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ON LOWER AND UPPER α-IRRESOLUTE INTUITIONISTIC FUZZY MULTIFUNCTIONS

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Abstract. The aim of this paper is to introduce the concepts of upper and lower intuitionistic fuzzy α-irresolute intuitionistic fuzzy multifunctions and to obtain some of their properties.
Keywords: Intuitionistic fuzzy sets, Intuitionistic fuzzy topology, Intuitionistic fuzzy multifunctions.

1. Introduction

After the introduction of fuzzy sets by Zadeh [32] in 1965 and fuzzy topology by Chang [7] in 1967, research was conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. The concept of intuitionistic fuzzy sets was introduced by Atanassov [2–4] as a generalization of fuzzy sets. In the last 27 years various concepts of fuzzy mathematics have been extended for intuitionistic fuzzy sets. In 1997 Coker [8] introduced the concept of intuitionistic fuzzy topological spaces as a generalization of fuzzy topological spaces. In 1999, Ozbakir and Coker [25] introduced the concept intuitionistic fuzzy multifunctions and studied their lower and upper intuitionistic fuzzy semi continuity from a topological space to an intuitionistic fuzzy topological space. In the present paper we extend the concepts of lower and upper α-irresolute multifunctions due to Neubrunn [20] to intuitionistic fuzzy multifunctions and obtain some of their characterizations and properties.

2. Preliminaries

Throughout this paper (X,τ) and (Y,Γ) represent a topological space and an intuitionistic fuzzy topological space, respectively.

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Definition 2.1. [14, 21] *A subset A of a topological space* (X, τ) *is called :*

(a) Semi-open if $A \subset Cl(Int(A))$.

(b) Semi-closed if its complement is semi-open.

(c) α*-open if* $A \subset Int(Cl(Int(A)))$.

(d) α*-closed if its complement is* α*-open.*

Remark 2.1. [21] *Every open (resp. closed) set is* α*-open (resp.* α*-closed) and every* α*-open (resp.* α*-closed) set is semi-open (resp. semi-closed) but the converses may not be true.*

The family of all α-open (resp. α-closed) subsets of topological space (X, τ) is denoted by $\alpha O(X)$ (resp. αC(X)). The intersection of all α-closed (resp. semi-closed) sets of X containing a set A of X is called the α-closure [16] (resp. semi-closure) of A. It is denoted by $\alpha Cl(A)$ (resp. sCl(A)). The union of all α-open (resp. semi-open) subsets of A of X is called the α-interior [16] (resp. semi-interior) of A .It is denoted by $\alpha Int(A)$ (resp. $sInt(A)$). A subset A of X is α-closed (resp. semi-closed) if and only if $A \supset Cl(Int(Cl(A)))$ (resp.$A \supset Int(Cl(A))$. A subset N of a topological space (X, τ) is called a α-neighborhood [15] of a point x of X if there exists a α-open set O of X such that $x \in O \subset N$. A is an α-open in X if and only if it is a α-neighborhood of each of its points. A subset V of X is called an α-neighborhood of a subset A of X if there exists $U \in \alpha O(X)$ such that $A \subset U \subset V$. A mapping f from a topological space (X, τ) to another topological space (X^*, τ^*) is said to be α-continuous [17, 18] (resp. α-irresolute [16]) if the inverse image of every open (resp. α-open) set of X^* is α-open in X. Every continuous (resp. α-irresolute) mapping is α-continuous but the converse may not be true [16, 17]. A multifunction F from a topological space (X, τ) to another topological space (X^*, τ^*) is said to be lower α-irresolute [20] (resp. upper α-irresolute [20]) at a point $x_0 \in X$ if for every α-neighborhood U of x_0 and for any α-open set W of X^* such that $F(x_0) \cap W \neq \phi$ (resp. $F(x_0) \subset W$) there is a α-neighborhood U of x_0 such that $F(x) \cap W \neq \phi$(resp. $F(x) \subset W$) for every $x \in U$.

Lemma 2.1. [27] *The following properties hold for a subset A of a topological space* (X, τ)*:*

(a) A is α*-closed in X* $\Leftrightarrow sInt(Cl(A) \subset A$*;*

(b) $sInt(Cl(A)) = Cl(Int(Cl(A)))$*;*

(c) $\alpha Cl(A) = A \cup Cl(Int(Cl(A)))$.

Lemma 2.2. [27] *The following are equivalent for a subset A of a topological space* (X, τ)*:*

(a) $A \in \alpha O(X)$,

(b) $U \subset A \subset Int(Cl(U))$ *for some open set U of X.*

(c) $U \subset A \subset sCl(U)$ *for some open set U of X.*

(d) $A \subset sCl(Int(A))$.

Definition 2.2. [2–4] *Let Y be a nonempty fixed set. An intuitionistic fuzzy set $\tilde{A}$ in Y is an object having the form*

$$\tilde{A} = \{< x, \mu_{\tilde{A}}(y), \nu_{\tilde{A}}(y) >: y \in Y\}$$

where the functions $\mu_{\tilde{A}}(y) : Y \to I$ and $\nu_{\tilde{A}}(y) : Y \to I$ denotes the degree of membership (namely $\mu_{\tilde{A}}(y)$) and the degree of non-membership (namely $\nu_{\tilde{A}}(y)$) of each element $y \in Y$ to the set $\tilde{A}$, respectively, and $0 \leq \mu_{\tilde{A}}(y) + \mu_{\tilde{A}}(y) \leq 1$ for each $y \in Y$.

Definition 2.3. [2–4] *Let Y be a nonempty set and the intuitionistic fuzzy sets $\tilde{A}$ and $\tilde{B}$ be in the form*

$$\tilde{A} = \{< x, \mu_{\tilde{A}}(y), \nu_{\tilde{A}}(y) >: y \in Y\}, \quad \tilde{B} = \{< x, \mu_{\tilde{B}}(y), \nu_{\tilde{B}}(y) >: y \in Y\}$$

and let $\tilde{B}_\alpha : \alpha \in \Lambda$ be an arbitrary family of intuitionistic fuzzy sets in Y. Then:

(a). $\tilde{A} \subseteq \tilde{B}$ *if* $\forall y \in Y\ [\mu_{\tilde{A}}(y) \leq \mu_{\tilde{B}}(y)$ *and* $\nu_{\tilde{A}}(y) \geq \nu_{\tilde{B}}(y)]$

(b). $\tilde{A} = \tilde{B}$ *if* $\tilde{A} \subseteq \tilde{B}$ *and* $\tilde{B} \subseteq \tilde{A}$;

(c). $\tilde{A}^c = \{< x, \nu_{\tilde{A}}(y), \mu_{\tilde{A}}(y) >: y \in Y\}$;

(d). $\tilde{0} = \{< y, 0, 1 >: y \in Y\}$ *and* $\tilde{1} = \{< y, 1, 0 >: y \in Y\}$

(e). $\cap \tilde{A}_\alpha = \{< x, \wedge \mu_{\tilde{A}}(y), \vee \nu_{\tilde{A}}(y) >: y \in Y\}$

(f). $\cup \tilde{A}_\alpha = \{< x, \vee \mu_{\tilde{A}}(y), \wedge \nu_{\tilde{A}}(y) >: y \in Y\}$

Definition 2.4. [9] *Two intuitionistic fuzzy sets $\tilde{A}$ and $\tilde{B}$ of Y are said to be quasi-coincident ($\tilde{A}q\tilde{B}$ for short) if $\exists y \in Y$ such that*

$$\mu_{\tilde{A}}(y) > \nu_{\tilde{B}}(y)$$

or

$$\nu_{\tilde{A}}(y) < \mu_{\tilde{B}}(y).$$

Lemma 2.3. [9] *For any two intuitionistic fuzzy sets $\tilde{A}$ and $\tilde{B}$ of Y,*

$$\sim (\tilde{A}q\tilde{B}) \Leftrightarrow \tilde{A} \subset \tilde{B}^c.$$

Definition 2.5. [8] *An intuitionistic fuzzy topology on a nonempty set Y is a family Γ of intuitionistic fuzzy sets in Y which satisfy the following axioms:*

O_1. $\tilde{0}, \tilde{1} \in \Gamma$,

O_2. $\tilde{A}_1 \cap \tilde{A}_2 \in \Gamma$ for any $\tilde{A}_1, \tilde{A}_2 \in \Gamma$,

O_3. $\cup \tilde{A}_\alpha \in \Gamma$ for arbitrary family $\{\tilde{A}_\alpha : \alpha \in \Lambda\} \in \Gamma$.

In this case the pair (Y, Γ) is called an intuitionistic fuzzy topological space and each intuitionistic fuzzy set in Γ, is known as an intuitionistic fuzzy open set in Y. The complement $\tilde{B}^c$ of an intuitionistic fuzzy open set $\tilde{B}$ is called an intuitionistic fuzzy closed set in Y.

Definition 2.6. [8] *Let (Y, Γ) be an intuitionistic fuzzy topological space and $\tilde{A}$ be an intuitionistic fuzzy set in Y. Then the interior and closure of $\tilde{A}$ are defined by*

$cl(\tilde{A})=\cap\{\tilde{K} : \tilde{K}$ *is an intuitionistic fuzzy closed set in Y and* $\{\tilde{A} \subseteq \tilde{K}\}$,
$Int(\tilde{A})=\cup\{\tilde{G} : \tilde{G}$ *is an intuitionistic fuzzy open set in Y and* $\{\tilde{G} \subseteq \tilde{A}\}$.

Lemma 2.4. [8] *For any intuitionistic fuzzy set $\tilde{A}$ in (Y, Γ) we have:*

(a) $\tilde{A}$ is an intuitionistic fuzzy closed set in $Y \Leftrightarrow Cl(\tilde{A}) = \tilde{A}$

(b) $\tilde{A}$ is an intuitionistic fuzzy open set in $Y \Leftrightarrow Int(\tilde{A}) = \tilde{A}$

(c) $Cl(\tilde{A}^c) = (Int(\tilde{A})^c)$

(d) $Int(\tilde{A}^c) = (Cl(\tilde{A})^c)$.

Definition 2.7. [25] *Let X and Y are two non empty sets. A function $F : (X, \tau) \to (Y, \Gamma)$ is called intuitionistic fuzzy multifunction if $F(x)$ is an intuitionistic fuzzy set in Y, $\forall x \in X$.*

Definition 2.8. [30] *Let $F : (X, \tau) \to (Y, \Gamma)$ is an intuitionistic fuzzy multifunction and A be a subset of X. Then $F(A) = \cup_{x \in A} F(x)$.*

Lemma 2.5. [30] *Let $F :(X, \tau) \to (Y, \Gamma)$ be an intuitionistic fuzzy multifunction. Then*

(a) $A \subseteq B \Rightarrow F(A) \subseteq F(B)$ for any subsets A and B of X.

(b) $F(A \cap B) \subseteq F(A) \cap F(B)$ for any subsets A and B of X.

(c) $F(\cup_{\alpha \in \Lambda} A_\alpha) = \cup\{F(A_\alpha) : \alpha \in \Lambda\}$ for any family of subsets $\{A_\alpha : \alpha \in \Lambda\}$ in X.

Definition 2.9. [25] *Let $F : (X, \tau) \to (Y, \Gamma)$ is an intuitionistic fuzzy multifunction. Then the upper inverse $F^+(\tilde{A})$ and lower inverse $F^-(\tilde{A})$ of an intuitionistic fuzzy set $\tilde{A}$ in Y are defined as follows:*

$F^+(\tilde{A}) = \{x \in X : F(x) \subseteq \tilde{A}\}$
$F^-(\tilde{A}) = \{x \in X : F(x) q \tilde{A}\}$.

Lemma 2.6. [30] *Let $F : (X, \tau) \to (Y, \Gamma)$ be an intuitionistic fuzzy multifunction and $\tilde{A}, \tilde{B}$ be intuitionistic fuzzy sets in Y. Then:*

(a) $F^{+}(\tilde{1}) = F^{-}(\tilde{1}) = X$

(b) $F^{+}(\tilde{A}) \subseteq F^{-}(\tilde{A})$

(c) $[F^{-}(\tilde{A})]^{c} = [F^{+}(\tilde{A})^{c}]$

(d) $[F^{+}(\tilde{A})]^{c} = [F^{-}(\tilde{A})^{c}]$

(e) If $\tilde{A} \subseteq \tilde{B}$ *, then* $F^{+}(\tilde{A}) \subseteq F^{+}(\tilde{B})$

(f) If $\tilde{A} \subseteq \tilde{B}$ *, then* $F^{-}(\tilde{A}) \subseteq F^{-}(\tilde{B})$.

Definition 2.10. [13] *A subset* $\tilde{A}$ *of an intuitionistic fuzzy topological space* (Y,Γ) *is called:*

(a) intuitionistic fuzzy Semi open if $\tilde{A} \subset Cl(Int(\tilde{A}))$.

(b) intuitionistic fuzzy Semi closed if its complement is semi open.

(c) intuitionistic fuzzy α*-open if* $\tilde{A} \subset Int(Cl(Int(\tilde{A})))$.

(d) intuitionistic fuzzy α*-closed if its complement is* α*-open.*

Definition 2.11. [7] *Let* (Y,Γ) *be an intuitionistic fuzzy topological space and* $\tilde{A}$ *be an intuitionistic fuzzy set in* Y*. Then the* α*-interior and* α*-closure of* $\tilde{A}$ *are defined by:*
$\alpha Cl(\tilde{A}) = \cap\{\tilde{K} : \tilde{K}$ *is an intuitionistic fuzzy* α*-closed set in* Y *and* $\tilde{A} \subseteq \tilde{K}\}$
$\alpha Int(\tilde{A}) = \cup\{\tilde{G} : \tilde{G}$ *is an intuitionistic fuzzy* α*-open set in* Y *and* $\tilde{G} \subseteq \tilde{A}\}$.

Definition 2.12. [25] *An intuitionistic fuzzy multifunction* $F : (X,\tau) \to (Y,\Gamma)$ *is said to be:*

(a) Intuitionistic fuzzy upper semi-continuous at a point $x_0 \in X$ *, if for any intuitionistic fuzzy open set* $\tilde{W} \subset Y$ *such that* $F(x_0) \subset \tilde{W}$ *there exists an open set* $U \subset X$ *containing* x_0 *such that* $F(U) \subset \tilde{W}$.

(b) Intuitionistic fuzzy lower semi-continuous at a point $x_0 \in X$ *, if for any intuitionistic fuzzy open set* $\tilde{W} \subset Y$ *such that* $F(x_0) q \tilde{W}$ *there exists an open set* $U \subset X$ *containing* x_0 *such that*

$$F(x) q \tilde{W}, \quad \forall x \in U.$$

(c) Intuitionistic fuzzy upper semi-continuous (intuitionistic fuzzy lower semi-continuous) if it is intuitionistic fuzzy upper semi-continuous (intuitionistic fuzzy lower semi-continuous) at each point of X.

3. Lower α-Irresolute Intuitionistic Fuzzy Multifunctions

Definition 3.1. *An intuitionistic fuzzy multifunction $F:(X,\tau)\to(Y,\Gamma)$ is said to be:*

(a) Intuitionistic fuzzy lower α-irresolute at a point $x_0 \in X$, if for any $\tilde{W} \in IF\alpha OY$ such that $F(x_0)q\tilde{W}$ there exists $U \in \alpha OX$ containing x_0 such that

$$F(x)q\tilde{W}, \forall x \in U.$$

(b) Intuitionistic fuzzy lower α-irresolute if it is intuitionistic fuzzy lower α-irresolute at each point of X.

Theorem 3.1. *Let $F:(X,\tau)\to(Y,\Gamma)$ be an intuitionistic fuzzy multifunction and let $x \in X$. Then the following statements are equivalent:*

(a) F is intuitionistic fuzzy lower α-irresolute at x.

(b) For each intuitionistic fuzzy α-open set $\tilde{B}$ of Y with $F(x)q\tilde{B}$, implies

$$x \in sCl(Int(F^{-}(\tilde{B}))).$$

(c) For any semi-open set U of X containing x and for any intuitionistic fuzzy α-open set $\tilde{B}$ of Y with $F(x)q\tilde{B}$, there exists a nonempty open set $V \subset U$ such that

$$F(v)q\tilde{B}, \quad \forall v \in V.$$

Proof. **(a)⇒(b)**. Let $x \in X$ and $\tilde{B}$ be any intuitionistic fuzzy α-open set of Y such that $F(x)q\tilde{B}$. Then by (a) $\exists U \in \alpha O(X)$ such that $x \in U$ and $F(v)q\tilde{B}$, $\forall v \in U$. Thus $x \in U \subset F^{-}(\tilde{B})$. Since $U \in \alpha O(X)$ by Lemma 2.2, $U \subset sCl(Int(U))$. Hence $x \in sCl(IntF^{-}(\tilde{B}))$.

(b)⇒(c). Let $\tilde{B}$ be any intuitionistic fuzzy α-open set of Y such that $F(x)q\tilde{B}$, then $x \in sCl(IntF^{-}(\tilde{B}))$. Let U be any semi-open set of X containing x. Then $U \cap Int(F^{-}(\tilde{B})) \neq \phi$ and $U \cap Int(F^{-}(\tilde{B}))$ is semi-open in X. Put

$$V = Int(U \cap Int(F^{-}(\tilde{B})),$$

then V is an open set of X, $V \subset U, V \neq \phi$ and $F(v)q\tilde{B}$, $\forall v \in V$.

(c)⇒(a). Let $\{U_x\}$ be the system of the semi−open sets in X containing x. For any semi−open set U of X such that $x \in U$ and any intuitionistic fuzzy α-open set $\tilde{B}$ of Y such that $F(x)q\tilde{B}$, there exists a nonempty open set $B_U \subset U$ such that $F(v)q\tilde{B}, \forall v \in B_U$. Let $W = \cup B_U : U \in \{U_x\}$, then W is open in X, $x \in sCl(W)$ and $F(v)q\tilde{B}, \forall v \in W$. Put $S = W \cup x$, then $W \subset S \subset sCl(W)$. Thus $S \in \alpha O(X)$, $x \in S$ and $F(v)q\tilde{B}, \forall v \in S$. Hence F is intuitionistic fuzzy lower α-irresolute at x. □

Definition 3.2. [26] *Let X and Y are two nonempty sets. A multifunction $F: X \to Y$ is called fuzzy multifunction if $F(x)$ is a fuzzy set in Y, $\forall x \in X$.*

Corollary 3.1. *Let F be a fuzzy multifunction form a topological space $(X, ⅂)$ into a fuzzy topological space (Y, σ) and let $x \in X$. Then the following statements are equivalent:*

(a) F is fuzzy lower α-irresolute at x.

(b) For each fuzzy α-open set B of Y with $F(x)qB$, implies

$$x \in sCl(Int(F^{-}B)).$$

(c) For any semi-open set U of X containing x and for any fuzzy α-open set B of Y with $F(x)qB$, there exists a nonempty open set $V \subset U$ such that

$$F(v)qB, \quad \forall v \in V.$$

Corollary 3.2. *For a multifunction $F : X \to Y$ and a point $x \in X$. Then the following statements are equivalent:*

(a) F is lower α-irresolute at x.

(b) For each α-open set B of Y with $F(x) \cap B \neq \phi$, implies $x \in sCl(Int(F^{-}B))$.

(c) For any semi-open set U of X containing x and for any α-open set B of Y with $F(x) \cap B \neq \phi$, there exists a nonempty open set $V \subset U$ such that $F(x) \cap B \neq \phi, \forall v \in V$.

Theorem 3.2. *Let $F : (X, \tau) \to (Y, \Gamma)$ be an intuitionistic fuzzy multifunction. Then the following statements are equivalent:*

(a) F is intuitionistic fuzzy lower α-irresolute.

(b) $F^{-}(\tilde{G}) \in \alpha O(X)$, for every intuitionistic fuzzy α-open set $\tilde{G}$ of Y.

(c) $F^{+}(\tilde{V}) \in \alpha C(X)$, for every intuitionistic fuzzy α-closed set $\tilde{V}$ of Y.

(d) $sInt(Cl(F^{+}(\tilde{B}))) \subset F^{+}(\alpha Cl(\tilde{B}))$, for each intuitionistic fuzzy set $\tilde{B}$ of Y.

(e) $F(sInt(Cl(A))) \subset \alpha Cl(F(A))$, for each subset A of X.

(f) $F(\alpha Cl(A)) \subset \alpha Cl(F(A))$, for each subset A of X,

(g) $\alpha Cl(F^{+}(\tilde{B})) \subset F^{+}(\alpha Cl(\tilde{B}))$, for each intuitionistic fuzzy set $\tilde{B}$ of Y.

(h) $F(Cl(Int(Cl(A)))) \subset \alpha Cl(F(A))$, for each subset A of X.

Proof. **(a)⇒(b)**. Let $\tilde{G}$ be any intuitionistic fuzzy α-open set of Y and $x \in F^{-}(\tilde{G})$, so $F(x)q\tilde{G}$, since F is Intuitionistic fuzzy lower α-irresolute, by Theorem 3.1 it follows that $x \in sCl(Int(F^{-}(\tilde{G})))$. As x is chosen arbitrarily in $F^{-}(\tilde{G})$, we have $F^{-}(\tilde{G}) \subset sCl(IntF^{-}(\tilde{G}))$ and thus $F^{-}(\tilde{G}) \in \alpha O(X)$.

(b)⇒(a). Let x be arbitrarily chosen in X and $\tilde{G}$ be any intuitionistic fuzzy α-open set of Y such that $F(x)q\tilde{G}$, so $x \in F^{-}(\tilde{G})$. By hypothesis $F^{-}(\tilde{G}) \in \alpha O(X)$, we have

$x \in F^{-}(\tilde{G}) \subset sCl(Int(F^{-}(\tilde{G})))$ and thus F is intuitionistic fuzzy lower α-irresolute at x according to Theorem 3.1. As x was arbitrarily chosen, F is intuitionistic fuzzy lower α-irresolute.

(b)⇔(c). Obvious.

(c)⇒(d). Let $\tilde{B}$ be any arbitrary intuitionistic fuzzy set of Y. Since $\alpha Cl(\tilde{B})$ is intuitionistic fuzzy α-closed set in Y by hypothesis, $F^{+}(Cl(\tilde{B})) \in \alpha C(X)$. Hence by lemma 2.1, we obtain

$$F^{+}(\alpha Cl(\tilde{B})) \supset sInt(Cl(F^{+}(Cl(\tilde{B})))) \supset sInt(Cl(F^{+}(\tilde{B}))).$$

(d)⇒(e). Suppose that (d) holds, and let A be an arbitrary subset of X. Let us put $\tilde{B} = F(A)$, then $A \subset F^{+}(\tilde{B})$. Therefore, by hypothesis, we have

$$sInt(Cl(A)) \subset sInt(Cl(F^{+}(\tilde{B}))) \subset F^{+}(\alpha Cl(\tilde{B})).$$

Therefore, $F(sInt(Cl(A))) \subset F(F^{+}(\alpha Cl(\tilde{B}))) \subset \alpha Cl(\tilde{B}) = \alpha Cl(F(A))$.

(e)⇒(c). Suppose that (e) holds, and let $\tilde{B}$ be any intuitionistic fuzzy α-closed set of Y. Put $A = F^{+}(\tilde{B})$, then $F(A) \subset \tilde{B}$. Therefore, by hypothesis, we have

$$F(sInt(Cl(A))) \subset \alpha Cl(F(A)) \subset \alpha Cl(\tilde{B}) = \tilde{B}$$

and

$$F^{+}(F(sInt(Cl(A)))) \subset F^{+}(\tilde{B}).$$

Since we always have

$$F^{+}(F(sInt(Cl(A)))) \supset sInt(Cl(A)),$$

one can verify

$$F^{+}(\tilde{B}) \supset sInt(Cl(F^{+}(\tilde{B}))).$$

Hence by lemma, 2.1, $F^{+}(\tilde{B}) \in \alpha C(X)$.

(c)⇒(f). Since $A \subset F^{+}(F(A))$, we have $A \subset F^{+}(Cl(F(A)))$. Now $\alpha Cl(F(A))$ is an intuitionistic fuzzy α-closed set in Y and so by hypothesis

$$F^{+}(Cl(F(A))) \in \alpha C(X).$$

Thus

$$\alpha Cl(A) \subset F^{+}(\alpha Cl(F(A))).$$

Consequently,

$$F(\alpha Cl(A)) \subset F(F^{+}(\alpha Cl(F(A)))) \subset \alpha Cl(F(A)).$$

(f)⇒(c). Let $\tilde{B}$ be any intuitionistic fuzzy α-closed set of Y. Replacing A by $F^{+}(\tilde{B})$ we get by(f),

$$F(\alpha Cl(F^{+}(\tilde{B}))) \subset \alpha Cl(F(F^{+}(\tilde{B}))) \subset \alpha Cl(\tilde{B}) = \tilde{B}.$$

Consequently, $\alpha Cl(F^{+}(\tilde{B})) \subset F^{+}(\tilde{B})$. But $F^{+}(\tilde{B}) \subset \alpha Cl(F^{+}(\tilde{B}))$. And so, $\alpha Cl(F^{+}(\tilde{B})) = F^{+}(\tilde{B})$. Thus $F^{+}(\tilde{B}) \in \alpha C(X)$.

(f)⇒(g). Let $\tilde{B}$ be any intuitionistic fuzzy set of Y. Replacing A by $F^+(\tilde{B})$ we get by (f),

$$F(\alpha Cl(F^+(\tilde{B}))) \subset \alpha Cl(F(F^+(\tilde{B}))) \subset \alpha Cl(\tilde{B}).$$

Thus $\alpha Cl(F^+(\tilde{B})) \subset F^+(\alpha Cl(\tilde{B}))$.

(g)⇒(f). Replacing $\tilde{B}$ by $F(A)$, where A is a subset of X, we get by (g),

$$\alpha Cl(A) \subset \alpha Cl(F^+(F(A))) = \alpha Cl(F^+(\tilde{B})) = F^+(\alpha Cl(\tilde{B})) = F^+(\alpha Cl(F(A))).$$

Thus

$$F(\alpha Cl(A)) \subset F(F^+(\alpha Cl(F(A))) \subset \alpha Cl(F(A)).$$

(e)⇒(h). Follows from by Lemma 2.6.

(h)⇒(a). Let $x \in X$ and $\tilde{V}$ be any intuitionistic fuzzy α-open set in Y such that $F(x)q\tilde{V}$. Then $x \in F^-(\tilde{V})$. We shall show that $F^-(\tilde{V}) \in \alpha O(X)$. By the hypothesis, we have

$$F(Cl(Int(Cl(F^+(\tilde{V}^c))))) \subset \alpha Cl(F(F^+(\tilde{V}^c))) \subset (\tilde{V}^c),$$

which implies

$$Cl(Int(Cl(F^+(\tilde{V}^c))))) \subset F^+(\tilde{V}^c) \subset (F^-(\tilde{V}))^c.$$

Therefore, we obtain $F^-(\tilde{V}) \subset Int(Cl(Int(F^-(\tilde{V}))))$. Hence $F^-(\tilde{V}) \in \alpha O(X)$. Put $U = F^-(\tilde{V})$. Then $x \in U \in \alpha O(X)$ and $F(u)q\tilde{V}$ for every $u \in U$ thus F is intuitionistic fuzzy lower α-irresolute. □

Corollary 3.3. *Let F be a fuzzy multifunction form a topological space $(X, \urcorner)$ into a fuzzy topological space (Y, σ). Then the following statements are equivalent:*

(a) F is fuzzy lower α-irresolute.

(b) $F^-(G) \in \alpha O(X)$, for every fuzzy α-open set G of Y.

(c) $F^+(V) \in \alpha C(X)$, for every fuzzy α-closed set V of Y.

(d) $sInt(Cl(F^+(B))) \subset F^+(\alpha Cl(B))$, for each fuzzy set B of Y.

(e) $F(sInt(Cl(A))) \subset \alpha Cl(F(A))$, for each subset A of X.

(f) $F(\alpha Cl(A)) \subset \alpha Cl(F(A))$, for each subset A of X,

(g) $\alpha Cl(F^+(B)) \subset F^+(\alpha Cl(B))$, for each fuzzy set B of Y.

(h) $F(Cl(Int(Cl(A)))) \subset \alpha Cl(F(A))$, for each subset A of X.

Corollary 3.4. *Let F be a multifunction from a topological space $(X, \urcorner)$ into another topological space (Y, ζ). Then the following statements are equivalent:*

(a) F is lower α-irresolute.

(b) $F^-(G) \in \alpha O(X)$, for every α-open set G of Y.

(c) $F^+(V) \in \alpha C(X)$, *for every* α*-closed set* V *of* Y.

(d) $sInt(Cl(F^+(B))) \subset F^+(\alpha Cl(B))$, *for each set* B *of* Y.

(e) $F(sInt(Cl(A))) \subset \alpha Cl(F(A))$, *for each subset* A *of* X.

(f) $F(\alpha Cl(A)) \subset \alpha Cl(F(A))$, *for each subset* A *of* X,

(g) $\alpha Cl(F^+(B)) \subset F^+(\alpha Cl(B))$, *for each set* B *of* Y.

(h) $F(Cl(Int(Cl(A)))) \subset \alpha Cl(F(A))$, *for each subset* A *of* X.

4. Upper α-Irresolute Intuitionistic Fuzzy Multifunctions

Definition 4.1. *An intuitionistic fuzzy multifunction* $F : (X,\tau) \to (Y,\Gamma)$ *is said to be:*

(a) Intuitionistic fuzzy upper α*-irresolute at a point* $x_0 \in X$ *, if for any intuitionistic fuzzy* α*-open set* $\tilde{W}$ *of* Y*, such that* $F(x_0) \subset \tilde{W}$ *there exists* $U \in \alpha OX$ *containing* x_0 *such that* $F(U) \subset \tilde{W}$.

(b) Intuitionistic fuzzy upper α*-irresolute if it has this property at each point of* X.

Theorem 4.1. *Let* $F : (X,\tau) \to (Y,\Gamma)$ *be an intuitionistic fuzzy multifunction and let* $x \in X$*. Then the following statements are equivalent:*

(a) F *is intuitionistic fuzzy Upper* α*-irresolute at* x.

(b) For each intuitionistic fuzzy α*-open set* $\tilde{G}$ *of* Y *with* $F(x) \subset \tilde{G}$*, there result the relation* $x \in sCl(Int(F^-(\tilde{G})))$.

(c) For any semi-open set U *of* X *containing* x *and for any intuitionistic fuzzy* α*-open set* $\tilde{G}$ *of* Y *with* $F(x) \subset \tilde{G}$*, there exists a nonempty open set* $V \subset U$ *such that* $F(V) \subset \tilde{G}$.

Proof. **(a)⇒(b)**. Let $x \in X$ and $\tilde{G}$ be any intuitionistic fuzzy α-open set of Y such that $F(x) \subset \tilde{G}$, there is a $U \in \alpha O(X)$ such that $x \in U$ and $F(v) \subset \tilde{G}, \forall v \in U$. Thus $x \in U \subset F^+(\tilde{G})$. Since

$$U \in \alpha O(X), U \subset sCl(Int(U)) \subset sCl(Int(F^+(\tilde{G}))).$$

Hence, $x \in sCl(Int(F^+(\tilde{G})))$.

(b)⇒(c). Let $\tilde{G}$ be any intuitionistic fuzzy α-open set of Y such that $F(x) \subset \tilde{G}$, then $x \in sCl(Int(F^+(\tilde{G})))$. Let $U \subset X$ be any semi-open set such that $x \in U$, then $U \cap Int(F^+(\tilde{G})) \neq \phi$. Put $V = Int(U \cap Int(F^+(\tilde{G})))$, then V is an open set in X, $V \subset U$, $V \neq \phi$ and $F(V) \subset \tilde{G}$.

(c)⇒(a). Let $\{U_x\}$ be the system of the semi-open sets in X containing x. For any semi-open set $U \subset X$ such that $x \in U$ and $\tilde{G}$ be any intuitionistic fuzzy α-open set of

Y such that $F(x) \subset \tilde{G}$, there exists a nonempty open set $G_U \subset U$ such that $F(G_U) \subset \tilde{G}$. Let $W = \bigcup G_U$:U$\in \{U_x\}$. Then W is open, $x \in sCl(W)$ and $F(w) \subset \tilde{G}, \forall w \in W$. Put $S = W \cup x$, then $W \subset S \subset sCl(W)$. Thus $S \in \alpha O(X)$, $x \in S$ and $F(w) \subset \tilde{G}, \forall w \in S$. Hence F is intuitionistic fuzzy upper α-irresolute at x. □

Corollary 4.1. *Let F be a fuzzy multifunction form a topological space $(X, \urcorner)$ into a fuzzy topological space (Y, σ) and let $x \in X$. Then the following statements are equivalent:*

(a) F is fuzzy upper α-irresolute at x.

(b) For each fuzzy α-open set G of Y with $F(x) \subset G$, there exists the relation $x \in sCl(Int(F^{-}(G)))$.

(c) For any semi-open set U of X containing x and for any fuzzy α-open set G of Y with $F(x) \subset G$, there exists a nonempty open set $V \subset U$ such that $F(V) \subset G$.

Corollary 4.2. *Let F be a multifunction from a topological space $(X, \urcorner)$ into another topological space (Y, ζ) and let $x \in X$. Then the following statements are equivalent:*

(a) F is Upper α-irresolute at x.

(b) For each α-open set G of Y with $F(x) \subset G$, there exists the relation $x \in sCl(Int(F^{-}(G)))$.

(c) For any semi-open set U of X containing x and for any α-open set G of Y with $F(x) \subset G$, there exists a nonempty open set $V \subset U$ such that $F(V) \subset G$.

Definition 4.2. *Let $\tilde{A}$ be an intuitionistic fuzzy set of an intuitionistic fuzzy topological space (Y, Γ). Then $\tilde{V}$ is said to be an α-neighbourhood of $\tilde{A}$ in Y if there exists an intuitionistic fuzzy α-open set $\tilde{U} \subset Y$ such that $\tilde{A} \subset \tilde{U} \subset \tilde{V}$.*

Theorem 4.2. *For an intuitionistic fuzzy multifunction $F : (X, \tau) \to (Y, \Gamma)$ the following statements are equivalent:*

(a) F is intuitionistic fuzzy upper α-irresolute.

(b) $F^{+}(\tilde{G}) \in \alpha O(X)$, for every intuitionistic fuzzy α-open set $\tilde{G}$ of Y.

(c) $F^{-}(\tilde{B}) \in \alpha C(X)$, for each intuitionistic fuzzy α-closed set $\tilde{B}$ of Y.

(d) For each point $x \in X$ and for each α-neighborhood $\tilde{V}$ of $F(x)$ in Y, $F^{+}(\tilde{V})$ is an α-neighborhood of x.

(e) For each point $x \in X$ and for each α-neighborhood $\tilde{V}$ of $F(x)$ in Y, there is an α-neighborhood U of x such that $F(U) \subset \tilde{V}$.

(f) $\alpha Cl(F^{-}(\tilde{B})) \subset F^{-}(\alpha Cl(\tilde{B}))$ for each intuitionistic fuzzy set $\tilde{B}$ of Y.

(g) $sInt(Cl(F^{-}(\tilde{B}))) \subset F^{-}(\alpha Cl(\tilde{B}))$ for any intuitionistic fuzzy set $\tilde{B}$ of Y.

Proof. **(a)⇒(b)**. Let $\tilde{V}$ be any intuitionistic fuzzy α-open set of Y and $x \in F^{+}(\tilde{V})$. By Theorem4.1, $x \in sCl(IntF^{+}(\tilde{B}))$. Therefore, we obtain

$$F^{+}(\tilde{V}) \subset sCl(IntF^{+}(\tilde{B})).$$

Hence by Lemma 2.2, $F^{+}(\tilde{V}) \in \alpha O(X)$.

(b)⇒(a). Let x be arbitrarily chosen in X and $\tilde{G}$ be any intuitionistic fuzzy α-open set of Y such that $F(x) \subset \tilde{G}$, so $x \in F^{+}(\tilde{G})$. By hypothesis $F^{+}(\tilde{G}) \in \alpha O(X)$, we have

$$x \in F^{+}(\tilde{G}) \subset sCl(Int(F^{+}(\tilde{G})))$$

and thus F is intuitionistic fuzzy upper α-irresolute at x according to Theorem 4.1. As x was arbitrarily chosen, F is intuitionistic fuzzy upper α-irresolute.

(b)⇒(c). This follows from Lemma 2.6 that $[F^{-}(\tilde{A})]^{c} = [F^{+}(\tilde{A})^{c}]$.

(c)⇒(f). Let $\tilde{B}$ be any intuitionistic fuzzy α-open set of Y. Then by (c), $F^{-}(\alpha Cl(\tilde{B}))$ is an α-closed set in X. Thus by Lemma 2.1 we have

$$\begin{aligned} F^{-}(\alpha Cl(\tilde{B})) &\supset sInt(Cl(F^{-}(Cl(\tilde{B})))) \supset sInt(Cl(F^{-}(\tilde{B}))) \\ &\supset F^{-}(\tilde{B}) \cup sInt(Cl(F^{-}(\tilde{B}))) \supset \alpha Cl(F^{-}(\tilde{B})). \end{aligned}$$

(f)⇒(g). Let $\tilde{B}$ be any intuitionistic fuzzy α-open set of Y. By Lemma 2.1, we have

$$\alpha Cl(F^{-}(\tilde{B})) = F^{-}(\tilde{B}) \cup sInt(Cl(F^{-}(\tilde{B}))) \subset F^{-}(\alpha Cl(\tilde{B})).$$

(g)⇒(c). Let $\tilde{B}$ be any intuitionistic fuzzy α-closed set of Y. Then by (g) we have,

$$sInt(Cl(F^{-}(\tilde{B}))) \subset F^{-}(\tilde{B}) \cup sInt(Cl(F^{-}(\tilde{B}))) \subset F^{-}(\alpha Cl(\tilde{B})) = F^{-}(\tilde{B}).$$

Hence By Lemma 2.1, $F^{-}(\tilde{B}) \in \alpha C(X)$.

(b)⇒(d). Let $x \in X$ and $\tilde{V}$ be an α-neighborhood of $F(x)$ in Y. Then there is an intuitionistic fuzzy α-open set $\tilde{G}$ of Y such that $F(x) \subset \tilde{G} \subset \tilde{V}$. Hence, $x \in F^{+}(\tilde{G}) \subset F^{+}(\tilde{V})$. Now by hypothesis $F^{+}(\tilde{G}) \in \alpha O(X)$, and thus $F^{+}(\tilde{V})$ is an α-neighborhood of x.

(d)⇒(e). Let $x \in X$ and $\tilde{V}$ be an α-neighborhood of $F(x)$ in Y. Put $U = F^{+}(\tilde{V})$. Then U is an α-neighborhood of x and $F(U) \subset \tilde{V}$.

(e)⇒(a). Let $x \in X$ and $\tilde{V}$ be an intuitionistic fuzzy set in Y such that $F(x) \subset \tilde{V}$. $\tilde{V}$, being an intuitionistic fuzzy α-open set in Y , is an α-neighborhood of $F(x)$ and according to the hypothesis there is an α-neighborhood U of x such that $F(U) \subset \tilde{V}$. Therefore there is $A \in \alpha O(X)$ such that $x \in A \subset U$ and hence $F(A) \subset F(U) \subset \tilde{V}$. Hence F is intuitionistic fuzzy upper α-irresolute at x. □

Corollary 4.3. *Let F be a fuzzy multifunction from a topological space $(X, \urcorner)$ into a fuzzy topological space (Y, σ) and let $x \in X$. Then the following statements are equivalent:*

(a) F is fuzzy upper α-irresolute.

(b) $F^{+}(G) \in \alpha O(X)$*, for every fuzzy* α*-open set* G *of* Y.

(c) $F^{-}(B) \in \alpha C(X)$*, for each fuzzy* α*-closed set* B *of* Y.

(d) For each point $x \in X$ *and for each* α*-neighborhood* V *of* $F(x)$ *in* Y, $F^{+}(V)$ *is an* α*-neighborhood of* x.

(e) For each point $x \in X$ *and for each* α*-neighborhood* V *of* $F(x)$ *in* Y*, there is an* α*-neighborhood* U *of* x *such that* $F(U) \subset V$.

(f) $\alpha Cl(F^{-}(B)) \subset F^{-}(\alpha Cl(B))$ *for each fuzzy set* B *of* Y.

(g) $sInt(Cl(F^{-}(B))) \subset F^{-}(\alpha Cl(B))$ *for any fuzzy set* B *of* Y.

Corollary 4.4. *: Let* F *be a multifunction form a topological space* $(X, \urcorner)$ *into another topological space* (Y, ζ) *and let* $x \in X$*. Then the following statements are equivalent:*

(a) F *is upper* α*-irresolute.*

(b) $F^{+}(G) \in \alpha O(X)$*, for every* α*-open set* G *of* Y.

(c) $F^{-}(B) \in \alpha C(X)$*, for each* α*-closed set* B *of* Y.

(d) For each point $x \in X$ *and for each* α*-neighborhood* V *of* $F(x)$ *in* Y, $F^{+}(V)$ *is an* α*-neighborhood of* x.

(e) For each point $x \in X$ *and for each* α*-neighborhood* V *of* $F(x)$ *in* Y*, there is an* α*-neighborhood* U *of* x *such that* $F(U) \subset V$.

(f) $\alpha Cl(F^{-}(B)) \subset F^{-}(\alpha Cl(B))$ *for each set* B *of* Y.

(g) $sInt(Cl(F^{-}(B))) \subset F^{-}(\alpha Cl(B))$ *for any set* B *of* Y.

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