma) = G$ and two vertices $x, y \in V(\Gamma)$ are adjacent if and only if $yx^{-1} \in S$. It is a well-known fact that a Cayley graph is connected if and only if $G = \langle S \rangle$. Also a Cayley graph is a regular graph (every vertex has the same degree).

A derangement is a permutation with no fixed points. The set $\mathcal{D}$ of permutation group is derangement if all elements of $\mathcal{D}$ are derangements. Suppose G is a permutation group and $\mathcal{D} \subseteq G$ is a derangement set. The derangement graph $\Gamma_G = Cay(G, \mathcal{D})$ has the elements of G as its vertices and two vertices are adjacent if and only if they do not intersect.

Suppose G is a permutation group of degree n. A subset S of G is said to be intersecting if for any pair of permutations $\sigma, \tau \in S$ there exists $i \in \{1, 2, \ldots, n\}$ such that $\sigma\tau^{-1}(i) = i$. A group G has the Erdös-Ko-Rado (***ekr***) property, if for any intersecting subset $S \subseteq G$, $|S|$ is bounded above by the size of the largest point stabilizer in G. The maximal intersecting set is one with maximum size. A group can have the property under one action while it fails to have this property under

Received January 24, 2019; accepted July 05, 2019
2010 *Mathematics Subject Classification.* Primary 05C35; Secondary 05E30, 05B10

another action. We refer to [1, 2, 8, 9, 13, 17] for background information about the history of this intresting problem.

Section 2 includes the ***ekr*** properties of well-known groups. In section 3, the derangement set of well-known groups are studied.

2. Erdös-Ko-Rado property

For the subgroup H of group G and the element $g \in G$, the conjugate of subgroup H in G is denoted by $H^g = g^{-1}Hg$. Suppose $G \leq Sym(n)$ is a transitive permutation group, then G is called a Frobenius group if it has a non-trivial subgroup H, where $H \cap H^g = \{1\}$, for all $g \in G \setminus H$. The kernel of Frobenius group G is defined as

$$K = (G \setminus \cup_{g \in G} H^g) \cup \{1\}.$$

It is not difficult to see that all non-identity elements of K are all derangement elements of G. In other words, let G be a non-trivial permutation group and $G^* = G - \{1\}$. If G is a Frobenius group then for all $g \in G^*$, $|fix(g)| \leq 1$ and at least there exist an element $g_0 \in G^*$ such that $|fix(g_0)| = 1$.

Theorem 2.1. *[16] (Frobenius Theorem) Suppose H is a proper non-identity subgroup of G such that for all $g \in G \setminus H$, we have $H \cap g^{-1}Hg = \{1\}$. Let $K = G \setminus \cup_{g \in G} g^{-1}(H \setminus \{1\})g$, then $K \lhd G$, $G = KH$ and $H \cap K = \{1\}$.*

Proposition 2.1. *[2] Every Frobenius group has the **ekr** property.*

Theorem 2.2. *Let $G \leq Sym(n)$ and the derangement graph $Cay(G, \mathcal{D})$ be the disjoint union of n-cliques. Then G has the **ekr** property.*

Proof. Let $\{k_1, k_2, \ldots, k_{n-1}\}$ be the set of derangements of G and $\{g_i, g_ik_1, \ldots, g_i k_{n-1}\}$ be the vertices of the i-th clique in derangemen graph $Cay(G, \mathcal{D})$, where $g_i \in G$. Since each clique has size n and G acts on n elements, every elemen of each clique has exactly one fixed point and every pair of elements in a clique has no same fixed point. Let H be the set of all vertices in $Cay(G, \mathcal{D})$ that fixes point x. Suppose $1 \neq g_rk_t \in H$ and $(g_rk_t)^g \in H$, where $g \in G - H$. So $g^{-1}g_rk_tg(x) = x$ and thus $g_rk_tg(x) = g(x)$. This means that g_rk_t fixes $g(x)$ while $g(x) \neq x$, a contradiction. The proof is completed. □

A group G acting on a set X is transitive if for every pair of points $(a, b) \in X$ there exist $x \in G$ such that $x.a = b$. The permutation group G is regular if G acts transitively on X and for all $x \in X$, $G_x = 1$. A group G is 2–transitive if for any two ordered pairs $(a, r), (b, s) \in X$, with $a \neq r$ and $b \neq s$ there exists $x \in G$ such that $x.a = b$ and $x.r = s$. We say that G is sharply 2-transitive if G is 2-transitive and for any two points $x, y \in X$, $G_{x,y} = 1$. In this paper by, $(G|X)$ we mean that the group G acts on the set X.

Theorem 2.3. *[5] Let $(G|X)$ be transitive and $x \in X$. Then $(G|X)$ is 2-transitive if and only if G_x acts transitively on the set $X - \{x\}$.*

Theorem 2.4. *[5] (The orbit-stabilizer property) Let $(G|X)$ and $x \in X$. If G is finite, then $|x^G||G_x| = |G|$.*

Theorem 2.5. *[5] (Galois Theorem). Let $(G|X)$ be a transitive permutation group of degree a prime number. Then the group G is solvable if and only if for all $x, y \in X, x \neq y$, we have $G_{x,y} = 1$.*

Theorem 2.6. *Let $(G|X)$ be a 2-transitive permutation group of degree n and $(x_1, x_2) \in X^2$. Then $|G| = n(n-1)|G_{x_1,x_2}|$.*

Proof. Suppose the group G acts on X, transitively. So the action of G on X has one orbit. Then by Theorem 2.4, $|G| = n|G_{x_1}|$. On the other hand, by Theorem 2.3 group G_{x_1} acts transitively on the set $X - \{x_1\}$, and by the orbit-stabilizer property $|G_{x_1}| = (n-1)|G_{x_1,x_2}|$. This completes the proof. □

Theorem 2.7. *Let $(G|X)$ be a transitive non-regular group of degree a prime number. If G is solvable then G has the **ekr** property.*

Proof. Since G is non-regular, there exist $x \in X$ such that $G_x \neq 1$. By Theorem 2.5, for $x, y \in X$ we have $G_{x,y} = 1$ and this means that every non-identity element of G fixes at most one element. If every non-identity element of G fixes no element of X, then $|G| = |X|$ and it is contradict with the non-regularity of G. So there exist at least one $1 \neq x \in X$ such that $|G_x| = 1$. Hence, G is Frobenius group and by Proposition 2.1, it has the ***ekr*** property. □

Theorem 2.8. *Let $(G|X)$ be a transitive permutation group such that the action G is non-regular and for all $x, y \in X, x \neq y$, we have $G_{x,y} = 1$. Then G has the **ekr** property.*

Proof. Similar to the proof of theorm 2.7, we can conclude that G is Frobenius group and the result follows. □

Theorem 2.9. *[5] Let $(G|X)$ and the act of G be 2-transitive. Then the action of G on X is sharply 2-transitive if and only if $|G| = n(n-1)$.*

Theorem 2.10. *Let $(G|X)$ be 2-transitive non-regular permutation group of degree n such that $|G| = n(n-1)$. Then G has the **ekr** property.*

Proof. By Theorem 2.9, G is a sharply 2-transitive group and so for $x, y \in X(x \neq y)$, we have $G_{x,y} = 1$. Now, similar to the proof of Theorem 2.7, G is a Frobenius group and thus it has the ***ekr*** property. □

Let $\rho : G \to GL(n, \mathbb{F})$ be a representation with $\rho(g) = [g]_\beta$. The character $\chi_\rho : G \to \mathbb{C}$ of ρ is defined as $\chi_\rho(g) = tr([g]_\beta)$ for some basis β. The character χ of an irreducible representation is called the irreducible character and χ is linear, if $\chi(1) = 1$. The set of all irreducible characters of group G is denoted by $Irr(G)$.

Let $(G|X)$ and $fix(g) = \{x \in X | g(x) = x\}$. The character π such that $\pi(g) = |fix(g)|$ is called permutation character and the character $\chi = |fix(g)| - 1$ is called standard character.

Theorem 2.11. *[12] Let G be 2-transitive group, then the standard character of G is irreducible character.*

Theorem 2.12. *[6] Let G be a finite group with a normal symmetric subset S. Let A be the adjacency matrix of graph $Cay(G, S)$. Then the eigenvalues of A are given by*

$$[\lambda_\chi]^{\chi(1)^2}, \ \chi \in Irr(G)$$

where $\lambda_\chi = \frac{1}{\chi(1)} \sum_{s \in S} \chi(s)$.

Theorem 2.13. *The derangement graph of any 2-transitive group is not a bipartite graph.*

Proof. Let G acts 2-transitive on n elementsa and complete bipartite graph $K_{r,s}$ be the derangement graph of G. Since the derangement graph is a regular graph, we have $r = s$. The eigenvalues of $K_{r,r}$ are $\{[-r]^1, [0]^{2r-2}, [r]^1\}$. On the other hand by Theorem 2.11, the standard character π of a 2-transitive group is irreducible. So by Theorem 2.12, we have $\lambda_\chi = \frac{-|\mathcal{D}|}{\chi(1)} = \frac{-r}{n-1}$. Since the rational eigenvalues of a graph are integers, we have $n = 2$ and then $G \cong \mathbb{Z}_2$ or $G \cong \{1\}$. $\square$

3. Partial difference set

Let G be a finite group and $D \subseteq G$. Then D is a (n, k, λ, μ)-partial difference set (PDS) in G if and only if $DD^{-1} = \gamma 1_G + \lambda D + \mu(G - D)$, where $\gamma = k - \mu$ if $1_G \notin D$ and $\gamma = k - \lambda$ if $1_G \in D$. We will usually assume that $1_G \notin D$ and $D^{(-1)} = D$, in which case, we have

$$D^2 = (k - \mu)1_G + (\lambda - \mu)D + \mu G.$$

Partial difference sets were named by I. M. Chakravarti, 1969 [4], but introduced by Bose and Cameron, 1965 [3] in their studies of calibration designs and the bridge tournament problem. D is called abelian if G is abelian. It is well known that a PDS D with $1 \notin D$ and $\{d^{-1} \ : d \in D\} = D$ is equivalent to a strongly regular Cayley graph, such a PDS is called regular. The study of partial difference sets is closely related to partial geometries, Schur rings, strongly regular Cayley graphs and two-weight codes. Asurvey of Ma [15] contains very detailed descriptions of these connections.

Theorem 3.1. *Let $G = HK \leq Sym(n)$ be a Frobenius group with kernel K. The derangement set of G is a $(n|H|, n-1, n-2, 0)$-PDS.*

Proof. We know that $|K| = n$. Every non-identity element of kernel G is a derangement of G and $\mathcal{D} \cup \{1\}$ is a subgroup. This implies that the derangement set of G is a $(n|H|, n-1, n-2, 0)$-PDS. □

Theorem 3.2. *Consider the dihedral group D_{2n} with derangement set $\mathcal{D}$. If n is odd, then $\mathcal{D}$ is a PDS and if n is even, then $\mathcal{D}$ is not a PDS.*

Proof. Consider the dihedral group $D_{2n} = \langle a, b | a^n = b^2 = 1, aba^{-1} = a^{-1} \rangle$. If n is odd, then D_{2n} is a Frobenius group and by Theorem 3.1 the derangement set is a PDS. Now, let n be even. Suppose that $a = (1, 2, 3, \ldots, n)$ and $b = (1, 2)(3, n) \ldots (\frac{n}{2} + 1, \frac{n}{2} + 2)$ is permutation presentation of generators of D_{2n}. The derangement set of D_{2n} is

$$\mathcal{D} = \{a, a^2, \ldots, a^{n-1}, b, a^2b, a^4b, \ldots, a^{n-2}b\}.$$

If $a^i a^{-j} = a^2$, then $i - j \equiv 2(mod\ n)$ and $\{(3, 1), (4, 2), \ldots, (n-1, n-3)\}$ are $n-3$ solutions for (i, j). On the other hand, if $(a^i b)(a^j b)^{-1} = a^2 (i, j$ are even), then $a^i a^{-j} = a^2$ and so $i - j \equiv 2(mod\ n)$. Thus $\{(4, 2), (6, 4), \ldots, (n-2, n-4)\}$ are $n/2 - 2$ solutions for (i, j). One can see that $a(a^{n-1})^{-1} = a^2$, $b(a^{n-2}b)^{-1} = a^2$ and $(a^2 b)b^{-1} = a^2$. Let $(a^i b)a^{-j} = a^2$, by using the relation of group, we have $a^{i-j}b = a^2$ and this is impossible. The equation $a^i(a^j b)^{-1} = a^2$ is impossible, too. So if $d_i, d_j \in \mathcal{D}$, then $d_i d_j^{-1} = a^2$ has $(3n/2) - 2$ solutions. If $a^i a^{-j} = a$, then $i - j \equiv 1(mod\ n)$ and $\{(2, 1), (3, 2), \ldots, (n-1, n-2)\}$ are the solutions for (i, j). By the relation of D_{2n}, there is no other solutions for $d_i d_j^{-1} = a$. So in this case there are $n-2$ solutions. Then we conclude that the derangement set of dihedral group in this case is not a PDS. □

Consider the dicyclic group T_{4n}, U_{6n} and V_{8n} by the following presentations:

$$T_{4n} = \langle a, b | a^{2n} = e, a^n = b^2, b^{-1}ab = a^{-1} \rangle,$$

$$U_{6n} = \langle a, b | a^{2n} = b^3 = e, a^n = b^2, a^{-1}ba = b^{-1} \rangle,$$

$$V_{8n} = \langle a, b | a^{2n} = b^4 = e, aba = b^{-1}, ab^{-1}a = b^{-1} \rangle.$$

Theorem 3.3. *The derangement set of dicyclic group T_{4n} is a $(4n, 4n-1, 4n-2, 0)$-PDS.*

Proof. In [7] Darafsheh proved that two elements $a = (1, 2, 3, \ldots, 2n)(2n+1, 2n+2, 2n+3, \ldots, 4n)$ and $b = (1, 2n+1, n+1, 3n+1)(2, 4n, n+2, 3n)(3, 4n-1n+3, 3n-1), \ldots, (n-1, 3n+3, 2n-1, 2n+3)(n, 3n+2, 2n, 2n+2)$ are the generators of T_{4n}. All elements of T_{4n} have no fixed point. Then $\mathcal{D} = T_{4n} - \{e\}$ which is a $(4n, 4n-1, 4n-2, 0)$-PDS. □

Theorem 3.4. *The derangement set of* $U_{6n}(n \geq 4)$ *is not a PDS set.*

Proof. Let $a = (1, 2, 3, \ldots, 2n)(2n+1, 2n+2)$ and $b = (2n+1, 2n+2, 2n+3)$ be the permutation peresentations of generators of U_{6n} [7]. One can see that the derangement set of U_{6n} is $\mathcal{D} = \{a^ib, a^ib^2 | 2 \leq i \leq 2n-2$ and i is even$\}$. Let $a^ib^j, a^rb^s \in \mathcal{D}$ and $(a^ib^j)(a^rb^s)^{-1} = b$. Then we have $a^ib^{j-s}a^{-r} = b$ and so $a^{-i}ba^r = b^{j-s}$. Thus $a^{r-i}a^{-r}ba^r = b^{j-s}$ and by using the relation of U_{6n}, we have $a^{r-i}b^{(-1)^r} = b^{j-s}$. This yields that

$$\begin{cases} r \equiv i \ (mod\ 2n) \\ j - s = 1 \end{cases}.$$

Hence the relation $(a^ib^j)(a^rb^s)^{-1} = b$ has $n-1$ solutions. On the other hand $(a^ib^j)(a^rb^s)^{-1} = a$ has no solution and thus $\mathcal{D}$ is not a PDS set. □

Theorem 3.5. *The derangement set of* V_{8n} $(n \geq 3)$ *is not a PDS set.*

Proof. For group V_{8n} we can consider two following cases:

• Case 1. Suppose n is an odd number. Let $a = (1, 2, 3, \ldots, 2n)(2n+1, 2n+2, \ldots, 4n)$ and $b = (1, 2, 2n+1, 2n+2)(3, 2n, 2n+3, 4n)(4, 4n-1, 2n+4, 2n-1) \ldots (n+1, 3n+2, 3n+1, n+2)$ be the permutation peresentations of generators of V_{8n} [7]. One can see that the derangement set of V_{8n} is

$$\mathcal{D} = \{a, a^2, \ldots, a^{2n-1}, b, b^2, b^3, a^ib, a^ib^2, a^ib^3, a^rb^2\},$$

where $2 \leq i \leq 2n-2$ (i is even) and $1 \leq r \leq 2n-1$ (r is odd).

We are going to show that the number of elements of $A = \{d_i, d_j \in \mathcal{D} | d_id_j^{-1} = a\}$ and $B = \{d_i, d_j \in \mathcal{D} | d_i, d_j^{-1} = a^2\}$ are not equal. By considering $i - j \equiv 1 \ (mod\ 2n)$, the equation $a^i(a^j)^{-1} = a$ has $2n-2$ solutions. Similarly, the equation $(a^ib^2)(a^jb^2)^{-1} = a$ has $2n-2$ solutions. On the other hand, we have $b^2(a^{2n-1}b^2)^{-1} = a$ and $(ab^2)(b^2)^{-1} = a$. So the set A has $4n-2$ elements. Now, we compute the elements of the set B. By considering $i - j \equiv 2 \ (mod\ 2n)$, the equation $a^i(a^j)^{-1} = a^2$ has $2n-3$ solutions. Also, $(a^ib^2)(a^jb^2)^{-1} = a^2$ has $2n-3$ solutions. Suppose that $4 \leq i \leq 2n-2$ (i is even) and $j \equiv i-2 \ (mod\ 2n)$, then we have $(a^ib)(a^jb)^{-1} = a^2$ and $(a^ib^3)(a^jb^3)^{-1} = a^2$. This means that each of this equations has $n-2$ solutions. On can see that $b^i(a^{2n-2}b^i)^{-1} = a^2$ for $i = 1, 2, 3$. On the other hand, we have $(a^2b^i)(b^{-i}) = a^2 (i = 1, 2, 3)$, $(ab^2)(a^{2n-1}b^2) = a^2$ and $a(a^{2n-1})^{-1} = a^2$. Then the set B has $6n-2$ elements and the derangement set of V_{8n}(n is odd) is not a PDS set.

• Case 2. Suppose n is even number. Let $a = (1, 2, 3, \ldots, 2n)(2n+1, 2n+2, \ldots, 4n)$ and $b = (1, 2, 2n+1, 2n+2)(3, 2n, 2n+3, 4n)(4, 4n-1, 2n+4, 2n-1) \ldots (n, 3n+3, 3n, n+3)(n+1, n+2, 3n+1, 3n+2)$ be the permutation peresentations of generators of V_{8n} [7]. One can see that the derangement set of V_{8n} is

$$\mathcal{D} = \{a, a^2, \ldots, a^{2n-1}, b, b^2, b^3, a^ib, a^ib^2, a^ib^3, a^rb, a^rb^2, a^sb^2, a^sb^3\},$$

where $2 \leq i \leq 2n-2$ (i is even), $r \in \{1, 5, 9, \ldots, 2n-3\}$ and $s \in \{3, 7, 11, \ldots, 2n-1\}$.

Now, we show that the number of elements of $E = \{d_i, d_j \in \mathcal{D} | d_i d_j^{-1} = a\}$ and $F = \{d_i, d_j \in \mathcal{D} | d_i d_j^{-1} = a^4\}$ are not equal. By regarding $i - j \equiv 1 \ (mod\ 2n)$, the equation $a^i(a^j)^{-1} = a$ has $2n-2$ solutions. If $j \equiv i-1 \ (mod\ n)$ and $i \in \{2, 5, 6, 9, 10, \ldots, 2n-2\}$, then the equation $(a^i b^s)(a^j b^s)^{-1} = a$, where $s \in \{1, 2\}$ has $n-1$ solutions. If $j \equiv i-1 \ (mod\ n)$ and $i \in \{3, 4, 7, 8, 11, \ldots, 2n-1\}$, then the equation $(a^i b^s)(a^j b^s)^{-1} = a$, where $s \in \{2, 3\}$ has $n-1$ solutions. One can see that $(ab^t)(b^t)^{-1} = a$, where $t \in \{1, 2\}$ and $b^t(a^{2n-1}b^t)^{-1} = a$, where $t \in \{2, 3\}$. Then the set E has $6n-2$ elements. Now, we compute the elements of the set F. By considering $i - j \equiv 4 \ (mod\ 2n)$ the equation $a^i(a^j)^{-1} = a^4$ has $2n-5$ solutions. It is clear that $a^1(a^{2n-3})^{-1} = a^2(a^{2n-2})^{-1} = a^3(a^{2n-1})^{-1} = a^4$. One can see that if $t \in \{1, 2, 3\}$ then $(a^4 b^t)(b^t)^{-1} = a^4$, and $b^t(a^{2n-4}b^t)^{-1} = a^4$. Let i, j be even, $i - j \equiv 4 \ (mod\ 2n)$ and $r \in \{1, 2, 3\}$. Then $(a^i b^r)(a^j b^r)^{-1} = a^4$ yields $3(n-1)$ solutions. Let i be odd, $i - j \equiv 4 \ (mod\ 2n)$ and $r \in \{5, 9, 13, \ldots, 2n-3\}$. Then by using $(a^i b^r)(a^j b^r)^{-1} = a^4$ we get $n-2$ solutions for this equation. Let i be odd, $i - j \equiv 4 \ (mod\ 2n)$ and $r \in \{7, 11, 15, \ldots, 2n-1\}$. Again by $(a^i b^r)(a^j b^r)^{-1} = a^4$ we acheive $n-2$ solutions. If $i \in \{1, 2\}$ then $(ab^i)(a^{2n-3}b^i)^{-1} = a^4$. If $i \in \{2, 3\}$ then $(a^3 b^i)(a^{2n-1}b^i)^{-1} = a^4$ and if $i \in \{1, 2, 3\}$ then $(a^2 b^i)(a^{2n-2}b^i)^{-1} = a^4$. So the set F has $7n-2$ elements. Then the derangement set of V_{8n}(n is odd) is not a PDS set. □

REFERENCES

1. B. Ahmadi: *Maximum intersecting families of permutations*. Ph.D. thesis. University of Regina, Regina, 2013.
2. B. Ahmadi and K. Meagher: *The Erdös-Ko-Rado property for some permutation groups*. Austr. J. Comb. **61** (2015), 23–41.
3. R. C. Bose and J. M. Cameron: *The bridge tournament problem and calibration designs for comparing pairs of objects*. Journal of Research of the NBS-B, Maths. and Math.Phys. **69** (1965), 323–332.
4. I. M. Chakravarti: *Partial difference sets, calibration designs and error correcting codes*. Bull. Inter. Stat. Inst. **43** (1969), 104–106.
5. J. D. Dixon and B. Mortimer: *Permutation Groups*. Springer, 1966.
6. P. Diaconis and M. Shahshahani: *Generating a random permutation with random transpositions*. Zeit. für Wahrscheinlichkeitstheorie verw. Gebiete. **57** (1981), 159—179.
7. M. R. Darafsheh and N. S. Poursalavati: *On the existence of the orthogonal basis of the symmetry classes of tensors associated with certain groups*. SUT J. Math. **37** (2001), 1–17. Zbl 0999.20006
8. P. Erdös, Chao Ko and R. Rado: *Intersection theorems for systems of finite sets*. Quart. J. Math. **12** (1961), 313-–320.
9. P. Frankl and M. Deza: *The maximum number of permutations with given maximal or minimal distance*. J. Combin. Theory Ser. A. **22** (1977), 352-–360.

10. M. Ghorbani and M. Rajabi-Parsa: *On the Erdös-Ko-Rado property of finite groups of order a product of primes*. Electronic journal of graph theory and applications. **7** (2019), 31–42.
11. M. Ghorbani and M. Rajabi-Parsa: *On the spectrum of derangement graphs of order a product of three primes*. **6** (2019), 81–89.
12. I. M. Isaacs: *Character Theory of Finite Groups*. University of Wiaconslr. New York. Academic Press, 1976.
13. M. Jalali-Rad and A. R. Ashrafi: *Erdös-Ko-Rado properties of some finite groups*. Sib. Electron. Math. Reports. **13** (2016), 1249–1257.
14. G. James and M. Liebeck: *Representation and characters of groups*. Cambridge University Press. Cambridge, 1993.
15. S. L. Ma: *A survey of partial difference sets*. Designs, Codes and Cryptogr. **4** (1994), 221–261.
16. J.S. Rose: *A Course on Group Theory*. Cambridge University Press, 1978.
17. J. Wang and S. J. Zhang: *An Erdös-Ko-Rado-type theorem in Coxeter groups*. European J. Combin. **29** (2008), 1112–1115.

Modjtaba Ghorbani
Department of Mathematics
Faculty of Science
Shahid Rajaee Teacher Training University
Tehran, 16785-136, I. R. Iran
mghorbani@sru.ac.ir

Mina Rajabi-Parsa
Department of Mathematics
Faculty of Science
Shahid Rajaee Teacher Training University
Tehran, 16785-136, I. R. Iran
mina.rparsa@gmail.com