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AUTHOMATED METHOD FOR DESIGNING FUZZY SYSTEMS *

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Abstract. The paper presents a method for building fuzzy systems using the input-output data that can be obtained from the examples. Using this method, a rule-based system is created, where fuzzy logic depends on the opinions and preferences of decision-makers involved in the process. Some advantages of the proposed method are highlighted. We have provided a practical example to illustrate the application of the process.
Keywords: fuzzy systems; rule-based system; fuzzy rule; membership function.

1. Introduction

Zadeh's fuzzy rule based systems deal with fuzzy rules instead of classical logic rules. Nowadays, they have been successfully used for modeling and control in different fields and industries [1, 2, 5, 15, 16].

Fuzzy rule based systems with fuzzifier and defuzzifier introduced by Mamdani [9, 10] are commonly known as fuzzy logic controllers. Mamdani fuzzy rule based systems deal with real-valued inputs and outputs, and therefore, they can be used in a wide range of real-world applications. The behavior of the system is guided by linguistic rules with the "IF-THEN" form whose premises and consequents are composed of fuzzy logic statements [3, 12, 14]. More on linguistic Mamdani-type fuzzy rule-based systems can be found in [17].

One of drawbacks of Mamdani fuzzy rule based systems can be viewed in a fact that good performance on input-output training data do not nonsensically led to good performance on novel inputs [4, 6, 11]. Therefore, a construction of fuzzy functions and corresponding base of rules based on inclusion of expert knowledge into the process is proposed.

The model presented in this paper is shown to be very good, because of its flexibility, therefore it can be very easy for implementing and application in various

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fields. In this work, trough one illustrative example, it will be shown that greater number of functions for presenting the input data will give much better results, and therefore, the flexibility od the model is limited by the lower bound in the number of functions for presenting input data.

2. Automated method for designing fuzzy systems based on learning from example

The input of the observed fuzzy system is a set of N input-output pairs, of the form

$$\{(X_0^p, y_0^p)\}, \quad p \in 1, \dots, N, \tag{2.1}$$

where $X_0^p \in U = [\alpha_1, \beta_1] \times \cdots \times [\alpha_n, \beta_n] \subset R^n$ and $y_0^p \in V = [\alpha_y, \beta_y] \subset R$. Clearly, the input of a fuzzy system (2.1) is the collection of data given by Table 2.1.

Table 2.1: Input-output data collection

Input	C_1	C_2	...	C_i	...	C_n	Output
X_0^1	x_{01}^1	x_{02}^1	...	x_{0i}^1	...	x_{0n}^1	y_0^1
X_0^2	x_{01}^2	x_{02}^2	...	x_{0i}^2	...	x_{0n}^2	y_0^2
			...		...		
X_0^p	x_{01}^p	x_{02}^p	...	x_{0i}^p	...	x_{0n}^p	y_0^p
			...		...		
X_0^N	x_{01}^N	x_{02}^N	...	x_{0i}^N	...	x_{0n}^N	y_0^N

Designing the fuzzy system based on these input-output data collection can be described in the the following five steps.

Step 1. *Experts opinion*

For each $i \in \{1, 2, \dots, n\}$ and corresponding attribute C_i, values represented in the ith column of Table 2.1 can have different importance to a decision expert. Some values are extremely important, while others are totally unacceptable. On the other hand, different decision experts can have different intuition and preferences on what's important. Therefore, N_i decision experts are involved to express their preference on attribute C_i.

For each $i \in \{1, 2, \dots, n\}$ and $j \in \{1, 2, \dots, N_i\}$, the jth expert on attribute C_i choose four elements $a_i^j, b_i^j, c_i^j, d_i^j \in [\alpha_i, \beta_i]$.

Table 2.2: Expert's preference on atributes

a_i^j	b_i^j	c_i^j	d_i^j

For example, let attribute C_i represent price of some article (or service) which can generally range between α_i and β_i. Given values a_i^j, b_i^j, c_i^j and d_i^j have the following meaning: If the price is lower than a_i or it is higher than d_i, then we are not interested in buying that article (too cheap or too expensive items are not interesting to us). If the price is between b_i^j and c_i^j, then we are absolutely interested in buying the article (shopping surely). As price goes from a_i to b_i, we are increasingly interested for buying it, and if price goes from c_i to d_i our interest in the purchase of item drops.

In this way, for each attribute C_i $(i = 1, 2, \ldots, n)$, we have determined N_i fuzzy sets

$$A_i^j : [\alpha_i, \beta_i] \to [0, 1], \qquad j = 1, 2, \ldots, N_i, \tag{2.2}$$

as follows:

$$A_i^j(x) = \begin{cases} 0, & \alpha_i \leqslant x \leq a_i^j \ \text{ or } \ d_i^j \leqslant x \leq \beta_i; \\ \frac{x - a_i^j}{b_i^j - a_i^j}, & a_i^j \leqslant x \leq b_i^j; \\ 1, & b_i^j \leqslant x \leq c_i^j; \\ \frac{x - d_i^j}{c_i^j - d_i^j}, & c_i^j \leqslant x \leq d_i^j. \end{cases}$$

It is assumed that, for each $i = 1, 2, \ldots, n$, the set of fuzzy functions (2.2) is complete in $[\alpha_i, \beta_i]$, i.e., for every $x_i \in [\alpha_i, \beta_i]$, there exists A_i^j such that $\mu_{A_i^j}(x_i) \neq 0$.

With similar arguments, N_y decision experts are involved to express their preference on output column $(y_0^1, y_0^2, \ldots, y_0^N)^T$.

Table 2.3: Expert's preference on output

a_i	b_i	c_i	d_i

Consequently, N_y fuzzy sets

$$B^j : [\alpha_y, \beta_y] \to [0, 1], \qquad j = 1, 2, \ldots, N_y, \tag{2.3}$$

are defined in the following way:

$$B^j(x) = \begin{cases} 0, & \alpha_y \leqslant x \leq a^j \ \text{ or } \ d^j \leqslant x \leqslant \beta_y; \\ \frac{x - a^j}{b^j - a^j}, & a^j \leqslant x \leq b^j; \\ 1, & b^j \leqslant x \leq c^j; \\ \frac{x - d^j}{c^j - d^j}, & c^j \leqslant x \leq d^j, \end{cases}$$

Again, the assumption is that they are complete in $[\alpha_y, \beta_y]$.

One can notice that in the case of incompleteness of obtained fuzzy sets (2.2) or (2.3), the number of experts being examined must increase. Also, let us notice that obtained fuzzy sets are trapezoidal, and in a naturally way they can be transformed to triangular fuzzy sets or singletons.

Step 2. *Rules generated by input-output data*

In this step, for every input-output pair

$$(X_0^p, y_0^p), \qquad p = 1, 2, \ldots, N,$$

and corresponding inputs and output

$$x_{0i}^p, \quad i = 1, 2, \ldots, n \quad \text{and} \quad y_0^p,$$

we will determine the membership values

$$A_i^j(x_{0i}^p), \qquad j = 1, 2, \ldots, N_i,$$

and the membership values

$$B^l(y_0^p), \qquad l = 1, 2, \ldots, N_y.$$

Then for every input variable x_{0i}^p, $i = 1, 2, \ldots, n$, we will determine the fuzzy set in which x_{0i}^p has the largest membership value, that is, we will determine A_i^{j*} such that

$$A_i^{j*}(x_{0i}^p) \geqslant A_i^j(x_{0i}^p), \quad j = 1, 2, \ldots, N_i.$$

Similarly, we will determine B^{l*} such that

$$B^{l*}(y_0^p) \geqslant B^l(y_0^p), \quad 1 = 1, 2, \ldots, N_y.$$

Finally, we obtain a fuzzy IF-THEN rule as

$$\text{IF} \quad x_1 = A_1^{j*} \quad \text{and} \quad \cdots \quad \text{and} \quad x_n = A_n^{j*} \quad \text{THEN} \quad y = B^{l*}. \tag{2.4}$$

Step 3. *Degrees of fuzzy rules*

Since the number of input-output pairs is usually large, and for every pair one rule is generated, it is highly likely that there are conflicting rules, i.e., there are rules with the same IF part and different THEN part. In order to overcome this conflict, the degree to each rule generated in Step 2 is assigned and only one rule from a conflicting group that has the maximum degree is chosen. That procedure resolves the conflict problem, but also reduced the number of rules.

The degree of the rule, denoted by D, is defined as follows: Let the rule (2.4) be generated by a pair (X_0^p, y_0^p), then its degree is defined by:

$$D(rule) = \prod_{i=1}^{n} A_1^{j*}(x_{0i}^p) \cdot B^{l*}(y_0^p) \tag{2.5}$$

If the input-output pairs have different reliability and we can determine a number to asses it, we may incorporate this information into the degrees of the rules.

Specifically, suppose the input-output pair (X_0^p, y_0^p) has the degree $\mu^p \in [0,1]$, then the degree of the rule generated by a pair (X_0^p, y_0^p) is defined by:

$$D(rule) = \prod_{i=1}^{n} A_1^{j*}(x_{0i}^p) \cdot B^{l*}(y_0^p) \cdot \mu^p. \tag{2.6}$$

In practice, an expert may check the data (if the number of input-output pairs is small) and estimate the degree μ^p. If we cannot tell the difference among the input-output pairs, we simply choose all μ^p value 1, in that way (2.6) is reduced to (2.5).

Step 4. *Fuzzy rule base*

The fuzzy rule base consists of the following set of rules:

1. The rules generated in Step 2 that do not conflict with any other rules;
2. The rule from a conflicting group that has the maximum degree, where a group of conflicting rules consists of rules with the same IF parts;
3. Linguistic rules from human experts (due to conscious knowledge).

Step 5. *Fuzzy system*

In this step of algorithm, the fuzzy system is constructed based on the fuzzy rule base obtained in Step 4 (see [13, 17]).

In the sequel, we present an simple example, with a small amount of input-output data, which will illustrate working of the previous procedure and problems that may occur when using this method.

3. Example

In order to rate the quality of service offered by the hotel, one hotel booking site measures two components - cleanliness and comfort. Cleanliness takes values from interval $[0, 6]$, while comfort takes values from interval $[0, 11]$. According to these components, as a result the rating of hotel, which takes values from 1 to 5, is obtained. The following table presents the rate of the quality that customers specified based on the ratings they gave for cleanliness and comfort:

For this two input - one output space system, we will present how using of automated method for designing fuzzy systems based on learning from example works. Moreover, we will compare the results for certain value, which is obtained when for the same system, we change only the number of membership functions used for presenting the input data.

Table 3.1: Input-output data of example

Id	Clean	Comfort	Rate
1	2.8	2	2
2	3.9	8.2	4
3	1.2	5	2
4	2	8.4	3
5	5	10.3	5
6	5	9.2	4
7	4	4	3
8	3.7	1	1
9	4	9.8	5
10	4	8.7	4
11	2	9	3
12	1.3	5.7	2
13	0.8	4.1	1
14	3	9.4	3
15	3.1	9.9	4

For presenting the cleanliness and the comfort the trapezoid functions will be used. The input space is $U_x = [0,6] \times [0,11]$. For presenting the rate, singleton functions will be used, and the output space is $U_y = \{1,..,5\}$.

In the first case, we will have 4 membership functions for the cleanliness and 6 for comfort. Trapezoid functions $A_1, A_2, A_3, A_4 : [0,6] \to [0,1]$ for cleanliness:

$$A_1(x) = \begin{cases} 1, & 0 \leqslant x \leqslant 1; \\ \frac{2-x}{1}, & 1 \leqslant x \leqslant 2; \\ 0, & \text{otherwise.} \end{cases} \qquad A_2(x) = \begin{cases} \frac{x-1}{0.5}, & 1 \leqslant x \leqslant 1.5; \\ 1, & 1.5 \leqslant x \leqslant 2.5; \\ \frac{3-x}{0.5}, & 2.5 \leqslant x \leqslant 3; \\ 0, & \text{otherwise.} \end{cases}$$

$$A_3(x) = \begin{cases} \frac{x-2}{1}, & 2 \leqslant x \leqslant 3; \\ 1, & 3 \leqslant x \leqslant 4; \\ \frac{5-x}{1}, & 4 \leqslant x \leqslant 5; \\ 0, & \text{otherwise.} \end{cases} \qquad A_4(x) = \begin{cases} \frac{x-3}{2}, & 3 \leqslant x \leqslant 5; \\ 1, & 5 \leqslant x \leqslant 6; \\ 0, & \text{otherwise.} \end{cases}$$

Trapezoid functions $B_1, .., B_6 : [0,11] \to [0,1]$ for comfort:

$$B_1(x_2) = \begin{cases} 1, & 0 \leqslant x_2 \leqslant 1; \\ \frac{2.5-x}{1.5}, & 1 \leqslant x_2 \leqslant 2.5; \\ 0, & \text{otherwise.} \end{cases} \qquad B_2(x_2) = \begin{cases} \frac{x-0.5}{1.5}, & 0.5 \leqslant x_2 \leqslant 2; \\ 1, & 2 \leqslant x_2 \leqslant 3; \\ \frac{4.5-x}{1.5}, & 3 \leqslant x_2 \leqslant 4.5; \\ 0, & \text{otherwise.} \end{cases}$$

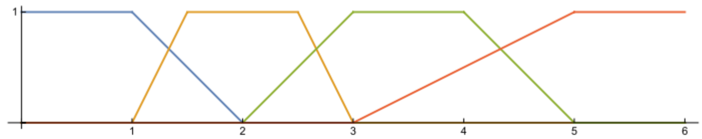

FIG. 3.1: Membership functions A_1 (blue), A_2 (yellow), A_3 (green) and A_4 (red).

$$B_3(x_2) = \begin{cases} \frac{x_2-2.5}{1.5}, & 2.5 \leqslant x_2 \leqslant 4; \\ 1, & 4 \leqslant x_2 \leqslant 5; \\ \frac{6.5-x_2}{1.5}, & 5 \leqslant x_2 \leqslant 6.5; \\ 0, & \text{otherwise.} \end{cases} \quad B_4(x_2) = \begin{cases} \frac{x_2-4.5}{1.5}, & 4.5 \leqslant x_2 \leqslant 6; \\ 1, & 6 \leqslant x_2 \leqslant 7; \\ \frac{6.5-x_2}{1.5}, & 6.5 \leqslant x \leqslant 8; \\ 0, & \text{otherwise.} \end{cases}$$

$$B_5(x_2) = \begin{cases} \frac{x_2-6.5}{1.5}, & 6.5 \leqslant x_2 \leqslant 8; \\ 1, & 8 \leqslant x_2 \leqslant 9; \\ \frac{10.5-x_2}{1.5}, & 9 \leqslant x_2 \leqslant 10.5; \\ 0, & \text{otherwise.} \end{cases} \quad B_6(x_2) = \begin{cases} \frac{x_2-8.5}{1.5}, & 8.5 \leqslant x_2 \leqslant 10; \\ 1, & 10 \leqslant x_2 \leqslant 11; \\ 0, & \text{otherwise.} \end{cases}$$

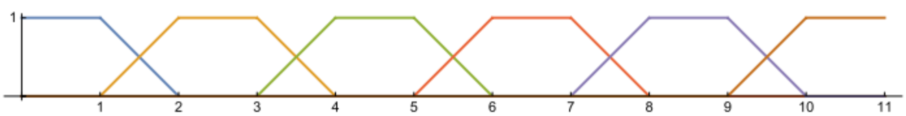

FIG. 3.2: Membership functions B_1 (blue), B_2 (yellow), B_3 (green), B_4 (red), B_5 (purple) and B_6 (brown).

In the second case, we will have 2 membership functions for the cleanliness and 3 for comfort. Trapezoid functions $A'_1, A'_2 : [0,6] \to [0,1]$ for cleanliness:

$$A'_1(x_1) = \begin{cases} 1, & 0 \leqslant x_1 \leqslant 2; \\ \frac{3.5-x_1}{1.5}, & 2 \leqslant x_1 \leqslant 3.5; \\ 0, & \text{otherwise.} \end{cases} \quad A'_2(x_1) = \begin{cases} \frac{x_1-1.5}{1.5}, & 1.5 \leqslant x_1 \leqslant 3; \\ 1, & 3 \leqslant x_1 \leqslant 5; \\ 0, & \text{otherwise.} \end{cases}$$

Trapezoid functions $B'_1, .., B'_3 : [0,11] \to [0,1]$ for comfort:

$$B'_1(x_2) = \begin{cases} 1, & 0 \leqslant x_2 \leqslant 3; \\ \frac{6-x_2}{3}, & 3 \leqslant x_2 \leqslant 6; \\ 0, & \text{otherwise.} \end{cases} \quad B'_2(x_2) = \begin{cases} \frac{x_2-2}{3}, & 2 \leqslant x_2 \leqslant 5; \\ 1, & 5 \leqslant x_2 \leqslant 7; \\ \frac{10-x_2}{3}, & 7 \leqslant x_2 \leqslant 10; \\ 0, & \text{otherwise.} \end{cases}$$

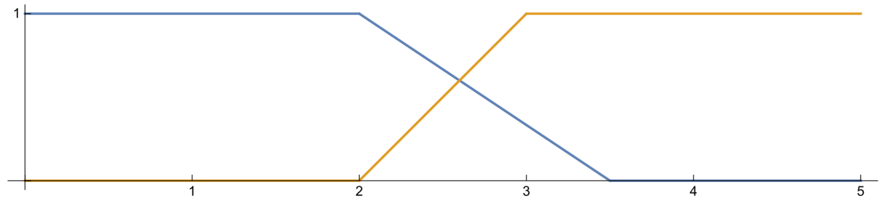

FIG. 3.3: Membership functions A'_1 (blue) and A'_2 (yellow).

$$B'_3(x_2) = \begin{cases} \frac{x_2-6}{3}, & 6 \leqslant x_2 \leqslant 9; \\ 1, & 9 \leqslant x_2 \leqslant 11; \\ 0, & \text{otherwise.} \end{cases}$$

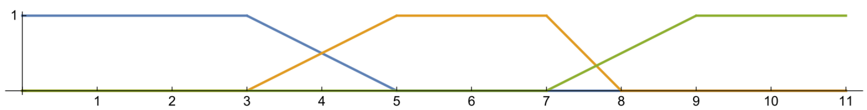

FIG. 3.4: Membership functions B'_1 (blue), B'_2 (yellow) and B'_3 (green).

For presenting the rate of the hotel, in the both cases, 5 singleton functions $C_i : U_y \to \{0,1\}$, $i \in \{1,..,5\}$ will be used:

$$C_i(y) = \begin{cases} 1, & y = i; \\ 0, & \text{otherwise.} \end{cases}$$

Fuzzy rule base, constructed from input-output data, in the first case is given by Table 3.2, and in the second case, fuzzy rule base constructed from input-output data is given by Table 3.3.

As we can see from Table 3.3, there is a lot of conflict rules. When we solve all the conflicts, and get rid of double rules, we obtain reduced Model 2., presented in Table 3.4

Fuzzy inference engine used here is Minimum Inference Engine, that is: individual-rule based inference with union combination, Mamdani's minimum implication, and min for all the t-norm operators and max for all the s-norm operators [7, 8]:

$$O(y) = max_{l=1}^{M}[sup_{(x_1,x_2)\in U_x} \min(I(x_1,x_2), A^l(x_1), B^l(x_2), C^l(y))].$$

where M is the number of rules. Fuzzifier $I(x_1, x_2)$, used here, is the the singleton fuzzifier, i.e. for the given input (x_1^0, x_2^0):

$$I(x_1,x_2) = \begin{cases} 1, & (x_1,x_2) = (x_1^0, x_2^0); \\ 0, & \text{otherwise.} \end{cases}$$

Table 3.2: Model 1.

Rule 1	IF x_1 is A_2 and x_2 is B_2 then y is C_2
Rule 2	IF x_1 is A_3 and x_2 is B_5 then y is C_4
Rule 3	IF x_1 is A_1 and x_2 is B_3 then y is C_2
Rule 4	IF x_1 is A_2 and x_2 is B_5 then y is C_3
Rule 5	IF x_1 is A_4 and x_2 is B_6 then y is C_5
Rule 6	IF x_1 is A_4 and x_2 is B_5 then y is C_4
Rule 7	IF x_1 is A_3 and x_2 is B_3 then y is C_3
Rule 8	IF x_1 is A_3 and x_2 is B_1 then y is C_1
Rule 9	IF x_1 is A_3 and x_2 is B_6 then y is C_5
Rule 10	IF x_1 is A_3 and x_2 is B_5 then y is C_4
Rule 11	IF x_1 is A_2 and x_2 is B_5 then y is C_3
Rule 12	IF x_1 is A_1 and x_2 is B_4 then y is C_2
Rule 13	IF x_1 is A_1 and x_2 is B_3 then y is C_1
Rule 14	IF x_1 is A_2 and x_2 is B_5 then y is C_3
Rule 15	IF x_1 is A_2 and x_2 is B_6 then y is C_4

The outputs obtained by both methods are given in Table 3.5. As we can see from Table 3.5 and Table 3.1, better approximation is obtained by Model 1. For example, in the first row of input output Table 3.1 cleanliness is valued by 2.8, comfort by 2 and the overall impression rate is 2, and in the second row of Table 3.2 cleanliness is valued by 2, comfort by 3 and the overall impression rates are 2 and 1, by Model 1 and Model 2, respectively. Similarly, fifth row of Table 3.1 corresponds to fourth row of Table 3.4, and again we can see that better approximation is achieved by Model 1. The reason for this lies in the fact that Model 1 consider higher number of fuzzy functions for cleanliness and comfort (in Model 1 there are four fuzzy functions for cleanliness and six fuzzy functions for comfort, while in Model 2 we have only two fuzzy functions for cleanliness and tree for comfort). Therefore Model 1 provides sophisticated and finer fuzzy partition of the universe of the discourse. On the other hand, the Model 2, due to insufficient number of input functions, will never rate the quality of a hotel with a rating of 2 or 5, which is a serious disadvantage of this model. So, the suggestion is that there must be a lower bound on the number of functions that represent the input data-set.

Table 3.3: Model 2.

Rule 1	IF x_1 is A'_2 and x_2 is B'_2 then y is C_2
Rule 2	IF x_1 is A'_2 and x_2 is B'_3 then y is C_4
Rule 3	IF x_1 is A'_1 and x_2 is B'_2 then y is C_2
Rule 4	IF x_1 is A'_2 and x_2 is B'_2 then y is C_3
Rule 5	IF x_1 is A'_2 and x_2 is B'_3 then y is C_5
Rule 6	IF x_1 is A'_2 and x_2 is B'_3 then y is C_4
Rule 7	IF x_1 is A'_2 and x_2 is B'_2 then y is C_3
Rule 8	IF x_1 is A'_2 and x_2 is B'_1 then y is C_1
Rule 9	IF x_1 is A'_2 and x_2 is B'_3 then y is C_5
Rule 10	IF x_1 is A'_2 and x_2 is B'_3 then y is C_4
Rule 11	IF x_1 is A'_1 and x_2 is B'_3 then y is C_3
Rule 12	IF x_1 is A'_1 and x_2 is B'_2 then y is C_1
Rule 13	IF x_1 is A'_1 and x_2 is B'_2 then y is C_1
Rule 14	IF x_1 is A'_2 and x_2 is B'_3 then y is C_3
Rule 15	IF x_1 is A'_2 and x_2 is B'_3 then y is C_4

Table 3.4: Reduced model 2.

Rule 1	IF x_1 is A'_2 and x_2 is B'_2 then y is C_3
Rule 2	x_1 is A'_1 and x_2 is B'_2 then y is C_1
Rule 3	x_1 is A'_2 and x_2 is B'_3 then y is C_4
Rule 4	x_1 is A'_2 and x_2 is B'_1 then y is C_1
Rule 5	x_1 is A'_1 and x_2 is B'_3 then y is C_3

Table 3.5: Output of the algorithm.

Id	Input	Model 1	Model 2
1	(2, 3)	2	1
2	(2.5, 1.9)	2	1
3	(3, 8)	3	4
4	(5, 10.6)	5	4
5	(4, 6)	3	3

4. Conclusion

The paper presents an algorithm for designing fuzzy systems based on learning from examples. The concept uses Mamdani's fuzzy rule systems with a fuzzifier and defuzzifier. Particular attention has been given to the preferences of decision makers involved in the process as experts with extensive practical experience. Based on their opinion, corresponding fuzzy functions that express the importance of attributes in the model are defined. An example to illustrate the process has also been provided. Moreover, through this example, the importance of determining the lower number of functions which represent the input data set is highlighted. In other words, it is shown that the number of these functions significantly influences the quality of the solution.

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REFERENCES

1. A. Bardossy and L. Duckstein: *Fuzzy rule-based modeling with application to geophysical, biological and engineering systems.* CRC Press, Boca Raton, 1995.
2. Z. Chi, H. Yan and T. Pham: *Fuzzy algorithms: with applications to image processing and pattern recognition.* World Scientific, 1996.
3. D. Dubois and H. Prade: *What are fuzzy rules and how to use them*, Fuzzy Sets Syst. **84** (1996), 169–185.
4. S. Guillaume: *Designing Fuzzy Inference Systems from Data: An Interpretability-Oriented Review.* IEEE Trans. on Fuzzy Syst. **9(3)** (2001), 426–443.
5. K. Hirota: *Industrial applications of fuzzy technology.* Springer-Verlag, 1993.
6. S. S. Izquierd and L. R. Izquierdo: *Mamdani Fuzzy Systems for Modelling and Simulation: A Critical Assessment.* Journal of Artificial Societies and Social Simulation **21(3)**, 2018.
7. E.P. Klement, R. Mesiar and E. Pap: *Triangular Norms.* Dordrecht: Kluwer, 2000.
8. W. Van Leekwijck and E.E. Kerre: *Defuzzification: criteria and classification.* Fuzzy Sets Syst. **108** (1999), 159–178.
9. E.H. Mamdani: *Applications of fuzzy algorithm for control a simple dynamic plant.* In: Proc. of the IEEE **121** (1974), 585–1588.
10. E.H. Mamdani and S. Assilian: *An experiment in linguistic synthesis with a fuzzy logic controller.* Int J Man-Mach Stud **7(1)** (1975), 1–13.
11. C. Moraga: *An Essay on the Interpretability of Mamdani Systems*, in: E. Trillas, P.P. Bonissone, L. Magdalena, J. Kacprzyk (Eds.), Comb. Exp. Theory SE - 5, Springer Berlin Heidelberg, 2012, 61–72.

12. V. Novak: *Reasoning about mathematical fuzzy logic and its future* Fuzzy Sets Syst. **192** (2012), 25–44.
13. V. Novak, I. Perfilieva and A.Dvorak: *Insite into fuzzy modeling.* John Wiley& Sons, Inc., 2015.
14. S.K. Pal and D.P. Mandal: *Fuzzy Logic and Approximate Reasoning: An Overview.* IETE J. Res. **37** (2015), 548–560.
15. W. Pedrycz: *Fuzzy Modelling: Paradigms and Practice.* Kluwer Academic Press, 1996.
16. T. Takagi and M. Sugeno: *Fuzzy identification of systems and its applications to modeling and control.* IEEE Trans Syst Man Cybern **15(1)**(1985), 116–132.
17. L.-X. Wang: *A Course In Fuzzy Systems and Control.* Prentice Hall PTR, 1997.

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