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# SOME NEW TYPES OF CONTINUITY IN ASYMMETRIC METRIC SPACES

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**Abstract.** Using the notions of forward and backward arithmetic convergence in asymmetric metric spaces, we have defined arithmetic $ff$-continuity and arithmetic $fb$-continuity and prove some interesting results. Moreover, we have introduced the concepts of forward and backward arithmetic compactness and obtained the related results in the setting of asymmetric metric space.
**Keywords**: asymmetric metric spaces; forward and backward arithmetic compactness; forward and backward arithmetic convergence; arithmetic $ff$-continuity.

## 1. Introduction and Preliminaries

In 1931, Wilson [18] first introduced asymmetric metric spaces as quasi-metric spaces, and afterwards they were studied by many other authors (see [1, 14, 15, 16]). An asymmetric metric space is a generalization of a metric space but the symmetry axiom is eliminated in the definition of metric spaces. We can come up with some troubles in several classical statements of symmetric analysis without the symmetry property in the definition of such spaces. In asymmetric metric spaces, some notions such as convergence, completeness and compactness are different from the metric case. There are two notions for each of them, namely forward and backward ones, since we have two topologies which are the forward topology and the backward topology in the same space (see [13]). Collins and Zimmer [10] studied these notions in the asymmetric context.

An example that asymmetric metrics are common in real life is taxicab geometry topology including one-way streets, where a path from point $A$ to point $B$ contains a different set of streets than a path from $B$ to $A$. Also, the examples of the latest applications of asymmetric metric spaces in the field of pure and applied

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mathematics and material science are as in [8]. In [9], Cobzas gave the basic results on asymmetric normed spaces.

Ruckle [17] introduced the notion of*arithmetic convergence* as a sequence $x = (x_k)$ defined on $\mathbb{N}$, and it is said to be arithmetic convergent if for each $\varepsilon > 0$ there is an integer $n$ such that for every integer $m$ we have $|x_m - x_{<m,n>}| < \varepsilon$. Here and henceforth, $< m, n >$ denotes the greatest common divisor of $m$ and $n$. Çakalli [4] gave another definition of arithmetic convergence of a sequence $(x_k)$ as a sequence $x = (x_k)$ is said to be arithmetically convergent if for each $\varepsilon > 0$ there is an integer $n_0$ such that $|x_m - x_{<m,n>}| < \varepsilon$ for every integers $m, n$ satisfying $< m, n > \geq n_0$. Throughout the article, we follow the definition given by Çakalli in his corrigendum to the paper [4]. For more details on arithmetic convergence and arithmetic continuity, we refer to [4, 19, 20, 21, 22, 23]. For different types of continuity and $b$- metric spaces, we refer to [2, 3, 5, 6, 7, 11, 12].

In this article, we will first introduce the concepts of forward and backward arithmetic convergence and using these notions we will define forward and backward arithmetic continuity in asymmetric metric spaces and establish some interesting results. In the last section, we will introduce forward and backward arithmetic compactness and obtain related results.

## 2. Asymmetric Metric Spaces

Let us recall some definitions and results on asymmetric metric spaces which were given in [10].

**Definition 2.1.** A function $d : X \times X \to R$ is an asymmetric metric and $(X, d)$ is an asymmetric metric space if

(i) $d(x, y) \geq 0$ and $d(x, y) = 0$ holds if and only if $x = y$ for every $x, y \in X$.

(ii) $d(x, z) \leq d(x, y) + d(y, z)$; for every $x, y, z \in X$.

**Definition 2.2.** The *forward topology* $\tau_+$ induced by $d$ is the topology generated by the forward open balls $B^+(x, \varepsilon) = \{y \in X : d(x, y) < \varepsilon\}$ for $x \in X; \varepsilon > 0$.

Likewise, the *backward topology* $\tau_-$ induced by $d$ is the topology generated by the backward open balls $B^-(x, \varepsilon) = \{y \in X : d(y, x) < \varepsilon\}$ for $x \in X; \varepsilon > 0$.

**Definition 2.3.** A set $S \subset X$ is *forward bounded* (resp. *backward bounded*), if there exists $x \in X$ and $\varepsilon > 0$ such that $S \subset B^+(x, \varepsilon)$ (resp. $S \subset B^-(x, \varepsilon)$).

**Definition 2.4.** A sequence $(x_n)$ is said to be *forward convergent* to $x \in X$ (*backward convergent* to $x \in X$) if and only if

$$\lim_{n\to\infty} d(x, x_n) = 0 \ (\lim_{n\to\infty} d(x_n, x) = 0)$$

and is denoted by $x_n \xrightarrow{f} x$ ($x_n \xrightarrow{b} x$).

**Definition 2.5.** A sequence $(x_n)$ in an asymmetric metric space $(X, d)$ is *forward Cauchy (backward Cauchy)* if for each $\varepsilon > 0$ there exists a $N \in \mathbb{N}$ such that for $k \geqslant n \geqslant N$; $d(x_n, x_k) < \varepsilon$ ($d(x_k, x_n) < \varepsilon$) holds.

**Definition 2.6.** [Sequential definition of continuity] Let $(X, d_X)$ and $(Y, d_Y)$ be asymmetric metric spaces. A function $f : X \to Y$ is $ff-$*continuous* at $x \in X$ if and only if whenever $x_n \xrightarrow{f} x$ in $(X, d_X)$ we have $f(x_n) \xrightarrow{f} f(x)$ in $(Y, d_Y)$.

The statement holds analogously for the other types. Note that the forward uniform continuity is same as the backward uniform continuity.

**Definition 2.7.** A set $S \subset X$ is

(i) *forward compact* if every open cover of $S$ in the forward topology has a finite subcover.

(ii) *forward relatively compact* if $\overline{S}$ is forward compact, where $\overline{S}$ denotes the closure of $S$ in the forward topology.

(iii) *forward sequentially compact* if every sequence in $X$ contains a forward convergent subsequence.

(iv) *forward complete* if every forward Cauchy sequence is forward convergent.

**Definition 2.8.** Let $(f_n)$ be a function sequence and $f$ be a function from $X$ to $Y$. We say that the sequence $(f_n)$ is *forward convergent uniformly (backward convergent uniformly)* to limit $f$ if for every $\varepsilon > 0$ there exists a positive number $N$ such that for all $x \in X$ and all $n \geq N$ we have $d(f(x), f_n(x)) < \varepsilon$ ($d(f_n(x), f(x)) < \varepsilon$).

## 3. Arithmetic Continuity in Asymmetric Metric Spaces

In this section, we introduce the concepts of forward and backward arithmetic convergence and forward and backward arithmetic continuity in asymmetric metric spaces and prove some results using these notions.

**Definition 3.1.** A sequence $x = (x_k)$ is called forward arithmetic convergent (resp. backward arithmetic convergent) in an asymmetric metric space $(X, d)$ if for each $\varepsilon > 0$ there is an integer $N$ such that $d(x_{<m,n>}, x_m) < \varepsilon$ (resp. $d(x_m, x_{<m,n>}) < \varepsilon$), for every integers $m, n$ satisfying $<m, n> \geq N$. We shall denote it by writing $x_m \xrightarrow{af} x_{<m,n>}$ (resp. $x_m \xrightarrow{ab} x_{<m,n>}$).

**Definition 3.2.** Let $(X, d_X)$ and $(Y, d_Y)$ be two asymmetric metric spaces. A function $f : X \to Y$ is arithmetic $ff-$ *continuous* (respectively arithmetic $fb-$*continuous*), iff it transforms forward arithmetic convergent sequence in $(X, d_X)$ to forward arithmetic convergent sequence (respectively backward arithmetic convergent sequence) in $(Y, d_Y)$.

**Theorem 3.1.** *Let $(X, d_X)$ and $(Y, d_Y)$ be asymmetric metric spaces. If $f : X \to Y$ is uniformly continuous then it is arithmetic $ff$-continuous.*

*Proof.* Let $f : X \to Y$ be uniformly continuous and $(x_n)$ be any forward arithmetic convergence sequence in $X$. Since $f$ is uniformly continuous, for a given $\varepsilon > 0$ there exists $\delta > 0$ such that for every $x, y \in X$ with $d_X(x, y) < \delta$, $d_Y(f(x), f(y)) < \varepsilon$. Again, the sequence $(x_n)$ is forward arithmetic convergent in $X$, hence for the same $\delta > 0$ there exists a positive integer $m_0$ such that for all integers $m, n$ satisfying $< m, n >\geq 0$,

$$\begin{aligned} d_X(x_{<n,m>}, x_n) < \delta \text{ for each } n \quad &\Rightarrow \quad d_Y(f(x_{<n,m>}), f(x_n)) < \varepsilon \text{ for each } n \\ &\Rightarrow \quad \text{the sequence } (f(x_n)) \text{ is forward arithmetic convergent} \\ &\Rightarrow \quad \text{the function } f \text{ is arithmetic } \textit{ff-continuous}. \end{aligned}$$

This completes the proof. □

**Definition 3.3.** A sequence of functions $(f_n)$ from an asymmetric metric space $(X, d_X)$ to an asymmetric metric space $(Y, d_Y)$ is said to be forward arithmetic convergent (resp. backward arithmetic convergent) if for any $\varepsilon > 0$ and $\forall\ x \in X$ there exists a positive integer $m_0$ such that for all integers $m, n$ satisfying $< m, n >\geq 0$,

$$d_Y(f_{<n,m>}(x), f_n(x)) < \varepsilon \ (\textit{resp. } d_Y(f_n(x), f_{<n,m>}(x)) < \varepsilon).$$

**Theorem 3.2.** *If $(f_n)$ be a sequence of forward arithmetic convergent functions from an asymmetric metric space $(X, d_X)$ to an asymmetric metric space $(Y, d_Y)$ and $x_o$ is a point in $X$ such that*

$$\lim_{x \to x_o} f_n(x) = y_n,\ n = 1, 2, 3 \dots$$

*then $(y_n)$ is also forward arithmetic convergent.*

*Proof.* Since the sequence $(f_n)$ is forward arithmetic convergent, therefore, for $\varepsilon > 0$ and a positive integer $m_0$ such that for all integers $m, n$ satisfying $< m, n >\geq 0$

$$d_Y(f_{<n,m>}(x), f_n(x)) < \varepsilon\ \forall x \in X.$$

Keeping $n, m$ fixed and letting $x \to x_o$,

$$d_Y(y_{<n,m>}, y_n) < \varepsilon.$$

Hence, the sequence $(y_n)$ is forward arithmetic convergent. □

**Remark 3.1.** *The same result can be written for backward arithmetic convergence.*

**Theorem 3.3.** *If $(f_n)$ is a sequence of arithmetic $ff-$continuous functions from asymmetric metric space $(X, d_X)$ to asymmetric metric space $(Y, d_Y)$ with forward convergence equivalent to backward convergence in $Y$ and $(f_n)$ forward converges uniformly to a function $f$, then $f$ is arithmetic $ff$-continuous.*

*Proof.* Let $\varepsilon > 0$ and $(x_n)$ be any forward arithmetic convergent sequence in $X$. Since $f_n \xrightarrow{f} f$ uniformly, we can choose $N_1 \in \mathbb{N}$ so that $d_Y(f(x), f_n(x)) < \frac{\varepsilon}{3}$ for all $n \geq N_1$ and $x \in X$. Now, in particular, $f_n(x_{<n,m>}) \xrightarrow{f} f(x_{<n,m>})$ and so $f_n(x_{<n,m>}) \xrightarrow{b} f(x_{<n,m>})$. Thus, we can find $N_2 \in \mathbb{N}$ so that $d_Y(f_n(x_{<n,m>}), f(x_{<n,m>})) < \frac{\varepsilon}{3}$ for all $n \geq N_2$. Let $N = \max\{N_1, N_2\}$. Further, $(f_n)$ is given to be a sequence of arithmetic $ff$-continuous functions. In particular, $f_N$ is arithmetic $ff$-continuous function, and thus arithmetic $fb$-continuous by equivalence of forward and backward convergence in $Y$. So there exists an integer $n_0$, greater than $N$ and $\delta > 0$ such that

$$d_Y(f_N(x_n), f_N(x_{<n,m>})) < \frac{\varepsilon}{3} \text{ for } d_X(x_{<n,m>}, x_n) < \delta,$$

for all integers $m, n$ satisfying $< m, n >\geq n_0$. Consequently, whenever $d_X(x_{<n,m>}, x_n) < \delta$ and $< m, n >\geq n_0$, we have

$$\begin{aligned} d_Y(f(x_n), f(x_{<n,m>})) &\leq d_Y(f(x_n), f_N(x_n)) + d_Y(f_N(x_n), f_N(x_{<n,m>})) \\ &\quad + d_Y(f_N(x_{<n,m>}), f(x_{<n,m>})) \\ &< \frac{\varepsilon}{3} + \frac{\varepsilon}{3} + \frac{\varepsilon}{3} = \varepsilon. \end{aligned}$$

Therefore $f$ is arithmetic $fb$-continuous and by equivalence of convergence it is also arithmetic $ff$-continuous. □

**Theorem 3.4.** *Let $(X, d_X)$ and $(Y, d_Y)$ be two asymmetric metric spaces. Then the set of all arithmetic $ff$-continuous functions from $X$ to $Y$, with forward convergence equivalent to backward convergence in $Y$, is a closed subset of all continuous functions from $X$ to $Y$ i.e. $\mathbb{A}^{ff}(X, Y) = \overline{\mathbb{A}^{ff}(X, Y)}$ where $\mathbb{A}^{ff}(X, Y)$ is the set of all arithmetic $ff$-continuous functions from $X$ to $Y$ and $\overline{\mathbb{A}^{ff}(X, Y)}$ denotes the closure of $\mathbb{A}^{ff}(X, Y)$.*

*Proof.* Let $f \in \overline{\mathbb{A}^{ff}(X, Y)}$. Then there exists a sequence of points in $\mathbb{A}^{ff}(X, Y)$ such that $f_n \xrightarrow{f} f$ as $n \to \infty$. Let $\varepsilon > 0$ and $(x_n)$ be any forward arithmetic convergent sequence in $X$. Since $f_n \xrightarrow{f} f$ uniformly, we can choose $N_1 \in \mathbb{N}$ so that $d_Y(f(x), f_n(x)) < \frac{\varepsilon}{3}$ for all $n \geq N_1$ and $x \in X$. In particular, $f_n(x_{<n,m>}) \xrightarrow{f} f(x_{<n,m>})$ and so $f_n(x_{<n,m>}) \xrightarrow{b} f(x_{<n,m>})$. Therefore, we can find $N_2 \in \mathbb{N}$ so that $d_Y(f_n(x_{<n,m>}), f(x_{<n,m>})) < \frac{\varepsilon}{3}$ for all $n \geq N_2$. Assume that $N = \max\{N_1, N_2\}$. Moreover, $(f_n)$ is given to be a sequence of arithmetic $ff$-continuous functions.

In particular, $f_N$ is arithmetic $ff$-continuous function, and thus arithmetic $fb$-continuous by equivalence of forward and backward convergence in $Y$. So there exists an integer $n_0$ greater than $N$ and $\delta > 0$ such that

$$d_Y(f_N(x_n), f_N(x_{<n,m>})) < \frac{\varepsilon}{3} \text{ for } d_X(x_{<n,m>}, x_n) < \delta$$

for all integers $m, n$ satisfying $< m, n >\geq n_0$. Thus, whenever $d_X(x_{<n,m>}, x_n) < \delta$ and $< m, n >\geq n_0$,, we have

$$\begin{aligned} d_Y(f(x_n), f(x_{<n,m>})) &\leq d_Y(f(x_n), f_N(x_n)) + d_Y(f_N(x_n), f_N(x_{<n,m>})) \\ &\quad + d_Y(f_N(x_{<n,m>}), f(x_{<n,m>})) \\ &< \frac{\varepsilon}{3} + \frac{\varepsilon}{3} + \frac{\varepsilon}{3} = \varepsilon. \end{aligned}$$

Therefore $f$ is arithmetic $fb$-continuous and by equivalence of convergence it is also arithmetic $ff$-continuous. So $f \in \mathbb{A}^{ff}(X, Y)$. This completes the prove of the theorem. □

In [4], Çakalli introduced the notion of $(cAC)$-continuity as follows: a function $f$ is said to be $(cAC)$-*continuous* (or $f \in (cAC)$) if $f$ transforms convergent sequences to arithmetic convergent sequences. We define this notion in the sense of arithmetic forward (or backward) convergence as follows:

A function $f$ from asymmetric metric space $X$ to asymmetric metric space $Y$ is said to be *forward* $(cAC)$-*continuous* if it transforms forward convergent sequences in $X$ to forward arithmetic convergent sequences in $Y$, i.e. $(x_n)$ is forward convergent in $X$ implies $f(x_n)$ is forward arithmetic convergent in $Y$.

**Theorem 3.5.** *Let $(X, d_X)$ and $(Y, d_Y)$ be two asymmetric metric spaces, with forward convergence equivalent to backward convergence in $Y$. If $(f_n)$ is a sequence of forward $(cAC)$-continuous functions from $X$ to $Y$ and $(f_n)$ forward converges uniformly to a function $f$, then $f$ is forward $(cAC)$-continuous.*

*Proof.* Let $\varepsilon > 0$ be given and $(x_k)$ be any forward convergent sequence in $X$. Since $f_n$ forward converges uniformly to $f$, there exists a positive integer $N_1$ such that $d_Y(f(x), f_n(x)) < \frac{\varepsilon}{3}$ for all $x \in X$ and $n \geq N_1$. In particular, $f_n(x_n) \xrightarrow{f} f(x_n)$ and so $f_n(x_n) \xrightarrow{b} f(x_n)$. Thus we can find $N_2 \in \mathbb{N}$ so that $d_Y(f_n(x_n), f(x_n)) < \frac{\varepsilon}{3}$ for all $n \geq N_2$. Assume that $N = \max\{N_1, N_2\}$. By hypothesis, $f_n$ is forward $(cAC)$-continuous. In particular $f_N$ is forward $(cAC)$-continuous, so there exists an integer $n_0$, greater than $N$ such that $d_Y(f_N(x_{<m,n>}), f_N(x_n)) < \frac{\varepsilon}{3}$ for all $x \in X$ and for all integers $m, n$ satisfying $< m, n >\geq n_0$. Thus, it follows that

$$\begin{aligned} d_Y(f(x_{<m,n>}), f(x_n)) &\leq d_Y(f(x_{<m,n>}), f_N(x_{<m,n>})) \\ &\quad + d_Y(f_N(x_{<m,n>}), f_N(x_n)) + d_Y(f_N(x_n), f(x_n)) \\ &< \frac{\varepsilon}{3} + \frac{\varepsilon}{3} + \frac{\varepsilon}{3} = \varepsilon. \end{aligned}$$

This establishes the result. □

**Theorem 3.6.** *Let $(X, d_X)$ and $(Y, d_Y)$ be two asymmetric metric spaces. Then the set of all forward $(cAC)$-continuous functions from $X$ to $Y$, with forward convergence equivalent to backward convergence in $Y$, is a closed subset of the set of all continuous functions from $X$ to $Y$.*

*Proof.* The result immediately follows from the previous theorem. □

## 4. Compactness in Asymmetric Metric Spaces

We will first introduce forward arithmetic compactness and backward arithmetic compactness in the setting of asymmetric metric space as follows:

**Definition 4.1.** A subset $A$ of an asymmetric metric space $(X, d_X)$ is said to be

(i) *forward arithmetic compact* if every sequence in $A$ has forward arithmetic convergent subsequence.

(ii) *backward arithmetic compact* if every sequence in $A$ has backward arithmetic convergent subsequence.

**Theorem 4.1.** *An arithmetic $ff$-continuous image of an forward arithmetic compact subset of an asymmetric metric space $(X, d)$ is forward arithmetic compact.*

*Proof.* Let $(X, d_X)$ and $(Y, d_Y)$ be asymmetric metric spaces. Let $f : X \to Y$ be an arithmetic $ff$-continuous function and $A \subset X$ be forward arithmetic compact. Let $(y_n)$ be a sequence in $f(A)$. Then we can write $y_n = f(x_n)$ where $x_n \in X$ for each $n \in \mathbb{N}$. Since $A$ is forward arithmetic compact, there exists an forward arithmetic convergent subsequence $(x_{n_k})$ of $(x_n)$. Again, it is given that $f$ is arithmetic $ff$-continuous, this implies that $f(x_{n_k})$ is forward arithmetic convergent subsequence of $f(x_n)$. Hence, $f(A)$ is forward arithmetic compact. □

**Theorem 4.2.** *An arithmetic $fb$-continuous image of a backward arithmetic compact subset of an asymmetric metric space $(X, d)$ is backward arithmetic compact.*

*Proof.* The proof is the same as in the previous theorem. □

**Theorem 4.3.** *Any closed subset of a forward arithmetic compact subset of an asymmetric metric space $(X, d)$ is forward arithmetic compact.*

*Proof.* Let $A$ be any forward arithmetic compact subset of $X$ and $B$ be a closed subset of $A$. Let $x = (x_n)$ be any sequence of points in $B$. Then $x = (x_n)$ is a sequence of points in $A$. Since $A$ is forward arithmetic compact, there exists an forward arithmetic convergent subsequence $(x_{n_k})$ of the sequence $x$. Since $B$ is closed, so any sequence $x = (x_n)$ of points in $B$ has forward arithmetic convergent subsequence in $B$. Hence the result. □

**Theorem 4.4.** *Any closed subset of a backward arithmetic compact subset of an asymmetric metric space $(X, d)$ is backward arithmetic compact.*

*Proof.* The proof is the same as in the previous theorem. □

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