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SOME FIXED POINT RESULTS FOR CONVEX CONTRACTION MAPPINGS ON $\mathcal{F}-$METRIC SPACES

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Abstract. In this paper, we have established some fixed point theorems for convex contraction mappings in $\mathcal{F}$-metric spaces. Also, we have ntroduced the concept of $(\alpha, \beta)-$convex contraction mapping in $\mathcal{F}$-metric spaces and give some fixed point results for such contractions. Moreover, some examples are given to support our theoretical results.

Keywords: $\mathcal{F}$-Complete; Convex contraction; Fixed point; $\mathcal{F}$-Metric space; Orbital continuity.

1. Introduction

Fixed point theory plays a pivotal role in functional and nonlinear analysis. The Banach contraction principle is an important result of the fixed point theory. In recent years, various extensions of metric spaces have been introduced (see e.g. [1, 4, 6, 9, 10, 12, 16, 18] and references therein). The notion of a $\mathcal{F}$-metric space was firstly introduced and studied by Jleli and Samet in [17] (see e.g. [13, 20] and references therein). We recall some of the basic definitions and results in the sequel.
Let $\mathcal{F}$ be the set of functions $f : (0, +\infty) \to \mathbb{R}$ such that
$\mathcal{F}_1$) f is non-decreasing, i.e., $0 < s < t$ implies $f(s) \leq f(t)$.
$\mathcal{F}_2$) For every sequence $\{t_n\} \subset (0, \infty)$, we have

$$\lim_{n\to+\infty} t_n = 0 \;\; \textit{if and only if} \; \lim_{n\to+\infty} f(t_n) = -\infty.$$

Definition 1.1. [17] Let X be a (nonempty) set. A function $D : X \times X \to [0, \infty)$ is called a $\mathcal{F}$-metric on X if there exists $(f, \alpha) \in \mathcal{F} \times [0, \infty)$ such that for all $x, y \in X$ the following conditions hold:
(D_1) $D(x, y) = 0$ *if and only if* $x = y$.

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(D_2) $D(x,y) = D(y,x)$.
(D_3) For every $N \in \mathbb{N}, N \geq 2$ and for every $\{u_i\}_i^N \subset X$ with $(u_1, u_N) = (x,y)$, we have

$$D(x,y) > 0 \textit{ implies } f(D(x,y)) \leq f(\sum_{i=1}^{N-1} D(u_i, u_{i+1})) + \alpha.$$

In this case, the pair (X,D) is called a $\mathcal{F}$-metric space.

Example 1.1. [17] Let $X = \mathbb{R}$ and $D : X \times X \to [0,\infty)$ be defined as follows:

$$D(x,y) = \begin{cases} (x-y)^2, & (x,y) \in [0,3] \times [0,3] \\ |x-y|, & \text{otherwise,} \end{cases}$$

and let $f(t) = ln(t)$ for all $t > 0$ and $\alpha = ln(3)$. Then, D is a $\mathcal{F}$-metric on X. Since $D(1,3) = 4 \geq D(1,2) + D(2,3) = 2$, Then D is not a metric on X.

Example 1.2. [17] Let $X = \mathbb{R}$ and $D : X \times X \to [0,\infty)$ be defined as follows:

$$D(x,y) = \begin{cases} e^{|x-y|}, & x \neq y \\ 0, & x = y \end{cases}$$

Then, D is a $\mathcal{F}$-metric on X. Since $D(1,3) = e^2 \geq D(1,2) + D(2,3) = 2e$, Then D is not a metric on X.

Definition 1.2. [17] Let (X,D) be an $\mathcal{F}$-metric space and $\{x_n\}$ be a sequence in X.
1) A sequence $\{x_n\}$ is called $\mathcal{F}$-convergent to $x \in X$, if $\lim_{n\to+\infty} D(x_n, x) = 0$.
2) A sequence $\{x_n\}$ is $\mathcal{F}$-Cauchy, if and only if $\lim_{n,m\to+\infty} D(x_n, x_m) = 0$.
3) A $\mathcal{F}$-metric space (X,D) is said to be $\mathcal{F}$-complete, if every $\mathcal{F}$-Cauchy sequence in X is $\mathcal{F}$-convergent to some element in X.

Istratescu [14] introduced the notion of convex contraction and proved that if (X,d) is a complete metric space, then every convex contraction mapping on X has a unique fixed point.

Definition 1.3. [14] Let (X,d) be a metric space. The continuous selfmap T on X is called a convex contraction of order 2 whenever there exist $a_i \in (0,1)$, $i = 1,2$, with $a_1 + a_2 < 1$ such that for all $x,y \in X$,

$$d(T^2x, T^2y) \leq a_1 d(Tx, Ty) + a_2 d(dx, y). \tag{1.1}$$

Theorem 1.1. *[14] Let (X,d) be a complete metric space. Then any convex contraction mapping of order 2 has a fixed point which is unique.*

Definition 1.4. [14] Let (X,d) be a metric space. The continuous selfmap T on X is called a two-sided convex contraction mapping if there exist $a_i, b_i \in (0,1)$, $i = 1,2$, with $a_1 + a_2 + b_1 + b_2 < 1$ such that for all $x,y \in X$,

$$d(T^2x, T^2y) \leq a_1 d(x, Tx) + a_2 d(Tx, T^2x) + b_1 d(y, Ty) + b_2 d(Ty, T^2y). \tag{1.2}$$

Theorem 1.2. *[14] Let (X, d) be a complete metric space. Then any two-sided convex contraction mapping has a unique fixed point.*

Remark 1.1. [5] The assumption of continuity condition of Theorem 1.1 and Theorem 1.2 can be replaced by a relatively weaker condition of orbital continuity.

Definition 1.5. [5] Let (X, d) is a metric space. A self maping T on X is called orbitally continuous at a point $x^* \in X$, if for any $\{x_n\} \subseteq O(x, T)$ we have

$$x_n \to x^* \text{ implies } Tx_n \to Tx^* \text{ as } n \to +\infty,$$

where $O(x, T) = \{T^n x \mid n = 0, 1, 2, ...\}$.

Recently, a number of fixed point theorems for convex contraction mapping have been obtained by various authors (see e.g. [2, 3, 5, 7, 8, 11, 15, 19, 21] and references therein).

2. Convex Contraction Mappings on $\mathcal{F}-$Metric Spaces

In this section, we prove several fixed point theorems for convex contractions mappings defined on a $\mathcal{F}$-metric space.

Theorem 2.1. *Let (X, D) be an $\mathcal{F}$-complete $\mathcal{F}$-metric space. Let T be a convex contraction of order 2 on X. Then T has a unique fixed point.*

Proof. Let x_0 be an arbitrary point in X. We can define a sequence $\{x_n\}$ such that $x_{n+1} = Tx_n$ for each $n \in \mathbb{N} \cup \{0\}$. In case $x_m = x_{m+1}$ for some $m \in \mathbb{N} \cup \{0\}$, then it is clear that x_m is a fixed point of T. So assume that $x_n \neq x_{n+1}$ for all $n \in \mathbb{N} \cup \{0\}$. Set $v = \max\{D(x_0, Tx_0), D(Tx_0, T^2x_0)\}$. Using (1.1), we have the following relations:

$$D(T^3x_0, T^2x_0) \leq a_1 D(T^2x_0, Tx_0) + a_2 D(Tx_0, x_0) \leq v(a_1 + a_2),$$

similarly,

$$\begin{aligned} D(T^4x_0, T^3x_0) &\leq a_1 D(T^3x_0, T^2x_0) + a_2 D(T^2x_0, Tx_0) \\ &\leq a_1 v(a_1 + a_2) + a_2 v \\ &\leq v(a_1 + a_2), \end{aligned}$$

as well as

$$\begin{aligned} D(T^5x_0, T^4x_0) &\leq a_1 D(T^4x_0, T^3x_0) + a_2 D(T^3x_0, T^2x_0) \\ &\leq a_1 v(a_1 + a_2) + a_2 v(a_1 + a_2) \\ &= v(a_1 + a_2)^2. \end{aligned}$$

An induction argument shows that

$$D(T^{2m+1}x_0, T^{2m}x_0) \leq v(a_1 + a_2)^m, \tag{2.1}$$

and

$$D(T^{2m-1}x_0, T^{2m}x_0) \leq v(a_1 + a_2)^{m-1}, \tag{2.2}$$

for all $m \in \mathbb{N}$. Now, we show that $\{T^n x_0\}$ is a $\mathcal{F}$-Cauchy sequence. Let $(f, \alpha) \in \mathcal{F} \times [0, \infty)$ be such that D_3 is satisfied. Let $\varepsilon > 0$ be fixed. From $(\mathcal{F}_2)$, there exists $\delta > 0$ such that

$$0 < t < \delta \textit{ implies } f(t) < f(\varepsilon) - \alpha. \tag{2.3}$$

Let $m, n \in \mathbb{N}$ and $n > m$. If $m = 2k$ or $m = 2k + 1$, from (2.1) and (2.2), we have

$$\sum_{i=m}^{n-1} D(T^i x_0, T^{i+1} x_0) \leq 2v(a_1 + a_2)^k (\frac{1}{1 - (a_1 + a_2)}).$$

Since $a_1 + a_2 < 1$, we have

$$\lim_{k \to +\infty} 2v(a_1 + a_2)^k (\frac{1}{1 - (a_1 + a_2)}) = 0.$$

Then there exists some $N \in \mathbb{N}$ such that

$$0 < 2v(a_1 + a_2)^k (\frac{1}{1 - (a_1 + a_2)}) < \delta$$

for all $k \geq N$. Using (2.3) and $(\mathcal{F}_1)$, we get

$$\begin{aligned} f(\sum_{i=m}^{n-1} D(T^i x_0, T^{i+1} x_0)) &\leq f(2v(a_1 + a_2)^k (\frac{1}{1 - (a_1 + a_2)})) \\ &< f(\varepsilon) - \alpha. \end{aligned} \tag{2.4}$$

From (D_3) and (2.4), for $n > m \geq N$, we have

$$f(D(T^m x_0, T^n x_0)) \leq (\sum_{i=m}^{n-1} D(T^i x_0, T^{i+1} x_0)) + \alpha < f(\varepsilon).$$

Using $(\mathcal{F}_1)$, we obtain $D(T^m x_0, T^n x_0) < \varepsilon$, $n > m \geq N$. So $\{x_n\}$ is $\mathcal{F}$-Cauchy in the $\mathcal{F}$-complete $\mathcal{F}$-metric space X, so there exists $x^* \in X$ such that, $\lim_{n \to \infty} D(x_n, x^*) = 0$. Since T is $\mathcal{F}$-continuous, then, we have

$$Tx^* = T(\lim_{n \to +\infty} x_n) = \lim_{n \to +\infty} Tx_n = x^*,$$

so x^* is the fixed point of T. Finally, we shall show that the fixed point is unique. To this end, we assume that there exists another fixed point z^* and $D(x^*, z^*) > 0$. From (1.1), we have

$$\begin{aligned} D(x^*, z^*) &= D(T^2 x^*, T^2 z^*) \leq a_1 D(Tx^*, Tz^*) + a_2 D(x^*, z^*) \\ &= (a_1 + a_2) D(x^*, z^*). \end{aligned}$$

Since $a_1 + a_2 < 1$, we get $x^* = z^*$. □

Example 2.1. Let $X = [0, \infty)$ be endowed with the $\mathcal{F}$-metric given in Example 1.1. Define $T : X \to X$ by $Tx = \frac{x}{2} + 1$. Hence, for $a_1 = 0$ and $a_2 = \frac{1}{2}$, all the conditions of Theorem 2.1 are satisfied and T has a unique fixed point in X.

Theorem 2.2. *Let (X, D) be $\mathcal{F}$-complete $\mathcal{F}$-metric space and T be a two-sided convex contraction mapping on X. Then T has a unique fixed point.*

Proof. Let x_0 be an arbitrary point in X. We define the Picard iteration sequence $\{x_n\}$ by $x_{n+1} = Tx_n$ for all $n \in \mathbb{N} \cup \{0\}$. Without loss of generality, we assume that $x_n \neq x_{n+1}$ for all $n \in \mathbb{N} \cup \{0\}$. Set $v = \max\{D(x_0, Tx_0), D(Tx_0, T^2x_0)\}$. From (1.2), we have

$$\begin{aligned} D(T^3x_0, T^2x_0) &\leq a_1 D(Tx_0, T^2x_0) + a_2 D(T^2x_0, T^3x_0) \\ &+ b_1 D(x_0, Tx_0) + b_2 D(Tx_0, T^2x_0), \end{aligned}$$

then, we have

$$D(T^3x_0, T^2x_0) \leq (\frac{\lambda}{\gamma})v,$$

where $\lambda = a_1 + b_1 + b_2$ and $\gamma = 1 - a_2$. Similarly we obtain the following relation,

$$D(T^4x_0, T^3x_0) \leq (\frac{\lambda}{\gamma})v,$$

and

$$D(T^5x_0, T^4x_0) \leq (\frac{\lambda}{\gamma})^2 v.$$

Continuing this process, we obtain

$$D(T^{2m+1}x_0, T^{2m}x_0) \leq (\frac{\lambda}{\gamma})^m v, \tag{2.5}$$

and

$$D(T^{2m-1}x_0, T^{2m}x_0) \leq (\frac{\lambda}{\gamma})^{m-1} v. \tag{2.6}$$

Now, we show that $\{T^n x_0\}$ is a $\mathcal{F}$-Cauchy sequence. Let $m, n \in \mathbb{N}$ and $n > m$. If $m = 2k$ or $m = 2k + 1$, from (2.5) and (2.6), we have

$$\sum_{i=m}^{n-1} D(T^i x_0, T^{i+1}x_0) \leq 2v(\frac{\lambda}{\gamma})^k (\frac{1}{1 - \frac{\lambda}{\gamma}}).$$

Since $\frac{\lambda}{\gamma} < 1$, we have

$$\lim_{k \to +\infty} 2v(\frac{\lambda}{\gamma})^k (\frac{1}{1 - \frac{\lambda}{\gamma}}) = 0.$$

Using a similar technique to that in the proof of Theorem 2.1, it is easy to see that $\{x_n\}$ is a $\mathcal{F}$-Cauchy sequence in $\mathcal{F}$-complete $\mathcal{F}$-metric. Then, there exists x^* such that, $\lim_{n\to\infty} D(x_n, x^*) = 0$. Since T is $\mathcal{F}$-continuous, we have

$$Tx^* = T(\lim_{n\to+\infty} x_n) = \lim_{n\to+\infty} Tx_n = x^*,$$

so x^* is the fixed point T. For the uniqueness of the fixed point x^*, assume z^* is another fixed point of T and $D(x^*, z^*) > 0$. From (1.2), we have

$$\begin{aligned} D(x^*, z^*) = D(T^2x^*, T^2z^*) &\leq a_1D(x^*, Tx^*) + a_2D(Tx^*, T^2x^*) \\ &+ b_1D(z^*, Tz^*) + b_2D(Tz^*, T^2z^*) \\ &\leq (a_1 + a_2 + b_1 + b_2)D(x^*, z^*). \end{aligned}$$

Since $a_1 + a_2 + b_1 + b_2 < 1$, we obtain $D(x^*, z^*) = 0$ that is $x^* = z^*$. □

Example 2.2. Let $X = \{0, 1, 2\}$ be endowed with the $\mathcal{F}$-metric given in Example 1.2. Define $T : X \to X$ by $T0 = T2 = 0$ and $T1 = 2$. If $x = 0$ and $y = 1$, then for all $\lambda \in (0, 1)$, we have

$$D(T0, T1) = D(0, 2) = e^2 > \lambda e = \lambda D(0, 1).$$

Hence, T does not satisfy the condition of Banach contraction principle [17]. Since $T^2x = 0$ for all $x \in X$, then, all assumption of Theorem 2.2 are satisfied. Hence T has a unique fixed point.

Now, we give the definitions of $(\alpha, \beta)-$admissible convex contraction of order 2 and two-sided convex contraction mappings in the setting of $\mathcal{F}$-metric space and prove several fixed point theorems for such mappings on $\mathcal{F}$-metric spaces.

Definition 2.1. Let (X, D) be an $\mathcal{F}$-metric space and $T : X \to X$ be a cyclic $(\alpha, \beta)-$admissible mapping. We say that T is a $(\alpha, \beta)-$admissible convex contraction of order 2 if T is orbitally continuous and there exist $a_i \in (0, 1)$, $i = 1, 2$, such that

(2.7) $\alpha(x)\beta(y) \geq 1$ *implies* $D(T^2x, T^2y) \leq a_1D(Tx, Ty) + a_2D(x, y)$,

where $a_1 + a_2 < 1$ for all $x, y \in X$.

Theorem 2.3. *Let (X, D) be an $\mathcal{F}-$complete $\mathcal{F}-$metric space and $T : X \to X$ be a $(\alpha, \beta)-$admissible convex contraction mapping of order 2. Assume, there exists $x_0 \in X$ such that $\alpha(x_0) \geq 1$ and $\beta(x_0) \geq 1$. Then T has a fixed point. Moreover, if $\alpha(x) \geq 1$ and $\beta(y) \geq 1$ for all $x, y \in Fix(T)$, then T has a unique fixed point.*

Proof. Let $x_0 \in X$ be arbitrary in X. Define the sequence $\{x_n\}$ by $x_{n+1} = Tx_n$ for all $n \in \mathbb{N} \cup \{0\}$. If $x_{m+1} = x_m$ for some $m \in X$, then x_m is a fixed point of T. So, assume that $x_n \neq x_{n+1}$ for all $n \in \mathbb{N} \cup \{0\}$. Since $\beta(x_0) \geq 1$ and $T : X \to X$ is a $(\alpha, \beta)-$admissible mapping then $\alpha(Tx_0) = \alpha(x_1) \geq 1$ which implies $\beta(T^2x_0) = \beta(x_2) \geq 1$. By continuing this process, we have $\beta(T^{2n}x_0) \geq 1$ and

$\alpha(T^{2n-1}x_0) \geq 1$ for all $n \in \mathbb{N}$. Again, since T is a cyclic $(\alpha, \beta)-$admissible mapping and $\alpha(x_0) \geq 1$ by similarly, it can be shown that, $\beta(T^{2n-1}x_0) \geq 1$ and $\alpha(T^{2n}x_0) \geq 1$ for all $n \in \mathbb{N}$. Then, we obtain $\alpha(T^n x_0) = \alpha(x_n) \geq 1$ and $\beta(T^n x_0) = \beta(x_n) \geq 1$ for all $n \in \mathbb{N}$. Let $v = \max\{D(x_0, Tx_0), D(T^2x_0, Tx_0)\}$. Since $\alpha(Tx_0)\beta(x_0) \geq 1$, from the inequlity (2.7), we have

$$\begin{aligned} D(T^3x_0, T^2x_0) &= D(T^2(Tx_0), T^2x_0) \\ &\leq a_1 D(T^2x_0, Tx_0) + a_2 D(Tx_0, x_0) \\ &\leq (a_1 + a_2)v. \end{aligned}$$

Again, $\alpha(T^2x_0)\beta(Tx_0) \geq 1$, then we have

$$\begin{aligned} D(T^4x_0, T^3x_0) &= D(T^2(T^2x_0), T^2(Tx_0)) \\ &\leq a_1 D(T^3x_0, T^2x_0) + a_2 D(T^2x_0, Tx_0) \\ &\leq (a_1 + a_2)v. \end{aligned}$$

By continuing this process and using a similar technique to that in the proof of Theorem 2.1, it is easy to see that

$$D(T^{2m+1}x_0, T^{2m}x_0) \leq v(a_1 + a_2)^m,$$

and

$$D(T^{2m-1}x_0, T^{2m}x_0) \leq v(a_1 + a_2)^{m-1},$$

for all $m \in \mathbb{N}$ and $\{x_n\}$ is $\mathcal{F}$-Cauchy in the $\mathcal{F}$-complete $\mathcal{F}$-metric space X. Then, there exists $x^* \in X$ such that $\lim_{n\to+\infty} D(x_n, x^*) = 0$. Since T is orbitally $\mathcal{F}$-continuous, we have

$$Tx^* = \lim_{n\to+\infty} Tx_n = \lim_{n\to+\infty} x_{n+1} = x^*.$$

We claim that the fixed point of T is unique. Assume that, on contrary, there exists another fixed point $z^* \in X$ of T such that $D(x^*, z^*) > 0$. Since $\alpha(x^*)\beta(z^*) \geq 1$, it follows from (2.7) that

$$D(x^*, z^*) = D(T^2x^*, T^2z^*) \leq a_1 D(Tx^*, Tz^*) + a_2 D(x^*, z^*) = (a_1 + a_2)D(x^*, z^*).$$

Since $a_1 + a_2 < 1$, it follows that $x^* = z^*$. Consequently, T has a unique fixed point. This completes proof of the Theorem 2.3. □

Definition 2.2. Let (X, D) be an $\mathcal{F}$-metric space and $T : X \to X$ be a cyclic $(\alpha, \beta)-$admissible mapping. We say that T is a $(\alpha, \beta)-$admissible two-sided convex contraction if T is orbitally continuous and there exist $a_i, b_i \in (0, 1)$, $i = 1, 2$, such that

$$\begin{aligned} (2.8) \quad \alpha(x)\beta(y) \geq 1 \textit{ implies } D(T^2x, T^2y) &\leq a_1 D(x, Tx) + a_2 D(Tx, T^2x) \\ &+ b_1 D(y, Ty) + b_2 D(Ty, T^2y), \end{aligned}$$

where $a_1 + a_2 + b_1 + b_2 < 1$ for all $x, y \in X$.

Theorem 2.4. *Let (X, D) be an $\mathcal{F}-$complete $\mathcal{F}-$metric space and $T : X \to X$ be a $(\alpha, \beta)-$admissible two-sided convex contraction. Assume, there exists $x_0 \in X$ such that $\alpha(x_0) \geq 1$ and $\beta(x_0) \geq 1$. Then T has a fixed point. Moreover, if $\alpha(x) \geq 1$ and $\beta(y) \geq 1$ for all $x, y \in Fix(T)$, then T has a unique fixed point.*

Proof. The proof is similar to Theorem 2.2 and Theorem 2.3, therefore we omit it. □

Example 2.3. Consider the $\mathcal{F}$-metric space given in Example 1.1. Let

$$Tx = \begin{cases} -\frac{x}{3}, & x \in [-3, 3] \\ x^3, & \text{otherwise} \end{cases}$$

and $\alpha, \beta : X \to [0, +\infty)$ be given by

$$\alpha(x) = \begin{cases} 1, & x \in [-3, 0] \\ 0, & \text{otherwise} \end{cases} \qquad \beta(x) = \begin{cases} 1, & x \in [0, 3] \\ 0, & \text{otherwise} \end{cases}$$

First, we show that T is an (α, β)-admissible mapping. Let $x \in X$, if $\alpha(x) \geq 1$, then $x \in [-3, 0]$ and so $Tx \in [0, 3]$, that is $\beta(Tx) \geq 1$. Also, if $\beta(x) \geq 1$, then $\alpha(Tx) \geq 1$. Thus T is a cyclic (α, β)-admissible mapping. Let $x, y \in X$ and $\alpha(x)\beta(y) \geq 1$. Then $x \in [-3, 0]$ and $y \in [0, 3]$. Then, we get

$$D(T^2x, T^2y) = D(\frac{x}{9}, \frac{y}{9}) = |\frac{x}{9} - \frac{y}{9}| = \frac{1}{9}D(x, y) \leq \frac{1}{2}D(x, y).$$

Then, all assumption of Theorem 2.3 for $a_1 = 0$ and $a_2 = \frac{1}{2}$ are satisfied. Hence T has a fixed point.

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