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EIGHTY ONE RICCI-TYPE IDENTITIES *

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Abstract. In this manuscript, the identities of Ricci Type with respect to a non-symmetric affine connection space are obtained and simplified. The components of commutation formulae are discussed.

Key words: covariant derivative; identities of Ricci Type; commutation formula.

1. Introduction

An N-dimensional manifold $\mathcal{M}_N$ equipped with an affine connection with torsion ∇ is the non-symmetric affine connection space $\mathbb{GA}_N$ $\big($see L. P. Eisenhart [1], S. M. Minčić [4–6, 6–8]$\big)$, M. S. Stanković [13], Lj. S. Velimirović [10, 11], M. Lj. Zlatanović [13, 14], M. Z. Petrović [9–11]. The non-symmetric affine connection spaces are subjects of research for many other authors but our aim is to examine some basic facts about these spaces in this paper.

The affine connection coefficients for the affine connection ∇ are L^i_{jk}. These coefficients are non-symmetric by indices j and k. Hence, their symmetric and anti-symmetric parts are defined as

$$L^i_{\underline{jk}} = \frac{1}{2}\big(L^i_{jk} + L^i_{kj}\big) \quad \text{and} \quad L^i_{\underset{\vee}{jk}} = \frac{1}{2}\big(L^i_{jk} - L^i_{kj}\big). \tag{1.1}$$

Four kinds of covariant derivatives with respect to the non-symmetric affine connection ∇ are defined. Coordinately, these four types $\big($for a tensor a^i_j of the type $(1,1)$ $\big)$ are [4–11, 13, 14]

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$$
(1.2)\qquad
\begin{aligned}
&a^i_{j\underset{1}{|}k} = a^i_{j,k} + L^i_{\alpha k} a^\alpha_j - L^\alpha_{jk} a^i_\alpha, \quad a^i_{j\underset{2}{|}k} = a^i_{j,k} + L^i_{k\alpha} a^\alpha_j - L^\alpha_{kj} a^i_\alpha,\\
&a^i_{j\underset{3}{|}k} = a^i_{j,k} + L^i_{\alpha k} a^\alpha_j - L^\alpha_{kj} a^i_\alpha, \quad a^i_{j\underset{4}{|}k} = a^i_{j,k} + L^i_{k\alpha} a^\alpha_j - L^\alpha_{jk} a^i_\alpha.
\end{aligned}
$$

In the case of $L^i_{\underset{\vee}{jk}} = 0$, the four kinds of covariant derivatives (1.2) reduce to one kind [2, 12]

$$
(1.3)\qquad a^i_{j|k} = a^i_{j\underset{0}{|}k} = a^i_{j,k} + L^i_{\underline{\alpha k}} a^\alpha_j - L^\alpha_{\underline{jk}} a^i_\alpha,
$$

Proposition 1.1. *The fourth kind of the covariant derivative expressed in* (1.2) *and the covariant derivative with respect to the symmetric affine connection given by* (1.3) *satisfy the equalities*

$$
(1.4)\qquad
\begin{aligned}
a^i_{j\underset{4}{|}k} &= a^i_{j\underset{1}{|}k} + a^i_{j\underset{2}{|}k} - a^i_{j\underset{3}{|}k},\\
a^i_{j|k} &= \frac{1}{2} a^i_{j\underset{1}{|}k} + \frac{1}{2} a^i_{j\underset{2}{|}k}.
\end{aligned}
$$

If $L^i_{\underset{\vee}{jk}} \not\equiv 0$, the geometrical objects $a^i_{j\underset{1}{|}k}$, $a^i_{j\underset{2}{|}k}$, $a^i_{j\underset{3}{|}k}$ are linearly independent.

Proof. With respect to the equalities $L^i_{jk} = L^i_{\underline{jk}} + L^i_{\underset{\vee}{jk}}$, $L^i_{\underset{\vee}{jk}} = -L^i_{\underset{\vee}{kj}}$ and the equation (1.3), one gets

$$
(1.5)\qquad
\begin{aligned}
&a^i_{j\underset{1}{|}k} = a^i_{j|k} + L^i_{\underset{\vee}{\alpha k}} a^\alpha_j - L^\alpha_{\underset{\vee}{jk}} a^i_\alpha, \quad a^i_{j\underset{2}{|}k} = a^i_{j|k} - L^i_{\underset{\vee}{\alpha k}} a^\alpha_j + L^\alpha_{\underset{\vee}{jk}} a^i_\alpha,\\
&a^i_{j\underset{3}{|}k} = a^i_{j|k} + L^i_{\underset{\vee}{\alpha k}} a^\alpha_j + L^\alpha_{\underset{\vee}{jk}} a^i_\alpha, \quad a^i_{j\underset{4}{|}k} = a^i_{j|k} - L^i_{\underset{\vee}{\alpha k}} a^\alpha_j - L^\alpha_{\underset{\vee}{jk}} a^i_\alpha,
\end{aligned}
$$

From the expressions (1.5), one obtains [9, 10]

$$
a^i_{j\underset{4}{|}k} = a^i_{j\underset{1}{|}k} + a^i_{j\underset{2}{|}k} - a^i_{j\underset{3}{|}k} \quad \text{and} \quad a^i_{j|k} = \frac{1}{2} a^i_{j\underset{1}{|}k} + \frac{1}{2} a^i_{j\underset{2}{|}k},
$$

which proves the first part of this proposition.

Furthermore, the geometrical objects $a^i_{j\underset{1}{|}k}$, $a^i_{j\underset{2}{|}k}$, $a^i_{j\underset{3}{|}k}$ expressed as in the equation (1.5) may be considered as the vectors $v_1 = (1, 1, -1)$, $v_2 = (1, -1, 1)$, $v_3 = (1, 1, 1)$. These vectors are linearly independent, which completes the proof for this proposition. □

Curvatures of the space $\mathbb{GA}_N$ are $a^i_{j\underset{v_1}{|}m\underset{w_1}{|}n} - a^i_{j\underset{v_2}{|}n\underset{w_2}{|}m}$, for $v_1, v_2, w_1, w_2 \in \{0,1,2,3,4\}$. We will study the curvatures of the space $\mathbb{GA}_N$ obtained with respect to the first three kinds of covariant derivatives (1.2) in this paper.

Our purpose is to coordinately express the curvatures of the space $\mathbb{GA}_N$ with respect to first three kinds of covariant derivatives (1.2) in this paper. We will obtain the coordinates of the differences $a^i_{j\underset{v_1}{|}m\underset{w_1}{|}n} - a^i_{j\underset{v_2}{|}n\underset{w_2}{|}m}$, for $v_1, v_2, w_1, w_2 \in \{1,2,3\}$. The pseudocurvature tensors as possible components of these differences will be discussed. The number of linearly independent geometrical objects $a^i_{j\underset{v_1}{|}m\underset{w_1}{|}n} - a^i_{j\underset{v_2}{|}n\underset{w_2}{|}m}$, $v_1, v_2, w_1, w_2 \in \{1,2,3\}$, will be obtained. At the end of the paper, we will list all of the commutation formulae with respect to $a^i_{j\underset{v_1}{|}m\underset{w_1}{|}n} - a^i_{j\underset{v_2}{|}n\underset{w_2}{|}m}$, $v_1, v_2, w_1, w_2 \in \{1,2,3\}$.

2. Identities of Ricci type

With respect to the equations (1.3, 1.5), one gets

$$a^i_{j\underset{v}{|}k} = a^i_{j|k} + c_v L^i_{\underset{\vee}{\alpha k}} a^\alpha_j + d_v L^\alpha_{\underset{\vee}{jk}} a^i_\alpha, \tag{2.1}$$

for $v = 0, \dots, 4$ and $c_0 = 0$, $c_1 = 1$, $c_2 = -1$, $c_3 = 1$, $c_4 = -1$, $d_0 = 0$, $d_1 = -1$, $d_2 = 1$, $d_3 = 1$, $d_4 = -1$.

Moreover, it holds the equation

$$\begin{aligned} a^i_{j\underset{v}{|}m\underset{w}{|}n} &= a^i_{j|m|n} + c_v L^i_{\underset{\vee}{\alpha m}} a^\alpha_{j|n} + c_w L^i_{\underset{\vee}{\alpha n}} a^\alpha_{j|m} + d_v L^\alpha_{\underset{\vee}{jm}} a^i_{\alpha|n} + d_w L^\alpha_{\underset{\vee}{jn}} a^i_{\alpha|m} + d_w L^\alpha_{\underset{\vee}{mn}} a^i_{j|\alpha} \\ &+ a^\alpha_j \big(c_v L^i_{\underset{\vee}{\alpha m}|n} + c_v c_w L^\beta_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} + c_v(c_w + d_w) L^\beta_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}} - c_v d_w L^\beta_{\underset{\vee}{mn}} L^i_{\underset{\vee}{\beta\alpha}} \big) \\ &- a^i_\alpha \big(- d_v L^\alpha_{\underset{\vee}{jm}|n} - d_v(c_w + d_w) L^\beta_{\underset{\vee}{jm}} L^\alpha_{\underset{\vee}{\beta n}} - d_v d_w L^\beta_{\underset{\vee}{jn}} L^\alpha_{\underset{\vee}{\beta m}} + d_v d_w L^\beta_{\underset{\vee}{mn}} L^\alpha_{\underset{\vee}{\beta j}} \big) \\ &+ a^\alpha_\beta \big(c_w d_v L^\beta_{\underset{\vee}{jm}} L^i_{\underset{\vee}{\alpha n}} + c_v d_w L^\beta_{\underset{\vee}{jn}} L^i_{\underset{\vee}{\alpha m}} \big), \end{aligned} \tag{2.2}$$

for $v, w \in \{0,1,2,3,4\}$.

The next theorem holds.

Theorem 2.1. First Ricci-Type Identities Theorem *The family of identities of the Ricci Type with respect to a non-symmetric affine connection* ∇ *is*

$$
\begin{aligned}
a^i_{j\underset{v_1}{|}m\underset{w_1}{|}n}-a^i_{j\underset{v_2}{|}n\underset{w_2}{|}m} &= (c_{v_1}-c_{w_2})L^i_{\underset{\vee}{\alpha m}}a^\alpha_{j|n}+(c_{w_1}-c_{v_2})L^i_{\underset{\vee}{\alpha n}}a^\alpha_{j|m}+(d_{v_1}-d_{w_2})L^\alpha_{\underset{\vee}{jm}}a^i_{\alpha|n}\\
&+(d_{w_1}-d_{v_2})L^\alpha_{\underset{\vee}{jn}}a^i_{\alpha|m}+(d_{w_1}+d_{w_2})L^\alpha_{\underset{\vee}{mn}}a^i_{j|\alpha}\\
&+a^\alpha_j\Big\{R^i_{\alpha mn}+c_{v_1}L^i_{\underset{\vee}{\alpha m}|n}-c_{v_2}L^i_{\underset{\vee}{\alpha n}|m}\\
&\qquad+\big[c_{v_1}c_{w_1}-c_{v_2}(c_{w_2}+d_{w_2})\big]L^\beta_{\underset{\vee}{\alpha m}}L^i_{\underset{\vee}{\beta n}}\\
&\qquad+\big[c_{v_1}(c_{w_1}+d_{w_1})-c_{v_2}c_{w_2}\big]L^\beta_{\underset{\vee}{\alpha n}}L^i_{\underset{\vee}{\beta m}}\\
&\qquad-(c_{v_1}d_{w_1}+c_{v_2}d_{w_2})L^\beta_{\underset{\vee}{mn}}L^i_{\underset{\vee}{\beta\alpha}}\Big\}\\
&-a^i_\alpha\Big\{R^\alpha_{jmn}-d_{v_1}L^\alpha_{\underset{\vee}{jm}|n}+d_{v_2}L^\alpha_{\underset{\vee}{jn}|m}\\
&\qquad-\big[d_{v_1}(c_{w_1}+d_{w_1})-d_{v_2}d_{w_2}\big]L^\beta_{\underset{\vee}{jm}}L^\alpha_{\underset{\vee}{\beta n}}\\
&\qquad-\big[d_{v_1}d_{w_1}-d_{v_2}(c_{w_2}+d_{w_2})\big]L^\beta_{\underset{\vee}{jn}}L^\alpha_{\underset{\vee}{\beta m}}\\
&\qquad+(d_{v_1}d_{w_1}+d_{v_2}d_{w_2})L^\beta_{\underset{\vee}{mn}}L^\alpha_{\underset{\vee}{\beta j}}\Big\}\\
&+a^\alpha_\beta\big\{(c_{w_1}d_{v_1}-c_{v_2}d_{w_2})L^\beta_{\underset{\vee}{jm}}L^i_{\underset{\vee}{\alpha n}}+(c_{v_1}d_{w_1}-c_{w_2}d_{v_2})L^\beta_{\underset{\vee}{jn}}L^i_{\underset{\vee}{\alpha m}}\big\},
\end{aligned}
\tag{2.3}
$$

for $v_1, v_2, w_1, w_2 \in \{0,1,2,3,4\}$. □

From this theorem, we obtain that just tensors are components of the curvatures for the space $\mathbb{GA}_N$.

The rank of the matrix of the type 81×19 whose rows are composed of the elements

$$
\begin{aligned}
&c_{v_1}-c_{w_2},\quad c_{w_1}-c_{v_2},\quad d_{v_1}-d_{w_2},\quad d_{w_1}-d_{v_2},\quad d_{w_1}+d_{w_2},\\
&1,\quad c_{v_1},\quad -c_{v_2},\quad c_{v_1}c_{w_1}-c_{v_2}(c_{w_2}+d_{w_2}),\quad c_{v_1}(c_{w_1}+d_{w_1})-c_{v_2}c_{w_2},\quad -(c_{v_1}d_{w_1}+c_{v_2}d_{w_2}),\\
&-1,\quad d_{v_1},\quad -d_{v_2},\quad d_{v_1}(c_{w_1}+d_{w_1})-d_{v_2}d_{w_2},\quad d_{v_1}d_{w_1}-d_{v_2}(c_{w_2}+d_{w_2}),\quad -(d_{v_1}d_{w_1}+d_{v_2}d_{w_2}),\\
&c_{w_1}d_{v_1}-c_{v_2}d_{w_2},\quad c_{v_1}d_{w_1}-c_{w_2}d_{v_2},
\end{aligned}
$$

for $v_1, v_2, w_1, w_2 \in \{1,2,3\}$, is 15.

In this way, we proved the next theorem.

Theorem 2.2. $1-2-3$-Commutation Formulae Theorem *Fifteen of the geometrical objects* $a^i_{j\underset{v_1}{|}m\underset{w_1}{|}n}-a^i_{j\underset{v_2}{|}n\underset{w_2}{|}m}$, *for* $v_1, v_2, w_1, w_2 \in \{1,2,3\}$, *are linearly independent.* □

One may check that the geometrical objects

$$
\begin{aligned}
\mathcal{B}^i_{(1).jmn} &= a^i_{j\underset{1}{|}m\underset{1}{|}n} - a^i_{j\underset{1}{|}n\underset{1}{|}m} \\
&= -2L^\alpha_{\underset{\vee}{mn}} a^i_{j|\alpha} + a^\alpha_j \big(R^i_{\alpha mn} + L^i_{\underset{\vee}{\alpha m}|n} - L^i_{\underset{\vee}{\alpha n}|m} + L^\beta_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} - L^\beta_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}} + 2L^\beta_{\underset{\vee}{mn}} L^i_{\underset{\vee}{\alpha j}}\big) \\
&\quad - a^i_\alpha \big(R^\alpha_{jmn} + L^\alpha_{\underset{\vee}{jm}|n} - L^\alpha_{\underset{\vee}{jn}|m} + L^\beta_{\underset{\vee}{jm}} L^\alpha_{\underset{\vee}{\alpha n}} - L^\beta_{\underset{\vee}{jn}} L^\alpha_{\underset{\vee}{\alpha m}} + 2L^\beta_{\underset{\vee}{mn}} L^\alpha_{\underset{\vee}{\beta j}}\big),
\end{aligned} \tag{2.4}
$$

$$
\begin{aligned}
\mathcal{B}^i_{(2).jmn} &= a^i_{j\underset{1}{|}m\underset{1}{|}n} - a^i_{j\underset{1}{|}n\underset{2}{|}m} \\
&= 2L^i_{\underset{\vee}{\alpha m}} a^\alpha_{j|n} - 2L^\alpha_{\underset{\vee}{jm}} a^i_{\alpha|n} - 2a^\alpha_\beta \big(L^\beta_{\underset{\vee}{jm}} L^i_{\underset{\vee}{\alpha n}} + L^\beta_{\underset{\vee}{jn}} L^i_{\underset{\vee}{\alpha m}}\big) \\
&\quad + a^\alpha_j \big(R^i_{\alpha mn} + L^i_{\underset{\vee}{\alpha m}|n} - L^i_{\underset{\vee}{\alpha n}|m} + L^\beta_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} + L^i_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}}\big) \\
&\quad - a^i_\alpha \big(R^\alpha_{jmn} + L^\alpha_{\underset{\vee}{jm}|n} - L^\alpha_{\underset{\vee}{jn}|m} - L^\beta_{\underset{\vee}{jm}} L^\alpha_{\underset{\vee}{\beta n}} - L^\beta_{\underset{\vee}{jn}} L^\alpha_{\underset{\vee}{\beta m}}\big)
\end{aligned} \tag{2.5}
$$

$$
\begin{aligned}
\mathcal{B}^i_{(3).jmn} &= a^i_{j\underset{1}{|}m\underset{1}{|}n} - a^i_{j\underset{1}{|}n\underset{3}{|}m} \\
&= -2L^\alpha_{\underset{\vee}{jm}} a^i_{\alpha|n} - 2a^\alpha_\beta L^\beta_{\underset{\vee}{jm}} L^i_{\underset{\vee}{\alpha n}} \\
&\quad + a^\alpha_j \big(R^i_{\alpha mn} + L^i_{\underset{\vee}{\alpha m}|n} - L^i_{\underset{\vee}{\alpha n}|m} - L^\beta_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} - L^\beta_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}}\big) \\
&\quad - a^i_\alpha \big(R^\alpha_{jmn} + L^\alpha_{\underset{\vee}{jm}|n} - L^\alpha_{\underset{\vee}{jn}|m} - L^\beta_{\underset{\vee}{jm}} L^\alpha_{\underset{\vee}{\beta n}} - 3L^\beta_{\underset{\vee}{jn}} L^\alpha_{\underset{\vee}{\beta m}}\big),
\end{aligned} \tag{2.6}
$$

$$
\begin{aligned}
\mathcal{B}^i_{(4).jmn} &= a^i_{j\underset{1}{|}m\underset{1}{|}n} - a^i_{j\underset{2}{|}n\underset{1}{|}m} \\
&= 2L^i_{\underset{\vee}{\alpha n}} a^\alpha_{j|m} - 2L^\alpha_{\underset{\vee}{jn}} a^i_{\alpha|m} - 2L^\alpha_{\underset{\vee}{mn}} a^i_{j|\alpha} - 2a^\alpha_\beta \big(L^\beta_{\underset{\vee}{jm}} L^i_{\underset{\vee}{\alpha n}} + L^\beta_{\underset{\vee}{jn}} L^i_{\underset{\vee}{\alpha m}}\big) \\
&\quad + a^\alpha_j \big(R^i_{\alpha mn} + L^i_{\underset{\vee}{\alpha m}|n} + L^i_{\underset{\vee}{\alpha n}|m} + L^\beta_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} + L^\beta_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}}\big) \\
&\quad - a^i_\alpha \big(R^\alpha_{jmn} + L^\alpha_{\underset{\vee}{jm}|n} + L^\alpha_{\underset{\vee}{jn}|m} - L^\beta_{\underset{\vee}{jm}} L^\alpha_{\underset{\vee}{\beta n}} - L^\beta_{\underset{\vee}{jn}} L^\alpha_{\underset{\vee}{\beta m}}\big),
\end{aligned} \tag{2.7}
$$

$$
\begin{aligned}
\mathcal{B}^i_{(5).jmn} &= a^i_{j\underset{1}{|}m\underset{1}{|}n} - a^i_{j\underset{2}{|}n\underset{2}{|}m} \\
&= 2L^i_{\underset{\vee}{\alpha m}} a^\alpha_{j|n} + 2L^i_{\underset{\vee}{\alpha n}} a^\alpha_{j|m} - 2L^\alpha_{\underset{\vee}{jm}} a^i_{\alpha|n} - 2L^\alpha_{\underset{\vee}{jn}} a^i_{\alpha|m} \\
&\quad + a^\alpha_j \big(R^i_{\alpha mn} + L^i_{\underset{\vee}{\alpha m}|n} + L^i_{\underset{\vee}{\alpha n}|m} + L^\beta_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} - L^\beta_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}} + 2L^\beta_{\underset{\vee}{mn}} L^i_{\underset{\vee}{\beta\alpha}}\big) \\
&\quad - a^i_\alpha \big(R^\alpha_{jmn} + L^\alpha_{\underset{\vee}{jm}|n} + L^\alpha_{\underset{\vee}{jn}|m} + L^\beta_{\underset{\vee}{jm}} L^\alpha_{\underset{\vee}{\beta n}} - L^\beta_{\underset{\vee}{jn}} L^\alpha_{\underset{\vee}{\beta m}} + 2L^\beta_{\underset{\vee}{mn}} L^\alpha_{\underset{\vee}{\beta j}}\big),
\end{aligned} \tag{2.8}
$$

$$
\begin{aligned}
\mathcal{B}^i_{(6).jmn} &= a^i_{j\underset{1}{|}m\underset{1}{|}n} - a^i_{j\underset{2}{|}n\underset{3}{|}m} \\
&= 2L^i_{\underset{\vee}{\alpha n}} a^\alpha_{j|m} - 2L^\alpha_{\underset{\vee}{jm}} a^i_{\alpha|n} - 2L^\alpha_{\underset{\vee}{jn}} a^i_{\alpha|m} - 2a^\alpha_\beta L^\beta_{\underset{\vee}{jn}} L^i_{\underset{\vee}{\alpha m}} \\
&\quad + a^\alpha_j \big(R^i_{\alpha mn} + L^i_{\underset{\vee}{\alpha m}|n} + L^i_{\underset{\vee}{\alpha n}|m} + 3L^\beta_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} + L^\beta_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}} + 2L^\beta_{\underset{\vee}{mn}} L^i_{\underset{\vee}{\beta\alpha}}\big) \\
&\quad - a^i_\alpha \big(R^\alpha_{jmn} + L^\alpha_{\underset{\vee}{jm}|n} + L^\alpha_{\underset{\vee}{jn}|m} + L^\beta_{\underset{\vee}{jm}} L^\alpha_{\underset{\vee}{\beta n}} + L^\beta_{\underset{\vee}{jn}} L^\alpha_{\underset{\vee}{\beta m}} + 2L^\beta_{\underset{\vee}{mn}} L^\alpha_{\underset{\vee}{\beta j}}\big),
\end{aligned} \tag{2.9}
$$

$$\begin{aligned}
\mathcal{B}^{i}_{(7).jmn} &= a^{i}_{j\underset{1}{|}m\underset{1}{|}n} - a^{i}_{j\underset{3}{|}n\underset{1}{|}m}\\
&= -2L^{\alpha}_{\underset{\vee}{jn}}a^{i}_{\alpha|m} - 2L^{\alpha}_{\underset{\vee}{mn}}a^{i}_{j|\alpha} - 2a^{\alpha}_{\beta}L^{\beta}_{\underset{\vee}{jn}}L^{i}_{\underset{\vee}{\alpha m}}\\
&+ a^{\alpha}_{j}\big(R^{i}_{\alpha mn} + L^{i}_{\underset{\vee}{\alpha m}|n} - L^{i}_{\underset{\vee}{\alpha n}|m} + L^{\beta}_{\underset{\vee}{\alpha m}}L^{i}_{\underset{\vee}{\beta n}} - L^{\beta}_{\underset{\vee}{\alpha n}}L^{i}_{\underset{\vee}{\beta m}} + 2L^{\beta}_{\underset{\vee}{mn}}L^{i}_{\underset{\vee}{\beta\alpha}}\big)\\
&- a^{i}_{\alpha}\big(R^{\alpha}_{jmn} + L^{\alpha}_{\underset{\vee}{jm}|n} + L^{\alpha}_{\underset{\vee}{jn}|m} - L^{\beta}_{\underset{\vee}{jm}}L^{\alpha}_{\underset{\vee}{\beta n}} - L^{\beta}_{\underset{\vee}{jn}}L^{\alpha}_{\underset{\vee}{\beta m}}\big),
\end{aligned}\tag{2.10}$$

$$\begin{aligned}
\mathcal{B}^{i}_{(8).jmn} &= a^{i}_{j\underset{1}{|}m\underset{1}{|}n} - a^{i}_{j\underset{3}{|}n\underset{2}{|}m}\\
&= 2L^{i}_{\underset{\vee}{\alpha m}}a^{\alpha}_{j|n} - 2L^{\alpha}_{\underset{\vee}{jm}}a^{i}_{\alpha|n} - 2L^{\alpha}_{\underset{\vee}{jn}}a^{i}_{\alpha|m} - 2a^{\alpha}_{\beta}L^{\beta}_{\underset{\vee}{jm}}L^{i}_{\underset{\vee}{\alpha n}}\\
&+ a^{\alpha}_{j}\big(R^{i}_{\alpha mn} + L^{i}_{\underset{\vee}{\alpha m}|n} - L^{i}_{\underset{\vee}{\alpha n}|m} + L^{\beta}_{\underset{\vee}{\alpha m}}L^{i}_{\underset{\vee}{\beta n}} + L^{\beta}_{\underset{\vee}{\alpha n}}L^{i}_{\underset{\vee}{\beta m}}\big)\\
&- a^{i}_{\alpha}\big(R^{\alpha}_{jmn} + L^{\alpha}_{\underset{\vee}{jm}|n} + L^{\alpha}_{\underset{\vee}{jn}|m} + L^{\beta}_{\underset{\vee}{jm}}L^{\alpha}_{\underset{\vee}{\beta n}} - L^{\beta}_{\underset{\vee}{jn}}L^{\alpha}_{\underset{\vee}{\beta m}} + 2L^{\beta}_{\underset{\vee}{mn}}L^{\alpha}_{\underset{\vee}{\beta j}}\big),
\end{aligned}\tag{2.11}$$

$$\begin{aligned}
\mathcal{B}^{i}_{(9).jmn} &= a^{i}_{j\underset{1}{|}m\underset{1}{|}n} - a^{i}_{j\underset{3}{|}n\underset{3}{|}m}\\
&= -2L^{\alpha}_{\underset{\vee}{jm}}a^{i}_{\alpha|n} - 2L^{\alpha}_{\underset{\vee}{jn}}a^{i}_{\alpha|m} - 2a^{\alpha}_{\beta}\big(L^{\beta}_{\underset{\vee}{jm}}L^{i}_{\underset{\vee}{\alpha n}} + L^{\beta}_{\underset{\vee}{jn}}L^{i}_{\underset{\vee}{\alpha m}}\big)\\
&+ a^{\alpha}_{j}\big(R^{i}_{\alpha mn} + L^{i}_{\underset{\vee}{\alpha m}|n} - L^{i}_{\underset{\vee}{\alpha n}|m} - L^{\beta}_{\underset{\vee}{\alpha m}}L^{i}_{\underset{\vee}{\beta n}} - L^{\beta}_{\underset{\vee}{\alpha n}}L^{i}_{\underset{\vee}{\beta m}}\big)\\
&- a^{i}_{\alpha}\big(R^{\alpha}_{jmn} + L^{\alpha}_{\underset{\vee}{jm}|n} + L^{\alpha}_{\underset{\vee}{jn}|m} + L^{\beta}_{\underset{\vee}{jm}}L^{\alpha}_{\underset{\vee}{\beta n}} + L^{\beta}_{\underset{\vee}{jn}}L^{\alpha}_{\underset{\vee}{\beta m}} + 2L^{\beta}_{\underset{\vee}{mn}}L^{\alpha}_{\underset{\vee}{\beta j}}\big),
\end{aligned}\tag{2.12}$$

$$\begin{aligned}
\mathcal{B}^{i}_{(10).jmn} &= a^{i}_{j\underset{1}{|}m\underset{2}{|}n} - a^{i}_{j\underset{1}{|}n\underset{1}{|}m}\\
&= -2L^{i}_{\underset{\vee}{\alpha n}}a^{\alpha}_{j|m} + 2L^{\alpha}_{\underset{\vee}{jn}}a^{i}_{\alpha|m} + 2a^{\alpha}_{\beta}\big(L^{\beta}_{\underset{\vee}{jm}}L^{i}_{\underset{\vee}{\alpha n}} + L^{\beta}_{\underset{\vee}{jn}}L^{i}_{\underset{\vee}{\alpha m}}\big)\\
&+ a^{\alpha}_{j}\big(R^{i}_{\alpha mn} + L^{i}_{\underset{\vee}{\alpha m}|n} - L^{i}_{\underset{\vee}{\alpha n}|m} - L^{\beta}_{\underset{\vee}{\alpha m}}L^{i}_{\underset{\vee}{\beta n}} - L^{\beta}_{\underset{\vee}{\alpha n}}L^{i}_{\underset{\vee}{\beta m}}\big)\\
&- a^{i}_{\alpha}\big(R^{\alpha}_{jmn} + L^{\alpha}_{\underset{\vee}{jm}|n} - L^{\alpha}_{\underset{\vee}{jn}|m} + L^{\beta}_{\underset{\vee}{jm}}L^{\alpha}_{\underset{\vee}{\beta n}} + L^{\beta}_{\underset{\vee}{jn}}L^{\alpha}_{\underset{\vee}{\beta m}}\big),
\end{aligned}\tag{2.13}$$

$$\begin{aligned}
\mathcal{B}^{i}_{(11).jmn} &= a^{i}_{j\underset{1}{|}m\underset{3}{|}n} - a^{i}_{j\underset{1}{|}n\underset{1}{|}m}\\
&= 2L^{\alpha}_{\underset{\vee}{jn}}a^{i}_{\alpha|m} + 2a^{\alpha}_{\beta}L^{\beta}_{\underset{\vee}{jn}}L^{i}_{\underset{\vee}{\alpha m}}\\
&+ a^{\alpha}_{j}\big(R^{i}_{\alpha mn} + L^{i}_{\underset{\vee}{\alpha m}|n} - L^{i}_{\underset{\vee}{\alpha n}|m} + L^{\beta}_{\underset{\vee}{\alpha m}}L^{i}_{\underset{\vee}{\beta n}} + L^{\beta}_{\underset{\vee}{\alpha n}}L^{i}_{\underset{\vee}{\beta m}}\big)\\
&- a^{i}_{\alpha}\big(R^{\alpha}_{jmn} + L^{\alpha}_{\underset{\vee}{jm}|n} - L^{\alpha}_{\underset{\vee}{jn}|m} + 3L^{\beta}_{\underset{\vee}{jm}}L^{\alpha}_{\underset{\vee}{\beta n}} + L^{\beta}_{\underset{\vee}{jn}}L^{\alpha}_{\underset{\vee}{\beta m}}\big),
\end{aligned}\tag{2.14}$$

$$\begin{aligned}
\mathcal{B}^{i}_{(12).jmn} &= a^{i}_{j\underset{2}{|}m\underset{1}{|}n} - a^{i}_{j\underset{1}{|}n\underset{1}{|}m}\\
&= -2L^{i}_{\underset{\vee}{\alpha m}}a^{\alpha}_{j|n} + 2L^{\alpha}_{\underset{\vee}{jm}}a^{i}_{\alpha|n} - 2L^{\alpha}_{\underset{\vee}{mn}}a^{i}_{j|\alpha} + 2a^{\alpha}_{\beta}\big(L^{\beta}_{\underset{\vee}{jm}}L^{i}_{\underset{\vee}{\alpha n}} + L^{\beta}_{\underset{\vee}{jn}}L^{i}_{\underset{\vee}{\alpha m}}\big)\\
&+ a^{\alpha}_{j}\big(R^{i}_{\alpha mn} - L^{i}_{\underset{\vee}{\alpha m}|n} - L^{i}_{\underset{\vee}{\alpha n}|m} - L^{\beta}_{\underset{\vee}{\alpha m}}L^{i}_{\underset{\vee}{\beta n}} - L^{\beta}_{\underset{\vee}{\alpha n}}L^{i}_{\underset{\vee}{\beta m}}\big)\\
&- a^{i}_{\alpha}\big(R^{\alpha}_{jmn} - L^{\alpha}_{\underset{\vee}{jm}|n} - L^{\alpha}_{\underset{\vee}{jn}|m} + L^{\beta}_{\underset{\vee}{jm}}L^{\alpha}_{\underset{\vee}{\beta n}} + L^{\beta}_{\underset{\vee}{jn}}L^{\alpha}_{\underset{\vee}{\beta m}}\big),
\end{aligned}\tag{2.15}$$

$$
\begin{aligned}
\mathcal{B}^i_{(13).jmn} &= a^i_{j\underset{2}{|}m\underset{2}{|}n} - a^i_{j\underset{1}{|}n\underset{1}{|}m} \\
&= -2L^i_{\underset{\vee}{\alpha m}} a^\alpha_{j|n} - 2L^i_{\underset{\vee}{\alpha n}} a^\alpha_{j|m} + 2L^\alpha_{\underset{\vee}{jm}} a^i_{\alpha|n} + 2L^\alpha_{\underset{\vee}{jn}} a^i_{\alpha|m} \\
&+ a^\alpha_j \big(R^i_{\alpha mn} - L^i_{\underset{\vee}{\alpha m}|n} - L^i_{\underset{\vee}{\alpha n}|m} + L^\beta_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} - L^\beta_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}} + 2L^\beta_{\underset{\vee}{mn}} L^i_{\underset{\vee}{\beta\alpha}}\big) \\
&- a^i_\alpha \big(R^\alpha_{jmn} - L^\alpha_{\underset{\vee}{jm}|n} - L^\alpha_{\underset{\vee}{jn}|m} + L^\beta_{\underset{\vee}{jm}} L^\alpha_{\underset{\vee}{\beta n}} - L^\beta_{\underset{\vee}{jn}} L^\alpha_{\underset{\vee}{\beta m}} + 2L^\beta_{\underset{\vee}{mn}} L^\alpha_{\underset{\vee}{\beta j}}\big),
\end{aligned} \tag{2.16}
$$

$$
\begin{aligned}
\mathcal{B}^i_{(14).jmn} &= a^i_{j\underset{2}{|}m\underset{3}{|}n} - a^i_{j\underset{1}{|}n\underset{1}{|}m} \\
&= -2L^i_{\underset{\vee}{\alpha m}} a^\alpha_{j|n} + 2L^\alpha_{\underset{\vee}{jm}} a^i_{\alpha|n} + 2L^\alpha_{\underset{\vee}{jn}} a^i_{\alpha|m} + 2a^\alpha_\beta L^\beta_{\underset{\vee}{jm}} L^i_{\underset{\vee}{\alpha n}} \\
&+ a^\alpha_j \big(R^i_{\alpha mn} - L^i_{\underset{\vee}{\alpha m}|n} - L^i_{\underset{\vee}{\alpha n}|m} - L^\beta_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} - 3L^\beta_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}} + 2L^\beta_{\underset{\vee}{mn}} L^i_{\underset{\vee}{\beta\alpha}}\big) \\
&- a^i_\alpha \big(R^\alpha_{jmn} - L^\alpha_{\underset{\vee}{jm}|n} - L^\alpha_{\underset{\vee}{jn}|m} - L^\beta_{\underset{\vee}{jm}} L^\alpha_{\underset{\vee}{\beta n}} - L^\beta_{\underset{\vee}{jn}} L^\alpha_{\underset{\vee}{\beta m}} + 2L^\beta_{\underset{\vee}{mn}} L^\alpha_{\underset{\vee}{\beta j}}\big),
\end{aligned} \tag{2.17}
$$

$$
\begin{aligned}
\mathcal{B}^i_{(15).jmn} &= a^i_{j\underset{3}{|}m\underset{1}{|}n} - a^i_{j\underset{1}{|}n\underset{1}{|}m} \\
&= a^i_{j\underset{1}{|}m\underset{1}{|}n} - a^i_{j\underset{1}{|}n\underset{1}{|}m} \\
&= 2L^\alpha_{\underset{\vee}{jm}} a^i_{\alpha|n} - 2L^\alpha_{\underset{\vee}{mn}} a^i_{j|\alpha} + 2a^\alpha_\beta L^\beta_{\underset{\vee}{jm}} L^i_{\underset{\vee}{\alpha n}} \\
&+ a^\alpha_j \big(R^i_{\alpha mn} + L^i_{\underset{\vee}{\alpha m}|n} - L^i_{\underset{\vee}{\alpha n}|m} + L^\beta_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} - L^\beta_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}} + 2L^\beta_{\underset{\vee}{mn}} L^i_{\underset{\vee}{\beta\alpha}}\big) \\
&- a^i_\alpha \big(R^\alpha_{jmn} - L^\alpha_{\underset{\vee}{jm}|n} - L^\alpha_{\underset{\vee}{jn}|m} + L^\beta_{\underset{\vee}{jm}} L^\alpha_{\underset{\vee}{\beta n}} + L^\beta_{\underset{\vee}{jn}} L^\alpha_{\underset{\vee}{\beta m}}\big),
\end{aligned} \tag{2.18}
$$

are a base of the vector spaces generated by the differences $a^i_{j\underset{v_1}{|}m\underset{w_1}{|}n} - a^i_{j\underset{v_2}{|}n\underset{w_2}{|}m}$, $v_1, v_2, w_1, w_2 \in \{1, 2, 3\}$.

With respect to the equation (2.3), we obtain that many curvature tensors but no one curvature pseudotensor may be obtained with respect to the identities of Ricci Type presented in the First Ricci-Type Identities Theorem.

Vice versa, any linear combination of the geometrical objects $b^i_{(k)jmn}$, $k = 1, \dots, 16$, corresponds to infinitely many linear combinations of the differences $a^i_{j\underset{v_1}{|}m\underset{w_1}{|}n} - a^i_{j\underset{v_2}{|}n\underset{w_2}{|}m}$, $v_1, v_2, w_1, w_2 \in \{0, 1, 2, 3, 4\}$.

To obtain curvature pseudotensors for the space $\mathbb{GA}_N$, we need to consider the base $(c^i_{(k)jmn}) = (b^i_{(k)jmn} + \mathcal{L}^i_{(k)jmn})$, $k = 1, \dots, 16$, where the geometrical objects $\mathcal{L}^i_{(k)jmn}$ are linear combinations of the products $L^i_{\underline{\alpha n}} L^\alpha_{\underset{\vee}{jm}}$, $L^i_{\underline{\alpha m}} L^\alpha_{\underset{\vee}{jn}}$, $L^i_{\underline{\alpha j}} L^\alpha_{\underset{\vee}{mn}}$, $L^i_{\underset{\vee}{\alpha n}} L^\alpha_{\underline{jm}}$, $L^i_{\underset{\vee}{\alpha m}} L^\alpha_{\underline{jn}}$, $L^i_{\underset{\vee}{\alpha j}} L^\alpha_{\underline{mn}}$.

Any linear combination of the geometrical objects $c^i_{(k)jmn}$ does not correspond to a linear combination of the differences $a^i_{j\underset{v_1}{|}m\underset{w_1}{|}n} - a^i_{j\underset{v_2}{|}n\underset{w_2}{|}m}$, $v_1, v_2, w_1, w_2 \in \{0, 1, 2, 3, 4\}$.

For this reason, the geometrical objects $b^i_{(k)jmn}$ are components of a base for the space of differences $a^i_{j\underset{v_1}{|}m\underset{w_1}{|}n} - a^i_{j\underset{v_2}{|}n\underset{w_2}{|}m}$, $v_1, v_2, w_1, w_2 \in \{0, 1, 2, 3, 4\}$ unlike the geometrical objects $c^i_{(k)jmn}$.

Remark 2.1. *Any identity of Ricci Type where the curvature pseudotensors of the space $\mathbb{GA}_N$ are obtained may be simplified and reduced to the form* (2.3).

2.1. Eighty one Ricci-Type identities

With respect to the First Ricci-Type Identities Theorem, and for $v_1, v_2, w_1, w_2 \in \{1, 2, 3\}$, we obtain the next identities of Ricci Type.

$$
\begin{aligned}
a^i_{j\underset{1}{|}m\underset{1}{|}n} - a^i_{j\underset{1}{|}n\underset{1}{|}m} &= -2L^\alpha_{\underset{\vee}{mn}}a^i_{j|\alpha}\\
&+ a^\alpha_j\big(R^i_{\alpha mn} + L^i_{\underset{\vee}{\alpha m}|n} - L^i_{\underset{\vee}{\alpha n}|m} + L^\beta_{\underset{\vee}{\alpha m}}L^i_{\underset{\vee}{\beta n}} - L^\beta_{\underset{\vee}{\alpha n}}L^i_{\underset{\vee}{\beta m}} - 2L^i_{\underset{\vee}{\alpha\beta}}L^\beta_{\underset{\vee}{mn}}\big)\\
&- a^i_\alpha\big(R^\alpha_{jmn} + L^\alpha_{\underset{\vee}{jm}|n} - L^\alpha_{\underset{\vee}{jn}|m} + L^\alpha_{\underset{\vee}{\beta n}}L^\beta_{\underset{\vee}{jm}} - L^\alpha_{\underset{\vee}{\beta m}}L^\beta_{\underset{\vee}{jn}} - 2L^\alpha_{\underset{\vee}{j\beta}}L^\beta_{\underset{\vee}{mn}}\big),\\
a^i_{j\underset{1}{|}m\underset{2}{|}n} - a^i_{j\underset{1}{|}n\underset{1}{|}m} &= 2L^\alpha_{\underset{\vee}{jn}}a^i_{\alpha|m} - 2L^i_{\underset{\vee}{\alpha n}}a^\alpha_{j|m}\\
&+ a^\alpha_j\big(R^i_{\alpha mn} + L^i_{\underset{\vee}{\alpha m}|n} - L^i_{\underset{\vee}{\alpha n}|m} - L^\beta_{\underset{\vee}{\alpha m}}L^i_{\underset{\vee}{\beta n}} - L^\beta_{\underset{\vee}{\alpha n}}L^i_{\underset{\vee}{\beta m}}\big)\\
&- a^i_\alpha\big(R^\alpha_{jmn} + L^\alpha_{\underset{\vee}{jm}|n} - L^\alpha_{\underset{\vee}{jn}|m} + L^\alpha_{\underset{\vee}{\beta n}}L^\beta_{\underset{\vee}{jm}} + L^\alpha_{\underset{\vee}{\beta m}}L^\beta_{\underset{\vee}{jn}}\big)\\
&+ 2a^\alpha_\beta\big(L^i_{\underset{\vee}{\alpha m}}L^\beta_{\underset{\vee}{jn}} + L^i_{\underset{\vee}{\alpha n}}L^\beta_{\underset{\vee}{jm}}\big),\\
a^i_{j\underset{1}{|}m\underset{3}{|}n} - a^i_{j\underset{1}{|}n\underset{1}{|}m} &= 2L^\alpha_{\underset{\vee}{jn}}a^i_{\alpha|m}\\
&+ a^\alpha_j\big(R^i_{\alpha mn} + L^i_{\underset{\vee}{\alpha m}|n} - L^i_{\underset{\vee}{\alpha n}|m} + L^\beta_{\underset{\vee}{\alpha m}}L^i_{\underset{\vee}{\beta n}} - L^\beta_{\underset{\vee}{\alpha n}}L^i_{\underset{\vee}{\beta m}}\big)\\
&- a^i_\alpha\big(R^\alpha_{jmn} + L^\alpha_{\underset{\vee}{jm}|n} - L^\alpha_{\underset{\vee}{jn}|m} + L^\alpha_{\underset{\vee}{\beta n}}L^\beta_{\underset{\vee}{jm}} + L^\alpha_{\underset{\vee}{\beta m}}L^\beta_{\underset{\vee}{jn}}\big)\\
&+ 2a^\alpha_\beta L^i_{\underset{\vee}{\alpha m}}L^\beta_{\underset{\vee}{jn}},\\
a^i_{j\underset{2}{|}m\underset{1}{|}n} - a^i_{j\underset{1}{|}n\underset{1}{|}m} &= 2L^\alpha_{\underset{\vee}{jm}}a^i_{\alpha|n} - 2L^\alpha_{\underset{\vee}{mn}}a^i_{j|\alpha} - 2L^i_{\underset{\vee}{\alpha m}}a^\alpha_{j|n}\\
&+ a^\alpha_j\big(R^i_{\alpha mn} - L^i_{\underset{\vee}{\alpha m}|n} - L^i_{\underset{\vee}{\alpha n}|m} - L^\beta_{\underset{\vee}{\alpha m}}L^i_{\underset{\vee}{\beta n}} - L^\beta_{\underset{\vee}{\alpha n}}L^i_{\underset{\vee}{\beta m}}\big)\\
&- a^i_\alpha\big(R^\alpha_{jmn} - L^\alpha_{\underset{\vee}{jm}|n} - L^\alpha_{\underset{\vee}{jn}|m} + L^\alpha_{\underset{\vee}{\beta n}}L^\beta_{\underset{\vee}{jm}} + L^\alpha_{\underset{\vee}{\beta m}}L^\beta_{\underset{\vee}{jn}}\big)\\
&+ 2a^\alpha_\beta\big(L^i_{\underset{\vee}{\alpha n}}L^\beta_{\underset{\vee}{jm}} + L^i_{\underset{\vee}{\alpha m}}L^\beta_{\underset{\vee}{jn}}\big),\\
a^i_{j\underset{2}{|}m\underset{2}{|}n} - a^i_{j\underset{1}{|}n\underset{1}{|}m} &= 2L^\alpha_{\underset{\vee}{jm}}a^i_{\alpha|n} + 2L^\alpha_{\underset{\vee}{jn}}a^i_{\alpha|m} - 2L^i_{\underset{\vee}{\alpha m}}a^\alpha_{j|n} - 2L^i_{\underset{\vee}{\alpha n}}a^\alpha_{j|m}\\
&+ a^\alpha_j\big(R^i_{\alpha mn} - L^i_{\underset{\vee}{\alpha m}|n} - L^i_{\underset{\vee}{\alpha n}|m} + L^\beta_{\underset{\vee}{\alpha m}}L^i_{\underset{\vee}{\beta n}} - L^\beta_{\underset{\vee}{\alpha n}}L^i_{\underset{\vee}{\beta m}} - 2L^i_{\underset{\vee}{\alpha\beta}}L^\beta_{\underset{\vee}{mn}}\big)\\
&- a^i_\alpha\big(R^\alpha_{jmn} - L^\alpha_{\underset{\vee}{jm}|n} - L^\alpha_{\underset{\vee}{jn}|m} + L^\alpha_{\underset{\vee}{\beta n}}L^\beta_{\underset{\vee}{jm}} - L^\alpha_{\underset{\vee}{\beta m}}L^\beta_{\underset{\vee}{jn}} - 2L^\alpha_{\underset{\vee}{j\beta}}L^\beta_{\underset{\vee}{mn}}\big),
\end{aligned}
$$

$$
\begin{aligned}
a^i_{j\underset{2}{|}m\underset{3}{|}n} - a^i_{j\underset{1}{|}n\underset{1}{|}m} &= 2L^{\alpha}_{\underset{\vee}{jm}} a^i_{\alpha|n} + 2L^{\alpha}_{\underset{\vee}{jn}} a^i_{\alpha|m} - 2L^i_{\underset{\vee}{\alpha m}} a^{\alpha}_{j|n} \\
&\quad + a^{\alpha}_j \big(R^i_{\alpha mn} - L^i_{\underset{\vee}{\alpha m}|n} - L^i_{\underset{\vee}{\alpha n}|m} - L^{\beta}_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} - L^{\beta}_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}} - 2L^i_{\underset{\vee}{\alpha\beta}} L^{\beta}_{\underset{\vee}{mn}}\big) \\
&\quad - a^i_{\alpha} \big(R^{\alpha}_{jmn} - L^{\alpha}_{\underset{\vee}{jm}|n} - L^{\alpha}_{\underset{\vee}{jn}|m} + L^{\alpha}_{\underset{\vee}{\beta n}} L^{\beta}_{\underset{\vee}{jm}} - L^{\alpha}_{\underset{\vee}{\beta m}} L^{\beta}_{\underset{\vee}{jn}} - 2L^{\alpha}_{\underset{\vee}{j\beta}} L^{\beta}_{\underset{\vee}{mn}}\big) \\
&\quad + 2a^{\alpha}_{\beta} L^i_{\underset{\vee}{\alpha n}} L^{\beta}_{\underset{\vee}{jm}}, \\
a^i_{j\underset{3}{|}m\underset{1}{|}n} - a^i_{j\underset{1}{|}n\underset{1}{|}m} &= 2L^{\alpha}_{\underset{\vee}{jm}} a^i_{\alpha|n} - 2L^{\alpha}_{\underset{\vee}{mn}} a^i_{j|\alpha} \\
&\quad + a^{\alpha}_j \big(R^i_{\alpha mn} + L^i_{\underset{\vee}{\alpha m}|n} - L^i_{\underset{\vee}{\alpha n}|m} + L^{\beta}_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} - L^{\beta}_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}} - 2L^i_{\underset{\vee}{\alpha\beta}} L^{\beta}_{\underset{\vee}{mn}}\big) \\
&\quad - a^i_{\alpha} \big(R^{\alpha}_{jmn} - L^{\alpha}_{\underset{\vee}{jm}|n} - L^{\alpha}_{\underset{\vee}{jn}|m} + L^{\alpha}_{\underset{\vee}{\beta n}} L^{\beta}_{\underset{\vee}{jm}} + L^{\alpha}_{\underset{\vee}{\beta m}} L^{\beta}_{\underset{\vee}{jn}}\big) \\
&\quad + 2a^{\alpha}_{\beta} L^i_{\underset{\vee}{\alpha n}} L^{\beta}_{\underset{\vee}{jm}}, \\
a^i_{j\underset{3}{|}m\underset{2}{|}n} - a^i_{j\underset{1}{|}n\underset{1}{|}m} &= 2L^{\alpha}_{\underset{\vee}{jm}} a^i_{\alpha|n} + 2L^{\alpha}_{\underset{\vee}{jn}} a^i_{\alpha|m} - 2L^i_{\underset{\vee}{\alpha n}} a^{\alpha}_{j|m} \\
&\quad + a^{\alpha}_j \big(R^i_{\alpha mn} + L^i_{\underset{\vee}{\alpha m}|n} - L^i_{\underset{\vee}{\alpha n}|m} - L^{\beta}_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} - L^{\beta}_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}}\big) \\
&\quad - a^i_{\alpha} \big(R^{\alpha}_{jmn} - L^{\alpha}_{\underset{\vee}{jm}|n} - L^{\alpha}_{\underset{\vee}{jn}|m} + L^{\alpha}_{\underset{\vee}{\beta n}} L^{\beta}_{\underset{\vee}{jm}} - L^{\alpha}_{\underset{\vee}{\beta m}} L^{\beta}_{\underset{\vee}{jn}} - 2L^{\alpha}_{\underset{\vee}{j\beta}} L^{\beta}_{\underset{\vee}{mn}}\big) \\
&\quad + 2a^{\alpha}_{\beta} L^i_{\underset{\vee}{\alpha m}} L^{\beta}_{\underset{\vee}{jn}}, \\
a^i_{j\underset{3}{|}m\underset{3}{|}n} - a^i_{j\underset{1}{|}n\underset{1}{|}m} &= 2L^{\alpha}_{\underset{\vee}{jm}} a^i_{\alpha|n} + 2L^{\alpha}_{\underset{\vee}{jn}} a^i_{\alpha|m} \\
&\quad + a^{\alpha}_j \big(R^i_{\alpha mn} + L^i_{\underset{\vee}{\alpha m}|n} - L^i_{\underset{\vee}{\alpha n}|m} + L^{\beta}_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} - L^{\beta}_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}}\big) \\
&\quad - a^i_{\alpha} \big(R^{\alpha}_{jmn} - L^{\alpha}_{\underset{\vee}{jm}|n} - L^{\alpha}_{\underset{\vee}{jn}|m} + L^{\alpha}_{\underset{\vee}{\beta n}} L^{\beta}_{\underset{\vee}{jm}} - L^{\alpha}_{\underset{\vee}{\beta m}} L^{\beta}_{\underset{\vee}{jn}} - 2L^{\alpha}_{\underset{\vee}{j\beta}} L^{\beta}_{\underset{\vee}{mn}}\big) \\
&\quad + 2a^{\alpha}_{\beta} \big(L^i_{\underset{\vee}{\alpha n}} L^{\beta}_{\underset{\vee}{jm}} + L^i_{\underset{\vee}{\alpha m}} L^{\beta}_{\underset{\vee}{jn}}\big), \\
a^i_{j\underset{1}{|}m\underset{1}{|}n} - a^i_{j\underset{1}{|}n\underset{2}{|}m} &= -2L^{\alpha}_{\underset{\vee}{jm}} a^i_{\alpha|n} + 2L^i_{\underset{\vee}{\alpha m}} a^{\alpha}_{j|n} \\
&\quad + a^{\alpha}_j \big(R^i_{\alpha mn} + L^i_{\underset{\vee}{\alpha m}|n} - L^i_{\underset{\vee}{\alpha n}|m} + L^{\beta}_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} + L^{\beta}_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}}\big) \\
&\quad - a^i_{\alpha} \big(R^{\alpha}_{jmn} + L^{\alpha}_{\underset{\vee}{jm}|n} - L^{\alpha}_{\underset{\vee}{jn}|m} - L^{\alpha}_{\underset{\vee}{\beta n}} L^{\beta}_{\underset{\vee}{jm}} - L^{\alpha}_{\underset{\vee}{\beta m}} L^{\beta}_{\underset{\vee}{jn}}\big) \\
&\quad - 2a^{\alpha}_{\beta} \big(L^i_{\underset{\vee}{\alpha m}} L^{\beta}_{\underset{\vee}{jn}} + L^i_{\underset{\vee}{\alpha n}} L^{\beta}_{\underset{\vee}{jm}}\big), \\
a^i_{j\underset{1}{|}m\underset{2}{|}n} - a^i_{j\underset{1}{|}n\underset{2}{|}m} &= -2L^{\alpha}_{\underset{\vee}{jm}} a^i_{\alpha|n} + 2L^{\alpha}_{\underset{\vee}{jn}} a^i_{\alpha|m} + 2L^{\alpha}_{\underset{\vee}{mn}} a^i_{j|\alpha} + 2L^i_{\underset{\vee}{\alpha m}} a^{\alpha}_{j|n} - 2L^i_{\underset{\vee}{\alpha n}} a^{\alpha}_{j|m} \\
&\quad + a^{\alpha}_j \big(R^i_{\alpha mn} + L^i_{\underset{\vee}{\alpha m}|n} - L^i_{\underset{\vee}{\alpha n}|m} - L^{\beta}_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} + L^{\beta}_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}} + 2L^i_{\underset{\vee}{\alpha\beta}} L^{\beta}_{\underset{\vee}{mn}}\big) \\
&\quad - a^i_{\alpha} \big(R^{\alpha}_{jmn} + L^{\alpha}_{\underset{\vee}{jm}|n} - L^{\alpha}_{\underset{\vee}{jn}|m} - L^{\alpha}_{\underset{\vee}{\beta n}} L^{\beta}_{\underset{\vee}{jm}} + L^{\alpha}_{\underset{\vee}{\beta m}} L^{\beta}_{\underset{\vee}{jn}} + 2L^{\alpha}_{\underset{\vee}{j\beta}} L^{\beta}_{\underset{\vee}{mn}}\big), \\
a^i_{j\underset{1}{|}m\underset{3}{|}n} - a^i_{j\underset{1}{|}n\underset{2}{|}m} &= -2L^{\alpha}_{\underset{\vee}{jm}} a^i_{\alpha|n} + 2L^{\alpha}_{\underset{\vee}{jn}} a^i_{\alpha|m} + 2L^{\alpha}_{\underset{\vee}{mn}} a^i_{j|\alpha} + 2L^i_{\underset{\vee}{\alpha m}} a^{\alpha}_{j|n} \\
&\quad + a^{\alpha}_j \big(R^i_{\alpha mn} + L^i_{\underset{\vee}{\alpha m}|n} - L^i_{\underset{\vee}{\alpha n}|m} + L^{\beta}_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} + L^{\beta}_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}} + 2L^i_{\underset{\vee}{\alpha\beta}} L^{\beta}_{\underset{\vee}{mn}}\big) \\
&\quad - a^i_{\alpha} \big(R^{\alpha}_{jmn} + L^{\alpha}_{\underset{\vee}{jm}|n} - L^{\alpha}_{\underset{\vee}{jn}|m} - L^{\alpha}_{\underset{\vee}{\beta n}} L^{\beta}_{\underset{\vee}{jm}} + L^{\alpha}_{\underset{\vee}{\beta m}} L^{\beta}_{\underset{\vee}{jn}} + 2L^{\alpha}_{\underset{\vee}{j\beta}} L^{\beta}_{\underset{\vee}{mn}}\big) \\
&\quad - 2a^{\alpha}_{\beta} L^i_{\underset{\vee}{\alpha n}} L^{\beta}_{\underset{\vee}{jm}},
\end{aligned}
$$

$$
\begin{aligned}
a^i_{j\underset{2}{|}m\underset{1}{|}n} - a^i_{j\underset{1}{|}n\underset{2}{|}m} &= a^\alpha_j\big(R^i_{\alpha mn} - L^i_{\underset{\vee}{\alpha m}|n} - L^i_{\underset{\vee}{\alpha n}|m} - L^\beta_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} + L^\beta_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}} + 2L^i_{\underset{\vee}{\alpha\beta}} L^\beta_{\underset{\vee}{mn}}\big)\\
&\quad - a^i_\alpha\big(R^\alpha_{jmn} - L^\alpha_{\underset{\vee}{jm}|n} - L^\alpha_{\underset{\vee}{jn}|m} - L^\alpha_{\underset{\vee}{\beta n}} L^\beta_{\underset{\vee}{jm}} + L^\alpha_{\underset{\vee}{\beta m}} L^\beta_{\underset{\vee}{jn}} + 2L^\alpha_{\underset{\vee}{j\beta}} L^\beta_{\underset{\vee}{mn}}\big),\\
a^i_{j\underset{2}{|}m\underset{2}{|}n} - a^i_{j\underset{1}{|}n\underset{2}{|}m} &= 2L^\alpha_{\underset{\vee}{jn}} a^i_{\alpha|m} + 2L^\alpha_{\underset{\vee}{mn}} a^i_{j|\alpha} - 2L^i_{\underset{\vee}{\alpha n}} a^\alpha_{j|m}\\
&\quad + a^\alpha_j\big(R^i_{\alpha mn} - L^i_{\underset{\vee}{\alpha m}|n} - L^i_{\underset{\vee}{\alpha n}|m} + L^\beta_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} + L^\beta_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}}\big)\\
&\quad - a^i_\alpha\big(R^\alpha_{jmn} - L^\alpha_{\underset{\vee}{jm}|n} - L^\alpha_{\underset{\vee}{jn}|m} - L^\alpha_{\underset{\vee}{\beta n}} L^\beta_{\underset{\vee}{jm}} - L^\alpha_{\underset{\vee}{\beta m}} L^\beta_{\underset{\vee}{jn}}\big)\\
&\quad - 2a^\alpha_\beta\big(L^i_{\underset{\vee}{\alpha m}} L^\beta_{\underset{\vee}{jn}} + L^i_{\underset{\vee}{\alpha n}} L^\beta_{\underset{\vee}{jm}}\big),\\
a^i_{j\underset{2}{|}m\underset{3}{|}n} - a^i_{j\underset{1}{|}n\underset{2}{|}m} &= 2L^\alpha_{\underset{\vee}{jn}} a^i_{\alpha|m} + 2L^\alpha_{\underset{\vee}{mn}} a^i_{j|\alpha}\\
&\quad + a^\alpha_j\big(R^i_{\alpha mn} - L^i_{\underset{\vee}{\alpha m}|n} - L^i_{\underset{\vee}{\alpha n}|m} - L^\beta_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} + L^\beta_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}}\big)\\
&\quad - a^i_\alpha\big(R^\alpha_{jmn} - L^\alpha_{\underset{\vee}{jm}|n} - L^\alpha_{\underset{\vee}{jn}|m} - L^\alpha_{\underset{\vee}{\beta n}} L^\beta_{\underset{\vee}{jm}} - L^\alpha_{\underset{\vee}{\beta m}} L^\beta_{\underset{\vee}{jn}}\big)\\
&\quad - 2a^\alpha_\beta L^i_{\underset{\vee}{\alpha m}} L^\beta_{\underset{\vee}{jn}},\\
a^i_{j\underset{3}{|}m\underset{1}{|}n} - a^i_{j\underset{1}{|}n\underset{2}{|}m} &= 2L^i_{\underset{\vee}{\alpha m}} a^\alpha_{j|n}\\
&\quad + a^\alpha_j\big(R^i_{\alpha mn} + L^i_{\underset{\vee}{\alpha m}|n} - L^i_{\underset{\vee}{\alpha n}|m} + L^\beta_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} + L^\beta_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}}\big)\\
&\quad - a^i_\alpha\big(R^\alpha_{jmn} - L^\alpha_{\underset{\vee}{jm}|n} - L^\alpha_{\underset{\vee}{jn}|m} - L^\alpha_{\underset{\vee}{\beta n}} L^\beta_{\underset{\vee}{jm}} + L^\alpha_{\underset{\vee}{\beta m}} L^\beta_{\underset{\vee}{jn}} + 2L^\alpha_{\underset{\vee}{j\beta}} L^\beta_{\underset{\vee}{mn}}\big)\\
&\quad - 2a^\alpha_\beta L^i_{\underset{\vee}{\alpha m}} L^\beta_{\underset{\vee}{jn}},\\
a^i_{j\underset{3}{|}m\underset{2}{|}n} - a^i_{j\underset{1}{|}n\underset{2}{|}m} &= 2L^\alpha_{\underset{\vee}{jn}} a^i_{\alpha|m} + 2L^\alpha_{\underset{\vee}{mn}} a^i_{j|\alpha} + 2L^i_{\underset{\vee}{\alpha m}} a^\alpha_{j|n} - 2L^i_{\underset{\vee}{\alpha n}} a^\alpha_{j|m}\\
&\quad + a^\alpha_j\big(R^i_{\alpha mn} + L^i_{\underset{\vee}{\alpha m}|n} - L^i_{\underset{\vee}{\alpha n}|m} - L^\beta_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} + L^\beta_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}} + 2L^i_{\underset{\vee}{\alpha\beta}} L^\beta_{\underset{\vee}{mn}}\big)\\
&\quad - a^i_\alpha\big(R^\alpha_{jmn} - L^\alpha_{\underset{\vee}{jm}|n} - L^\alpha_{\underset{\vee}{jn}|m} - L^\alpha_{\underset{\vee}{\beta n}} L^\beta_{\underset{\vee}{jm}} - L^\alpha_{\underset{\vee}{\beta m}} L^\beta_{\underset{\vee}{jn}}\big)\\
&\quad - 2a^\alpha_\beta L^i_{\underset{\vee}{\alpha n}} L^\beta_{\underset{\vee}{jm}},\\
a^i_{j\underset{3}{|}m\underset{3}{|}n} - a^i_{j\underset{1}{|}n\underset{2}{|}m} &= 2L^\alpha_{\underset{\vee}{jn}} a^i_{\alpha|m} + 2L^\alpha_{\underset{\vee}{mn}} a^i_{j|\alpha} + 2L^i_{\underset{\vee}{\alpha m}} a^\alpha_{j|n}\\
&\quad + a^\alpha_j\big(R^i_{\alpha mn} + L^i_{\underset{\vee}{\alpha m}|n} - L^i_{\underset{\vee}{\alpha n}|m} + L^\beta_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} + L^\beta_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}} + 2L^i_{\underset{\vee}{\alpha\beta}} L^\beta_{\underset{\vee}{mn}}\big)\\
&\quad - a^i_\alpha\big(R^\alpha_{jmn} - L^\alpha_{\underset{\vee}{jm}|n} - L^\alpha_{\underset{\vee}{jn}|m} - L^\alpha_{\underset{\vee}{\beta n}} L^\beta_{\underset{\vee}{jm}} - L^\alpha_{\underset{\vee}{\beta m}} L^\beta_{\underset{\vee}{jn}}\big),\\
a^i_{j\underset{1}{|}m\underset{1}{|}n} - a^i_{j\underset{1}{|}n\underset{3}{|}m} &= -2L^\alpha_{\underset{\vee}{jm}} a^i_{\alpha|n}\\
&\quad + a^\alpha_j\big(R^i_{\alpha mn} + L^i_{\underset{\vee}{\alpha m}|n} - L^i_{\underset{\vee}{\alpha n}|m} + L^\beta_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} - L^\beta_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}}\big)\\
&\quad - a^i_\alpha\big(R^\alpha_{jmn} + L^\alpha_{\underset{\vee}{jm}|n} - L^\alpha_{\underset{\vee}{jn}|m} - L^\alpha_{\underset{\vee}{\beta n}} L^\beta_{\underset{\vee}{jm}} - L^\alpha_{\underset{\vee}{\beta m}} L^\beta_{\underset{\vee}{jn}}\big)\\
&\quad - 2a^\alpha_\beta L^i_{\underset{\vee}{\alpha n}} L^\beta_{\underset{\vee}{jm}},
\end{aligned}
$$

$$
\begin{aligned}
a^i_{j\underset{1}{|}m\underset{2}{|}n} - a^i_{j\underset{1}{|}n\underset{3}{|}m} &= -2\underset{\vee}{L}{}^{\alpha}_{jm} a^i_{\alpha|n} + 2\underset{\vee}{L}{}^{\alpha}_{jn} a^i_{\alpha|m} + 2\underset{\vee}{L}{}^{\alpha}_{mn} a^i_{j|\alpha} - 2\underset{\vee}{L}{}^{i}_{\alpha n} a^\alpha_{j|m} \\
&\quad + a^\alpha_j\big(R^i_{\alpha mn} + \underset{\vee}{L}{}^{i}_{\alpha m|n} - \underset{\vee}{L}{}^{i}_{\alpha n|m} - \underset{\vee}{L}{}^{\beta}_{\alpha m}\underset{\vee}{L}{}^{i}_{\beta n} - \underset{\vee}{L}{}^{\beta}_{\alpha n}\underset{\vee}{L}{}^{i}_{\beta m} + 2\underset{\vee}{L}{}^{i}_{\alpha\beta}\underset{\vee}{L}{}^{\beta}_{mn}\big) \\
&\quad - a^i_\alpha\big(R^\alpha_{jmn} + \underset{\vee}{L}{}^{\alpha}_{jm|n} - \underset{\vee}{L}{}^{\alpha}_{jn|m} - \underset{\vee}{L}{}^{\alpha}_{\beta n}\underset{\vee}{L}{}^{\beta}_{jm} + \underset{\vee}{L}{}^{\alpha}_{\beta m}\underset{\vee}{L}{}^{\beta}_{jn} + 2\underset{\vee}{L}{}^{\alpha}_{j\beta}\underset{\vee}{L}{}^{\beta}_{mn}\big) \\
&\quad + 2a^\alpha_\beta \underset{\vee}{L}{}^{i}_{\alpha m}\underset{\vee}{L}{}^{\beta}_{jn}, \\
a^i_{j\underset{1}{|}m\underset{3}{|}n} - a^i_{j\underset{1}{|}n\underset{3}{|}m} &= -2\underset{\vee}{L}{}^{\alpha}_{jm} a^i_{\alpha|n} + 2\underset{\vee}{L}{}^{\alpha}_{mn} a^i_{j|\alpha} + 2\underset{\vee}{L}{}^{\alpha}_{jn} a^i_{\alpha|m} \\
&\quad + a^\alpha_j\big(R^i_{\alpha mn} + \underset{\vee}{L}{}^{i}_{\alpha m|n} - \underset{\vee}{L}{}^{i}_{\alpha n|m} + \underset{\vee}{L}{}^{\beta}_{\alpha m}\underset{\vee}{L}{}^{i}_{\beta n} - \underset{\vee}{L}{}^{\beta}_{\alpha n}\underset{\vee}{L}{}^{i}_{\beta m} + 2\underset{\vee}{L}{}^{i}_{\alpha\beta}\underset{\vee}{L}{}^{\beta}_{mn}\big) \\
&\quad - a^i_\alpha\big(R^\alpha_{jmn} + \underset{\vee}{L}{}^{\alpha}_{jm|n} - \underset{\vee}{L}{}^{\alpha}_{jn|m} - \underset{\vee}{L}{}^{\alpha}_{\beta n}\underset{\vee}{L}{}^{\beta}_{jm} + \underset{\vee}{L}{}^{\alpha}_{\beta m}\underset{\vee}{L}{}^{\beta}_{jn} + 2\underset{\vee}{L}{}^{\alpha}_{j\beta}\underset{\vee}{L}{}^{\beta}_{mn}\big) \\
&\quad + 2a^\alpha_\beta\big(\underset{\vee}{L}{}^{i}_{\alpha m}\underset{\vee}{L}{}^{\beta}_{jn} - \underset{\vee}{L}{}^{i}_{\alpha n}\underset{\vee}{L}{}^{\beta}_{jm}\big), \\
a^i_{j\underset{2}{|}m\underset{1}{|}n} - a^i_{j\underset{1}{|}n\underset{3}{|}m} &= -2\underset{\vee}{L}{}^{i}_{\alpha m} a^\alpha_{j|n} \\
&\quad + a^\alpha_j\big(R^i_{\alpha mn} - \underset{\vee}{L}{}^{i}_{\alpha m|n} - \underset{\vee}{L}{}^{i}_{\alpha n|m} - \underset{\vee}{L}{}^{\beta}_{\alpha m}\underset{\vee}{L}{}^{i}_{\beta n} - \underset{\vee}{L}{}^{\beta}_{\alpha n}\underset{\vee}{L}{}^{i}_{\beta m} + 2\underset{\vee}{L}{}^{i}_{\alpha\beta}\underset{\vee}{L}{}^{\beta}_{mn}\big) \\
&\quad - a^i_\alpha\big(R^\alpha_{jmn} - \underset{\vee}{L}{}^{\alpha}_{jm|n} - \underset{\vee}{L}{}^{\alpha}_{jn|m} - \underset{\vee}{L}{}^{\alpha}_{\beta n}\underset{\vee}{L}{}^{\beta}_{jm} + \underset{\vee}{L}{}^{\alpha}_{\beta m}\underset{\vee}{L}{}^{\beta}_{jn} + 2\underset{\vee}{L}{}^{\alpha}_{j\beta}\underset{\vee}{L}{}^{\beta}_{mn}\big) \\
&\quad + 2a^\alpha_\beta \underset{\vee}{L}{}^{i}_{\alpha m}\underset{\vee}{L}{}^{\beta}_{jn}, \\
a^i_{j\underset{2}{|}m\underset{2}{|}n} - a^i_{j\underset{1}{|}n\underset{3}{|}m} &= 2\underset{\vee}{L}{}^{\alpha}_{jn} a^i_{\alpha|m} + 2\underset{\vee}{L}{}^{\alpha}_{mn} a^i_{j|\alpha} - 2\underset{\vee}{L}{}^{i}_{\alpha m} a^\alpha_{j|n} - 2\underset{\vee}{L}{}^{i}_{\alpha n} a^\alpha_{j|m} \\
&\quad + a^\alpha_j\big(R^i_{\alpha mn} - \underset{\vee}{L}{}^{i}_{\alpha m|n} - \underset{\vee}{L}{}^{i}_{\alpha n|m} + \underset{\vee}{L}{}^{\beta}_{\alpha m}\underset{\vee}{L}{}^{i}_{\beta n} - \underset{\vee}{L}{}^{\beta}_{\alpha n}\underset{\vee}{L}{}^{i}_{\beta m}\big) \\
&\quad - a^i_\alpha\big(R^\alpha_{jmn} - \underset{\vee}{L}{}^{\alpha}_{jm|n} - \underset{\vee}{L}{}^{\alpha}_{jn|m} - \underset{\vee}{L}{}^{\alpha}_{\beta n}\underset{\vee}{L}{}^{\beta}_{jm} - \underset{\vee}{L}{}^{\alpha}_{\beta m}\underset{\vee}{L}{}^{\beta}_{jn}\big) \\
&\quad - 2a^\alpha_\beta \underset{\vee}{L}{}^{i}_{\alpha n}\underset{\vee}{L}{}^{\beta}_{jm}, \\
a^i_{j\underset{2}{|}m\underset{3}{|}n} - a^i_{j\underset{1}{|}n\underset{3}{|}m} &= 2\underset{\vee}{L}{}^{\alpha}_{jn} a^i_{\alpha|m} + 2\underset{\vee}{L}{}^{\alpha}_{mn} a^i_{j|\alpha} - 2\underset{\vee}{L}{}^{i}_{\alpha m} a^\alpha_{j|n} \\
&\quad + a^\alpha_j\big(R^i_{\alpha mn} - \underset{\vee}{L}{}^{i}_{\alpha m|n} - \underset{\vee}{L}{}^{i}_{\alpha n|m} - \underset{\vee}{L}{}^{\beta}_{\alpha m}\underset{\vee}{L}{}^{i}_{\beta n} - \underset{\vee}{L}{}^{\beta}_{\alpha n}\underset{\vee}{L}{}^{i}_{\beta m}\big) \\
&\quad - a^i_\alpha\big(R^\alpha_{jmn} - \underset{\vee}{L}{}^{\alpha}_{jm|n} - \underset{\vee}{L}{}^{\alpha}_{jn|m} - \underset{\vee}{L}{}^{\alpha}_{\beta n}\underset{\vee}{L}{}^{\beta}_{jm} - \underset{\vee}{L}{}^{\alpha}_{\beta m}\underset{\vee}{L}{}^{\beta}_{jn}\big), \\
a^i_{j\underset{3}{|}m\underset{1}{|}n} - a^i_{j\underset{1}{|}n\underset{3}{|}m} &= a^\alpha_j\big(R^i_{\alpha mn} + \underset{\vee}{L}{}^{i}_{\alpha m|n} - \underset{\vee}{L}{}^{i}_{\alpha n|m} + \underset{\vee}{L}{}^{\beta}_{\alpha m}\underset{\vee}{L}{}^{i}_{\beta n} - \underset{\vee}{L}{}^{\beta}_{\alpha n}\underset{\vee}{L}{}^{i}_{\beta m}\big) \\
&\quad - a^i_\alpha\big(R^\alpha_{jmn} - \underset{\vee}{L}{}^{\alpha}_{jm|n} - \underset{\vee}{L}{}^{\alpha}_{jn|m} - \underset{\vee}{L}{}^{\alpha}_{\beta n}\underset{\vee}{L}{}^{\beta}_{jm} + \underset{\vee}{L}{}^{\alpha}_{\beta m}\underset{\vee}{L}{}^{\beta}_{jn} + 2\underset{\vee}{L}{}^{\alpha}_{j\beta}\underset{\vee}{L}{}^{\beta}_{mn}\big), \\
a^i_{j\underset{3}{|}m\underset{2}{|}n} - a^i_{j\underset{1}{|}n\underset{3}{|}m} &= 2\underset{\vee}{L}{}^{\alpha}_{jn} a^i_{\alpha|m} + 2\underset{\vee}{L}{}^{\alpha}_{mn} a^i_{j|\alpha} - 2\underset{\vee}{L}{}^{i}_{\alpha n} a^\alpha_{j|m} \\
&\quad + a^\alpha_j\big(R^i_{\alpha mn} + \underset{\vee}{L}{}^{i}_{\alpha m|n} - \underset{\vee}{L}{}^{i}_{\alpha n|m} - \underset{\vee}{L}{}^{\beta}_{\alpha m}\underset{\vee}{L}{}^{i}_{\beta n} - \underset{\vee}{L}{}^{\beta}_{\alpha n}\underset{\vee}{L}{}^{i}_{\beta m} + 2\underset{\vee}{L}{}^{i}_{\alpha\beta}\underset{\vee}{L}{}^{\beta}_{mn}\big) \\
&\quad - a^i_\alpha\big(R^\alpha_{jmn} - \underset{\vee}{L}{}^{\alpha}_{jm|n} - \underset{\vee}{L}{}^{\alpha}_{jn|m} - \underset{\vee}{L}{}^{\alpha}_{\beta n}\underset{\vee}{L}{}^{\beta}_{jm} - \underset{\vee}{L}{}^{\alpha}_{\beta m}\underset{\vee}{L}{}^{\beta}_{jn}\big) \\
&\quad + 2a^\alpha_\beta\big(\underset{\vee}{L}{}^{i}_{\alpha m}\underset{\vee}{L}{}^{\beta}_{jn} - \underset{\vee}{L}{}^{i}_{\alpha n}\underset{\vee}{L}{}^{\beta}_{jm}\big),
\end{aligned}
$$

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\begin{aligned}
a^{i}_{j\underset{3}{|}m\underset{3}{|}n} - a^{i}_{j\underset{1}{|}n\underset{3}{|}m} &= 2L^{\alpha}_{\underset{\vee}{jn}}a^{i}_{\alpha|m} + 2L^{\alpha}_{\underset{\vee}{mn}}a^{i}_{j|\alpha}\\
&+ a^{\alpha}_{j}\big(R^{i}_{\alpha mn} + L^{i}_{\underset{\vee}{\alpha m}|n} - L^{i}_{\underset{\vee}{\alpha n}|m} + L^{\beta}_{\underset{\vee}{\alpha m}}L^{i}_{\underset{\vee}{\beta n}} - L^{\beta}_{\underset{\vee}{\alpha n}}L^{i}_{\underset{\vee}{\beta m}} + 2L^{i}_{\underset{\vee}{\alpha\beta}}L^{\beta}_{\underset{\vee}{mn}}\big)\\
&- a^{i}_{\alpha}\big(R^{\alpha}_{jmn} - L^{\alpha}_{\underset{\vee}{jm}|n} - L^{\alpha}_{\underset{\vee}{jn}|m} - L^{\alpha}_{\underset{\vee}{\beta n}}L^{\beta}_{\underset{\vee}{jm}} - L^{\alpha}_{\underset{\vee}{\beta m}}L^{\beta}_{\underset{\vee}{jn}}\big)\\
&+ 2a^{\alpha}_{\beta}L^{i}_{\underset{\vee}{\alpha m}}L^{\beta}_{\underset{\vee}{jn}},\\
a^{i}_{j\underset{1}{|}m\underset{1}{|}n} - a^{i}_{j\underset{2}{|}n\underset{1}{|}m} &= -2L^{\alpha}_{\underset{\vee}{jn}}a^{i}_{\alpha|m} - 2L^{\alpha}_{\underset{\vee}{mn}}a^{i}_{j|\alpha} + 2L^{i}_{\underset{\vee}{\alpha n}}a^{\alpha}_{j|m}\\
&+ a^{\alpha}_{j}\big(R^{i}_{\alpha mn} + L^{i}_{\underset{\vee}{\alpha m}|n} + L^{i}_{\underset{\vee}{\alpha n}|m} + L^{\beta}_{\underset{\vee}{\alpha m}}L^{i}_{\underset{\vee}{\beta n}} + L^{\beta}_{\underset{\vee}{\alpha n}}L^{i}_{\underset{\vee}{\beta m}}\big)\\
&- a^{i}_{\alpha}\big(R^{\alpha}_{jmn} + L^{\alpha}_{\underset{\vee}{jm}|n} + L^{\alpha}_{\underset{\vee}{jn}|m} - L^{\alpha}_{\underset{\vee}{\beta n}}L^{\beta}_{\underset{\vee}{jm}} - L^{\alpha}_{\underset{\vee}{\beta m}}L^{\beta}_{\underset{\vee}{jn}}\big)\\
&- 2a^{\alpha}_{\beta}\big(L^{i}_{\underset{\vee}{\alpha n}}L^{\beta}_{\underset{\vee}{jm}} + L^{i}_{\underset{\vee}{\alpha m}}L^{\beta}_{\underset{\vee}{jn}}\big),\\
a^{i}_{j\underset{1}{|}m\underset{2}{|}n} - a^{i}_{j\underset{2}{|}n\underset{1}{|}m} &= a^{\alpha}_{j}\big(R^{i}_{\alpha mn}+L^{i}_{\underset{\vee}{\alpha m}|n}+L^{i}_{\underset{\vee}{\alpha n}|m}-L^{\beta}_{\underset{\vee}{\alpha m}}L^{i}_{\underset{\vee}{\beta n}}+L^{\beta}_{\underset{\vee}{\alpha n}}L^{i}_{\underset{\vee}{\beta m}}+2L^{i}_{\underset{\vee}{\alpha\beta}}L^{\beta}_{\underset{\vee}{mn}}\big)\\
&- a^{i}_{\alpha}\big(R^{\alpha}_{jmn}+L^{\alpha}_{\underset{\vee}{jm}|n}+L^{\alpha}_{\underset{\vee}{jn}|m}-L^{\alpha}_{\underset{\vee}{\beta n}}L^{\beta}_{\underset{\vee}{jm}}+L^{\alpha}_{\underset{\vee}{\beta m}}L^{\beta}_{\underset{\vee}{jn}}+2L^{\alpha}_{\underset{\vee}{j\beta}}L^{\beta}_{\underset{\vee}{mn}}\big),\\
a^{i}_{j\underset{1}{|}m\underset{3}{|}n} - a^{i}_{j\underset{2}{|}n\underset{1}{|}m} &= 2L^{i}_{\underset{\vee}{\alpha n}}a^{\alpha}_{j|m}\\
&+ a^{\alpha}_{j}\big(R^{i}_{\alpha mn}+L^{i}_{\underset{\vee}{\alpha m}|n}+L^{i}_{\underset{\vee}{\alpha n}|m}+L^{\beta}_{\underset{\vee}{\alpha m}}L^{i}_{\underset{\vee}{\beta n}}+L^{\beta}_{\underset{\vee}{\alpha n}}L^{i}_{\underset{\vee}{\beta m}}+2L^{i}_{\underset{\vee}{\alpha\beta}}L^{\beta}_{\underset{\vee}{mn}}\big)\\
&- a^{i}_{\alpha}\big(R^{\alpha}_{jmn}+L^{\alpha}_{\underset{\vee}{jm}|n}+L^{\alpha}_{\underset{\vee}{jn}|m} - L^{\alpha}_{\underset{\vee}{\beta n}}L^{\beta}_{\underset{\vee}{jm}} + L^{\alpha}_{\underset{\vee}{\beta m}}L^{\beta}_{\underset{\vee}{jn}} + 2L^{\alpha}_{\underset{\vee}{j\beta}}L^{\beta}_{\underset{\vee}{mn}}\big)\\
&- 2a^{\alpha}_{\beta}L^{i}_{\underset{\vee}{\alpha n}}L^{\beta}_{\underset{\vee}{jm}},\\
a^{i}_{j\underset{2}{|}m\underset{1}{|}n} - a^{i}_{j\underset{2}{|}n\underset{1}{|}m} &= 2L^{\alpha}_{\underset{\vee}{jm}}a^{i}_{\alpha|n} - 2L^{\alpha}_{\underset{\vee}{jn}}a^{i}_{\alpha|m} - 2L^{\alpha}_{\underset{\vee}{mn}}a^{i}_{j|\alpha} - 2L^{i}_{\underset{\vee}{\alpha m}}a^{\alpha}_{j|n} + 2L^{i}_{\underset{\vee}{\alpha n}}a^{\alpha}_{j|m}\\
&+ a^{\alpha}_{j}\big(R^{i}_{\alpha mn} - L^{i}_{\underset{\vee}{\alpha m}|n} + L^{i}_{\underset{\vee}{\alpha n}|m} - L^{\beta}_{\underset{\vee}{\alpha m}}L^{i}_{\underset{\vee}{\beta n}} + L^{\beta}_{\underset{\vee}{\alpha n}}L^{i}_{\underset{\vee}{\beta m}} + 2L^{i}_{\underset{\vee}{\alpha\beta}}L^{\beta}_{\underset{\vee}{mn}}\big)\\
&- a^{i}_{\alpha}\big(R^{\alpha}_{jmn} - L^{\alpha}_{\underset{\vee}{jm}|n} + L^{\alpha}_{\underset{\vee}{jn}|m} - L^{\alpha}_{\underset{\vee}{\beta n}}L^{\beta}_{\underset{\vee}{jm}} + L^{\alpha}_{\underset{\vee}{\beta m}}L^{\beta}_{\underset{\vee}{jn}} + 2L^{\alpha}_{\underset{\vee}{j\beta}}L^{\beta}_{\underset{\vee}{mn}}\big),\\
a^{i}_{j\underset{2}{|}m\underset{2}{|}n} - a^{i}_{j\underset{2}{|}n\underset{1}{|}m} &= 2L^{\alpha}_{\underset{\vee}{jm}}a^{i}_{\alpha|n} - 2L^{i}_{\underset{\vee}{\alpha m}}a^{\alpha}_{j|n}\\
&+ a^{\alpha}_{j}\big(R^{i}_{\alpha mn} - L^{i}_{\underset{\vee}{\alpha m}|n} + L^{i}_{\underset{\vee}{\alpha n}|m} + L^{\beta}_{\underset{\vee}{\alpha m}}L^{i}_{\underset{\vee}{\beta n}} + L^{\beta}_{\underset{\vee}{\alpha n}}L^{i}_{\underset{\vee}{\beta m}}\big)\\
&- a^{i}_{\alpha}\big(R^{\alpha}_{jmn} - L^{\alpha}_{\underset{\vee}{jm}|n} + L^{\alpha}_{\underset{\vee}{jn}|m} - L^{\alpha}_{\underset{\vee}{\beta n}}L^{\beta}_{\underset{\vee}{jm}} - L^{\alpha}_{\underset{\vee}{\beta m}}L^{\beta}_{\underset{\vee}{jn}}\big)\\
&- 2a^{\alpha}_{\beta}\big(L^{i}_{\underset{\vee}{\alpha m}}L^{\beta}_{\underset{\vee}{jn}} + L^{i}_{\underset{\vee}{\alpha n}}L^{\beta}_{\underset{\vee}{jm}}\big),\\
a^{i}_{j\underset{2}{|}m\underset{3}{|}n} - a^{i}_{j\underset{2}{|}n\underset{1}{|}m} &= 2L^{\alpha}_{\underset{\vee}{jm}}a^{i}_{\alpha|n} - 2L^{i}_{\underset{\vee}{\alpha m}}a^{\alpha}_{j|n} + 2L^{i}_{\underset{\vee}{\alpha n}}a^{\alpha}_{j|m}\\
&+ a^{\alpha}_{j}\big(R^{i}_{\alpha mn} - L^{i}_{\underset{\vee}{\alpha m}|n} + L^{i}_{\underset{\vee}{\alpha n}|m} - L^{\beta}_{\underset{\vee}{\alpha m}}L^{i}_{\underset{\vee}{\beta n}} + L^{\beta}_{\underset{\vee}{\alpha n}}L^{i}_{\underset{\vee}{\beta m}}\big)\\
&- a^{i}_{\alpha}\big(R^{\alpha}_{jmn} - L^{\alpha}_{\underset{\vee}{jm}|n} + L^{\alpha}_{\underset{\vee}{jn}|m} - L^{\alpha}_{\underset{\vee}{\beta n}}L^{\beta}_{\underset{\vee}{jm}} - L^{\alpha}_{\underset{\vee}{\beta m}}L^{\beta}_{\underset{\vee}{jn}}\big)\\
&- 2a^{\alpha}_{\beta}L^{i}_{\underset{\vee}{\alpha m}}L^{\beta}_{\underset{\vee}{jn}},
\end{aligned}
$$

$$\begin{aligned}
a^i_{j\underset{3}{|}m\underset{1}{|}n} - a^i_{j\underset{2}{|}n\underset{1}{|}m} &= 2L^\alpha_{\underset{\vee}{jm}} a^i_{\alpha|n} - 2L^\alpha_{\underset{\vee}{jn}} a^i_{\alpha|m} - 2L^\alpha_{\underset{\vee}{mn}} a^i_{j|\alpha} + 2L^i_{\underset{\vee}{\alpha n}} a^\alpha_{j|m} \\
&+ a^\alpha_j \big(R^i_{\alpha mn} + L^i_{\underset{\vee}{\alpha m}|n} + L^i_{\underset{\vee}{\alpha n}|m} + L^\beta_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} + L^\beta_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}}\big) \\
&- a^i_\alpha \big(R^\alpha_{jmn} - L^\alpha_{\underset{\vee}{jm}|n} + L^\alpha_{\underset{\vee}{jn}|m} - L^\alpha_{\underset{\vee}{\beta n}} L^\beta_{\underset{\vee}{jm}} + L^\alpha_{\underset{\vee}{\beta m}} L^\beta_{\underset{\vee}{jn}} + 2L^\alpha_{\underset{\vee}{j\beta}} L^\beta_{\underset{\vee}{mn}}\big) \\
&- 2a^\alpha_\beta L^i_{\underset{\vee}{\alpha m}} L^\beta_{\underset{\vee}{jn}},
\end{aligned}$$

$$\begin{aligned}
a^i_{j\underset{3}{|}m\underset{2}{|}n} - a^i_{j\underset{2}{|}n\underset{1}{|}m} &= 2L^\alpha_{\underset{\vee}{jm}} a^i_{\alpha|n} \\
&+ a^\alpha_j \big(R^i_{\alpha mn} + L^i_{\underset{\vee}{\alpha m}|n} + L^i_{\underset{\vee}{\alpha n}|m} - L^\beta_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} + L^\beta_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}} + 2L^i_{\underset{\vee}{\alpha\beta}} L^\beta_{\underset{\vee}{mn}}\big) \\
&- a^i_\alpha \big(R^\alpha_{jmn} - L^\alpha_{\underset{\vee}{jm}|n} + L^\alpha_{\underset{\vee}{jn}|m} - L^\alpha_{\underset{\vee}{\beta n}} L^\beta_{\underset{\vee}{jm}} - L^\alpha_{\underset{\vee}{\beta m}} L^\beta_{\underset{\vee}{jn}}\big) \\
&- 2a^\alpha_\beta L^i_{\underset{\vee}{\alpha n}} L^\beta_{\underset{\vee}{jm}},
\end{aligned}$$

$$\begin{aligned}
a^i_{j\underset{3}{|}m\underset{3}{|}n} - a^i_{j\underset{2}{|}n\underset{1}{|}m} &= 2L^\alpha_{\underset{\vee}{jm}} a^i_{\alpha|n} + 2L^i_{\underset{\vee}{\alpha n}} a^\alpha_{j|m} \\
&+ a^\alpha_j \big(R^i_{\alpha mn} + L^i_{\underset{\vee}{\alpha m}|n} + L^i_{\underset{\vee}{\alpha n}|m} + L^\beta_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} + L^\beta_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}} + 2L^i_{\underset{\vee}{\alpha\beta}} L^\beta_{\underset{\vee}{mn}}\big) \\
&- a^i_\alpha \big(R^\alpha_{jmn} - L^\alpha_{\underset{\vee}{jm}|n} + L^\alpha_{\underset{\vee}{jn}|m} - L^\alpha_{\underset{\vee}{\beta n}} L^\beta_{\underset{\vee}{jm}} - L^\alpha_{\underset{\vee}{\beta m}} L^\beta_{\underset{\vee}{jn}}\big),
\end{aligned}$$

$$\begin{aligned}
a^i_{j\underset{1}{|}m\underset{1}{|}n} - a^i_{j\underset{2}{|}n\underset{2}{|}m} &= -2L^\alpha_{\underset{\vee}{jm}} a^i_{\alpha|n} - 2L^\alpha_{\underset{\vee}{jn}} a^i_{\alpha|m} + 2L^i_{\underset{\vee}{\alpha m}} a^\alpha_{j|n} + 2L^i_{\underset{\vee}{\alpha n}} a^\alpha_{j|m} \\
&+ a^\alpha_j \big(R^i_{\alpha mn} + L^i_{\underset{\vee}{\alpha m}|n} + L^i_{\underset{\vee}{\alpha n}|m} + L^\beta_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} - L^\beta_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}} - 2L^i_{\underset{\vee}{\alpha\beta}} L^\beta_{\underset{\vee}{mn}}\big) \\
&- a^i_\alpha \big(R^\alpha_{jmn} + L^\alpha_{\underset{\vee}{jm}|n} + L^\alpha_{\underset{\vee}{jn}|m} + L^\alpha_{\underset{\vee}{\beta n}} L^\beta_{\underset{\vee}{jm}} - L^\alpha_{\underset{\vee}{\beta m}} L^\beta_{\underset{\vee}{jn}} - 2L^\alpha_{\underset{\vee}{j\beta}} L^\beta_{\underset{\vee}{mn}}\big),
\end{aligned}$$

$$\begin{aligned}
a^i_{j\underset{1}{|}m\underset{2}{|}n} - a^i_{j\underset{2}{|}n\underset{2}{|}m} &= -2L^\alpha_{\underset{\vee}{jm}} a^i_{\alpha|n} + 2L^\alpha_{\underset{\vee}{mn}} a^i_{j|\alpha} + 2L^i_{\underset{\vee}{\alpha m}} a^\alpha_{j|n} \\
&+ a^\alpha_j \big(R^i_{\alpha mn} + L^i_{\underset{\vee}{\alpha m}|n} + L^i_{\underset{\vee}{\alpha n}|m} - L^\beta_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} - L^\beta_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}}\big) \\
&- a^i_\alpha \big(R^\alpha_{jmn} + L^\alpha_{\underset{\vee}{jm}|n} + L^\alpha_{\underset{\vee}{jn}|m} + L^\alpha_{\underset{\vee}{\beta n}} L^\beta_{\underset{\vee}{jm}} + L^\alpha_{\underset{\vee}{\beta m}} L^\beta_{\underset{\vee}{jn}}\big) \\
&+ 2a^\alpha_\beta \big(L^i_{\underset{\vee}{\alpha m}} L^\beta_{\underset{\vee}{jn}} + L^i_{\underset{\vee}{\alpha n}} L^\beta_{\underset{\vee}{jm}}\big),
\end{aligned}$$

$$\begin{aligned}
a^i_{j\underset{1}{|}m\underset{3}{|}n} - a^i_{j\underset{2}{|}n\underset{2}{|}m} &= -2L^\alpha_{\underset{\vee}{jm}} a^i_{\alpha|n} + 2L^\alpha_{\underset{\vee}{mn}} a^i_{j|\alpha} + 2L^i_{\underset{\vee}{\alpha m}} a^\alpha_{j|n} + 2L^i_{\underset{\vee}{\alpha n}} a^\alpha_{j|m} \\
&+ a^\alpha_j \big(R^i_{\alpha mn} + L^i_{\underset{\vee}{\alpha m}|n} + L^i_{\underset{\vee}{\alpha n}|m} + L^\beta_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} - L^\beta_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}}\big) \\
&- a^i_\alpha \big(R^\alpha_{jmn} + L^\alpha_{\underset{\vee}{jm}|n} + L^\alpha_{\underset{\vee}{jn}|m} + L^\alpha_{\underset{\vee}{\beta n}} L^\beta_{\underset{\vee}{jm}} + L^\alpha_{\underset{\vee}{\beta m}} L^\beta_{\underset{\vee}{jn}}\big) \\
&+ 2a^\alpha_\beta L^i_{\underset{\vee}{\alpha m}} L^\beta_{\underset{\vee}{jn}},
\end{aligned}$$

$$\begin{aligned}
a^i_{j\underset{2}{|}m\underset{1}{|}n} - a^i_{j\underset{2}{|}n\underset{2}{|}m} &= -2L^\alpha_{\underset{\vee}{jn}} a^i_{\alpha|m} + 2L^i_{\underset{\vee}{\alpha n}} a^\alpha_{j|m} \\
&+ a^\alpha_j \big(R^i_{\alpha mn} - L^i_{\underset{\vee}{\alpha m}|n} + L^i_{\underset{\vee}{\alpha n}|m} - L^\beta_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} - L^\beta_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}}\big) \\
&- a^i_\alpha \big(R^\alpha_{jmn} - L^\alpha_{\underset{\vee}{jm}|n} + L^\alpha_{\underset{\vee}{jn}|m} + L^\alpha_{\underset{\vee}{\beta n}} L^\beta_{\underset{\vee}{jm}} + L^\alpha_{\underset{\vee}{\beta m}} L^\beta_{\underset{\vee}{jn}}\big) \\
&+ 2a^\alpha_\beta \big(L^i_{\underset{\vee}{\alpha m}} L^\beta_{\underset{\vee}{jn}} + L^i_{\underset{\vee}{\alpha n}} L^\beta_{\underset{\vee}{jm}}\big),
\end{aligned}$$

$$
\begin{aligned}
a^i_{j\underset{2}{|}m\underset{2}{|}n} - a^i_{j\underset{2}{|}n\underset{2}{|}m} &= 2L^\alpha_{\underset{\vee}{mn}} a^i_{j|\alpha} \\
&+ a^\alpha_j \big(R^i_{\alpha mn} - L^i_{\underset{\vee}{\alpha m}|n} + L^i_{\underset{\vee}{\alpha n}|m} + L^\beta_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} - L^\beta_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}} - 2L^i_{\underset{\vee}{\alpha\beta}} L^\beta_{\underset{\vee}{mn}}\big) \\
&- a^i_\alpha \big(R^\alpha_{jmn} - L^\alpha_{\underset{\vee}{jm}|n} + L^\alpha_{\underset{\vee}{jn}|m} + L^\alpha_{\underset{\vee}{\beta n}} L^\beta_{\underset{\vee}{jm}} - L^\alpha_{\underset{\vee}{\beta m}} L^\beta_{\underset{\vee}{jn}} - 2L^\alpha_{\underset{\vee}{j\beta}} L^\beta_{\underset{\vee}{mn}}\big), \\
a^i_{j\underset{2}{|}m\underset{3}{|}n} - a^i_{j\underset{2}{|}n\underset{2}{|}m} &= 2L^\alpha_{\underset{\vee}{mn}} a^i_{j|\alpha} + 2L^i_{\underset{\vee}{\alpha n}} a^\alpha_{j|m} \\
&+ a^\alpha_j \big(R^i_{\alpha mn} - L^i_{\underset{\vee}{\alpha m}|n} + L^i_{\underset{\vee}{\alpha n}|m} - L^\beta_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} - L^\beta_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}} - 2L^i_{\underset{\vee}{\alpha\beta}} L^\beta_{\underset{\vee}{mn}}\big) \\
&- a^i_\alpha \big(R^\alpha_{jmn} - L^\alpha_{\underset{\vee}{jm}|n} + L^\alpha_{\underset{\vee}{jn}|m} + L^\alpha_{\underset{\vee}{\beta n}} L^\beta_{\underset{\vee}{jm}} - L^\alpha_{\underset{\vee}{\beta m}} L^\beta_{\underset{\vee}{jn}} - 2L^\alpha_{\underset{\vee}{j\beta}} L^\beta_{\underset{\vee}{mn}}\big) \\
&+ 2a^\alpha_\beta L^i_{\underset{\vee}{\alpha n}} L^\beta_{\underset{\vee}{jm}}, \\
a^i_{j\underset{3}{|}m\underset{1}{|}n} - a^i_{j\underset{2}{|}n\underset{2}{|}m} &= -2L^\alpha_{\underset{\vee}{jn}} a^i_{\alpha|m} + 2L^i_{\underset{\vee}{\alpha m}} a^\alpha_{j|n} + 2L^i_{\underset{\vee}{\alpha n}} a^\alpha_{j|m} \\
&+ a^\alpha_j \big(R^i_{\alpha mn} + L^i_{\underset{\vee}{\alpha m}|n} + L^i_{\underset{\vee}{\alpha n}|m} + L^\beta_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} - L^\beta_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}} - 2L^i_{\underset{\vee}{\alpha\beta}} L^\beta_{\underset{\vee}{mn}}\big) \\
&- a^i_\alpha \big(R^\alpha_{jmn} - L^\alpha_{\underset{\vee}{jm}|n} + L^\alpha_{\underset{\vee}{jn}|m} + L^\alpha_{\underset{\vee}{\beta n}} L^\beta_{\underset{\vee}{jm}} + L^\alpha_{\underset{\vee}{\beta m}} L^\beta_{\underset{\vee}{jn}}\big) \\
&+ 2a^\alpha_\beta L^i_{\underset{\vee}{\alpha n}} L^\beta_{\underset{\vee}{jm}}, \\
a^i_{j\underset{3}{|}m\underset{2}{|}n} - a^i_{j\underset{2}{|}n\underset{2}{|}m} &= 2L^\alpha_{\underset{\vee}{mn}} a^i_{j|\alpha} + 2L^i_{\underset{\vee}{\alpha m}} a^\alpha_{j|n} \\
&+ a^\alpha_j \big(R^i_{\alpha mn} + L^i_{\underset{\vee}{\alpha m}|n} + L^i_{\underset{\vee}{\alpha n}|m} - L^\beta_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} - L^\beta_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}}\big) \\
&- a^i_\alpha \big(R^\alpha_{jmn} - L^\alpha_{\underset{\vee}{jm}|n} + L^\alpha_{\underset{\vee}{jn}|m} + L^\alpha_{\underset{\vee}{\beta n}} L^\beta_{\underset{\vee}{jm}} - L^\alpha_{\underset{\vee}{\beta m}} L^\beta_{\underset{\vee}{jn}} - 2L^\alpha_{\underset{\vee}{j\beta}} L^\beta_{\underset{\vee}{mn}}\big) \\
&+ 2a^\alpha_\beta L^i_{\underset{\vee}{\alpha m}} L^\beta_{\underset{\vee}{jn}}, \\
a^i_{j\underset{3}{|}m\underset{3}{|}n} - a^i_{j\underset{2}{|}n\underset{2}{|}m} &= 2L^\alpha_{\underset{\vee}{mn}} a^i_{j|\alpha} + 2L^i_{\underset{\vee}{\alpha m}} a^\alpha_{j|n} + 2L^i_{\underset{\vee}{\alpha n}} a^\alpha_{j|m} \\
&+ a^\alpha_j \big(R^i_{\alpha mn} + L^i_{\underset{\vee}{\alpha m}|n} + L^i_{\underset{\vee}{\alpha n}|m} + L^\beta_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} - L^\beta_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}}\big) \\
&- a^i_\alpha \big(R^\alpha_{jmn} - L^\alpha_{\underset{\vee}{jm}|n} + L^\alpha_{\underset{\vee}{jn}|m} + L^\alpha_{\underset{\vee}{\beta n}} L^\beta_{\underset{\vee}{jm}} - L^\alpha_{\underset{\vee}{\beta m}} L^\beta_{\underset{\vee}{jn}} - 2L^\alpha_{\underset{\vee}{j\beta}} L^\beta_{\underset{\vee}{mn}}\big) \\
&+ 2a^\alpha_\beta \big(L^i_{\underset{\vee}{\alpha n}} L^\beta_{\underset{\vee}{jm}} + L^i_{\underset{\vee}{\alpha m}} L^\beta_{\underset{\vee}{jn}}\big), \\
a^i_{j\underset{1}{|}m\underset{1}{|}n} - a^i_{j\underset{2}{|}n\underset{3}{|}m} &= -2L^\alpha_{\underset{\vee}{jm}} a^i_{\alpha|n} - 2L^\alpha_{\underset{\vee}{jn}} a^i_{\alpha|m} + 2L^i_{\underset{\vee}{\alpha n}} a^\alpha_{j|m} \\
&+ a^\alpha_j \big(R^i_{\alpha mn} + L^i_{\underset{\vee}{\alpha m}|n} + L^i_{\underset{\vee}{\alpha n}|m} + L^\beta_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} + L^\beta_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}} - 2L^i_{\underset{\vee}{\alpha\beta}} L^\beta_{\underset{\vee}{mn}}\big) \\
&- a^i_\alpha \big(R^\alpha_{jmn} + L^\alpha_{\underset{\vee}{jm}|n} + L^\alpha_{\underset{\vee}{jn}|m} + L^\alpha_{\underset{\vee}{\beta n}} L^\beta_{\underset{\vee}{jm}} - L^\alpha_{\underset{\vee}{\beta m}} L^\beta_{\underset{\vee}{jn}} - 2L^\alpha_{\underset{\vee}{j\beta}} L^\beta_{\underset{\vee}{mn}}\big) \\
&- 2a^\alpha_\beta L^i_{\underset{\vee}{\alpha m}} L^\beta_{\underset{\vee}{jn}}, \\
a^i_{j\underset{1}{|}m\underset{2}{|}n} - a^i_{j\underset{2}{|}n\underset{3}{|}m} &= -2L^\alpha_{\underset{\vee}{jm}} a^i_{\alpha|n} + 2L^\alpha_{\underset{\vee}{mn}} a^i_{j|\alpha} \\
&+ a^\alpha_j \big(R^i_{\alpha mn} + L^i_{\underset{\vee}{\alpha m}|n} + L^i_{\underset{\vee}{\alpha n}|m} - L^\beta_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} + L^\beta_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}}\big) \\
&- a^i_\alpha \big(R^\alpha_{jmn} + L^\alpha_{\underset{\vee}{jm}|n} + L^\alpha_{\underset{\vee}{jn}|m} + L^\alpha_{\underset{\vee}{\beta n}} L^\beta_{\underset{\vee}{jm}} + L^\alpha_{\underset{\vee}{\beta m}} L^\beta_{\underset{\vee}{jn}}\big) \\
&+ 2a^\alpha_\beta L^i_{\underset{\vee}{\alpha n}} L^\beta_{\underset{\vee}{jm}},
\end{aligned}
$$

$$
\begin{aligned}
a^i_{j\underset{1}{|}m\underset{3}{|}n} - a^i_{j\underset{2}{|}n\underset{3}{|}m} &= -2L^\alpha_{\underset{\vee}{jm}} a^i_{\alpha|n} + 2L^\alpha_{\underset{\vee}{mn}} a^i_{j|\alpha} + 2L^i_{\underset{\vee}{\alpha n}} a^\alpha_{j|m} \\
&\quad + a^\alpha_j \big(R^i_{\alpha mn} + L^i_{\underset{\vee}{\alpha m}|n} + L^i_{\underset{\vee}{\alpha n}|m} + L^\beta_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} + L^\beta_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}}\big) \\
&\quad - a^i_\alpha \big(R^\alpha_{jmn} + L^\alpha_{\underset{\vee}{jm}|n} + L^\alpha_{\underset{\vee}{jn}|m} + L^\alpha_{\underset{\vee}{\beta n}} L^\beta_{\underset{\vee}{jm}} + L^\alpha_{\underset{\vee}{\beta m}} L^\beta_{\underset{\vee}{jn}}\big), \\
a^i_{j\underset{2}{|}m\underset{1}{|}n} - a^i_{j\underset{2}{|}n\underset{3}{|}m} &= -2L^\alpha_{\underset{\vee}{jn}} a^i_{\alpha|m} - 2L^i_{\underset{\vee}{\alpha m}} a^\alpha_{j|n} + 2L^i_{\underset{\vee}{\alpha n}} a^\alpha_{j|m} \\
&\quad + a^\alpha_j \big(R^i_{\alpha mn} - L^i_{\underset{\vee}{\alpha m}|n} + L^i_{\underset{\vee}{\alpha n}|m} - L^\beta_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} + L^\beta_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}}\big) \\
&\quad - a^i_\alpha \big(R^\alpha_{jmn} - L^\alpha_{\underset{\vee}{jm}|n} + L^\alpha_{\underset{\vee}{jn}|m} + L^\alpha_{\underset{\vee}{\beta n}} L^\beta_{\underset{\vee}{jm}} + L^\alpha_{\underset{\vee}{\beta m}} L^\beta_{\underset{\vee}{jn}}\big) \\
&\quad + 2a^\alpha_\beta L^i_{\underset{\vee}{\alpha n}} L^\beta_{\underset{\vee}{jm}}, \\
a^i_{j\underset{2}{|}m\underset{2}{|}n} - a^i_{j\underset{2}{|}n\underset{3}{|}m} &= 2L^\alpha_{\underset{\vee}{mn}} a^i_{j|\alpha} - 2L^i_{\underset{\vee}{\alpha m}} a^\alpha_{j|n} \\
&\quad + a^\alpha_j \big(R^i_{\alpha mn} - L^i_{\underset{\vee}{\alpha m}|n} + L^i_{\underset{\vee}{\alpha n}|m} + L^\beta_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} + L^\beta_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}} - 2L^i_{\underset{\vee}{\alpha\beta}} L^\beta_{\underset{\vee}{mn}}\big) \\
&\quad - a^i_\alpha \big(R^\alpha_{jmn} - L^\alpha_{\underset{\vee}{jm}|n} + L^\alpha_{\underset{\vee}{jn}|m} + L^\alpha_{\underset{\vee}{\beta n}} L^\beta_{\underset{\vee}{jm}} - L^\alpha_{\underset{\vee}{\beta m}} L^\beta_{\underset{\vee}{jn}} - 2L^\alpha_{\underset{\vee}{j\beta}} L^\beta_{\underset{\vee}{mn}}\big) \\
&\quad - 2a^\alpha_\beta L^i_{\underset{\vee}{\alpha m}} L^\beta_{\underset{\vee}{jn}}, \\
a^i_{j\underset{2}{|}m\underset{3}{|}n} - a^i_{j\underset{2}{|}n\underset{3}{|}m} &= 2L^\alpha_{\underset{\vee}{mn}} a^i_{j|\alpha} - 2L^i_{\underset{\vee}{\alpha m}} a^\alpha_{j|n} + 2L^i_{\underset{\vee}{\alpha n}} a^\alpha_{j|m} \\
&\quad + a^\alpha_j \big(R^i_{\alpha mn} - L^i_{\underset{\vee}{\alpha m}|n} + L^i_{\underset{\vee}{\alpha n}|m} - L^\beta_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} + L^\beta_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}} - 2L^i_{\underset{\vee}{\alpha\beta}} L^\beta_{\underset{\vee}{mn}}\big) \\
&\quad - a^i_\alpha \big(R^\alpha_{jmn} - L^\alpha_{\underset{\vee}{jm}|n} + L^\alpha_{\underset{\vee}{jn}|m} + L^\alpha_{\underset{\vee}{\beta n}} L^\beta_{\underset{\vee}{jm}} - L^\alpha_{\underset{\vee}{\beta m}} L^\beta_{\underset{\vee}{jn}} - 2L^\alpha_{\underset{\vee}{j\beta}} L^\beta_{\underset{\vee}{mn}}\big) \\
&\quad - 2a^\alpha_\beta \big(L^i_{\underset{\vee}{\alpha m}} L^\beta_{\underset{\vee}{jn}} - L^i_{\underset{\vee}{\alpha n}} L^\beta_{\underset{\vee}{jm}}\big), \\
a^i_{j\underset{3}{|}m\underset{1}{|}n} - a^i_{j\underset{2}{|}n\underset{3}{|}m} &= -2L^\alpha_{\underset{\vee}{jn}} a^i_{\alpha|m} + 2L^i_{\underset{\vee}{\alpha n}} a^\alpha_{j|m} \\
&\quad + a^\alpha_j \big(R^i_{\alpha mn} + L^i_{\underset{\vee}{\alpha m}|n} + L^i_{\underset{\vee}{\alpha n}|m} + L^\beta_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} + L^\beta_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}} - 2L^i_{\underset{\vee}{\alpha\beta}} L^\beta_{\underset{\vee}{mn}}\big) \\
&\quad - a^i_\alpha \big(R^\alpha_{jmn} - L^\alpha_{\underset{\vee}{jm}|n} + L^\alpha_{\underset{\vee}{jn}|m} + L^\alpha_{\underset{\vee}{\beta n}} L^\beta_{\underset{\vee}{jm}} + L^\alpha_{\underset{\vee}{\beta m}} L^\beta_{\underset{\vee}{jn}}\big) \\
&\quad - 2a^\alpha_\beta \big(L^i_{\underset{\vee}{\alpha m}} L^\beta_{\underset{\vee}{jn}} - L^i_{\underset{\vee}{\alpha n}} L^\beta_{\underset{\vee}{jm}}\big), \\
a^i_{j\underset{3}{|}m\underset{2}{|}n} - a^i_{j\underset{2}{|}n\underset{3}{|}m} &= 2L^\alpha_{\underset{\vee}{mn}} a^i_{j|\alpha} \\
&\quad + a^\alpha_j \big(R^i_{\alpha mn} + L^i_{\underset{\vee}{\alpha m}|n} + L^i_{\underset{\vee}{\alpha n}|m} - L^\beta_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} + L^\beta_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}}\big) \\
&\quad - a^i_\alpha \big(R^\alpha_{jmn} - L^\alpha_{\underset{\vee}{jm}|n} + L^\alpha_{\underset{\vee}{jn}|m} + L^\alpha_{\underset{\vee}{\beta n}} L^\beta_{\underset{\vee}{jm}} - L^\alpha_{\underset{\vee}{\beta m}} L^\beta_{\underset{\vee}{jn}} - 2L^\alpha_{\underset{\vee}{j\beta}} L^\beta_{\underset{\vee}{mn}}\big), \\
a^i_{j\underset{3}{|}m\underset{3}{|}n} - a^i_{j\underset{2}{|}n\underset{3}{|}m} &= 2L^\alpha_{\underset{\vee}{mn}} a^i_{j|\alpha} + 2L^i_{\underset{\vee}{\alpha n}} a^\alpha_{j|m} \\
&\quad + a^\alpha_j \big(R^i_{\alpha mn} + L^i_{\underset{\vee}{\alpha m}|n} + L^i_{\underset{\vee}{\alpha n}|m} + L^\beta_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} + L^\beta_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}}\big) \\
&\quad - a^i_\alpha \big(R^\alpha_{jmn} - L^\alpha_{\underset{\vee}{jm}|n} + L^\alpha_{\underset{\vee}{jn}|m} + L^\alpha_{\underset{\vee}{\beta n}} L^\beta_{\underset{\vee}{jm}} - L^\alpha_{\underset{\vee}{\beta m}} L^\beta_{\underset{\vee}{jn}} - 2L^\alpha_{\underset{\vee}{j\beta}} L^\beta_{\underset{\vee}{mn}}\big) \\
&\quad + 2a^\alpha_\beta L^i_{\underset{\vee}{\alpha n}} L^\beta_{\underset{\vee}{jm}}, \\
a^i_{j\underset{1}{|}m\underset{1}{|}n} - a^i_{j\underset{3}{|}n\underset{1}{|}m} &= -2L^\alpha_{\underset{\vee}{jn}} a^i_{\alpha|m} - 2L^\alpha_{\underset{\vee}{mn}} a^i_{j|\alpha} \\
&\quad + a^\alpha_j \big(R^i_{\alpha mn} + L^i_{\underset{\vee}{\alpha m}|n} - L^i_{\underset{\vee}{\alpha n}|m} + L^\beta_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} - L^\beta_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}} - 2L^i_{\underset{\vee}{\alpha\beta}} L^\beta_{\underset{\vee}{mn}}\big) \\
&\quad - a^i_\alpha \big(R^\alpha_{jmn} + L^\alpha_{\underset{\vee}{jm}|n} + L^\alpha_{\underset{\vee}{jn}|m} - L^\alpha_{\underset{\vee}{\beta n}} L^\beta_{\underset{\vee}{jm}} - L^\alpha_{\underset{\vee}{\beta m}} L^\beta_{\underset{\vee}{jn}}\big) \\
&\quad - 2a^\alpha_\beta L^i_{\underset{\vee}{\alpha m}} L^\beta_{\underset{\vee}{jn}},
\end{aligned}
$$

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\begin{aligned}
a^i_{j\underset{1}{|}m\underset{2}{|}n} - a^i_{j\underset{3}{|}n\underset{1}{|}m} &= -2L^i_{\underset{\vee}{\alpha n}}a^\alpha_{j|m}\\
&\quad + a^\alpha_j\big(R^i_{\alpha mn} + L^i_{\underset{\vee}{\alpha m}|n} - L^i_{\underset{\vee}{\alpha n}|m} - L^\beta_{\underset{\vee}{\alpha m}}L^i_{\underset{\vee}{\beta n}} - L^\beta_{\underset{\vee}{\alpha n}}L^i_{\underset{\vee}{\beta m}}\big)\\
&\quad - a^i_\alpha\big(R^\alpha_{jmn} + L^\alpha_{\underset{\vee}{jm}|n} + L^\alpha_{\underset{\vee}{jn}|m} - L^\alpha_{\underset{\vee}{\beta n}}L^\beta_{\underset{\vee}{jm}} + L^\alpha_{\underset{\vee}{\beta m}}L^\beta_{\underset{\vee}{jn}} + 2L^\alpha_{\underset{\vee}{j\beta}}L^\beta_{\underset{\vee}{mn}}\big)\\
&\quad + 2a^\alpha_\beta L^i_{\underset{\vee}{\alpha n}}L^\beta_{\underset{\vee}{jm}},\\
a^i_{j\underset{1}{|}m\underset{3}{|}n} - a^i_{j\underset{3}{|}n\underset{1}{|}m} &= a^\alpha_j\big(R^i_{\alpha mn} + L^i_{\underset{\vee}{\alpha m}|n} - L^i_{\underset{\vee}{\alpha n}|m} + L^\beta_{\underset{\vee}{\alpha m}}L^i_{\underset{\vee}{\beta n}} - L^\beta_{\underset{\vee}{\alpha n}}L^i_{\underset{\vee}{\beta m}}\big)\\
&\quad - a^i_\alpha\big(R^\alpha_{jmn} + L^\alpha_{\underset{\vee}{jm}|n} + L^\alpha_{\underset{\vee}{jn}|m} - L^\alpha_{\underset{\vee}{\beta n}}L^\beta_{\underset{\vee}{jm}} + L^\alpha_{\underset{\vee}{\beta m}}L^\beta_{\underset{\vee}{jn}} + 2L^\alpha_{\underset{\vee}{j\beta}}L^\beta_{\underset{\vee}{mn}}\big),\\
a^i_{j\underset{2}{|}m\underset{1}{|}n} - a^i_{j\underset{3}{|}n\underset{1}{|}m} &= 2L^\alpha_{\underset{\vee}{jm}}a^i_{\alpha|n} - 2L^\alpha_{\underset{\vee}{jn}}a^i_{\alpha|m} - 2L^\alpha_{\underset{\vee}{mn}}a^i_{j|\alpha} - 2L^i_{\underset{\vee}{\alpha m}}a^\alpha_{j|n}\\
&\quad + a^\alpha_j\big(R^i_{\alpha mn} - L^i_{\underset{\vee}{\alpha m}|n} - L^i_{\underset{\vee}{\alpha n}|m} - L^\beta_{\underset{\vee}{\alpha m}}L^i_{\underset{\vee}{\beta n}} - L^\beta_{\underset{\vee}{\alpha n}}L^i_{\underset{\vee}{\beta m}}\big)\\
&\quad - a^i_\alpha\big(R^\alpha_{jmn} - L^\alpha_{\underset{\vee}{jm}|n} + L^\alpha_{\underset{\vee}{jn}|m} - L^\alpha_{\underset{\vee}{\beta n}}L^\beta_{\underset{\vee}{jm}} + L^\alpha_{\underset{\vee}{\beta m}}L^\beta_{\underset{\vee}{jn}} + 2L^\alpha_{\underset{\vee}{j\beta}}L^\beta_{\underset{\vee}{mn}}\big)\\
&\quad + 2a^\alpha_\beta L^i_{\underset{\vee}{\alpha n}}L^\beta_{\underset{\vee}{jm}},\\
a^i_{j\underset{2}{|}m\underset{2}{|}n} - a^i_{j\underset{3}{|}n\underset{1}{|}m} &= 2L^\alpha_{\underset{\vee}{jm}}a^i_{\alpha|n} - 2L^i_{\underset{\vee}{\alpha m}}a^\alpha_{j|n} - 2L^i_{\underset{\vee}{\alpha n}}a^\alpha_{j|m}\\
&\quad + a^\alpha_j\big(R^i_{\alpha mn} - L^i_{\underset{\vee}{\alpha m}|n} - L^i_{\underset{\vee}{\alpha n}|m} + L^\beta_{\underset{\vee}{\alpha m}}L^i_{\underset{\vee}{\beta n}} - L^\beta_{\underset{\vee}{\alpha n}}L^i_{\underset{\vee}{\beta m}} - 2L^i_{\underset{\vee}{\alpha\beta}}L^\beta_{\underset{\vee}{mn}}\big)\\
&\quad - a^i_\alpha\big(R^\alpha_{jmn} - L^\alpha_{\underset{\vee}{jm}|n} + L^\alpha_{\underset{\vee}{jn}|m} - L^\alpha_{\underset{\vee}{\beta n}}L^\beta_{\underset{\vee}{jm}} - L^\alpha_{\underset{\vee}{\beta m}}L^\beta_{\underset{\vee}{jn}}\big)\\
&\quad - 2a^\alpha_\beta L^i_{\underset{\vee}{\alpha m}}L^\beta_{\underset{\vee}{jn}},\\
a^i_{j\underset{2}{|}m\underset{3}{|}n} - a^i_{j\underset{3}{|}n\underset{1}{|}m} &= 2L^\alpha_{\underset{\vee}{jm}}a^i_{\alpha|n} - 2L^i_{\underset{\vee}{\alpha m}}a^\alpha_{j|n}\\
&\quad + a^\alpha_j\big(R^i_{\alpha mn} - L^i_{\underset{\vee}{\alpha m}|n} - L^i_{\underset{\vee}{\alpha n}|m} - L^\beta_{\underset{\vee}{\alpha m}}L^i_{\underset{\vee}{\beta n}} - L^\beta_{\underset{\vee}{\alpha n}}L^i_{\underset{\vee}{\beta m}} - 2L^i_{\underset{\vee}{\alpha\beta}}L^\beta_{\underset{\vee}{mn}}\big)\\
&\quad - a^i_\alpha\big(R^\alpha_{jmn} - L^\alpha_{\underset{\vee}{jm}|n} + L^\alpha_{\underset{\vee}{jn}|m} - L^\alpha_{\underset{\vee}{\beta n}}L^\beta_{\underset{\vee}{jm}} - L^\alpha_{\underset{\vee}{\beta m}}L^\beta_{\underset{\vee}{jn}}\big)\\
&\quad - 2a^\alpha_\beta\big(L^i_{\underset{\vee}{\alpha m}}L^\beta_{\underset{\vee}{jn}} - L^i_{\underset{\vee}{\alpha n}}L^\beta_{\underset{\vee}{jm}}\big),\\
a^i_{j\underset{3}{|}m\underset{1}{|}n} - a^i_{j\underset{3}{|}n\underset{1}{|}m} &= 2L^\alpha_{\underset{\vee}{jm}}a^i_{\alpha|n} - 2L^\alpha_{\underset{\vee}{jn}}a^i_{\alpha|m} - 2L^\alpha_{\underset{\vee}{mn}}a^i_{j|\alpha}\\
&\quad + a^\alpha_j\big(R^i_{\alpha mn} + L^i_{\underset{\vee}{\alpha m}|n} - L^i_{\underset{\vee}{\alpha n}|m} + L^\beta_{\underset{\vee}{\alpha m}}L^i_{\underset{\vee}{\beta n}} - L^\beta_{\underset{\vee}{\alpha n}}L^i_{\underset{\vee}{\beta m}} - 2L^i_{\underset{\vee}{\alpha\beta}}L^\beta_{\underset{\vee}{mn}}\big)\\
&\quad - a^i_\alpha\big(R^\alpha_{jmn} - L^\alpha_{\underset{\vee}{jm}|n} + L^\alpha_{\underset{\vee}{jn}|m} - L^\alpha_{\underset{\vee}{\beta n}}L^\beta_{\underset{\vee}{jm}} + L^\alpha_{\underset{\vee}{\beta m}}L^\beta_{\underset{\vee}{jn}} + 2L^\alpha_{\underset{\vee}{j\beta}}L^\beta_{\underset{\vee}{mn}}\big)\\
&\quad - 2a^\alpha_\beta\big(L^i_{\underset{\vee}{\alpha m}}L^\beta_{\underset{\vee}{jn}} - L^i_{\underset{\vee}{\alpha n}}L^\beta_{\underset{\vee}{jm}}\big),\\
a^i_{j\underset{3}{|}m\underset{2}{|}n} - a^i_{j\underset{3}{|}n\underset{1}{|}m} &= 2L^\alpha_{\underset{\vee}{jm}}a^i_{\alpha|n} - 2L^i_{\underset{\vee}{\alpha n}}a^\alpha_{j|m}\\
&\quad + a^\alpha_j\big(R^i_{\alpha mn} + L^i_{\underset{\vee}{\alpha m}|n} - L^i_{\underset{\vee}{\alpha n}|m} - L^\beta_{\underset{\vee}{\alpha m}}L^i_{\underset{\vee}{\beta n}} - L^\beta_{\underset{\vee}{\alpha n}}L^i_{\underset{\vee}{\beta m}}\big)\\
&\quad - a^i_\alpha\big(R^\alpha_{jmn} - L^\alpha_{\underset{\vee}{jm}|n} + L^\alpha_{\underset{\vee}{jn}|m} - L^\alpha_{\underset{\vee}{\beta n}}L^\beta_{\underset{\vee}{jm}} - L^\alpha_{\underset{\vee}{\beta m}}L^\beta_{\underset{\vee}{jn}}\big),
\end{aligned}
$$

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\begin{aligned}
a^i_{j\underset{3}{|}m\underset{3}{|}n} - a^i_{j\underset{3}{|}n\underset{1}{|}m} &= 2L^{\alpha}_{\underset{\vee}{jm}} a^i_{\alpha|n} \\
&\quad + a^{\alpha}_j\big(R^i_{\alpha mn} + L^i_{\underset{\vee}{\alpha m}|n} - L^i_{\underset{\vee}{\alpha n}|m} + L^{\beta}_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} - L^{\beta}_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}}\big) \\
&\quad - a^i_{\alpha}\big(R^{\alpha}_{jmn} - L^{\alpha}_{\underset{\vee}{jm}|n} + L^{\alpha}_{\underset{\vee}{jn}|m} - L^{\alpha}_{\underset{\vee}{\beta n}} L^{\beta}_{\underset{\vee}{jm}} - L^{\alpha}_{\underset{\vee}{\beta m}} L^{\beta}_{\underset{\vee}{jn}}\big) \\
&\quad + 2a^{\alpha}_{\beta} L^i_{\underset{\vee}{\alpha n}} L^{\beta}_{\underset{\vee}{jm}}, \\
a^i_{j\underset{1}{|}m\underset{1}{|}n} - a^i_{j\underset{3}{|}n\underset{2}{|}m} &= -2L^{\alpha}_{\underset{\vee}{jm}} a^i_{\alpha|n} - 2L^{\alpha}_{\underset{\vee}{jn}} a^i_{\alpha|m} + 2L^i_{\underset{\vee}{\alpha m}} a^{\alpha}_{j|n} \\
&\quad + a^{\alpha}_j\big(R^i_{\alpha mn} + L^i_{\underset{\vee}{\alpha m}|n} - L^i_{\underset{\vee}{\alpha n}|m} + L^{\beta}_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} + L^{\beta}_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}}\big) \\
&\quad - a^i_{\alpha}\big(R^{\alpha}_{jmn} + L^{\alpha}_{\underset{\vee}{jm}|n} + L^{\alpha}_{\underset{\vee}{jn}|m} + L^{\alpha}_{\underset{\vee}{\beta n}} L^{\beta}_{\underset{\vee}{jm}} - L^{\alpha}_{\underset{\vee}{\beta m}} L^{\beta}_{\underset{\vee}{jn}} - 2L^{\alpha}_{\underset{\vee}{j\beta}} L^{\beta}_{\underset{\vee}{mn}}\big) \\
&\quad - 2a^{\alpha}_{\beta} L^i_{\underset{\vee}{\alpha n}} L^{\beta}_{\underset{\vee}{jm}}, \\
a^i_{j\underset{1}{|}m\underset{2}{|}n} - a^i_{j\underset{3}{|}n\underset{2}{|}m} &= -2L^{\alpha}_{\underset{\vee}{jm}} a^i_{\alpha|n} + 2L^{\alpha}_{\underset{\vee}{mn}} a^i_{j|\alpha} + 2L^i_{\underset{\vee}{\alpha m}} a^{\alpha}_{j|n} - 2L^i_{\underset{\vee}{\alpha n}} a^{\alpha}_{j|m} \\
&\quad + a^{\alpha}_j\big(R^i_{\alpha mn} + L^i_{\underset{\vee}{\alpha m}|n} - L^i_{\underset{\vee}{\alpha n}|m} - L^{\beta}_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} + L^{\beta}_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}} + 2L^i_{\underset{\vee}{\alpha\beta}} L^{\beta}_{\underset{\vee}{mn}}\big) \\
&\quad - a^i_{\alpha}\big(R^{\alpha}_{jmn} + L^{\alpha}_{\underset{\vee}{jm}|n} + L^{\alpha}_{\underset{\vee}{jn}|m} + L^{\alpha}_{\underset{\vee}{\beta n}} L^{\beta}_{\underset{\vee}{jm}} + L^{\alpha}_{\underset{\vee}{\beta m}} L^{\beta}_{\underset{\vee}{jn}}\big) \\
&\quad + 2a^{\alpha}_{\beta} L^i_{\underset{\vee}{\alpha m}} L^{\beta}_{\underset{\vee}{jn}}, \\
a^i_{j\underset{1}{|}m\underset{3}{|}n} - a^i_{j\underset{3}{|}n\underset{2}{|}m} &= -2L^{\alpha}_{\underset{\vee}{jm}} a^i_{\alpha|n} + 2L^{\alpha}_{\underset{\vee}{mn}} a^i_{j|\alpha} + 2L^i_{\underset{\vee}{\alpha m}} a^{\alpha}_{j|n} \\
&\quad + a^{\alpha}_j\big(R^i_{\alpha mn} + L^i_{\underset{\vee}{\alpha m}|n} - L^i_{\underset{\vee}{\alpha n}|m} + L^{\beta}_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} + L^{\beta}_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}} + 2L^i_{\underset{\vee}{\alpha\beta}} L^{\beta}_{\underset{\vee}{mn}}\big) \\
&\quad - a^i_{\alpha}\big(R^{\alpha}_{jmn} + L^{\alpha}_{\underset{\vee}{jm}|n} + L^{\alpha}_{\underset{\vee}{jn}|m} + L^{\alpha}_{\underset{\vee}{\beta n}} L^{\beta}_{\underset{\vee}{jm}} + L^{\alpha}_{\underset{\vee}{\beta m}} L^{\beta}_{\underset{\vee}{jn}}\big) \\
&\quad + 2a^{\alpha}_{\beta}\big(L^i_{\underset{\vee}{\alpha m}} L^{\beta}_{\underset{\vee}{jn}} - L^i_{\underset{\vee}{\alpha n}} L^{\beta}_{\underset{\vee}{jm}}\big), \\
a^i_{j\underset{2}{|}m\underset{1}{|}n} - a^i_{j\underset{3}{|}n\underset{2}{|}m} &= -2L^{\alpha}_{\underset{\vee}{jn}} a^i_{\alpha|m} \\
&\quad + a^{\alpha}_j\big(R^i_{\alpha mn} - L^i_{\underset{\vee}{\alpha m}|n} - L^i_{\underset{\vee}{\alpha n}|m} - L^{\beta}_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} + L^{\beta}_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}} + 2L^i_{\underset{\vee}{\alpha\beta}} L^{\beta}_{\underset{\vee}{mn}}\big) \\
&\quad - a^i_{\alpha}\big(R^{\alpha}_{jmn} - L^{\alpha}_{\underset{\vee}{jm}|n} + L^{\alpha}_{\underset{\vee}{jn}|m} + L^{\alpha}_{\underset{\vee}{\beta n}} L^{\beta}_{\underset{\vee}{jm}} + L^{\alpha}_{\underset{\vee}{\beta m}} L^{\beta}_{\underset{\vee}{jn}}\big) \\
&\quad + 2a^{\alpha}_{\beta} L^i_{\underset{\vee}{\alpha m}} L^{\beta}_{\underset{\vee}{jn}}, \\
a^i_{j\underset{2}{|}m\underset{2}{|}n} - a^i_{j\underset{3}{|}n\underset{2}{|}m} &= 2L^{\alpha}_{\underset{\vee}{mn}} a^i_{j|\alpha} - 2L^i_{\underset{\vee}{\alpha n}} a^{\alpha}_{j|m} \\
&\quad + a^{\alpha}_j\big(R^i_{\alpha mn} - L^i_{\underset{\vee}{\alpha m}|n} - L^i_{\underset{\vee}{\alpha n}|m} + L^{\beta}_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} + L^{\beta}_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}}\big) \\
&\quad - a^i_{\alpha}\big(R^{\alpha}_{jmn} - L^{\alpha}_{\underset{\vee}{jm}|n} + L^{\alpha}_{\underset{\vee}{jn}|m} + L^{\alpha}_{\underset{\vee}{\beta n}} L^{\beta}_{\underset{\vee}{jm}} - L^{\alpha}_{\underset{\vee}{\beta m}} L^{\beta}_{\underset{\vee}{jn}} - 2L^{\alpha}_{\underset{\vee}{j\beta}} L^{\beta}_{\underset{\vee}{mn}}\big) \\
&\quad - 2a^{\alpha}_{\beta} L^i_{\underset{\vee}{\alpha n}} L^{\beta}_{\underset{\vee}{jm}},
\end{aligned}
$$

$$
\begin{aligned}
a^i_{j\underset{2}{|}m\underset{3}{|}n}-a^i_{j\underset{3}{|}n\underset{2}{|}m}&=2L^{\alpha}_{\underset{\vee}{mn}}a^i_{j|\alpha}\\
&+a^{\alpha}_j\big(R^i_{\alpha mn}-L^i_{\underset{\vee}{\alpha m}|n}-L^i_{\underset{\vee}{\alpha n}|m}-L^{\beta}_{\underset{\vee}{\alpha m}}L^i_{\underset{\vee}{\beta n}}+L^{\beta}_{\underset{\vee}{\alpha n}}L^i_{\underset{\vee}{\beta m}}\big)\\
&-a^i_{\alpha}\big(R^{\alpha}_{jmn}-L^{\alpha}_{\underset{\vee}{jm}|n}+L^{\alpha}_{\underset{\vee}{jn}|m}+L^{\alpha}_{\underset{\vee}{\beta n}}L^{\beta}_{\underset{\vee}{jm}}-L^{\alpha}_{\underset{\vee}{\beta m}}L^{\beta}_{\underset{\vee}{jn}}-2L^{\alpha}_{\underset{\vee}{j\beta}}L^{\beta}_{\underset{\vee}{mn}}\big),\\
a^i_{j\underset{3}{|}m\underset{1}{|}n}-a^i_{j\underset{3}{|}n\underset{2}{|}m}&=-2L^{\alpha}_{\underset{\vee}{jn}}a^i_{\alpha|m}+2L^i_{\underset{\vee}{\alpha m}}a^{\alpha}_{j|n}\\
&+a^{\alpha}_j\big(R^i_{\alpha mn}+L^i_{\underset{\vee}{\alpha m}|n}-L^i_{\underset{\vee}{\alpha n}|m}+L^{\beta}_{\underset{\vee}{\alpha m}}L^i_{\underset{\vee}{\beta n}}+L^{\beta}_{\underset{\vee}{\alpha n}}L^i_{\underset{\vee}{\beta m}}\big)\\
&-a^i_{\alpha}\big(R^{\alpha}_{jmn}-L^{\alpha}_{\underset{\vee}{jm}|n}+L^{\alpha}_{\underset{\vee}{jn}|m}+L^{\alpha}_{\underset{\vee}{\beta n}}L^{\beta}_{\underset{\vee}{jm}}+L^{\alpha}_{\underset{\vee}{\beta m}}L^{\beta}_{\underset{\vee}{jn}}\big),\\
a^i_{j\underset{3}{|}m\underset{2}{|}n}-a^i_{j\underset{3}{|}n\underset{2}{|}m}&=2L^{\alpha}_{\underset{\vee}{mn}}a^i_{j|\alpha}+2L^i_{\underset{\vee}{\alpha m}}a^{\alpha}_{j|n}-2L^i_{\underset{\vee}{\alpha n}}a^{\alpha}_{j|m}\\
&+a^{\alpha}_j\big(R^i_{\alpha mn}+L^i_{\underset{\vee}{\alpha m}|n}-L^i_{\underset{\vee}{\alpha n}|m}-L^{\beta}_{\underset{\vee}{\alpha m}}L^i_{\underset{\vee}{\beta n}}+L^{\beta}_{\underset{\vee}{\alpha n}}L^i_{\underset{\vee}{\beta m}}+2L^i_{\underset{\vee}{\alpha\beta}}L^{\beta}_{\underset{\vee}{mn}}\big)\\
&-a^i_{\alpha}\big(R^{\alpha}_{jmn}-L^{\alpha}_{\underset{\vee}{jm}|n}+L^{\alpha}_{\underset{\vee}{jn}|m}+L^{\alpha}_{\underset{\vee}{\beta n}}L^{\beta}_{\underset{\vee}{jm}}-L^{\alpha}_{\underset{\vee}{\beta m}}L^{\beta}_{\underset{\vee}{jn}}-2L^{\alpha}_{\underset{\vee}{j\beta}}L^{\beta}_{\underset{\vee}{mn}}\big)\\
&+2a^{\alpha}_{\beta}\big(L^i_{\underset{\vee}{\alpha m}}L^{\beta}_{\underset{\vee}{jn}}-L^i_{\underset{\vee}{\alpha n}}L^{\beta}_{\underset{\vee}{jm}}\big),\\
a^i_{j\underset{3}{|}m\underset{3}{|}n}-a^i_{j\underset{3}{|}n\underset{2}{|}m}&=2L^{\alpha}_{\underset{\vee}{mn}}a^i_{j|\alpha}+2L^i_{\underset{\vee}{\alpha m}}a^{\alpha}_{j|n}\\
&+a^{\alpha}_j\big(R^i_{\alpha mn}+L^i_{\underset{\vee}{\alpha m}|n}-L^i_{\underset{\vee}{\alpha n}|m}+L^{\beta}_{\underset{\vee}{\alpha m}}L^i_{\underset{\vee}{\beta n}}+L^{\beta}_{\underset{\vee}{\alpha n}}L^i_{\underset{\vee}{\beta m}}+2L^i_{\underset{\vee}{\alpha\beta}}L^{\beta}_{\underset{\vee}{mn}}\big)\\
&-a^i_{\alpha}\big(R^{\alpha}_{jmn}-L^{\alpha}_{\underset{\vee}{jm}|n}+L^{\alpha}_{\underset{\vee}{jn}|m}+L^{\alpha}_{\underset{\vee}{\beta n}}L^{\beta}_{\underset{\vee}{jm}}-L^{\alpha}_{\underset{\vee}{\beta m}}L^{\beta}_{\underset{\vee}{jn}}-2L^{\alpha}_{\underset{\vee}{j\beta}}L^{\beta}_{\underset{\vee}{mn}}\big)\\
&+2a^{\alpha}_{\beta}L^i_{\underset{\vee}{\alpha m}}L^{\beta}_{\underset{\vee}{jn}},\\
a^i_{j\underset{1}{|}m\underset{1}{|}n}-a^i_{j\underset{3}{|}n\underset{3}{|}m}&=-2L^{\alpha}_{\underset{\vee}{jm}}a^i_{\alpha|n}-2L^{\alpha}_{\underset{\vee}{jn}}a^i_{\alpha|m}\\
&+a^{\alpha}_j\big(R^i_{\alpha mn}+L^i_{\underset{\vee}{\alpha m}|n}-L^i_{\underset{\vee}{\alpha n}|m}+L^{\beta}_{\underset{\vee}{\alpha m}}L^i_{\underset{\vee}{\beta n}}-L^{\beta}_{\underset{\vee}{\alpha n}}L^i_{\underset{\vee}{\beta m}}\big)\\
&-a^i_{\alpha}\big(R^{\alpha}_{jmn}+L^{\alpha}_{\underset{\vee}{jm}|n}+L^{\alpha}_{\underset{\vee}{jn}|m}+L^{\alpha}_{\underset{\vee}{\beta n}}L^{\beta}_{\underset{\vee}{jm}}-L^{\alpha}_{\underset{\vee}{\beta m}}L^{\beta}_{\underset{\vee}{jn}}-2L^{\alpha}_{\underset{\vee}{j\beta}}L^{\beta}_{\underset{\vee}{mn}}\big)\\
&-2a^{\alpha}_{\beta}\big(L^i_{\underset{\vee}{\alpha n}}L^{\beta}_{\underset{\vee}{jm}}+L^i_{\underset{\vee}{\alpha m}}L^{\beta}_{\underset{\vee}{jn}}\big),\\
a^i_{j\underset{1}{|}m\underset{2}{|}n}-a^i_{j\underset{3}{|}n\underset{3}{|}m}&=-2L^{\alpha}_{\underset{\vee}{jm}}a^i_{\alpha|n}+2L^{\alpha}_{\underset{\vee}{mn}}a^i_{j|\alpha}-2L^i_{\underset{\vee}{\alpha n}}a^{\alpha}_{j|m}\\
&+a^{\alpha}_j\big(R^i_{\alpha mn}+L^i_{\underset{\vee}{\alpha m}|n}-L^i_{\underset{\vee}{\alpha n}|m}-L^{\beta}_{\underset{\vee}{\alpha m}}L^i_{\underset{\vee}{\beta n}}-L^{\beta}_{\underset{\vee}{\alpha n}}L^i_{\underset{\vee}{\beta m}}+2L^i_{\underset{\vee}{\alpha\beta}}L^{\beta}_{\underset{\vee}{mn}}\big)\\
&-a^i_{\alpha}\big(R^{\alpha}_{jmn}+L^{\alpha}_{\underset{\vee}{jm}|n}+L^{\alpha}_{\underset{\vee}{jn}|m}+L^{\alpha}_{\underset{\vee}{\beta n}}L^{\beta}_{\underset{\vee}{jm}}+L^{\alpha}_{\underset{\vee}{\beta m}}L^{\beta}_{\underset{\vee}{jn}}\big),\\
a^i_{j\underset{1}{|}m\underset{3}{|}n}-a^i_{j\underset{3}{|}n\underset{3}{|}m}&=-2L^{\alpha}_{\underset{\vee}{jm}}a^i_{\alpha|n}+2L^{\alpha}_{\underset{\vee}{mn}}a^i_{j|\alpha}\\
&+a^{\alpha}_j\big(R^i_{\alpha mn}+L^i_{\underset{\vee}{\alpha m}|n}-L^i_{\underset{\vee}{\alpha n}|m}+L^{\beta}_{\underset{\vee}{\alpha m}}L^i_{\underset{\vee}{\beta n}}-L^{\beta}_{\underset{\vee}{\alpha n}}L^i_{\underset{\vee}{\beta m}}+2L^i_{\underset{\vee}{\alpha\beta}}L^{\beta}_{\underset{\vee}{mn}}\big)\\
&-a^i_{\alpha}\big(R^{\alpha}_{jmn}+L^{\alpha}_{\underset{\vee}{jm}|n}+L^{\alpha}_{\underset{\vee}{jn}|m}+L^{\alpha}_{\underset{\vee}{\beta n}}L^{\beta}_{\underset{\vee}{jm}}+L^{\alpha}_{\underset{\vee}{\beta m}}L^{\beta}_{\underset{\vee}{jn}}\big)\\
&-2a^{\alpha}_{\beta}L^i_{\underset{\vee}{\alpha n}}L^{\beta}_{\underset{\vee}{jm}},\\
a^i_{j\underset{2}{|}m\underset{1}{|}n}-a^i_{j\underset{3}{|}n\underset{3}{|}m}&=-2L^{\alpha}_{\underset{\vee}{jn}}a^i_{\alpha|m}-2L^i_{\underset{\vee}{\alpha m}}a^{\alpha}_{j|n}\\
&+a^{\alpha}_j\big(R^i_{\alpha mn}-L^i_{\underset{\vee}{\alpha m}|n}-L^i_{\underset{\vee}{\alpha n}|m}-L^{\beta}_{\underset{\vee}{\alpha m}}L^i_{\underset{\vee}{\beta n}}-L^{\beta}_{\underset{\vee}{\alpha n}}L^i_{\underset{\vee}{\beta m}}+2L^i_{\underset{\vee}{\alpha\beta}}L^{\beta}_{\underset{\vee}{mn}}\big)\\
&-a^i_{\alpha}\big(R^{\alpha}_{jmn}-L^{\alpha}_{\underset{\vee}{jm}|n}+L^{\alpha}_{\underset{\vee}{jn}|m}+L^{\alpha}_{\underset{\vee}{\beta n}}L^{\beta}_{\underset{\vee}{jm}}+L^{\alpha}_{\underset{\vee}{\beta m}}L^{\beta}_{\underset{\vee}{jn}}\big),
\end{aligned}
$$

$$
\begin{aligned}
a^i_{j\underset{2}{|}m\underset{2}{|}n} - a^i_{j\underset{3}{|}n\underset{3}{|}m} &= 2L^{\alpha}_{\underset{\vee}{mn}} a^i_{j|\alpha} - 2L^i_{\underset{\vee}{\alpha m}} a^{\alpha}_{j|n} - 2L^i_{\underset{\vee}{\alpha n}} a^{\alpha}_{j|m} \\
&+ a^{\alpha}_j \big(R^i_{\alpha mn} - L^i_{\underset{\vee}{\alpha m}|n} - L^i_{\underset{\vee}{\alpha n}|m} + L^{\beta}_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} - L^{\beta}_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}}\big) \\
&- a^i_{\alpha}\big(R^{\alpha}_{jmn} - L^{\alpha}_{\underset{\vee}{jm}|n} + L^{\alpha}_{\underset{\vee}{jn}|m} + L^{\alpha}_{\underset{\vee}{\beta n}} L^{\beta}_{\underset{\vee}{jm}} - L^{\alpha}_{\underset{\vee}{\beta m}} L^{\beta}_{\underset{\vee}{jn}} - 2L^{\alpha}_{\underset{\vee}{j\beta}} L^{\beta}_{\underset{\vee}{mn}}\big) \\
&- 2a^{\alpha}_{\beta}\big(L^i_{\underset{\vee}{\alpha n}} L^{\beta}_{\underset{\vee}{jm}} + L^i_{\underset{\vee}{\alpha m}} L^{\beta}_{\underset{\vee}{jn}}\big), \\
a^i_{j\underset{2}{|}m\underset{3}{|}n} - a^i_{j\underset{3}{|}n\underset{3}{|}m} &= 2L^{\alpha}_{\underset{\vee}{mn}} a^i_{j|\alpha} - 2L^i_{\underset{\vee}{\alpha m}} a^{\alpha}_{j|n} \\
&+ a^{\alpha}_j \big(R^i_{\alpha mn} - L^i_{\underset{\vee}{\alpha m}|n} - L^i_{\underset{\vee}{\alpha n}|m} - L^{\beta}_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} - L^{\beta}_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}}\big) \\
&- a^i_{\alpha}\big(R^{\alpha}_{jmn} - L^{\alpha}_{\underset{\vee}{jm}|n} + L^{\alpha}_{\underset{\vee}{jn}|m} + L^{\alpha}_{\underset{\vee}{\beta n}} L^{\beta}_{\underset{\vee}{jm}} - L^{\alpha}_{\underset{\vee}{\beta m}} L^{\beta}_{\underset{\vee}{jn}} - 2L^{\alpha}_{\underset{\vee}{j\beta}} L^{\beta}_{\underset{\vee}{mn}}\big) \\
&- 2a^{\alpha}_{\beta} L^i_{\underset{\vee}{\alpha m}} L^{\beta}_{\underset{\vee}{jn}}, \\
a^i_{j\underset{3}{|}m\underset{1}{|}n} - a^i_{j\underset{3}{|}n\underset{3}{|}m} &= -2L^{\alpha}_{\underset{\vee}{jn}} a^i_{\alpha|m} \\
&+ a^{\alpha}_j \big(R^i_{\alpha mn} + L^i_{\underset{\vee}{\alpha m}|n} - L^i_{\underset{\vee}{\alpha n}|m} + L^{\beta}_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} - L^{\beta}_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}}\big) \\
&- a^i_{\alpha}\big(R^{\alpha}_{jmn} - L^{\alpha}_{\underset{\vee}{jm}|n} + L^{\alpha}_{\underset{\vee}{jn}|m} + L^{\alpha}_{\underset{\vee}{\beta n}} L^{\beta}_{\underset{\vee}{jm}} + L^{\alpha}_{\underset{\vee}{\beta m}} L^{\beta}_{\underset{\vee}{jn}}\big) \\
&- 2a^{\alpha}_{\beta} L^i_{\underset{\vee}{\alpha m}} L^{\beta}_{\underset{\vee}{jn}}, \\
a^i_{j\underset{3}{|}m\underset{2}{|}n} - a^i_{j\underset{3}{|}n\underset{3}{|}m} &= 2L^{\alpha}_{\underset{\vee}{mn}} a^i_{j|\alpha} - 2L^i_{\underset{\vee}{\alpha n}} a^{\alpha}_{j|m} \\
&+ a^{\alpha}_j \big(R^i_{\alpha mn} + L^i_{\underset{\vee}{\alpha m}|n} - L^i_{\underset{\vee}{\alpha n}|m} - L^{\beta}_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} - L^{\beta}_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}} + 2L^i_{\underset{\vee}{\alpha\beta}} L^{\beta}_{\underset{\vee}{mn}}\big) \\
&- a^i_{\alpha}\big(R^{\alpha}_{jmn} - L^{\alpha}_{\underset{\vee}{jm}|n} + L^{\alpha}_{\underset{\vee}{jn}|m} + L^{\alpha}_{\underset{\vee}{\beta n}} L^{\beta}_{\underset{\vee}{jm}} - L^{\alpha}_{\underset{\vee}{\beta m}} L^{\beta}_{\underset{\vee}{jn}} - 2L^{\alpha}_{\underset{\vee}{j\beta}} L^{\beta}_{\underset{\vee}{mn}}\big) \\
&- 2a^{\alpha}_{\beta} L^i_{\underset{\vee}{\alpha n}} L^{\beta}_{\underset{\vee}{jm}}, \\
a^i_{j\underset{3}{|}m\underset{3}{|}n} - a^i_{j\underset{3}{|}n\underset{3}{|}m} &= 2L^{\alpha}_{\underset{\vee}{mn}} a^i_{j|\alpha} \\
&+ a^{\alpha}_j \big(R^i_{\alpha mn} + L^i_{\underset{\vee}{\alpha m}|n} - L^i_{\underset{\vee}{\alpha n}|m} + L^{\beta}_{\underset{\vee}{\alpha m}} L^i_{\underset{\vee}{\beta n}} - L^{\beta}_{\underset{\vee}{\alpha n}} L^i_{\underset{\vee}{\beta m}} + 2L^i_{\underset{\vee}{\alpha\beta}} L^{\beta}_{\underset{\vee}{mn}}\big) \\
&- a^i_{\alpha}\big(R^{\alpha}_{jmn} - L^{\alpha}_{\underset{\vee}{jm}|n} + L^{\alpha}_{\underset{\vee}{jn}|m} + L^{\alpha}_{\underset{\vee}{\beta n}} L^{\beta}_{\underset{\vee}{jm}} - L^{\alpha}_{\underset{\vee}{\beta m}} L^{\beta}_{\underset{\vee}{jn}} - 2L^{\alpha}_{\underset{\vee}{j\beta}} L^{\beta}_{\underset{\vee}{mn}}\big).
\end{aligned}
$$

3. Conclusion

This manuscript conducted the research of the components of curvatures for the non-symmetric affine connection space $\mathbb{GA}_N$ with respect to three of four plus one kinds of covariant derivatives (1.2, 1.3).

Here, it was elaborated that curvature pseudotensors are not components of the differences $a^i_{j\underset{v_1}{|}m\underset{w_1}{|}n} - a^i_{j\underset{v_2}{|}n\underset{w_2}{|}m}$, $v_1, v_2, w_1, w_2 \in \{0, 1, 2, 3, 4\}$.

In future work, we will study the commutation formulae obtained with respect to all triples of linearly independent geometrical objects $a^i_{j\underset{p}{|}k}$, $p = 0, \ldots, 4$.

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