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A TYPE OF SEMI-SYMMETRIC NON-METRIC CONNECTION ON NON-DEGENERATE HYPERSURFACES OF SEMI-RIEMANNIAN MANIFOLDS

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Abstract. The objective of the present paper is to study a non-degenerate hypersurface of a semi-Riemannian manifold with a semi-symmetric non-metric connection.

1. Introduction

Let $\widetilde{M}^{n+1}$ be a differentiable manifold of class C^∞ and M^n a differentiable manifold immersed in $\widetilde{M}$ by a differentiable immersion

$$i : M \to \widetilde{M}.$$

$i(M)$ identical to M, is said to be a hypersurface of $\widetilde{M}$. The differential di of the immersion i will be denoted by B so that a vector field X in M corresponds to a vector field BX in $\widetilde{M}$. We suppose that the manifold $\widetilde{M}$ is a semi-Riemannian manifold with the semi-Riemannian metric $\widetilde{g}$ of index ν, $0 \leq \nu \leq n+1$. Thus the index of $\widetilde{M}$ is the ν, which will be denoted by $ind\widetilde{M} = \nu$. If the induced metric tensor $g = \widetilde{g}|M$ defined by

$$g(X,Y) = \widetilde{g}(BX, BY), \quad for \ all \quad X, Y \quad in \quad \chi(M)$$

is non-degenerate, then the hypersurface M is called a non-degenerate hypersurface. Also M is a semi-Riemannian manifold with the induced semi-Riemannian metric g [15]. If the semi-Riemannian manifolds $\widetilde{M}$ and M are both orientable and we can choose a unit vector field N defined along M such that

$$\widetilde{g}(BX, N) = 0, \quad \widetilde{g}(N,N) = \epsilon = \begin{cases} +1, & for \ spacelike \ N \\ -1, & for \ timelike \ N, \end{cases}$$

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for all X in $\chi(M)$, where N is called the unit normal vector field to M and ind$M$ = ind$\widetilde{M}$ if $\epsilon = 1$, ind$M$ = ind$\widetilde{M} - 1$ if $\epsilon = -1$.

The hypersurface of a manifold have been studied by several authors such as De and Kamilya [9], De and Mondal [10], O'Neill [15], Yano and Kon [18], Yücesan and Ayyildiz [19], Yücesan and Yasar [20] and many others.

In 1924, Friedmann and Schouten [12] introduced the idea of semi-symmetric connection on a differentiable manifold. A linear connection $\widetilde{\nabla}$ on a differentiable manifold $(\widetilde{M}^{n+1}, g)$ with Riemannian connection $\widetilde{\nabla}$ is said to be a semi-symmetric connection if the torsion tensor $\widetilde{T}$ of the connection $\widetilde{\nabla}$ satisfies

$$\widetilde{T}(\widetilde{X}, \widetilde{Y}) = \widetilde{\eta}(\widetilde{Y})\widetilde{X} - \widetilde{\eta}(\widetilde{X})\widetilde{Y}, \tag{1.1}$$

where $\widetilde{\eta}$ is a 1-form and $\widetilde{\xi}$ is a vector field defined by

$$\widetilde{\eta}(\widetilde{X}) = \widetilde{g}(\widetilde{X}, \widetilde{\xi}), \tag{1.2}$$

for all vector fields $\widetilde{X} \in \chi(\widetilde{M}^{n+1})$, $\chi(\widetilde{M}^{n+1})$ is the set of all differentiable vector fields on $\widetilde{M}^{n+1}$.

In 1932, Hayden [13] introduced the idea of semi-symmetric connections on a Riemannian manifold $(\widetilde{M}^{n+1}, \widetilde{g})$. A semi-symmetric connection $\widetilde{\nabla}$ is said to be a semi-symmetric metric connection if

$$\widetilde{\nabla}\widetilde{g} = 0. \tag{1.3}$$

The study of semi-symmetric metric connection was further developed by Yano [17], Amur and Pujar [2], Chaki and Konar [4], De [5] and many others.

After a long gap the study of a semi-symmetric connection $\widetilde{\nabla}$ satisfying

$$\widetilde{\nabla}\widetilde{g} \neq 0. \tag{1.4}$$

was initiated by Prvanović [16] with the name pseudo-metric semi-symmetric connection and was just followed by Andonie [3].

In 1992, Agashe and Chafle [1] introduced and studied a semi-symmetric non-metric connection. In 1994, Liang [14] studied another type of semi-symmetric non-metric connection. The semi-symmetric non-metric connections was further developed by several authors such as De and Biswas [7], Biswas, De and Barua [6], De and Kamilya ([8], [9]) and many others.

A non-degenerate hypersurface of semi-Riemannian manifolds is said to be of constant curvature if the curvature tensor $\overline{R}$ of a non-degenerate hypersurface M satisfies the following condition

$$\overline{R}(X, Y, Z, U) = b'[g(X, Z)g(Y, U) - g(X, U)g(Y, Z)],$$

where b' is a constant.

In this paper we study non-degenerate hypersurfaces of a semi-Riemannian manifold admitting a semi-symmetric non-metric connection in the sense of Liang [14].

After introduction in section 2, we study a non-degenerate hypersurface of semi-Riemannian manifolds admitting a semi-symmetric non-metric connection. In section 3, we obtain the equations of Gauss and Codazzi-Mainardi with respect to the semi-symmetric non-metric connection. In this section we also derive the Ricci tensor and the scalar curvature of a non-degenerate hypersurface of semi-Riemannian manifolds with respect to the semi-symmetric non-metric connections. Finally, we observed that a totally geodesic non-degenerate hypersurface M of semi-Riemannian manifolds $\widetilde{M}$ whose curvature tensor vanishes with respect to the semi-symmetric non-metric connection M is conformally flat.

2. Semi-symmetric non-metric connection

Let $\widetilde{M}^{n+1}$ denotes a semi-Riemannian manifold with semi-Riemannian metric $\widetilde{g}$ of index ν, $0 \le \nu \le n+1$. A linear connection $\widetilde{\nabla}$ on $\widetilde{M}$ is called a semi-symmetric non-metric connection [14] if

$$(\widetilde{\nabla}_{\widetilde{X}}\widetilde{g})(\widetilde{Y},\widetilde{Z}) = 2\widetilde{\eta}(\widetilde{X})\widetilde{g}(\widetilde{Y},\widetilde{Z}). \tag{2.1}$$

Throughout the paper, we will denote by $\widetilde{M}$ the semi-Riemannian manifold admitting a semi-symmetric non-metric connection [14] given by

$$\widetilde{\nabla}_{\widetilde{X}}\widetilde{Y} = \widetilde{\nabla}^*_{\widetilde{X}}\widetilde{Y} - \widetilde{\eta}(\widetilde{X})\widetilde{Y}, \tag{2.2}$$

for any vector fields $\widetilde{X}$ and $\widetilde{Y}$ of $\widetilde{M}$. When M is a non-degenerate hypersurface, we have the following orthogonal direct sum :

$$\chi(\widetilde{M}) = \chi(M) \oplus \chi(M)^{\perp}. \tag{2.3}$$

According to (2.3), every vector field $\widetilde{X}$ on $\widetilde{M}$ is decomposed as

$$\widetilde{\xi} = B\xi + \mu N, \tag{2.4}$$

where μ is a scalar and a contravariant vector field ξ of the hypersurface M^n.

We denote by ∇^* the connection on the non-degenerate hypersurface M induced from the Levi-Civita connection $\widetilde{\nabla}^*$ on $\widetilde{M}$ with respect to the unit spacelike or timelike normal vector field N. We have the equality

$$\widetilde{\nabla}^*_{BX}BY = B(\nabla^*_X Y) + h^*(X,Y)N, \tag{2.5}$$

for arbitrary vector fields X and Y of M, where h^* is the second fundamental form of the non-degenerate hypersurface M. Let us define the connection ∇ on M which is induced by the semi-symmetric non-metric connection $\widetilde{\nabla}$ on $\tilde{M}$ with respect to the unit spacelike or timelike normal vector field N. We obtain the equation

$$\widetilde{\nabla}_{BX}BY = B(\nabla_X Y) + h(X,Y)N, \tag{2.6}$$

where h is the second fundamental form of the non-degenerate hypersurface M. If $h(X,Y) = 0$ (respectively, $h(X,Y) = a'g(X,Y)$, where a' is a scalar), then the hypersurface is called totally geodesic (respectively, totally umbilical) [10]. We call (2.6) the equation of Gauss with respect to the induced connection ∇.

According to (2.2), we have

$$\widetilde{\nabla}_{BX}BY = \widetilde{\nabla}^*_{BX}BY - \widetilde{\eta}(BX)BY. \tag{2.7}$$

Using (2.5) and (2.6) in (2.7), we get

$$\begin{aligned} B(\nabla_X Y) + h(X,Y)N = B(\nabla^*_X Y) + h^*(X,Y)N \\ -\widetilde{\eta}(BX)BY, \end{aligned} \tag{2.8}$$

which implies

$$\nabla_X Y = \nabla^*_X Y - \eta(X)Y, \tag{2.9}$$

where

$$\widetilde{\eta}(BY) = \eta(Y), \quad h(X,Y) = h^*(X,Y).$$

From (2.9), we conclude that

$$(\nabla_X g)(Y,Z) = 2\eta(X)g(Y,Z), \tag{2.10}$$

and

$$T(X,Y) = \eta(Y)X - \eta(X)Y, \tag{2.11}$$

for any X, Y, Z in $\chi(M)$.

From (2.10) and (2.11), we can state the following theorem:

Theorem 2.1. *The connection induced on a non-degenerate hypersurface of a semi-Riemannian manifold with a semi-symmetric non-metric connection with respect to the unit spacelike or timelike normal vector field is also a semi-symmetric non-metric connection.*

3. Equations of Gauss and Codazzi-Mainardi

We denote the curvature tensor of $\widetilde{M}$ with respect to the Levi-Civita connection $\widetilde{\nabla}^*$ by

$$\widetilde{R}^*(\widetilde{X},\widetilde{Y})\widetilde{Z} = \widetilde{\nabla}^*_{\widetilde{X}}\widetilde{\nabla}^*_{\widetilde{Y}}\widetilde{Z} - \widetilde{\nabla}^*_{\widetilde{Y}}\widetilde{\nabla}^*_{\widetilde{X}}\widetilde{Z} - \widetilde{\nabla}^*_{[\widetilde{X},\widetilde{Y}]}\widetilde{Z}$$

and that of M with respect to the Levi-Civita connection ∇^* by

$$R^*(X,Y)Z = \nabla^*_X\nabla^*_Y Z - \nabla^*_Y\nabla^*_X Z - \nabla^*_{[X,Y]}Z.$$

Then the equation of Gauss is given by

$$R^*(X,Y,Z,U) = \widetilde{R}^*(BX,BY,BZ,BU) + \epsilon\{h^*(X,U)h^*(Y,Z) - h^*(Y,U)h^*(X,Z)\},$$

where

$$\widetilde{R}^*(BX,BY,BZ,BU) = \widetilde{g}(\widetilde{R}^*(BX,BY)BZ,BU),$$

$$R^*(X,Y,Z,U) = g(R^*(X,Y)Z,U)$$

and the equation of Codazzi-Mainardi [15] is given by

$$\widetilde{R}^*(BX,BY,BZ,N) = \epsilon\{(\nabla^*_X h^*)(Y,Z) - (\nabla^*_Y h^*)(X,Z)\}.$$

We find the equation of Gauss and Codazzi-Mainardi with respect to the semi-symmetric non-metric connection. The curvature tensor $\widetilde{R}$ of the semi-symmetric non-metric connection $\widetilde{\nabla}$ of $\widetilde{M}$ is

$$\widetilde{R}(\widetilde{X},\widetilde{Y})\widetilde{Z} = \widetilde{\nabla}_{\widetilde{X}}\widetilde{\nabla}_{\widetilde{Y}}\widetilde{Z} - \widetilde{\nabla}_{\widetilde{Y}}\widetilde{\nabla}_{\widetilde{X}}\widetilde{Z} - \widetilde{\nabla}_{[\widetilde{X},\widetilde{Y}]}\widetilde{Z}. \tag{3.1}$$

The equation of Weingarten with respect to the Levi-Civita connection $\widetilde{\nabla}^*$ is

$$\widetilde{\nabla}^*_{BX}N = -B(HX), \tag{3.2}$$

where H is the second fundamental tensor field of type $(1,1)$ of M which is defined by

$$h^*(X,Y) = h(X,Y) = \epsilon g(HX,Y), \tag{3.3}$$

for any vector fields X and Y in M [15].

Using (2.2), we have

$$\widetilde{\nabla}_{BX}N = \widetilde{\nabla}^*_{BX}N - \eta(X)N. \tag{3.4}$$

Because of (2.4), we obtain

$$\widetilde{\eta}(BX) = \widetilde{g}(BX, \widetilde{\xi}) = \widetilde{g}(BX, B\xi + \mu N) = \widetilde{g}(BX, B\xi) = \eta(X).$$

Combining (3.2) and (3.4), we get

$$\widetilde{\nabla}_{BX} N = -B(HX) - \eta(X)N. \tag{3.5}$$

Putting $\widetilde{X} = BX$, $\widetilde{Y} = BY$, $\widetilde{Z} = BZ$ in (3.1) and using (2.6) and (3.5), we get

$$\begin{aligned} \widetilde{R}(BX, BY)BZ &= B[R(X, Y)Z + h(X, Z)HY - h(Y, Z)HX] \\ &+[(\nabla_X h)(Y, Z) - (\nabla_Y h)(X, Z) + h\{\eta(Y)X - \eta(X)Y, Z\}]N, \end{aligned} \tag{3.6}$$

where

$$R(X, Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X,Y]} Z$$

is the curvature tensor of the semi-symmetric non-metric connection ∇.

Combining (3.3) and (3.6), we obtain

$$\begin{aligned} \overline{\overline{R}}(BX, BY, BZ, BU) = \overline{R}(X, Y, Z, U) + \epsilon[h(X, Z)h(Y, U) \\ -h(Y, Z)h(X, U)] \end{aligned} \tag{3.7}$$

and

$$\begin{aligned} \overline{\overline{R}}(BX, BY, BZ, N) = \epsilon[(\nabla_X h)(Y, Z) - (\nabla_Y h)(X, Z) \\ +h\{\eta(Y)X - \eta(X)Y, Z\}], \end{aligned} \tag{3.8}$$

where

$$\overline{\overline{R}}(\widetilde{X}, \widetilde{Y}, \widetilde{Z}, \widetilde{U}) = \widetilde{g}(\widetilde{R}(\widetilde{X}, \widetilde{Y})\widetilde{Z}, \widetilde{U})$$

and

$$\overline{R}(X, Y, Z, U) = g(R(X, Y)Z, U).$$

The above equations (3.7) and (3.8) are Gauss and Codazzi-Mainardi with respect to the semi-symmetric non-metric connection respectively.

Now if we put $\overline{\overline{R}} = 0$ and $h(X, Y) = a' g(X, Y)$ in (3.7), we get

$$\begin{aligned} \overline{R}(X, Y, Z, U) = -\epsilon(a')^2[g(X, Z)g(Y, U) \\ -g(X, U)g(Y, Z)]. \end{aligned} \tag{3.9}$$

Therefore,

$$\overline{R}(X, Y, Z, U) = b'[g(X, Z)g(Y, U) - g(X, U)g(Y, Z)],$$

where $b' = -\epsilon(a')^2$.

This result shows that the non-degenerate hypersurface of a semi-Riemannian manifold is of constant curvature.

Hence we can state the following theorem.

Theorem 3.1. *Let M be a totally umbilical non-degenerate hypersurface of semi-Riemannian manifolds $\widetilde{M}$ with vanishing curvature tensor with respect to the semi-symmetric non-metric connections, then M is of constant curvature.*

Suppose that $\{Be_1, ..., Be_\nu, Be_{\nu+1}, ...Be_n, N\}$ is an orthonormal basis of $\chi(\widetilde{M})$. Then the Ricci tensor of $\widetilde{M}$ with respect to the semi-symmetric non-metric connection is

$$\widetilde{Ric}(BY, BZ) = \sum_{i=1}^{n} \epsilon_i \widetilde{g}(\widetilde{R}(Be_i, BY)BZ, Be_i) + \epsilon \widetilde{g}(\widetilde{R}(N, BY)BZ, N), \tag{3.10}$$

for all Y, Z in $\chi(M)$.

Putting $X = e_i$ and $U = e_i$ in (3.7) and using (3.3), we have

$$\sum_{i=1}^{n} \epsilon_i \overline{\overline{R}}(Be_i, BY, BZ, Be_i) = \sum_{i=1}^{n} \epsilon_i \widetilde{g}(\widetilde{R}(Be_i, BY)BZ, Be_i) = Ric(Y, Z) + \epsilon(1 - f)h(Y, Z), \tag{3.11}$$

where

$$f = \sum_{i=1}^{n} \epsilon_i h(e_i, e_i).$$

Combining (3.10) and (3.11), we obtain

$$\widetilde{Ric}(BY, BZ) = Ric(Y, Z) + \epsilon(1 - f)h(Y, Z) + \epsilon \widetilde{g}(\widetilde{R}(N, BY)BZ, N), \tag{3.12}$$

where $\widetilde{Ric}$ and Ric are the Ricci tensors with respect to $\widetilde{\nabla}$ and ∇ respectively.

The scalar curvature of $\widetilde{M}$ with respect to the semi-symmetric non-metric connection is

$$\widetilde{r} = \sum_{i=1}^{n} \epsilon_i \widetilde{Ric}(Be_i, Be_i) + \epsilon \widetilde{Ric}(N, N). \tag{3.13}$$

Putting $Y = e_i$ and $Z = e_i$ in (3.12), we get

$$\sum_{i=1}^{n} \epsilon_i \widetilde{Ric}(Be_i, Be_i) = r + \epsilon(1 - f)f + \epsilon \widetilde{Ric}(N, N), \tag{3.14}$$

where $\tilde{r}$ and r are the scalar curvatures with respect to $\widetilde{\nabla}$ and ∇ respectively.

Combining (3.14) and (3.13), we obtain

$$\widetilde{r} = r + \epsilon(1 - f)f + 2\epsilon\widetilde{Ric}(N, N). \tag{3.15}$$

Therefore, we can state the following theorem.

Theorem 3.2. *Let M be non-degenerate hypersurface of a semi-Riemannian manifold $\widetilde{M}$ with respect to the semi-symmetric non-metric connection, then the relation of the Ricci tensors with respect to ∇ and $\widetilde{\nabla}$ is*

$$\widetilde{Ric}(BY, BZ) = Ric(Y, Z) + \epsilon(1 - f)h(Y, Z) + \epsilon\widetilde{g}(\widetilde{R}(N, BY)BZ, N)$$

and also the relation of the scalar curvatures with respect to ∇ and $\widetilde{\nabla}$ is

$$\widetilde{r} = r + \epsilon(1 - f)f + 2\epsilon\widetilde{Ric}(N, N).$$

4. The Weyl Conformal curvature tensor of a non-degenerate hypersurface of a semi-Riemannian manifold with respect to the semi-symmetric non-metric connections

We denote the Weyl conformal curvature tensor $\overline{\overline{C}}$ of type $(0, 4)$ of semi-Riemannian manifolds $\widetilde{M}^{n+1}$ and the Weyl conformal curvature tensor $\overline{\mathbf{C}}$ of type $(0, 4)$ of a non-degenerate hypersurface M^n of semi-Riemannian manifolds with respect to the semi-symmetric non-metric connections $\widetilde{\nabla}$ and ∇ respectively, are given by

$$\begin{aligned}\overline{\overline{C}}(BX, BY, BZ, BU) = \overline{\overline{R}}(BX, BY, BZ, BU) - \frac{1}{n-1}[\widetilde{Ric}(BY, BZ)\widetilde{g}(BX, BU)\\ -\widetilde{Ric}(BX, BZ)\widetilde{g}(BY, BU) + \widetilde{Ric}(BX, BU)\widetilde{g}(BY, BZ)\\ -\widetilde{Ric}(BY, , BU)\widetilde{g}(BX, BZ)] + \frac{\widetilde{r}}{n(n-1)}[\widetilde{g}(BY, BZ)\widetilde{g}(BX, BU)\\ -\widetilde{g}(BX, BZ)\widetilde{g}(BY, BU)],\end{aligned} \tag{4.1}$$

and

$$\begin{aligned}\overline{\mathbf{C}}(X, Y, Z, U) = \overline{R}(X, Y, Z, U) - \frac{1}{n-2}[Ric(Y, Z)g(X, U) - Ric(X, Z)g(Y, U)\\ +Ric(X, U)g(Y, Z) - Ric(Y, U)g(X, Z)] + \frac{r}{(n-1)(n-2)}[g(Y, Z)g(X, U)\\ -g(X, Z)g(Y, U)],\end{aligned} \tag{4.2}$$

where

$$\overline{\overline{C}}(BX, BY, BZ, BU) = \widetilde{g}(\widetilde{C}(BX, BY)BZ, BU)$$

and

$$\overline{\mathbf{C}}(X, Y, Z, U) = g(\mathbf{C}(X, Y)Z, U).$$

The Weyl conformal curvature tensor is invariant under any conformal change of the metric. A flat manifold is conformally flat [11].

Using (3.7), (3.12), (3.15) and (4.2) in (4.1), we get

$$\begin{aligned}\overline{\mathbf{C}}(X, Y, Z, U) &= \overline{\overline{\widetilde{C}}}(BX, BY, BZ, BU) - \epsilon[h(X, Z)h(Y, U)\\ -h(Y, Z)h(X, U)] &- \frac{1}{(n-1)(n-2)}[\widetilde{Ric}(BY, BZ)\widetilde{g}(BX, BU)\\ &-\widetilde{Ric}(BX, BZ)\widetilde{g}(BY, BU) + \widetilde{Ric}(BX, BU)\widetilde{g}(BY, BZ)\\ -\widetilde{Ric}(BY, , BU)\widetilde{g}(BX, BZ)] &+ \frac{2\widetilde{r}}{n(n-1)(n-2)}[\widetilde{g}(BY, BZ)\widetilde{g}(BX, BU)\\ -\widetilde{g}(BX, BZ)\widetilde{g}(BY, BU)] &+ \frac{1}{n-2}[\epsilon(1-f)\{h(X, Z)\widetilde{g}(BY, BU)\\ &-h(Y, Z)\widetilde{g}(BX, BU) - h(X, U)\widetilde{g}(BY, BZ)\\ &+h(Y, U)\widetilde{g}(BX, BZ)\} - \epsilon\{\overline{\overline{R}}(N, BY, BZ, N)\widetilde{g}(BX, BU)\\ &-\overline{\overline{R}}(N, BX, BZ, N)\widetilde{g}(BY, BU) + \overline{\overline{R}}(N, BX, BU, N)\widetilde{g}(BY, BZ)\\ -\overline{\overline{R}}(N, BY, BU, N)\widetilde{g}(BX, BZ)\}] &- \frac{1}{(n-1)(n-2)}[\epsilon(1-f)f\\ &+2\epsilon\widetilde{S}(N, N)][\widetilde{g}(BY, BZ)\widetilde{g}(BX, BU)\\ &-\widetilde{g}(BX, BZ)\widetilde{g}(BY, BU)], \end{aligned} \tag{4.3}$$

where $f = h(e_i, e_i)$.

Suppose $\overline{\overline{R}} = 0$ and $h(X, Y) = 0$, then from (4.3), it follows that

$$\overline{\mathbf{C}}(X, Y, Z, U) = 0. \tag{4.4}$$

From (4.4), we obtain the following theorem.

Theorem 4.1. *A totally geodesic non-degenerate hypersurface M of semi-Riemannian manifolds $\widetilde{M}$ whose curvature tensor vanishes with respect to the semi-symmetric non-metric connection is conformally flat.*

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