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ON $\mathcal{I}_2$-CONVERGENCE AND $\mathcal{I}_2$-CAUCHY DOUBLE SEQUENCES OF FUNCTIONS IN 2-NORMED SPACES

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Abstract. In this study, firstly, we studied some properties of $\mathcal{I}_2$-convergence. Then, we introduced $\mathcal{I}_2$-Cauchy and $\mathcal{I}_2^*$-Cauchy sequence of double sequences of functions in 2-normed space. Also, we investigated the relationships between them for double sequences of functions in 2-normed spaces.
Keywords: $\mathcal{I}_2$-Convergence, $\mathcal{I}_2$-Cauchy, Double sequences of Functions, 2-normed Spaces.

1. Introduction and Background

Throughout the paper, $\mathbb{N}$ denotes the set of all positive integers and $\mathbb{R}$ the set of all real numbers. The concept of convergence of a sequence of real numbers has been extended to statistical convergence independently by Fast [15] and Schoenberg [36]. Gökhan et al. [20] introduced the notion of pointwise and uniform statistical convergence of double sequences of real-valued functions. The idea of $\mathcal{I}$-convergence was introduced by Kostyrko et al. [28] as a generalization of statistical convergence which is based on the structure of the ideal $\mathcal{I}$ of subset of $\mathbb{N}$ [15, 16]. Gezer and Karakuş [19] investigated $\mathcal{I}$-pointwise and uniform convergence and $\mathcal{I}^*$-pointwise and uniform convergence of function sequences and they examined the relation between them. Baláz et al. [5] investigated $\mathcal{I}$-convergence and $\mathcal{I}$-continuity of real functions. Das et al. [7] introduced the concept of $\mathcal{I}$-convergence of double sequences in a metric space and studied some properties of this convergence. Dündar and Altay [8, 10] studied the concepts of pointwise and uniformly $\mathcal{I}_2$-convergence and $\mathcal{I}_2^*$-convergence of double sequences of functions and investigated some properties about them. Furthermore, Dündar [13] investigated some results of $\mathcal{I}_2$-convergence of double sequences of functions. Also, a lot of development has been made about double sequences of functions (see [9, 11, 14, 30, 34, 40–42]).

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The concept of 2-normed spaces was initially introduced by Gähler [17, 18] in the 1960's. Statistical convergence and statistical Cauchy sequence of functions in 2-normed space were studied by Yegül and Dündar [43]. Yegül and Dündar [44] introduced concepts of pointwise and uniform convergence, statistical convergence and statistical Cauchy double sequences of functions in 2-normed space. Also, Yegül and Dündar [45] introduced concepts of $\mathcal{I}_2$-convergence and $\mathcal{I}_2^*$-convergence of double sequences of functions in 2-normed space. Recently, Arslan and Dündar [1,2] inroduced $\mathcal{I}$-convergence and $\mathcal{I}$-Cauchy sequences of functions in 2-normed spaces. Futhermore, there has been a lot of development in this area (see [3, 4, 6, 26, 27, 29, 31–33, 37–39]).

2. Definitions and Notations

Now, we recall the concept of density, statistical convergence, 2-normed space and some fundamental definitions and notations (See [1, 2, 7, 12, 16, 18–25, 28, 31, 35, 43–45]).

Let X be a real vector space of dimension d, where $2 \leq d < \infty$. A 2-norm on X is a function $\|\cdot,\cdot\| : X \times X \to \mathbb{R}$ which satisfies the following statements:

(i) $\|x, y\| = 0$ if and only if x and y are linearly dependent.

(ii) $\|x, y\| = \|y, x\|$.

(iii) $\|\alpha x, y\| = |\alpha|\|x, y\|$, $\alpha \in \mathbb{R}$.

(iv) $\|x, y + z\| \leq \|x, y\| + \|x, z\|$.

The pair $(X, \|\cdot,\cdot\|)$ is then called a 2-normed space. As an example of a 2-normed space we may take $X = \mathbb{R}^2$ being equipped with the 2-norm $\|x, y\| :=$ the area of the parallelogram based on the vectors x and y which may be given explicitly by the formula $\|x, y\| = |x_1y_2 - x_2y_1|$; $x = (x_1, x_2), y = (y_1, y_2) \in \mathbb{R}^2$.

In this study, we suppose X to be a 2-normed space having dimension d; where $2 \leq d < \infty$.

Throughout the paper, we let X and Y be two 2-normed spaces, $\{f_n\}_{n\in\mathbb{N}}$ and $\{g_n\}_{n\in\mathbb{N}}$ be two sequences of functions and f, g be two functions from X to Y.

The sequence of functions $\{f_n\}_{n\in\mathbb{N}}$ is said to be convergent to f if $f_n(x) \to f(x)(\|.,.\|_Y)$ for each $x \in X$. We write $f_n \to f(\|.,.\|_Y)$. This can be expressed by the formula $(\forall y \in Y)(\forall x \in X)(\forall \varepsilon > 0)(\exists n_0 \in \mathbb{N})(\forall n \geq n_0)\|f_n(x) - f(x), y\| < \varepsilon$.

A family of sets $\mathcal{I} \subseteq 2^{\mathbb{N}}$ is called an ideal if and only if

(i) $\emptyset \in \mathcal{I}$, **(ii)** For each $A, B \in \mathcal{I}$ we have $A \cup B \in \mathcal{I}$, **(iii)** For each $A \in \mathcal{I}$ and each $B \subseteq A$ we have $B \in \mathcal{I}$.

An ideal is called nontrivial if $\mathbb{N} \notin \mathcal{I}$ and nontrivial ideal is called admissible if $\{n\} \in \mathcal{I}$ for each $n \in \mathbb{N}$.

A family of sets $\mathcal{F} \subseteq 2^{\mathbb{N}}$ is called a filter if and only if

(i) $\emptyset \notin \mathcal{F}$, **(ii)** For each $A, B \in \mathcal{F}$ we have $A \cap B \in \mathcal{F}$, **(iii)** For each $A \in \mathcal{F}$ and each $B \supseteq A$ we have $B \in \mathcal{F}$.

$\mathcal{I}$ is nontrivial ideal in $\mathbb{N}$ if and only if $\mathcal{F}(\mathcal{I}) = \{M \subset \mathbb{N} : (\exists A \in \mathcal{I})(M = \mathbb{N}\backslash A)\}$ is a filter in $\mathbb{N}$.

A nontrivial ideal $\mathcal{I}_2$ of $\mathbb{N} \times \mathbb{N}$ is called strongly admissible ideal if $\{i\} \times \mathbb{N}$ and $\mathbb{N} \times \{i\}$ belong to $\mathcal{I}_2$ for each $i \in N$.

Throughout the paper we take $\mathcal{I}_2$ as a strongly admissible ideal in $\mathbb{N} \times \mathbb{N}$.

It is evident that a strongly admissible ideal is admissible also.

$\mathcal{I}_2^0 = \{A \subset \mathbb{N} \times \mathbb{N} : (\exists m(A) \in \mathbb{N})(i, j \geq m(A) \Rightarrow (i, j) \notin A)\}$. Then $\mathcal{I}_2^0$ is a strongly admissible ideal and clearly an ideal $\mathcal{I}_2$ is strongly admissible if and only if $\mathcal{I}_2^0 \subset \mathcal{I}_2$.

The sequence of functions $\{f_n\}$ is said to be $\mathcal{I}$-convergent (pointwise) to f, if for every $\varepsilon > 0$ and each nonzero $z \in Y$ $A(\varepsilon, z) = \{n \in \mathbb{N} : \|f_n(x) - f(x), z\| \geq \varepsilon\} \in \mathcal{I}$ or $\mathcal{I} - \lim_{n\to\infty} \|f_n(x) - f(x), z\|_Y = 0$, for each $x \in X$. This can be expressed by the formula $(\forall z \in Y)$ $(\forall \varepsilon > 0)$ $(\exists M \in \mathcal{I})$ $(\forall n_0 \in \mathbb{N}\backslash M)$ $(\forall x \in X)(\forall n \geq n_0)$ $\|f_n(x) - f(x), z\| \leq \varepsilon$. In this case, we write $f_n \to_{\mathcal{I}} f(\|.,.\|_Y)$.

The sequence of functions $\{f_n\}$ is said to be $\mathcal{I}^*$-convergent (pointwise sense) to f, if there exists a set $M \in \mathcal{F}(\mathcal{I})$, (i.e., $\mathbb{N}\backslash M \in \mathcal{I}$), $M = \{m_1 < m_2 < \cdots < m_k < \cdots\}$, such that for each $x \in X$ and each nonzero $z \in Y$ $\lim_{k\to\infty} \|f_{n_k}(x), z\| = \|f(x), z\|$ and we write $\mathcal{I}^* - \lim_{n\to\infty} \|f_n(x), z\| = \|f(x), z\|$ or $f_n \to_{\mathcal{I}^*} f(\|.,.\|_Y)$.

The sequence of functions $\{f_n\}$ is said to be $\mathcal{I}$-Cauchy sequence, if for every $\varepsilon > 0$ and each $x \in X$ there exists $s = s(\varepsilon, x) \in \mathbb{N}$ such that $\{n \in \mathbb{N} : \|f_n(x) - f_s(x), z\| \geq \varepsilon\} \in \mathcal{I}$, for each nonzero $z \in Y$.

The sequence of functions $\{f_n\}$ is said to be $\mathcal{I}^*$-Cauchy sequence, if there exists a set $M = \{m_1 < m_2 < \cdots < m_k < \cdots\} \subset \mathbb{N}$, such that the subsequence $\{f_M\} = \{f_{m_k}\}$ is a Cauchy sequence, i.e., $\lim_{k,p\to\infty} \|f_{m_k}(x) - f_{m_p}(x), z\| = 0$, for each $x \in X$ and each nonzero $z \in Y$.

An admissible ideal $\mathcal{I}_2 \subset 2^{\mathbb{N}\times\mathbb{N}}$ satisfies the property (AP2) if for every countable family of mutually disjoint sets $\{E_1, E_2, ...\}$ belonging to $\mathcal{I}_2$, there exists a countable family of sets $\{F_1, F_2, ...\}$ such that $E_j \Delta F_j \in \mathcal{I}_2^0$, i.e., $E_j \Delta F_j$ is included in the finite union of rows and columns in $\mathbb{N} \times \mathbb{N}$ for each $j \in \mathbb{N}$ and $F = \bigcup_{j=1}^{\infty} F_j \in \mathcal{I}_2$ (hence $F_j \in \mathcal{I}_2$ for each $j \in \mathbb{N}$).

Throughout the paper, we let $\mathcal{I}_2 \subset 2^{\mathbb{N}\times\mathbb{N}}$ be a strongly admissible ideal, X and Y be two 2-normed spaces, $\{f_{mn}\}_{(m,n)\in\mathbb{N}\times\mathbb{N}}$, $\{g_{mn}\}_{(m,n)\in\mathbb{N}\times\mathbb{N}}$ and $\{h_{mn}\}_{(m,n)\in\mathbb{N}\times\mathbb{N}}$ be three double sequences of functions, f, g and k be three functions from X to Y.

A double sequence $\{f_{mn}\}$ is said to be convergent (pointwise) to f if, for each point $x \in X$ and for each $\varepsilon > 0$, there exists a positive integer $k_0 = k_0(x, \varepsilon)$ such that for all $m, n \geq k_0$ implies $\|f_{mn}(x) - f(x), z\| < \varepsilon$, for every $z \in Y$. In this case, we write $f_{mn} \longrightarrow f(\|.,.\|_Y)$.

The double sequence of functions $\{f_{mn}\}$ is said to be $\mathcal{I}_2$-convergent (pointwise sense) to f, if for every $\varepsilon > 0$ and each nonzero $z \in Y$

$$A(\varepsilon, z) = \{(m,n) \in \mathbb{N} \times \mathbb{N} : \|f_{mn}(x) - f(x), z\| \geq \varepsilon\} \in \mathcal{I}_2,$$

for each $x \in X$. This can be expressed by the formula

$$(\forall z \in Y)\ (\forall x \in X)\ (\forall \varepsilon > 0)\ (\exists H \in \mathcal{I}_2)\ (\forall (m,n) \notin H)\ \|f_{mn}(x) - f(x), z\| < \varepsilon.$$

In this case, we write $\mathcal{I}_2 - \lim_{m,n\to\infty} \|f_{mn}(x), z\| = \|f(x), z\|$, *or* $f_{mn} \to_{\mathcal{I}_2} f(\|.,.\|_Y)$.

The double sequence of functions $\{f_{mn}\}$ in 2-normed space $(X, \|.,.\|)$ is said to be $\mathcal{I}_2^*$-convergent (pointwise) to f, if there exists a set $M \in \mathcal{F}(\mathcal{I}_2)$ $(H = \mathbb{N} \times \mathbb{N} \setminus M \in \mathcal{I}_2)$ such that for each $x \in X$, each nonzero $z \in Y$ and all $(m,n) \in M$ $\lim_{m,n\to\infty} \|f_{mn}(x), z\| = \|f(x), z\|$ and we write $\mathcal{I}_2^* - \lim_{m,n\to\infty} \|f_{mn}(x), z\| = \|f(x), z\|$ or $f_{mn} \to_{\mathcal{I}_2^*} f(\|.,.\|_Y)$.

Lemma 2.1. [45] *For each $x \in X$ and nonzero $z \in Y$,*

$$\mathcal{I}_2^* - \lim_{m,n\to\infty} \|f_{mn}(x), z\| = \|f(x), z\| \text{ implies } \mathcal{I}_2 - \lim_{m,n\to\infty} \|f_{mn}(x), z\| = \|f(x), z\|.$$

Lemma 2.2. [45] *Let $\mathcal{I}_2 \subset 2^{\mathbb{N}\times\mathbb{N}}$ be an admissible ideal having the property $(AP2)$. For each $x \in X$ and nonzero $z \in Y$,*

$$\mathcal{I}_2 - \lim_{m,n\to\infty} \|f_{mn}(x), z\| = \|f(x), z\| \text{ implies } \mathcal{I}_2^* - \lim_{m,n\to\infty} \|f_{mn}(x), z\| = \|f(x), z\|.$$

Lemma 2.3. [11] *Let $\{P_i\}_{i=1}^{\infty}$ be a countable collection of subsets of $\mathbb{N} \times \mathbb{N}$ such that $\{P_i\}_{i=1}^{\infty} \in F(\mathcal{I}_2)$ for each i, where $\mathcal{F}(\mathcal{I}_2)$ is a filter associate with a strongly admissible ideal $\mathcal{I}_2$ with the property (AP2). Then there exists a set $P \subset \mathbb{N} \times \mathbb{N}$ such that $P \in \mathcal{F}(\mathcal{I}_2)$ and the set $P \backslash P_i$ is finite for all i.*

Lemma 2.4. [45] *For each $x \in X$ and each nonzero $z \in Y$, If*

$\mathcal{I}_2 - \lim_{m,n\to\infty} \|f_{mn}(x), z\| = \|f(x), z\|$ *and* $\mathcal{I}_2 - \lim_{m,n\to\infty} \|g_{mn}(x), z\| = \|g(x), z\|$, *then*

(i) $\mathcal{I}_2 - \lim_{m,n\to\infty} \|f_{mn}(x) + g_{mn}(x), z\| = \|f(x) + g(x), z\|$,

(ii) $\mathcal{I}_2 - \lim_{m,n\to\infty} \|c.f_{mn}(x), z\| = \|c.f(x), z\|$, $c \in \mathbb{R}$,

(iii) $\mathcal{I}_2 - \lim_{m,n\to\infty} \|f_{mn}(x).g_{mn}(x), z\| = \|f(x).g(x), z\|$.

3. Main Results

In this study, firstly, we studied some properties of $\mathcal{I}_2$-convergence. Then, we introduced $\mathcal{I}_2$-Cauchy and $\mathcal{I}_2^*$-Cauchy sequence of double sequences of functions in 2-normed space. Also, were investigated relationships between them for double sequences of functions in 2-normed spaces.

Theorem 3.1. *Let $\mathcal{I}_2 \subset 2^{\mathbb{N}\times\mathbb{N}}$ be a strongly admissible ideal with the property $(AP2)$. Then, for each $x \in X$ and each nonzero $z \in Y$, following conditions are equivalent*

(i) $\mathcal{I}_2 - \lim_{m,n\to\infty} \|f_{mn}(x), z\| = \|f(x), z\|$

(ii) *There exists $\{g_{mn}(x)\}$ and $\{h_{mn}(x)\}$ be two sequences of functions from X to Y such that*

$$f_{mn}(x) = g_{mn}(x)+h_{mn}(x), \lim_{m,n\to\infty} \|g_{mn}(x), z\| = \|f(x), z\| \text{ and } supp\{h_{mn}(x)\} \in \mathcal{I}_2,$$

where supp $h_{mn}(x) = \{(m, n) \in \mathbb{N}\times\mathbb{N} : h_{mn}(x) \neq 0\}$.

Proof. **(i)** $\Rightarrow$ **(ii)**: $\mathcal{I}_2 - \lim_{m,n\to\infty} \|f_{mn}(x), z\| = \|f(x), z\|$, for each $x \in X$ and each nonzero $z \in Y$. Then, by Lemma 2.2 there exists a set $M \in \mathcal{F}(\mathcal{I}_2)$ (*i.e.*, $H = \mathbb{N}\times\mathbb{N} \setminus M \in \mathcal{I}_2$) such that for each $x \in X$, each nonzero $z \in Y$ and all $(m, n) \in M$

$$\lim_{m,n\to\infty} \|f_{mn}(x), z\| = \|f(x), z\|.$$

Let us define the sequence $\{g_{mn}(x)\}$ by

$$g_{mn}(x) = \begin{cases} f_{mn}(x), & (m, n) \in M, \\ f(x), & (m, n) \in \mathbb{N}\times\mathbb{N} \setminus M. \end{cases} \tag{3.1}$$

It is clear that $\{g_{mn}(x)\}$ is a double sequence of functions and $\lim_{m,n\to\infty} \|g_{mn}(x), z\| = \|f(x), z\|$ for each $x \in X$ and each nonzero $z \in Y$. Besides let

$$h_{mn}(x) = f_{mn}(x) - g_{mn}(x),\ (m, n) \in \mathbb{N}\times\mathbb{N} \tag{3.2}$$

for each $x \in X$. Since

$$\{(m, n) \in \mathbb{N}\times\mathbb{N} : f_{mn}(x) \neq g_{mn}(x)\} \subset \mathbb{N}\times\mathbb{N} \setminus M \in \mathcal{I}_2,$$

for each $x \in X$, so we have

$$\{(m, n) \in \mathbb{N}\times\mathbb{N} : h_{mn}(x) \neq 0\} \in \mathcal{I}_2.$$

It follows that supp $h_{mn}(x) \in \mathcal{I}_2$ and by (3.1) and (3.2) we get $f_{mn}(x) = g_{mn}(x) + h_{mn}(x)$, for each $x \in X$.

(ii)$\Rightarrow$**(i)**: Assume that there exist two sequences $\{g_{mn}\}$ and $\{h_{mn}\}$ such that

$$f_{mn}(x) = g_{mn}(x) + h_{mn}(x), \lim_{m,n\to\infty} \|g_{mn}(x), z\| = \|f(x), z\| \tag{3.3}$$
$$\text{and } supph_{mn}(x) = \{(m, n) \in \mathbb{N}\times\mathbb{N} : h_{mn}(x) \neq 0\} \in \mathcal{I}_2$$

for each $x \in X$ and each nonzero $z \in Y$. We show that $\mathcal{I}_2 - \lim_{m,n\to\infty} \|f_{mn}(x), z\| = \|f(x), z\|$ for each $x \in X$ and each nonzero $z \in Y$. Let

$$M = \{(m, n) \in \mathbb{N}\times\mathbb{N} : h_{mn}(x) = 0\} = \mathbb{N}\times\mathbb{N} \setminus supph_{mn}(x). \tag{3.4}$$

Since *supp* $h_{mn}(x) = \{(m,n) \in \mathbb{N}\times\mathbb{N} : h_{mn}(x) \neq 0\} \in \mathcal{I}_2$, then from (3.3) and (3.4), we have $M \in \mathcal{F}(\mathcal{I}_2)$ and $f_{mn}(x) = g_{mn}(x)$ for $(m,n) \in M$. Hence, we conclude that exists a set $M \in \mathcal{F}(\mathcal{I}_2)$, (*i.e.*, $H = \mathbb{N}\times\mathbb{N}\setminus M \in \mathcal{I}_2$) such that $\lim_{m,n\to\infty} \|f_{mn}(x), z\| = \|f(x), z\|$ and so

$$\mathcal{I}_2^* - \lim_{m,n\to\infty} \|f_{mn}(x), z\| = \|f(x), z\|$$

for $(m,n) \in M$, each $x \in X$ and each nonzero $z \in Y$. By Lemma 2.2 it follows that

$$\mathcal{I}_2 - \lim_{m,n\to\infty} \|f_{mn}(x), z\| = \|f(x), z\|,$$

for each $x \in X$ and each nonzero $z \in Y$. This completes the proof. □

Corollary 3.1. *Let $\mathcal{I}_2 \subset 2^{\mathbb{N}\times\mathbb{N}}$ be a strongly admissible ideal having the property $(AP2)$. Then, $\mathcal{I}_2 - \lim_{m,n\to\infty} \|f_{mn}(x), z\| = \|f(x), z\|$ if and only if there exist $\{g_{mn}\}$ and $\{h_{mn}\}$ be two sequences of functions from X to Y such that*

$$f_{mn}(x) = g_{mn}(x)+h_{mn}(x), \lim_{m,n\to\infty} \|g_{mn}(x), z\| = \|f(x), z\| \text{ and } \mathcal{I}_2 - \lim_{m,n\to\infty} \|h_{mn}(x), z\| = 0,$$

for each $x \in X$ and each nonzero $z \in Y$.

Proof. Let $\mathcal{I}_2 - \lim_{m,n\to\infty} \|f_{mn}(x), z\| = \|f(x), z\|$ and $\{g_{mn}(x)\}$ is sequence defined by (3.1). Consider the sequence

$$h_{mn}(x) = f_{mn}(x) - g_{mn}(x), \quad (m,n) \in \mathbb{N}\times\mathbb{N} \tag{3.5}$$

for each $x \in X$. Then, we have

$$\lim_{m,n\to\infty} \|g_{mn}(x), z\| = \|f(x), z\|$$

and since $\mathcal{I}_2$ is a strongly admissible ideal so

$$\mathcal{I}_2 - \lim_{m,n\to\infty} \|g_{mn}(x), z\| = \|f(x), z\|,$$

for each $x \in X$ and each nonzero $z \in Y$. By Lemma 2.4 and by (3.5) we have

$$\mathcal{I}_2 - \lim_{m,n\to\infty} \|h_{mn}(x), z\| = 0,$$

for each $x \in X$ and each nonzero $z \in Y$. Now, let

$$f_{mn}(x) = g_{mn}(x) + h_{mn}(x),$$

where

$$\lim_{m,n\to\infty} \|g_{mn}(x), z\| = \|f(x), z\| \text{ and } \mathcal{I}_2 - \lim_{m,n\to\infty} \|h_{mn}(x), z\| = 0,$$

for each $x \in X$ and each nonzero $z \in Y$. Since $\mathcal{I}_2$ is a strongly admissible ideal so

$$\mathcal{I}_2 - \lim_{m,n\to\infty} \|g_{mn}(x), z\| = \|f(x), z\|$$

and by Lemma 2.4 we get

$$\mathcal{I}_2 - \lim_{m,n\to\infty} \|f_{mn}(x), z\| = \|f(x), z\|,$$

for each $x \in X$ and each nonzero $z \in Y$. $\square$

Remark 3.1. In Theorem 3.1 if **(ii)** is satisfied then the admissible ideal $\mathcal{I}_2$ need not have the property $(AP2)$. Since for each $x \in X$ and each nonzero $z \in Y$,

$$\{(m,n) \in \mathbb{N}\times\mathbb{N} : \|h_{mn}(x), z\| \geq \varepsilon\} \subset \{(m,n) \in \mathbb{N}\times\mathbb{N} : h_{mn}(x) \neq 0\} \in \mathcal{I}_2,$$

for each $\varepsilon > 0$, then

$$\mathcal{I}_2 - \lim_{m,n\to\infty} \|h_{mn}(x), z\| = 0.$$

Hence, we have

$$\mathcal{I}_2 - \lim_{m,n\to\infty} \|f_{mn}(x), z\| = \|f(x), z\|,$$

for each $x \in X$ and each nonzero $z \in Y$.

Definition 3.1. A double sequence of functions $\{f_{mn}\}$ is said to be $\mathcal{I}_2$-Cauchy sequence, if for every $\forall\varepsilon > 0$ and each $x \in X$ there exist $s = s(\varepsilon, x)$, $t = t(\varepsilon, x) \in \mathbb{N}$ such that

$$\{(m,n) \in \mathbb{N}\times\mathbb{N} : \|f_{mn}(x) - f_{st}(x), z\| \geq \varepsilon\} \in \mathcal{I}_2,$$

for each nonzero $z \in Y$.

Theorem 3.2. *If $\{f_{mn}\}$ is $\mathcal{I}_2$-convergent if and only if it is $\mathcal{I}_2$-Cauchy sequence in 2-normed spaces.*

Proof. Assume that $\{f_{mn}\}$ is $\mathcal{I}_2$-convergent to f. Then, for $\varepsilon > 0$

$$A\left(\frac{\varepsilon}{2}, z\right) = \left\{(m,n) \in \mathbb{N}\times\mathbb{N} : \|f_{mn}(x) - f(x), z\| \geq \frac{\varepsilon}{2}\right\} \in \mathcal{I}_2,$$

for each $x \in X$ and each nonzero $z \in Y$. This implies that

$$A^c\left(\frac{\varepsilon}{2}, z\right) = \left\{(m,n) \in \mathbb{N}\times\mathbb{N} : \|f_{mn}(x) - f(x), z\| < \frac{\varepsilon}{2}\right\} \in \mathcal{F}(\mathcal{I}_2).$$

for each $x \in X$ and each nonzero $z \in Y$ and thus $A^c\left(\frac{\varepsilon}{2}, z\right)$ is non-empty. So we can select a positive integers k, l such that $(k,l) \notin A\left(\frac{\varepsilon}{2}, z\right)$ and $\|f_{kl}(x) - f(x), z\| < \frac{\varepsilon}{2}$. Now, we define the set

$$B(\varepsilon, z) = \{(m,n) \in \mathbb{N}\times\mathbb{N} : \|f_{mn}(x) - f_{kl}(x), z\| \geq \varepsilon\},$$

for each $x \in X$ and each nonzero $z \in Y$, such that we show that $B(\varepsilon, z) \subset A(\frac{\varepsilon}{2}, z)$. Let $(m,n) \in B(\varepsilon, z)$, then we have

$$\begin{aligned}\varepsilon \leq \|f_{mn}(x) - f_{kl}(x), z\| &\leq \|f_{mn}(x) - f(x), z\| + \|f_{kl}(x) - f(x), z\| \\ &< \|f_{mn}(x) - f(x), z\| + \frac{\varepsilon}{2},\end{aligned}$$

for each $x \in X$ and each nonzero $z \in Y$. This implies that $\frac{\varepsilon}{2} < \|f_{mn}(x) - f(x), z\|$ and so, $(m,n) \in A(\frac{\varepsilon}{2}, z)$. Hence, we have $B(\varepsilon, z) \subset A(\frac{\varepsilon}{2}, z)$ and so $\{f_{mn}\}$ is $\mathcal{I}_2$-Cauchy sequence.

Conversely, assume that $\{f_{mn}\}$ is $\mathcal{I}_2$-Cauchy sequence. We prove that $\{f_{mn}\}$ is $\mathcal{I}_2$-convergent. Let (ε_{pq}) be a strictly decreasing sequence of number converging to zero since $\{f_{mn}\}$ is $\mathcal{I}_2$-Cauchy sequence, there exist two strictly increasing sequences (k_p) and (l_q) of positive integers such that

$$A(\varepsilon_{pq}, z) = \{(m,n) \in \mathbb{N} \times \mathbb{N} : \|f_{mn}(x) - f_{k_p l_q}(x), z\| \geq \varepsilon_{pq}\} \in \mathcal{I}_2, (p,q = 1,2,...),$$

for each $x \in X$ and each nonzero $z \in Y$. This implies that

$$\emptyset \neq \{(m,n) \in \mathbb{N} \times \mathbb{N} : \|f_{mn}(x) - f_{k_p l_q}(x), z\| < \varepsilon_{pq}\} \in \mathcal{F}(\mathcal{I}_2), \tag{3.6}$$

$(p,q = 1,2,...)$, for each $x \in X$ and each nonzero $z \in Y$. Let p,q,s and t be four positive integers such that $p \neq q$ and $s \neq t$. By (3.6), both the sets

$$C(\varepsilon_{pq}, z) = \{(m,n) \in \mathbb{N} \times \mathbb{N} : \|f_{mn}(x) - f_{k_p l_q}(x), z\| < \varepsilon_{pq}\}$$

and

$$D(\varepsilon_{st}, z) = \{(m,n) \in \mathbb{N} \times \mathbb{N} : \|f_{mn}(x) - f_{k_s l_t}(x), z\| < \varepsilon_{st}\}$$

are non empty sets in $\mathcal{F}(\mathcal{I}_2)$, for each $x \in X$ and each nonzero $z \in Y$. Since $\mathcal{F}(\mathcal{I}_2)$ is a filter on $\mathbb{N} \times \mathbb{N}$, so

$$\emptyset \neq C(\varepsilon_{pq}, z) \cap D(\varepsilon_{st}, z) \in \mathcal{F}(\mathcal{I}_2).$$

Therefore, for each pair (p,q) and (s,t) of positive integers with $p \neq q$ and $s \neq t$, we can select a pair $(m_{(p,q),(s,t)}, n_{(p,q),(s,t)}) \in \mathbb{N} \times \mathbb{N}$ such that

$$\|f_{m_{pqst} n_{pqst}}(x) - f_{k_p l_q}(x), z\| < \varepsilon_{pq} \text{ and } \|f_{m_{pqst} n_{pqst}}(x) - f_{k_s l_t}(x), z\| < \varepsilon_{st},$$

for each $x \in X$ and each nonzero $z \in Y$. It follows that

$$\begin{aligned}\|f_{k_p l_q}(x) - f_{k_s l_t}(x), z\| &\leq \|f_{m_{pqst} n_{pqst}}(x) - f_{k_p l_q}(x), z\| \\ &\quad + \|f_{m_{pqst} n_{pqst}}(x) - f_{k_s l_t}(x), z\| \\ &\leq \varepsilon_{pq} + \varepsilon_{st} \to 0,\end{aligned}$$

as $p,q,s,t \to \infty$. This implies that $\{f_{k_p l_q}\}$ $(p,q = 1,2,...)$ is a Cauchy sequence and therefore it satisfies the Cauchy convergence criterion. Thus, the sequence $\{f_{k_p l_q}\}$ converges to a limit f (say) i,e.,

$$\lim_{p,q \to \infty} \|f_{k_p l_q}, z\| = \|f(x), z\|,$$

for each $x \in X$ and each nonzero $z \in Y$. Also, we have $\varepsilon_{pq} \to 0$ as $p, q \to \infty$, so for each $\varepsilon > 0$ we can choose positive integers p_0, q_0 such that

$$(3.7) \qquad \varepsilon_{p_0q_0} < \frac{\varepsilon}{2} \text{ and } \|f_{k_pl_q} - f(x), z\| < \frac{\varepsilon}{2}, \ \ (for\ p > p_0 \text{ and } q > q_0).$$

Now, we define the set

$$A(\varepsilon, z) = \{(m, n) \in \mathbb{N} \times \mathbb{N} : \|f_{mn}(x) - f(x), z\| \geq \varepsilon\},$$

for each $x \in X$ and each nonzero $z \in Y$. We prove that $A(\varepsilon, z) \subset A(\varepsilon_{p_0q_0}, z)$. Let $(m, n) \in A(\varepsilon, z)$, then by second half of (3.7) we have

$$\begin{aligned} \varepsilon \leq \|f_{mn}(x) - f(x), z\| &\leq \|f_{mn}(x) - f_{k_{p_0}l_{q_0}}(x), z\| + \|f_{k_{p_0}l_{q_0}}(x) - f(x), z\| \\ &\leq \|f_{mn}(x) - f_{k_{p_0}l_{q_0}}(x), z\| + \frac{\varepsilon}{2}, \end{aligned}$$

for each $x \in X$ and each nonzero $z \in Y$. This implies that

$$\frac{\varepsilon}{2} < \|f_{mn}(x) - f_{k_{p_0}l_{q_0}}(x), z\|$$

and therefore by first half of (3.7)

$$\varepsilon_{p_0q_0} < \|f_{mn}(x) - f_{k_{p_0}l_{q_0}}(x), z\|,$$

for each $x \in X$ and each nonzero $z \in Y$. Thus, we have $(m, n) \in A(\varepsilon_{p_0q_0}, z)$ and therefore $A(\varepsilon, z) \subset A(\varepsilon_{p_0q_0}, z)$. Since $A(\varepsilon_{p_0q_0}, z) \in \mathcal{I}_2$ so $A(\varepsilon, z) \in \mathcal{I}_2$ by property of ideal. Hence $\{f_{k_pl_q}\}$ is $\mathcal{I}_2$-convergent. □

Definition 3.2. A double sequence of functions $\{f_{mn}\}$ is said to be $\mathcal{I}_2^*$- Cauchy sequence, if there exists a set $M \in \mathcal{F}(\mathcal{I}_2)$ (*i.e.*, $H = \mathbb{N} \times \mathbb{N} \setminus M \in \mathcal{I}_2$) and for every $\varepsilon > 0$ and each $x \in X$, $k_0 = k_0(\varepsilon, x) \in \mathbb{N}$ such that for all $(m, n), (s, t) \in M$ and each $z \in Y$

$$\|f_{mn}(x) - f_{st}(x), z\| < \varepsilon,$$

whenever $m, n, s, t > k_0$. In this case, we write

$$\lim_{m,n,s,t\to\infty} \|f_{mn}(x) - f_{st}(x), z\| = 0.$$

Theorem 3.3. *If double sequence of functions $\{f_{mn}\}$ is a $\mathcal{I}_2^*$-Cauchy sequence, then it is $\mathcal{I}_2$-Cauchy sequence in 2-normed spaces.*

Proof. Let $\{f_{mn}\}$ is a $\mathcal{I}_2^*$-Cauchy sequence in 2-normed spaces. Then, by definition there exists a set $M \in \mathcal{F}(\mathcal{I}_2)$ (*i.e.*, $H = \mathbb{N} \times \mathbb{N} \setminus M \in \mathcal{I}_2$) and for every $\varepsilon > 0$ and each $x \in X$, $k_0 = k_0(\varepsilon, x) \in \mathbb{N}$ such that for all $(m, n), (s, t) \in M$ and each $z \in Y$

$$\|f_{mn}(x) - f_{st}(x), z\| < \varepsilon,$$

whenever $m, n, s, t > k_0$. Then, for each $x \in X$ and nonzero each $z \in Y$ we have

$$\begin{array}{rcl} A(\varepsilon, z) & = & \{(m,n) \in \mathbb{N} \times \mathbb{N} : \|f_{mn}(x) - f_{st}(x), z\| \geq \varepsilon\} \\ & \subset & H \cup [M \cap ((\{1,2,3,...,(k_0-1)\} \times \mathbb{N}) \cup (\mathbb{N} \times \{1,2,3,...,(k_0-1)\}))] \end{array}$$

Since $\mathcal{I}_2$ is an admissible ideal, then

$$H \cup [M \cap ((\{1,2,3,...,(k_0-1)\} \times \mathbb{N}) \cup (\mathbb{N} \times \{1,2,3,...,(k_0-1)\}))] \in \mathcal{I}_2.$$

Therefore, we have $A(\varepsilon, z) \in \mathcal{I}_2$ i.e., $\{f_{mn}\}$ is a $\mathcal{I}_2$-Cauchy sequence. □

Theorem 3.4. *If* $\mathcal{I}_2^* - \lim_{m,n\to\infty} \|f_{mn}(x) - f(x), z\| = 0$, *then* $\{f_{mn}\}$ *is* $\mathcal{I}_2$*-Cauchy sequence in 2-normed spaces.*

Proof. By assumption there exists a set $M \in \mathcal{F}(\mathcal{I}_2)$ $(i.e., H = \mathbb{N} \times \mathbb{N}\ M \in \mathcal{I}_2)$ such that $\lim_{m,n\to\infty} \|f_{mn}(x) - f(x), z\| = 0$ for each $x \in X$ and each $z \in Y$. It shows that for each $\varepsilon > 0$ there exists $k_0 = k_0(\varepsilon, x) \in \mathbb{N}$ such that for each $x \in X$, each $z \in Y$

$$\|f_{mn}(x) - f(x), z\| < \frac{\varepsilon}{2}$$

for all $(m, n) \in M$ and $m, n > k_0$. Since for each $\varepsilon > 0$,

$$\begin{array}{rcl} \|f_{mn}(x) - f_{st}(x), z\| & \leq & \|f_{mn}(x) - f(x), z\| + \|f_{st}(x) - f(x), z\| \\ & < & \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon, \end{array}$$

for each $x \in X$, each $z \in Y$ and $m, n, s, t \geq k_0$ we have

$$\lim_{m,n,s,t\to\infty} \|f_{mn}(x) - f_{st}(x), z\| = 0,$$

i.e., $\{f_{mn}\}$ is a $\mathcal{I}_2^*$-Cauchy sequence. Then, by Theorem 3.3 $\{f_{mn}\}$ is $\mathcal{I}_2$-Cauchy sequence. □

Theorem 3.5. *Let* $\mathcal{I}_2$ *be an admissible ideal with property* $(AP2)$ *and a double sequence of functions* $\{f_{mn}\}$*. Then, the concepts* $\mathcal{I}_2$*-Cauchy double sequence and* $\mathcal{I}_2^*$*-Cauchy double sequence of functions coincide in 2-normed spaces.*

Proof. By Theorem 3.3 $\mathcal{I}_2^*$-Cauchy sequence implies $\mathcal{I}_2$-Cauchy sequence (in this case $\mathcal{I}_2$ need not to have $(AP2)$ condition).

Now, it is sufficient to prove that a double sequence $\{f_{mn}\}$ is a $\mathcal{I}_2^*$-Cauchy double sequence under assumption that $\{f_{mn}\}$ is a $\mathcal{I}_2$-Cauchy double sequence. Let $\{f_{mn}\}$ is a $\mathcal{I}_2$-Cauchy double sequence. Then, for every $\varepsilon > 0$ and each $x \in X$ there exists $s = s(\varepsilon, z), t = t(\varepsilon, z) \in \mathbb{N}$ such that

$$A(\varepsilon, z) = \{(m,n) \in \mathbb{N} \times \mathbb{N} : \|f_{mn}(x) - f_{st}(x), z\| \geq \varepsilon\} \in \mathcal{I}_2$$

for each nonzero $z \in Y$. Let

$$P_i = \left\{(m,n) \in \mathbb{N}\times\mathbb{N} : \|f_{mn}(x) - f_{s_i t_i}(x), z\| < \frac{1}{i}\right\},\ (i = 1,2,...),$$

where $s = s(\frac{1}{i}),\ t = t(\frac{1}{i})$. It is clear that

$$P_i \in \mathcal{F}(\mathcal{I}_2),\ (i = 1,2,...).$$

Since $\mathcal{I}_2$ has $(AP2)$ property then by Lemma 2.3 there exists a set $P \subset \mathbb{N}\times\mathbb{N}$ such that $P \in F(\mathcal{I}_2)$ and $P\backslash P_i$ is finite for all i. Now we show that

$$\lim_{m,n,s,t\to\infty} \|f_{mn}(x) - f_{st}(x), z\| = 0$$

for each $x \in X$, $(m,n),(s,t) \in P$ and each nonzero $z \in Y$. Let $\varepsilon > 0$ and $j \in \mathbb{N}$ such that $j > \frac{2}{\varepsilon}$, if $(m,n),(s,t) \in P$ then $P\backslash P_i$ is a finite set, so there exists $k = k(j)$ such that $(m,n),(s,t) \in P_j$ for all $m,n,s,t > k(j)$. Therefore, for each $x \in X$

$$\|f_{mn}(x) - f_{s_j t_j}(x), z\| < \frac{1}{j} \text{ and } \|f_{st}(x) - f_{s_j t_j}(x), z\| < \frac{1}{j},$$

for each nonzero $z \in Y$ and all $m,n,s,t > k(j)$. Hence, for each $x \in X$ it follows that

$$\begin{aligned}\|f_{mn}(x) - f_{st}(x), z\| &\le \|f_{mn}(x) - f_{s_j t_j}(x), z\| + \|f_{st}(x) - f_{s_j t_j}(x), z\| \\ &< \frac{1}{j} + \frac{1}{j} = \frac{2}{j} < \varepsilon\end{aligned}$$

for all $m,n,s,t > k(j)$ and each nonzero $z \in Y$. Therefore, for any $\varepsilon > 0$ and each $x \in X$ there exists $k = k(\varepsilon, x)$ such that for $m,n,s,t > k$ and $(m,n),(s,t) \in P \in \mathcal{F}(\mathcal{I}_2)$

$$\|f_{mn}(x) - f_{st}(x), z\| < \varepsilon,$$

for each nonzero $z \in Y$ and so, the sequence $\{f_{mn}\}$ is a $\mathcal{I}_2^*$-Cauchy sequence in 2-normed space. □

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