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SOME STATISTICAL CONVERGENCE TYPES OF ORDER α FOR DOUBLE SET SEQUENCES

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Abstract. In this study, we have introduced the concepts of Wijsman statistical convergence of order α, Hausdorff statistical convergence of order α and Wijsman strongly p-Cesàro summability of order α for double set sequences. Also, we have investigated some properties of these concepts and examined the relationships among them.

Keywords: Statistical convergence; Cesàro summability; Double sequence; Order α; Wijsman convergence; Hausdorff convergence; Set sequences.

1. Introduction

The concept of statistical convergence was introduced by Steinhaus [30] and Fast [11], and later reintroduced by Schoenberg [28] independently. Moreover, many rearchers have studied this concept until recently (see, [5, 6, 12, 14, 15, 25, 31, 34]). The order of statistical convergence of a single sequence of numbers was given by Gadjiev and Orhan [13]. Then, the concepts of statistical convergence of order α and strongly p-Cesàro summability of order α were studied by Çolak [8] and Çolak and Bektaş [9].

In [24], Pringsheim introduced the concept of convergence for double sequences. Recently, Mursaleen and Edely [19] have extended this concept to statistical convergence. More developments on double sequences can be found in [4,7,16–18,20]. Very recently, the concepts of statistical convergence of order α and strongly p-Cesàro summability of order α for double sequences have been studied by Savaş [26] and Çolak and Altın [10].

The concepts of convergence for number sequences were transferred to the concepts of convergence for set sequences by many authors. In this study, the concepts of Wijsman convergence and Hausdorff convergence which are two of these transfers are considered (see, [1–3, 35]). Nuray and Rhoades [21] extended the concept of

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Wijsman convergence and Hausdorff convergence to statistical convergence for set sequences and gave some basic theorems. Very recently, the concept of Wijsman $\mathcal{I}$-statistical convergence of order α have been studied by Savaş [27] and Şengül and Et [32].

Nuray et al. [22] introduced the concepts of Wijsman convergence and Wijsman strongly p-Cesàro summability for double set sequences. Then, the concepts of Hausdorff convergence for double set sequences was studied by Sever et al. [29]. Also, the concepts of Wijsman statistical convergence and Hausdorff statistical convergence were studied by Nuray et al. [23] and Talo et al. [33], respectively.

In this study, we shall introduce the concepts of Wijsman statistical convergence of order α, Hausdorff statistical convergence of order α and Wijsman strongly p-Cesàro summability of order α for double set sequences. Also, we shall investigate some properties of these concepts and examine the relationships among them.

2. Definitions and Notations

Firstly, we recall the basic concepts that need for a good understanding of our study (see, [1–3, 19, 22–24, 29, 33, 35]).

A double sequence (x_{ij}) is said to be convergent to L in Pringsheim's sense if for every $\varepsilon > 0$, there exists $N_\varepsilon \in \mathbb{N}$ such that $|x_{ij} - L| < \varepsilon$ whenever $i, j > N_\varepsilon$.

A double sequence (x_{ij}) is said to be statistically convergent to L if for every $\varepsilon > 0$,

$$\lim_{m,n\to\infty} \frac{1}{mn} \Big| \big\{ (i,j) : \ i \leq n, j \leq m, \ |x_{ij} - L| \geq \varepsilon \big\} \Big| = 0.$$

Let X be any non-empty set. The function $f : \mathbb{N} \to P(X)$ is defined by $f(i) = U_i \in P(X)$ for each $i \in \mathbb{N}$, where $P(X)$ is power set of X. The sequence $\{U_i\} = (U_1, U_2, ...)$, which is the range's elements of f, is said to be set sequences.

Let (X, d) be a metric space. For any point $x \in X$ and any non-empty subset U of X, the distance from x to U is defined by

$$\rho(x, U) = \inf_{u \in U} d(x, u).$$

Throughout the study, we will take (X, d) be a metric space and U, U_{ij} be any non-empty closed subsets of X.

A double sequence $\{U_{ij}\}$ is said to be Wijsman convergent to U if for each $x \in X$,

$$\lim_{i,j\to\infty} \rho(x, U_{ij}) = \rho(x, U).$$

A double sequence $\{U_{ij}\}$ is said to be Hausdorff convergent to U if for each $x \in X$,

$$\lim_{i,j\to\infty} \sup_{x \in X} |\rho(x, U_{ij}) - \rho(x, U)| = 0.$$

A double sequence $\{U_{ij}\}$ is said to be Wijsman statistical convergent to U if for every $\varepsilon > 0$ and each $x \in X$,

$$\lim_{m,n\to\infty} \frac{1}{mn}\Big|\big\{(i,j):\ i \leq n, j \leq m,\ |\rho(x,U_{ij}) - \rho(x,U)| \geq \varepsilon\big\}\Big| = 0.$$

The class of all Wijsman statistical convergent sequences is simply denoted by $W(S_2)$.

A double sequence $\{U_{ij}\}$ is said to be Hausdorff statistical convergent to U if for every $\varepsilon > 0$ and each $x \in X$,

$$\lim_{m,n\to\infty} \frac{1}{mn}\Big|\big\{(i,j):\ i \leq n, j \leq m,\ \sup_{x\in X}|\rho(x,U_{ij}) - \rho(x,U)| \geq \varepsilon\big\}\Big| = 0.$$

The class of all Hausdorff statistical convergent sequences is simply denoted by $H(S_2)$.

A double sequence $\{U_{ij}\}$ is said to be Wijsman Cesàro summable to U if for each $x \in X$,

$$\lim_{m,n\to\infty} \frac{1}{mn}\sum_{i,j=1,1}^{m,n} \rho(x,U_{ij}) = \rho(x,U).$$

Let $0 < p < \infty$. A double sequence $\{U_{ij}\}$ is said to be Wijsman strongly p-Cesàro summable to U if for each $x \in X$,

$$\lim_{m,n\to\infty} \frac{1}{mn}\sum_{i,j=1,1}^{m,n} |\rho(x,U_{ij}) - \rho(x,U)|^p = 0.$$

The class of all Wijsman strongly p-Cesàro summable sequences is simply denoted by $W[C_2]^p$.

From now on, in short, we shall use $\rho_x(U)$ and $\rho_x(U_{ij})$ instead of $\rho(x,U)$ and $\rho(x,U_{ij})$, respectively.

3. Main Results

In this section, we shall introduce the concepts of Wijsman statistical convergence of order α, Hausdorff statistical convergence of order α and Wijsman strongly p-Cesàro summability of order α for double set sequences. Also, we shall investigate some properties of these concepts and examine the relationships among them.

Definition 3.1. Let $0 < \alpha \leq 1$. A double sequence $\{U_{ij}\}$ is Wijsman statistically convergent of order α to U or $W(S_2^\alpha)$-convergent to U if for every $\varepsilon > 0$ and each $x \in X$,

$$\lim_{m,n\to\infty} \frac{1}{(mn)^\alpha}\Big|\big\{(i,j):\ i \leq m, j \leq n,\ |\rho_x(U_{ij}) - \rho_x(U)| \geq \varepsilon\big\}\Big| = 0.$$

In this case, we write $U_{ij} \longrightarrow_{W(S_2^\alpha)} U$ or $U_{ij} \longrightarrow U\big(W(S_2^\alpha)\big)$.

The class of all $W(S_2^\alpha)$-convergent sequences will be simply denoted by $W(S_2^\alpha)$.

Example 3.1. Let $X = \mathbb{R}^2$ and a double sequence $\{U_{ij}\}$ be defined as following:

$$U_{ij} := \begin{cases} \{(x,y) \in \mathbb{R}^2 : (x-i)^2 + (y-j)^2 = 1\} & , \text{ if } i \text{ and } j \text{ are square integers} \\ \{(0,0)\} & , \text{ otherwise.} \end{cases}$$

Then, the double sequence $\{U_{ij}\}$ is Wijsman statistically convergent of order α to the set $U = \{(0,0)\}$.

Remark 3.1. For $\alpha = 1$, the concept of $W(S_2^\alpha)$-convergence coincides with the concept of Wijsman statistical convergence for double set sequences in [23].

Theorem 3.1. *If $0 < \alpha \leq \beta \leq 1$, then $W(S_2^\alpha) \subseteq W(S_2^\beta)$.*

Proof. Let $0 < \alpha \leq \beta \leq 1$ and suppose that $U_{ij} \longrightarrow_{W(S_2^\alpha)} U$. For every $\varepsilon > 0$ and each $x \in X$, we have

$$\frac{1}{(mn)^\beta}\Big|\big\{(i,j) : \ i \leq m, j \leq n, \ |\rho_x(U_{ij}) - \rho_x(U)| \geq \varepsilon\big\}\Big|$$
$$\leq \frac{1}{(mn)^\alpha}\Big|\big\{(i,j) : \ i \leq m, j \leq n, \ |\rho_x(U_{ij}) - \rho_x(U)| \geq \varepsilon\big\}\Big|.$$

Hence, by our assumption, we get $U_{ij} \longrightarrow_{W(S_2^\beta)} U$. Consequently, $W(S_2^\alpha) \subseteq W(S_2^\beta)$. $\square$

If we take $\beta = 1$ in Theorem 3.1, then we obtain the following corollary.

Corollary 3.1. *If a double sequence $\{U_{ij}\}$ is Wijsman statistically convergent of order α to U for some $0 < \alpha \leq 1$, then the double sequence is Wijsman statistically convergent to U, i.e., $W(S_2^\alpha) \subseteq W(S_2)$.*

Definition 3.2. Let $0 < \alpha \leq 1$. A double sequence $\{U_{ij}\}$ is Wijsman Cesàro summable of order α to U or $W(C_2^\alpha)$-summable to U if for each $x \in X$,

$$\lim_{m,n\to\infty} \frac{1}{(mn)^\alpha} \sum_{i,j=1,1}^{m,n} \rho_x(U_{ij}) = \rho_x(U).$$

In this case, we write $U_{ij} \longrightarrow_{W(C_2^\alpha)} U$ or $U_{ij} \longrightarrow U\big(W(C_2^\alpha)\big)$.

The class of all $W(C_2^\alpha)$-summable sequences will be simply denoted by $W(C_2^\alpha)$.

Definition 3.3. Let $0 < \alpha \leq 1$ and $0 < p < \infty$. A double sequence $\{U_{ij}\}$ is Wijsman strongly p-Cesàro summable of order α to U or $W[C_2^\alpha]^p$-summable to U if for each $x \in X$,

$$\lim_{m,n\to\infty} \frac{1}{(mn)^\alpha} \sum_{i,j=1,1}^{m,n} |\rho_x(U_{ij}) - \rho_x(U)|^p = 0.$$

In this case, we write $U_{ij} \longrightarrow_{W[C_2^\alpha]^p} U$ or $U_{ij} \longrightarrow U\big(W[C_2^\alpha]^p\big)$. If $p=1$, then a double sequence $\{U_{ij}\}$ is simply said to be Wijsman strongly Cesàro summable of order α to U and we write $U_{ij} \longrightarrow_{W[C_2^\alpha]} U$ or $U_{ij} \longrightarrow U\big(W[C_2^\alpha]\big)$.

The class of all $W[C_2^\alpha]^p$-summable sequences will be simply denoted by $W[C_2^\alpha]^p$.

Example 3.2. Let $X=\mathbb{R}^2$ and a double sequence $\{U_{ij}\}$ be defined as following:

$$U_{ij} := \begin{cases} \left\{(x,y)\in\mathbb{R}^2 : x^2+(y-1)^2=\frac{1}{ij}\right\} & , \text{ if } i \text{ and } j \text{ are square integers,} \\ \{(1,0)\} & , \text{ otherwise.} \end{cases}$$

Then, the double sequence $\{U_{ij}\}$ is Wijsman strongly Cesàro summable of order α to the set $U=\{(1,0)\}$.

Remark 3.2. For $\alpha=1$, the concepts of $W(C_2^\alpha)$-summability and $W[C_2^\alpha]^p$-summability coincides with the concepts of Wijsman Cesàro summability and Wijsman strongly p-Cesàro summability for double set sequences in [22], respectively.

Theorem 3.2. *If $0<\alpha\le\beta\le 1$, then $W[C_2^\alpha]^p \subseteq W[C_2^\beta]^p$.*

Proof. Let $0<\alpha\le\beta\le 1$ and suppose that $U_{ij} \longrightarrow_{W[C_2^\alpha]^p} U$. For each $x\in X$, we have

$$\frac{1}{(mn)^\beta}\sum_{i,j=1,1}^{m,n} |\rho_x(U_{ij})-\rho_x(U)|^p \le \frac{1}{(mn)^\alpha}\sum_{i,j=1,1}^{m,n} |\rho_x(U_{ij})-\rho_x(U)|^p.$$

Hence, by our assumption, we get $U_{ij} \longrightarrow_{W[C_2^\beta]^p} U$. Consequently, $W[C_2^\alpha]^p \subseteq W[C_2^\beta]^p$. $\square$

If we take $\beta=1$ in Theorem 3.2, then we obtain the following corollary.

Corollary 3.2. *If a double sequence $\{U_{ij}\}$ is Wijsman strongly p-Cesàro summable of order α to U for some $0<\alpha\le 1$, then the double sequence is Wijsman strongly p-Cesàro summable to U, i.e., $W[C_2^\alpha]^p \subseteq W[C_2]^p$.*

Now, without proof, we shall state a theorem that gives a relation between $W[C_2^\alpha]^p$ and $W[C_2^\alpha]^q$, where $0<\alpha\le 1$ and $0<p<q<\infty$.

Theorem 3.3. *Let $0<\alpha\le 1$ and $0<p<q<\infty$. Then, $W[C_2^\alpha]^q \subset W[C_2^\alpha]^p$.*

Theorem 3.4. *Let $0<\alpha\le\beta\le 1$ and $0<p<\infty$. If a double sequence $\{U_{ij}\}$ is Wijsman strongly p-Cesàro summable of order α to U, then the double sequence is Wijsman statistically convergent of order β to U.*

Proof. Let $0 < \alpha \leq \beta \leq 1$ and $0 < p < \infty$ and assume that the double sequence $\{U_{ij}\}$ is Wijsman strongly p-Cesàro summable of order α to U. For every $\varepsilon > 0$ and each $x \in X$, we have

$$\begin{aligned}\sum_{i,j=1,1}^{m,n} |\rho_x(U_{ij}) - \rho_x(U)|^p &= \sum_{i,j=1,1;\ |\rho_x(U_{ij})-\rho_x(U)|\geq\varepsilon}^{m,n} |\rho_x(U_{ij}) - \rho_x(U)|^p \\ &\quad + \sum_{i,j=1,1;\ |\rho_x(U_{ij})-\rho_x(U)|<\varepsilon}^{m,n} |\rho_x(U_{ij}) - \rho_x(U)|^p \\ &\geq \sum_{i,j=1,1;\ |\rho_x(U_{ij})-\rho_x(U)|\geq\varepsilon}^{m,n} |\rho_x(U_{ij}) - \rho_x(U)|^p \\ &\geq \varepsilon^p \Big|\big\{(i,j):\ i \leq m, j \leq n,\ |\rho_x(U_{ij}) - \rho_x(U)| \geq \varepsilon\big\}\Big|\end{aligned}$$

and so

$$\begin{aligned}&\frac{1}{(mn)^\alpha} \sum_{i,j=1,1}^{m,n} |\rho_x(U_{ij}) - \rho_x(U)|^p \\ &\qquad \geq \frac{\varepsilon^p}{(mn)^\alpha}\Big|\big\{(i,j):\ i \leq m, j \leq n,\ |\rho_x(U_{ij}) - \rho_x(U)| \geq \varepsilon\big\}\Big| \\ &\qquad \geq \frac{\varepsilon^p}{(mn)^\beta}\Big|\big\{(i,j):\ i \leq m, j \leq n,\ |\rho_x(U_{ij}) - \rho_x(U)| \geq \varepsilon\big\}\Big|.\end{aligned}$$

Hence, by our assumption, we get the double sequence $\{U_{ij}\}$ is Wijsman statistically convergent of order β to U. $\square$

If we take $\beta = \alpha$ in Theorem 3.4, then we obtain the following corollary.

Corollary 3.3. *Let $0 < \alpha \leq 1$ and $0 < p < \infty$. If a double sequence $\{U_{ij}\}$ is Wijsman strongly p-Cesàro summable of order α to U, then the double sequence is Wijsman statistically convergent of order α to U.*

Definition 3.4. Let $0 < \alpha \leq 1$. A double sequence $\{U_{ij}\}$ is Hausdorff statistically convergent of order α to U or $H(S_2^\alpha)$-convergent to U if for every $\varepsilon > 0$ and each $x \in X$,

$$\lim_{m,n\to\infty} \frac{1}{(mn)^\alpha}\Big|\big\{(i,j):\ i \leq m, j \leq n,\ \sup_{x\in X} |\rho_x(U_{ij}) - \rho_x(U)| \geq \varepsilon\big\}\Big| = 0.$$

In this case, we write $U_{ij} \longrightarrow_{H(S_2^\alpha)} U$ or $U_{ij} \longrightarrow U\big(H(S_2^\alpha)\big)$.

The class of all $H(S_2^\alpha)$-convergent sequences will be denoted by simply $H(S_2^\alpha)$.

Remark 3.3. For $\alpha = 1$, the concept of $H(S_2^\alpha)$-convergence coincides with the concept of Hausdorff statistical convergence for double set sequences in [33].

Theorem 3.5. *If $0 < \alpha \leq \beta \leq 1$, then $H(S_2^\alpha) \subseteq H(S_2^\beta)$.*

Proof. Let $0 < \alpha \leq \beta \leq 1$ and suppose that $U_{ij} \longrightarrow_{H(S_2^\alpha)} U$. For every $\varepsilon > 0$ and each $x \in X$, we have

$$\frac{1}{(mn)^\beta}\Big|\big\{(i,j):\ i \leq m, j \leq n,\ \sup_{x\in X}|\rho_x(U_{ij}) - \rho_x(U)| \geq \varepsilon\big\}\Big|$$
$$\leq \frac{1}{(mn)^\alpha}\Big|\big\{(i,j):\ i \leq m, j \leq n,\ \sup_{x\in X}|\rho_x(U_{ij}) - \rho_x(U)| \geq \varepsilon\big\}\Big|.$$

Hence, by our assumption, we get $U_{ij} \longrightarrow_{H(S_2^\beta)} U$. Consequently, $H(S_2^\alpha) \subseteq H(S_2^\beta)$. $\square$

If we take $\beta = 1$ in Theorem 3.5, then we obtain the following corollary.

Corollary 3.4. *If a double sequence $\{U_{ij}\}$ is Hausdorff statistically convergent of order α to U for some $0 < \alpha \leq 1$, then the double sequence is Hausdorff statistically convergent to U, i.e., $H(S_2^\alpha) \subseteq H(S_2)$.*

Theorem 3.6. *Let $0 < \alpha \leq \beta \leq 1$. If a double sequence $\{U_{ij}\}$ is Hausdorff statistically convergent of order α to U, then the double sequence is Wijsman statistically convergent of order β to U.*

Proof. Let $0 < \alpha \leq \beta \leq 1$ and assume that $U_{ij} \longrightarrow_{H(S_2^\alpha)} U$. For every $\varepsilon > 0$ and each $x \in X$, we have

$$\frac{1}{(mn)^\beta}\Big|\big\{(i,j):\ i \leq m, j \leq n,\ |\rho_x(U_{ij}) - \rho_x(U)| \geq \varepsilon\big\}\Big|$$
$$\leq \frac{1}{(mn)^\beta}\Big|\big\{(i,j):\ i \leq m, j \leq n,\ \sup_{x\in X}|\rho_x(U_{ij}) - \rho_x(U)| \geq \varepsilon\big\}\Big|$$
$$\leq \frac{1}{(mn)^\alpha}\Big|\big\{(i,j):\ i \leq m, j \leq n,\ \sup_{x\in X}|\rho_x(U_{ij}) - \rho_x(U)| \geq \varepsilon\big\}\Big|.$$

Hence, by our assumption, we achieve the desired result. $\square$

If we take $\beta = \alpha$ in Theorem 3.6, then we obtain the following corollary.

Corollary 3.5. *Let $0 < \alpha \leq 1$. If a double sequence $\{U_{ij}\}$ is Hausdorff statistically convergent of order α to U, then the double sequence is Wijsman statistically convergent of order α to U.*

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