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# INTUITIONISTIC FUZZY $I$-CONVERGENT DIFFERENCE SEQUENCE SPACES DEFINED BY COMPACT OPERATOR

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**Abstract.** In this paper, we have introduced and studied the intuitionistic fuzzy $I$-convergent difference sequence spaces $I^{(\mu,\upsilon)}(T,\Delta)$ and $I^{0(\mu,\upsilon)}(T,\Delta)$ used by compact operator. Also, we have introduced a new concept, called closed ball in these spaces. With the help of these notions, we have establised a new topological space and investigated some topological properties in intuitionistic fuzzy $I$-convergent difference sequence spaces $I^{(\mu,\upsilon)}(T,\Delta)$ and $I^{0(\mu,\upsilon)}(T,\Delta)$ used by compact operator.
**Key words:** intuitionistic fuzzy space, compact operator, topological space.

## 1. Introduction

Fuzzy set theory introduced by Zadeh [1] has been applied in various fields of mathematics such as the theory of functions [2] and the approximation theory [3]. Fuzzy topology plays an important role in fuzzy theory. It copes with such conditions where the classical theories break down. The intuitionistic fuzzy normed space and intuitionistic fuzzy $n$-normed space which were introduced in [4]-[5] are the most contemporary improvements in fuzzy topology. Recently, the definition of $I$-convergence in intuitionistic fuzzy zweier $I$-convergent sequence spaces and intuitionistic fuzzy zweier $I$-convergent double sequence spaces have been studied in [10]-[14].

The notion of statistical convergence was given by Steinhaus [15] and Fast [16] using the definition of density of the set of natural numbers. Many years later,

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statistical convergence was discussed by many researchers in the theory of fourier analysis, ergodic theory and number theory. Related studies can be found in [32]-[36]. Some statistical convergence types in intuitionistic fuzzy normed spaces and intuitionistic fuzzy $n$-normed spaces were studied in [6]-[9] and [28]-[37]. As an extended definition of statistical convergence, definition of $I$-convergence was introduced by Kostyrko, Salat and Wilczynski [17] by using the idea of $I$ of subsets of the set of natural numbers. Recently, $I$- and $I^*$- convergence of double sequences have been studied by Das et. al. [18].

New sequence spaces were introduced by means of various matrix transformations in [24]-[32]. Kızmaz [21] defined the difference sequence spaces with the difference matrix as follows:

$$X(\Delta) = \{x = (x_k) : \Delta x \in X\}$$

for $X = l_\infty$, $c$, $c_0$, where $\Delta x_k = x_k - x_{k+1}$ and $\Delta$ denotes the difference matrix $\Delta = (\Delta_{nk})$ defined by

$$\Delta_{nk} = \begin{cases} (-1)^{n-k}, & \text{if } n \leqslant k \leqslant n+1, \\ 0, & \text{if } 0 \leqslant k < n. \end{cases}$$

Recently, Kamber has studied intuitionistic fuzzy difference sequence spaces and intuitionistic fuzzy difference double sequence spaces in [26] and [27].

In this paper, we introduce the intuitionistic fuzzy $I$-convergent difference sequence spaces $I^{(\mu,\upsilon)}(T,\Delta)$ and $I^{0(\mu,\upsilon)}(T,\Delta)$ using by compact operator and investigate some topological properties of these new spaces.

## 2. Basic definitions

In this section, we give some definitions and notations which will be used for this study.

**Definition 2.1.** [19] A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is said to be a continuous $t$-norm if it satisfies the following conditions:

(i) $*$ is associative and commutative,

(ii) $*$ is continuous,

(iii) $a * 1 = a$ for all $a \in [0,1]$,

(iv) $a * b \leqslant c * d$ whenever $a \leqslant c$ and $b \leqslant d$ for each $a,\ b,\ c, d \in [0,1]$.

**Definition 2.2.** [19] A binary operation $\circ : [0,1] \times [0,1] \to [0,1]$ is said to be a continuous $t$-conorm if it satisfies the following conditions:

(i) $\circ$ is associative and commutative,

(ii) $\circ$ is continuous,

(iii) $a \circ 0 = a$ for all $a \in [0,1]$,

(iv) $a \circ b \leqslant c \circ d$ whenever $a \leqslant c$ and $b \leqslant d$ for each $a,\ b,\ c, d \in [0,1]$.

**Definition 2.3.** [4] The five-tuple $(X,\ \mu, \upsilon, *, \circ)$ is said to be intuitionistic fuzzy normed linear space (or shortly IFNLS) is where $X$ is a linear space over a field $F$, $*$ is a continuous $t$-norm, $\circ$ is a continuous $t$-conorm, $\mu$, $\upsilon$ are fuzzy sets on $X \times (0,\infty)$, $\mu$ denotes the degree of membership and $\upsilon$ denotes the degree of nonmembership of $(x,t) \in X \times (0,\infty)$ satisfying the following conditions for every $x, y \in X$ and $s, t > 0$:

(i) $\mu(x,t) + \upsilon(x,t) \leqslant 1$,

(ii) $\mu(x,t) > 0$,

(iii) $\mu(x,t) = 1$ if and only if $x = 0$,

(iv) $\mu(\alpha x,t) = \mu\left(x, \frac{t}{|\alpha|}\right)$ if $\alpha \neq 0$,

(v) $\mu(x,t) * \mu(y,s) \leqslant \mu(x+y, t+s)$,

(vi) $\mu(x,.) : (0,\infty) \to [0,1]$ is continuous,

(vii) $\lim\limits_{t\to\infty} \mu(x,t) = 1$ and $\lim\limits_{t\to 0} \mu(x,t) = 0$,

(viii) $\upsilon(x,t) < 1$,

(ix) $\upsilon(x,t) = 0$ if and only if $x = 0$,

(x) $\upsilon(\alpha x,t) = \upsilon\left(x, \frac{t}{|\alpha|}\right)$ if $\alpha \neq 0$,

(xi) $\upsilon(x,t) \circ \upsilon(y,s) \geqslant \upsilon(x+y, s+t)$,

(xii) $\upsilon(x,.) : (0,\infty) \to [0,1]$ is continuous,

(xiii) $\lim\limits_{t\to\infty} \upsilon(x,t) = 0$ and $\lim\limits_{t\to 0} \upsilon(x,t) = 1$.

In this case $(\mu, \upsilon)$ is called intuitionistic fuzzy norm.

**Example 2.1.** [4] Let $(\mathrm{X}, \|.\|)$ be a normed space, and let $a * b = ab$ and $a \circ b = \min\{a+b, 1\}$ for all $a, b \in [0,1]$. For all $x \in X$ and every $t > 0$, consider

$$\mu(x,t) := \tfrac{t}{t+\|x\|} \text{ and } \upsilon(x,t) := \tfrac{\|x\|}{t+\|x\|}.$$

Then $(X,\ \mu, \upsilon, *, \circ)$ is an IFNLS.

**Definition 2.4.** [4] Let $(X,\ \mu, \upsilon, *, \circ)$ be an IFNLS. A sequence $x = (x_k)$ in $X$ is convergent to $L \in X$ with respect to the intuitionistic fuzzy norm $(\mu, \upsilon)$ if, for every $\varepsilon > 0$ and $t > 0$, there exists $k_0 \in \mathbb{N}$ such that $\mu(x_k - L, t) > 1 - \varepsilon$ and $\upsilon(x_k - L, t) < \varepsilon$ for all $k \geqslant k_0$ where $k \in \mathbb{N}$. It is denoted by $(\mu, \upsilon) - \lim x = L$.

**Theorem 2.1.** [20] *Let $(X, \mu, \upsilon, *, \circ)$ be an IFNLS. Then, a sequence $x = (x_k)$ in $X$ is convergent to $L \in X$ if and only if $\lim\limits_{k\to\infty} \mu(x_k - L, t) = 1$ and $\lim\limits_{k\to\infty} \upsilon(x_k - L, t) = 0$.*

**Definition 2.5.** [17] If $X$ is a non-empty set, then a family of sets $I \subset P(X)$ is called an ideal in $X$ if and only if

(i) $\emptyset \in I$,

(ii) for each $A, B \in I$ implies that $A \cup B \in I$, and

(iii) for each $A \in I$ and $B \subset A$ we have $B \in I$,

where $P(X)$ is the power set of $X$.

**Definition 2.6.** [17] If $X$ is a non-empty set, then a non-empty family of sets $F \subset P(X)$ is called a filter on $X$ if and only if

(i) $\emptyset \notin F$,

(ii) for each $A, B \in F$ implies that $A \cap B \in F$, and

(iii) for each $A \in F$ and $B \supset A$, we have $B \in F$.

An ideal $I$ is called non-trivial if $I \neq \emptyset$ and $X \notin I$. A non-trivial ideal $I \subset P(X)$ is called an admissible ideal in $X$ if and only if $\{\{x\} : x \in X\} \subseteq I$.

A relation between the concepts of an ideal and a filter is given by the following proposition:

**Proposition 2.1.** [17] Let $I \subset P(X)$ be a non-trivial ideal. Then the class $F = F(I) = \{M \subset X : M = X - A,\ for\ some\ A \in I\}$ is a filter on $X$. $F = F(I)$ is called the filter associated with the ideal $I$.

**Definition 2.7.** [25] Let $I \subset P(\mathbb{N})$ be a non-trivial ideal and $(X, \mu, \upsilon, *, \circ)$ be an IFNLS. Then a sequence $x = (x_k)$ in $X$ is said to be $I$-convergent to $L \in X$ with respect to the intuitionistic fuzzy linear norm $(\mu, \upsilon)$ if, for every $\varepsilon > 0$ and $t > 0$, the set

$$\{k \in \mathbb{N} : \mu(x_k - L, t) \leqslant 1 - \varepsilon \text{ or } \upsilon(x_k - L, t) \geqslant \varepsilon\} \in I.$$

In this case, we write $I^{(\mu,\upsilon)} - \lim x = L$.

**Definition 2.8.** [14] Let $X$ and $Y$ be two normed linear spaces and $T : D(T) \to Y$ be a linear operator, where $D \subset X$. Then the operator $T$ is said to be bounded, if there exists a positive real $k$ such that $||Tx|| \leqslant k||x||$, for all $x \in D(T)$. The set of all bounded linear operators $B(X, Y)$ is a normed linear space normed by $||T|| = sup||Tx||(x \in X, ||x|| = 1)$ and $B(X, Y)$ is a Banach space if $Y$ is a Banach space.

**Definition 2.9.** [14] Let $X$ and $Y$ be two normed linear spaces. An operator $T : X \to Y$ is said to be a compact linear operator (or completely continuous linear operator,) if

(i) $T$ is linear,

(ii) $T$ maps every bounded sequence $(x_k)$ in $X$ onto a sequence $(T(x_k))$ in $Y$ which has a convergent subsequence. The set of all compact linear operator $C(X,Y)$ is a closed subspace of $B(X,Y)$ and $C(X,Y)$ is Banach space, if $Y$ is a Banach space.

## 3. Weighted Norlund-Euler $\lambda$-statistical convergence in IFNLS

In this paper, we defined a variant of ideal convergent sequence spaces called intuitionistic fuzzy ideal difference convergent sequence spaces using by compact operator and investigated some topological properties of these spaces.

Intuitionistic fuzzy $I$-convergent difference sequence spaces using by compact operator are introduced as:

$$I^{(\mu,\upsilon)}(T,\Delta) = \{(x_k) \in I^{(\mu,\upsilon)}(T,\Delta) : \\ \{k \in \mathbb{N} : \mu\left(T(\Delta x_k) - L, t\right) \leqslant 1-\varepsilon \text{ or } \upsilon\left(T(\Delta x_k) - L, t\right) \geqslant \varepsilon\} \in I\},$$

and

$$I^{0^{(\mu,\upsilon)}}(T,\Delta) = \{(x_k) \in I^{(\mu,\upsilon)}(T,\Delta) : \\ \{k \in \mathbb{N} : \mu\left(T(\Delta x_k), t\right) \leqslant 1-\varepsilon \text{ or } \upsilon\left(T(\Delta x_k), t\right) \geqslant \varepsilon\} \in I\}.$$

Moreover, an open ball $B_x(r,t)(T,\Delta)$ with centre $x \in I^{(\mu,\upsilon)}(T,\Delta)$ and radius $r \in (0,1)$ with respect to $t$, is defined as:

$$B_x(r,t)(T,\Delta) = \{y \in I^{(\mu,\upsilon)}(T,\Delta) : \\ \{k \in \mathbb{N} : \mu\left(T(\Delta x_k) - T(\Delta y_k), t\right) \leqslant 1-r \text{ or } \upsilon\left(T(\Delta x_k) - T(\Delta y_k), t\right) \geqslant r\} \in I\}.$$

**Theorem 3.1.** *$I^{(\mu,\upsilon)}(T,\Delta)$ and $I^{0^{(\mu,\upsilon)}}(T,\Delta)$ are linear spaces.*

*Proof.* We prove the result for $I^{(\mu,\upsilon)}(T,\Delta)$. Similarly, it can be proved for $I^{0^{(\mu,\upsilon)}}(T,\Delta)$. Let $(x_k),(y_k) \in I^{(\mu,\upsilon)}(T,\Delta)$ and $\alpha,\beta$ be scalars. The proof is trivial for $\alpha = 0$ and $\beta = 0$. Let $\alpha \neq 0$ and $\beta \neq 0$. For a given $\varepsilon > 0$, choose $s > 0$ such that $(1-\varepsilon) * (1-\varepsilon) > 1-s$ and $\varepsilon \circ \varepsilon < s$. Hence, we have

$$A_1 = \{k \in \mathbb{N} : \mu\left(T(\Delta x_k) - L_1, t/2|\alpha|\right) \leqslant 1-\varepsilon \text{or } \upsilon\left(T(\Delta x_k) - L_1, t/2|\alpha|\right) \geqslant \varepsilon\} \in I,$$

$$A_2 = \{k \in \mathbb{N} : \mu\left(T(\Delta y_k) - L_2, t/2|\beta|\right) \leqslant 1-\varepsilon \text{ or } \upsilon\left(T(\Delta y_k) - L_1, t/2|\beta|\right) \geqslant \varepsilon\} \in I,$$

$$\begin{aligned}A_1^c &= \{k \in \mathbb{N} : \mu\left(T(\Delta x_k) - L_1, t/2|\alpha|\right) > 1 - \varepsilon \text{and } \upsilon\left(T(\Delta x_k) - L_1, t/2|\alpha|\right) < \varepsilon\} \\ &\in F(I),\end{aligned}$$

and

$$\begin{aligned}A_2^c &= \{k \in \mathbb{N} : \mu\left(T(\Delta y_k) - L_2, t/2|\beta|\right) > 1 - \varepsilon \text{ and } \upsilon\left(T(\Delta y_k) - L_2, t/2|\beta|\right) < \varepsilon\} \\ &\in F(I).\end{aligned}$$

Let define the set $A_3 = A_1 \cup A_2$. Hence $A_3 \in I$. It follows that $A_3^c$ is a non-empty set in $F(I)$. We will prove that for every $(x_k), (y_k) \in I^{(\mu,\upsilon)}(T, \Delta)$,

$$A_3^c \subset \{k \in \mathbb{N} : \mu\left((\alpha.T(\Delta x_k) + \beta.T(\Delta y_k)) - (\alpha L_1 + \beta L_2), t\right) > 1 - s$$

and $\upsilon\left((\alpha.T(\Delta x_k) + \beta.T(\Delta y_k)) - (\alpha L_1 + \beta L_2), t\right) < s\}$. Let $m \in A_3^c$. In this case,

$$\mu\left(T(\Delta x_m) - L_1, t/2|\alpha|\right) > 1 - \varepsilon$$

and

$$\upsilon\left(T(\Delta x_m) - L_1, t/2|\alpha|\right) < \varepsilon$$

and

$$\mu\left(T(\Delta y_m) - L_2, t/2|\beta|\right) > 1 - \varepsilon$$

and

$$\upsilon\left(T(\Delta y_m) - L_2, t/2|\beta|\right) < \varepsilon.$$

Then,

$$\begin{aligned}&\mu((\alpha.T(\Delta x_m) + \beta.T(\Delta y_m) - (\alpha L_1 + \beta L_2), t) \geq \\ &\mu\left(\alpha.T(\Delta x_m) - \alpha L_1, t/2\right) * \mu\left(\beta.T(\Delta y_m) - \beta L_2, t/2\right) = \\ &\mu\left(T(\Delta x_m) - L_1, t/2|\alpha|\right) * \mu\left(T(\Delta y_m) - L_2, t/2|\beta|\right) \geq \\ &(1 - \varepsilon) * (1 - \varepsilon) > 1 - s\end{aligned}$$

and

$$\begin{aligned}&\upsilon((\alpha.T(\Delta x_m) + \beta.T(\Delta y_m)) - (\alpha L_1 + \beta L_2), t) \leq \\ &\upsilon\left(\alpha.T(\Delta x_m) - \alpha L_1, t/2\right) \circ \upsilon\left(\beta.T(\Delta y_m) - \beta L_2, t/2\right) = \\ &\upsilon\left(T(\Delta x_m) - L_1, t/2|\alpha|\right) \circ \upsilon\left(T(\Delta y_m) - L_2, t/2|\beta|\right) < \\ &\varepsilon \circ \varepsilon < s.\end{aligned}$$

This proves that

$$A_3^c \subset \{k \in \mathbb{N} : \mu\left((\alpha.T(\Delta x_k) + \beta.T(\Delta y_k)) - (\alpha L_1 + \beta L_2), t\right) > 1 - s$$

and $\upsilon\left((\alpha.T(\Delta x_k) + \beta.T(\Delta y_k)) - (\alpha L_1 + \beta L_2), t\right) < s\}$. Hence $I^{(\mu,\upsilon)}(T, \Delta)$ is a linear space. □

**Theorem 3.2.** *Every closed ball $B_x^c(r, t)(T, \Delta)$ is an open set in $I^{(\mu,\upsilon)}(T, \Delta)$.*

*Proof.* Let $B_x(r,t)(T,\Delta)$ be an open ball with centre $x \in I^{(\mu,\upsilon)}(T,\Delta)$ and radius $r \in (0,1)$ with respect to $t$, i.e.

$$B_x(r,t)(T,\Delta) = \{y \in I^{(\mu,\upsilon)}(T,\Delta) :$$
$$\{k \in \mathbb{N} : \mu\left(T(\Delta x_k) - T(\Delta y_k), t\right) \leqslant 1 - r \text{ or } \upsilon\left(T(\Delta x_k) - T(\Delta y_k), t\right) \geqslant r\} \in I\}.$$

Let $y \in B^c_x(r,t)(T,\Delta)$. Then $\mu\left(T(\Delta x) - T(\Delta y), t\right) > 1 - r$ and $\upsilon\left(T(\Delta x) - T(\Delta y), t\right) < r$. Since $\mu\left(T(\Delta x) - T(\Delta y), t\right) > 1-r$, there exists $t_0 \in (0,t)$ such that $\mu\left(T(\Delta x) - T(\Delta y), t_0\right) > 1 - r$ and $\upsilon\left(T(\Delta x) - T(\Delta y), t_0\right) < r$.

Let $r_0 = \mu\left(T(\Delta x) - T(\Delta y), t_0\right)$. Since $r_0 > 1 - r$, there exists $s \in (0,1)$ such that $r_0 > 1 - s > 1 - r$ and so there exists $r_1, r_2 \in (0,1)$ such that $r_0 * r_1 > 1 - s$ and $(1 - r_0) \circ (1 - r_2) < s$.

Let $r_3 = max\{r_1, r_2\}$. Then $1 - s < r_0 * r_1 \leqslant r_0 * r_3$ and $(1 - r_0) \circ (1 - r_3) \leqslant (1 - r_0) \circ (1 - r_2) < s$.

Consider the closed balls $B^c_y(1 - r_3, t - t_0)(T,\Delta)$ and $B^c_x(r,t)(T,\Delta)$. We prove that $B^c_y(1 - r_3, t - t_0)(T,\Delta) \subset B^c_x(r,t)(T,\Delta)$. Let $z \in B^c_y(1 - r_3, t - t_0)(T,\Delta)$. Then $\mu\left(T(\Delta y) - T(\Delta z), t - t_0\right) > r_3$ and $\upsilon\left(T(\Delta y) - T(\Delta z), t - t_0\right) < 1 - r_3$. Hence

$$\begin{aligned}&\mu\left(T(\Delta x) - T(\Delta z), t\right) \geqslant \\ &\mu\left(T(\Delta x) - T(\Delta y), t_0\right) * \mu\left(T(\Delta y) - T(\Delta z), t - t_0\right) > \\ &r_0 * r_3 \geqslant r_0 * r_1 > 1 - s > 1 - r\end{aligned}$$

and

$$\begin{aligned}&\upsilon\left(T(\Delta x) - T(\Delta z), t\right) \leqslant \\ &\upsilon\left(T(\Delta x) - T(\Delta y), t_0\right) \circ \upsilon\left(T(\Delta y) - T(\Delta z), t - t_0\right) < \\ &(1 - r_0) \circ (1 - r_3) < s < r.\end{aligned}$$

Thus $z \in B^c_x(r,t)(T,\Delta)$ and hence $B^c_y(1 - r_3, t - t_0)(T,\Delta) \subset B^c_x(r,t)(T,\Delta)$. Every closed ball $B^c_x(r,t)(T,\Delta)$ is an open set in $I^{(\mu,\upsilon)}(T,\Delta)$. It proves that $B^c_x(r,t)(T,\Delta)$ is an open set in $I^{(\mu,\upsilon)}(T,\Delta)$. □

**Remark 3.1.** It is clear that $I^{(\mu,\upsilon)}(T,\Delta)$ is an IFNLS. Define

$$\begin{aligned}&\tau^{(\mu,\upsilon)}(T,\Delta) = \\ &\Big\{A \subset I^{(\mu,\upsilon)}(T,\Delta) : \text{ for each } x \in A \text{ there exist } t > 0 \text{ and } r \in (0,1) \\ &\qquad \text{such that } B^c_x(r,t)(T,\Delta) \subset A\Big\}.\end{aligned}$$

Then $\tau^{(\mu,\upsilon)}(T,\Delta)$ is a topology on $I^{(\mu,\upsilon)}(T,\Delta)$.

**Theorem 3.3.** *The topology $\tau^{(\mu,\upsilon)}(T,\Delta)$ on $I^{0(\mu,\upsilon)}(T,\Delta)$ is first countable.*

*Proof.* It is clear that $\{B^c_x(\frac{1}{n}, \frac{1}{n})(T,\Delta) : n \in \mathbb{N}\}$ is a local base at $x \in I^{(\mu,\upsilon)}(T,\Delta)$. Then the topology $\tau^{(\mu,\upsilon)}(T,\Delta)$ on $I^{0(\mu,\upsilon)}(T,\Delta)$ is first countable. □

**Theorem 3.4.** *$I^{(\mu,\upsilon)}(T,\Delta)$ and $I^{0(\mu,\upsilon)}(T,\Delta)$ are Hausdorff spaces.*

*Proof.* Let $x, y \in I^{(\mu,\upsilon)}(T,\Delta)$ such that $x \neq y$. Then $0 < \mu\left(T(\Delta x) - T(\Delta y), t\right) < 1$ and $0 < \upsilon\left(T(\Delta x) - T(\Delta y), t\right) < 1$.

Let define $r_1, r_2$ and $r$ such that $r_1 = \mu\left(T(\Delta x) - T(\Delta y), t\right)$, $r_2 = \upsilon\left(T(\Delta x) - T(\Delta y), t\right)$ and $r = max\{r_1, 1 - r_2\}$. Then for each $r_0 \in (r, 1)$ there exist $r_3$ and $r_4$ such that $r_3 * r_4 \geqslant r_0$ and $(1 - r_3) \circ (1 - r_4) \leq (1 - r_0)$.

Let $r_5 = max\{r_3, (1 - r_4)\}$ and consider the closed balls $B^c_x(1 - r_5, \frac{t}{2})(T,\Delta)$ and $B^c_y(1 - r_5, \frac{t}{2})(T,\Delta)$. Then clearly $B^c_x(1 - r_5, \frac{t}{2})(T,\Delta) \cap B^c_y(1 - r_5, \frac{t}{2})(T,\Delta) = \emptyset$.

Suppose that $z \in B^c_x(1 - r_5, \frac{t}{2})(T,\Delta) \cap B^c_y(1 - r_5, \frac{t}{2})(T,\Delta)$. Then

$$\begin{aligned} & r_1 = \mu\left(T(\Delta x) - T(\Delta y), t\right) \geqslant \\ & \mu\left(T(\Delta x) - T(\Delta z), \frac{t}{2}\right) * \mu\left(T(\Delta y) - T(\Delta z), \frac{t}{2}\right) \geqslant \\ & r_5 * r_5 \geqslant r_3 * r_4 \geqslant r_0 > r \end{aligned}$$

and

$$\begin{aligned} & r_2 = \upsilon\left(T(\Delta x) - T(\Delta y), t\right) \leqslant \\ & \upsilon\left(T(\Delta x) - T(\Delta z), \frac{t}{2}\right) \circ \upsilon\left(T(\Delta y) - T(\Delta z), \frac{t}{2}\right) \leqslant \\ & (1 - r_5) \circ (1 - r_5) \leqslant (1 - r_3) \circ (1 - r_4) \leqslant (1 - r_0) < 1 - r, \end{aligned}$$

which is a contradiction. Hence $I^{(\mu,\upsilon)}(T,\Delta)$ is a Hausdorff space. $\square$

**Theorem 3.5.** *Let $I^{(\mu,\upsilon)}(T,\Delta)$ be an IFNLS, $\tau^{(\mu,\upsilon)}(T,\Delta)$ be a topology on $I^{(\mu,\upsilon)}(T,\Delta)$ and $(x_k)$ be a sequence in $I^{(\mu,\upsilon)}(T,\Delta)$. Then a sequence $(x_k)$ is $\Delta$-convergent to $\Delta x_0$ with respect to the intuitionistic fuzzy linear norm $(\mu,\upsilon)$ if and only if $\mu\left(T(\Delta x_k) - T(\Delta x_0), t\right) \longrightarrow 1$ and $\upsilon\left(T(\Delta x_k) - T(\Delta x_0), t\right) \longrightarrow 0$ as $k \longrightarrow \infty$.*

*Proof.* Let $B_{x_0}(r,t)(T,\Delta)$ be an open ball with centre $x_0 \in I^{(\mu,\upsilon)}(T,\Delta)$ and radius $r \in (0,1)$ with respect to $t$, i.e.

$$\begin{aligned} & B_{x_0}(r,t)(T,\Delta) = \{(x_k) \in I^{(\mu,\upsilon)}(T,\Delta) : \\ & \{k \in \mathbb{N} : \mu\left(T(\Delta x_k) - T(\Delta x_0), t\right) \leqslant 1 - r \text{ or } \upsilon\left(T(\Delta x_k) - T(\Delta x_0), t\right) \geqslant r\} \in I\}. \end{aligned}$$

Suppose that a sequence $(x_k)$ in $I^{(\mu,\upsilon)}(T,\Delta)$ is $\Delta$-convergent to $\Delta x_0$ with respect to the intuitionistic fuzzy linear norm $(\mu,\upsilon)$. Then for $r \in (0,1)$ and $t > 0$, there exists $k_0 \in \mathbb{N}$ such that $(x_k) \in B^c_{x_0}(r,t)(T,\Delta)$ for all $k \geqslant k_0$. Thus

$$\{k \in \mathbb{N} : \mu\left(T(\Delta x_k) - T(\Delta x_0), t\right) > 1 - r \text{ and } \upsilon\left(T(\Delta x_k) - T(\Delta x_0), t\right) < r\} \in F(I).$$

So $1 - \mu\left(T(\Delta x_k) - T(\Delta x_0), t\right) < r$ and $\upsilon\left(T(\Delta x_k) - T(\Delta x_0), t\right) < r$, for all $k \geqslant k_0$. Then $\mu\left(T(\Delta x_k) - T(\Delta x_0), t\right) \longrightarrow 1$ and $\upsilon\left(T(\Delta x_k) - T(\Delta x_0), t\right) \longrightarrow 0$ as $k \longrightarrow \infty$.

Conversely, for each $t > 0$ and $k \in \mathbb{N}$, suppose that $\mu\left(T(\Delta x_k) - T(\Delta x_0), t\right) \longrightarrow 1$ and $\upsilon\left(T(\Delta x_k) - T(\Delta x_0), t\right) \longrightarrow 0$ as $k \longrightarrow \infty$. Then for $r \in (0,1)$, there exists $k_0 \in \mathbb{N}$ such that $1 - \mu\left(T(\Delta x_k) - T(\Delta x_0), t\right) < r$ and $\upsilon\left(T(\Delta x_k) - T(\Delta x_0), t\right) < r$ for all $k \geqslant k_0$. So, $\mu\left(T(\Delta x_k) - T(\Delta x_0), t\right) > 1 - r$ and $\upsilon\left(T(\Delta x_k) - T(\Delta x_0), t\right) < r$ for all $k \geqslant k_0$. Hence $(x_k) \in B^c_{x_0}(r,t)(T)$ for all $k \geqslant k_0$. This proves that a sequence $(x_k)$ is $\Delta$-convergent to $\Delta x_0$ with respect to the intuitionistic fuzzy linear norm $(\mu, \upsilon)$. $\square$

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