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SOME FIXED POINT THEOREMS VIA CYCLIC CONTRACTIVE CONDITIONS IN S-METRIC SPACES

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Abstract. We present some fixed point theorems for mappings which satisfy certain cyclic contractive conditions in the setting of S-metric spaces. The results presented in this paper generalize or improve many existing fixed point theorems in the literature. At the end of the paper, we give some examples to demonstrate our results.
Key words: Fixed point, cyclic contraction, S-metric space.

1. Introduction

In the field of fixed point theory, to find the solution of fixed point problems, the contractive conditions on ambient functions play a significant role. The most fundamental result in metric fixed point theory is Banach Contraction Principle ([4]).

Let (X, d) be a complete metric space and let $T: X \to X$ be a self-mapping. If there exists $k \in [0, 1)$ such that

$$d(T(x), T(y)) \leq k\, d(x, y), \tag{1.1}$$

for all $x, y \in X$, then T has a unique fixed point $u \in X$. Moreover, for any $x_0 \in X$, the sequence $\{x_n\} \subset X$ defined by $x_{n+1} = Tx_n$, $n \in \mathbb{N}$, is convergent to the fixed point u. Inequality (1.1) also implies the continuity of T.

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Over the years, due to its importance and applications in different fields of science, several authors generalized the well-known Banach Contraction Principle by introducing a new ambient space or a contractive condition. It is no surprise that there is a great number of generalizations of this fundamental result.

Cyclic representation and cyclic contraction were introduced by Kirk et al. [14] in metric spaces and investigated the existence of proximity points and fixed points for cyclic contraction mappings and further used by several authors to obtain various fixed point results for not necessary continuous mappings in different spaces (see, e.g., [3, 6, 9, 11, 12, 13, 16, 17, 18, 19] and others).

Sedghi et al. [24] introduced the notion of S-metric spaces that generalized G-metric spaces and D^*-metric spaces. In [24] the authors proved some properties of S-metric spaces. They also obtained some fixed point theorems in the setting of S-metric spaces for a self-map.

Gupta [9] introduced the concept of cyclic contraction in S-metric spaces and proved some fixed theorems in the said spaces which are proper generalizations of the results of Sedghi et al. [24].

In this paper, we establish some fixed point theorems for cyclic contractive mappings in the setting of S-metric spaces. Our results generalize or improve several existing fixed point theorems in the literature.

2. Preliminaries

The notion of cyclic contraction is as follows:

Definition 2.1. ([14]) Let X be a nonempty set, $m \in \mathbb{N}$ and let $f\colon X \to X$ be a self-mapping. Then $X = \cup_{i=1}^{m} A_i$ is a cyclic representation of X with respect to f if

a) A_i, $i = 1, 2, \ldots, m$ are nonempty subsets of X;

b) $f(A_1) \subset A_2, f(A_2) \subset A_3, \ldots, f(A_{m-1}) \subset A_m, f(A_m) \subset A_1$.

Kirk et al. [14] proved the following fixed point result via cyclic contraction which is one of the extraordinary generalizations of the Banach's contraction principle.

Theorem 2.1. ([14]) *Let (X, d) be a complete metric space, $f\colon X \to X$ and let $X = \cup_{i=1}^{m} A_i$ be a cyclic representation of X with respect to f. Suppose that f satisfies the following condition:*

$$d(fx, fy) \leq \psi(d(x, y)), \tag{2.1}$$

for all $x \in A_i$, $y \in A_{i+1}$, $i \in \{1, 2, \ldots, m\}$, where $A_{m+1} = A_1$ and $\psi\colon [0, \infty) \to [0, \infty)$ is a function, upper semi-continuous from the right and $0 \leq \psi(t) < t$ for $t > 0$. Then f has a fixed point $z \in \cap_{i=1}^{m} A_i$.

In 2010, Păcurar and Rus [17] introduced the following notion of cyclic weaker φ-contraction.

Definition 2.2. ([17]) Let (X, d) be a metric space, $m \in \mathbb{N}$, $A_1, A_2, \dots, A_m$ be closed nonempty subsets of X and $X = \cup_{i=1}^{m} A_i$. An operator $f: X \to X$ is called a cyclic weaker φ-contraction if

1) $X = \cup_{i=1}^{m} A_i$ is a cyclic representation of X with respect to f;

2) there exists a continuous, nondecreasing function φ: $[0, 1) \to [0, 1)$ with $\varphi(t) > 0$ for $t \in (0, 1)$ and $\varphi(0) = 0$ such that

$$d(fx, fy) \le d(x, y) - \varphi(d(x, y)), \tag{2.2}$$

for any $x \in A_i$, $y \in A_{i+1}$, $i = 1, 2, \dots, m$, where $A_{m+1} = A_1$.

They proved the following result.

Theorem 2.2. ([17]) *Suppose f is a cyclic weaker φ-contraction on a complete metric space (X, d). Then f has a fixed point $z \in \cap_{i=1}^{m} A_i$.*

We need the following definitions and lemmas in the sequel.

Definition 2.3. ([24]) Let X be a nonempty set and $S: X^3 \to [0, \infty)$ be a function satisfying the following conditions for all x, y, z, $t \in X$:

$(S1)$ $S(x, y, z) = 0$ if and only if $x = y = z$;

$(S2)$ $S(x, y, z) \le S(x, x, t) + S(y, y, t) + S(z, z, t)$.

Then the function S is called an S-metric on X and the pair (X, S) is called an S-metric space or simply SMS.

Example 2.1. ([24]) Let $X = \mathbb{R}^n$ and $\|.\|$ a norm on X, then $S(x, y, z) = \|y + z - 2x\| + \|y - z\|$ is an S-metric on X.

Example 2.2. ([24]) Let $X = \mathbb{R}^n$ and $\|.\|$ a norm on X, then $S(x, y, z) = \|x - z\| + \|y - z\|$ is an S-metric on X.

Example 2.3. ([25]) Let $X = \mathbb{R}$ be the real line. Then $S(x, y, z) = |x - z| + |y - z|$ for all $x, y, z \in \mathbb{R}$ is an S-metric on X. This S-metric on X is called the usual S-metric on X.

Definition 2.4. ([24]) Let (X, S) be an S-metric space.

$(a1)$ A sequence $\{x_n\}$ in X converges to $x \in X$ if $S(x_n, x_n, x) \to 0$ as $n \to \infty$, that is, for each $\varepsilon > 0$, there exists $n_0 \in \mathbb{N}$ such that for all $n \ge n_0$ we have $S(x_n, x_n, x) < \varepsilon$. We denote this by $\lim_{n\to\infty} x_n = x$ or $x_n \to x$ as $n \to \infty$.

$(a2)$ A sequence $\{x_n\}$ in X is called a Cauchy sequence if $S(x_n, x_n, x_m) \to 0$ as $n, m \to \infty$, that is, for each $\varepsilon > 0$, there exists $n_0 \in \mathbb{N}$ such that for all $n, m \ge n_0$ we have $S(x_n, x_n, x_m) < \varepsilon$.

$(a3)$ The S-metric space (X, S) is called complete if every Cauchy sequence in X is convergent in X.

Definition 2.5. ([24]) Let (X, S) be an S-metric space. A mapping $T\colon X \to X$ is said to be a contraction if there exists a constant $0 \le L < 1$ such that

$$S(Tx, Ty, Tz) \le L\, S(x, y, z) \tag{2.3}$$

for all $x, y, z \in X$. If the S-metric space (X, S) is complete then the mapping defined as above has a unique fixed point.

Every S-metric on X defines a metric d_S on X by

$$d_S = S(x, x, y) + S(y, y, x) \;\; \forall\, x, y \in X. \tag{2.4}$$

Let τ be the set of all subsets A of X with $x \in A$ if and only if there exists $r > 0$ such that $B_S(x, r) \subset A$. Then τ is a topology on X. Also, a nonempty subset A in the S-metric space (X, S) is S-closed if $\bar{A} = A$.

Lemma 2.1. ([24, Lemma 2.5]) *In an S-metric space, we have* $S(x, x, y) = S(y, y, x)$ *for all* $x, y \in X$.

Lemma 2.2. ([24, Lemma 2.12]) *Let (X, S) be an S-metric space. If the sequence $\{x_n\}$ converges to x, that is, $x_n \to x$ as $n \to \infty$ and the sequence $\{y_n\}$ converges to y, that is, $y_n \to y$ as $n \to \infty$, then the sequence $\{S(x_n, x_n, y_n)\}$ converges to $S(x, x, y)$, that is, $S(x_n, x_n, y_n) \to S(x, x, y)$ as $n \to \infty$.*

Lemma 2.3. ([9, Lemma 8]) *Let (X, S) be an S-metric space and A is a nonempty subset of X. Then A is said to be S-closed if and only if for any sequence $\{x_n\}$ in A such that $x_n \to x$ as $n \to \infty$, then $x \in A$.*

3. Main Result

In this section, we shall prove some fixed point theorems via certain cyclic contractive conditions in the setting of complete S-metric spaces.

First of all, we shall denote Ψ the set of functions $\psi\colon [0, \infty) \to [0, \infty)$ satisfying the following conditions:

(Ψ_1) ψ is continuous; (Ψ_2) $\psi(t) < t$ for all $t > 0$.

Obviously, if $\psi \in \Psi$, then $\psi(0) = 0$ and $\psi(t) \le t$ for all $t \ge 0$.

Now, we introduce the notion of cyclic generalized g_ψ-contraction in S-metric space as follows.

Definition 3.1. Let (X, S) be a S-metric space. Let m be a positive integer, $A_1, A_2, \dots, A_m$ be nonempty subsets of X and $Y = \cup_{i=1}^m A_i$. An operator $g\colon Y \to Y$ is a cyclic generalized g_ψ-contraction for some $\psi \in \Psi$, if

(**I**) $Y = \cup_{i=1}^m A_i$ is a cyclic representation of Y with respect to g;

(**II**) there exists $b_1, b_2 \in [0,1)$ with $b_1 + b_2 < 1$ such that for all $(x,y,z) \in A_i \times A_i \times A_{i+1}$, $i = 1,2,\ldots,m$ (with $A_{m+1} = A_1$)

$$S(gx,gy,gz) \leq b_1\, \mathbf{n}(x,y,z) + b_2\, \mathbf{N}(x,y,z), \tag{3.1}$$

where

$$\mathbf{n}(x,y,z) = \psi\Big(S(gz,gz,z)\frac{1+S(gy,gy,y)}{1+S(x,y,z)}\Big),$$

and

$$\begin{aligned}\mathbf{N}(x,y,z) \quad = \quad \max\Big\{&\psi(S(x,y,z)), \psi(S(gx,gx,x)), \psi(S(gy,gy,y)),\\ &\psi\big(\frac{1}{2}[S(gx,gx,z)+S(gz,gz,x)]\big)\Big\}.\end{aligned}$$

Now, we are in a position to prove our main result.

Theorem 3.1. *Let (X,S) be a complete S-metric space, $m \in \mathbb{N}$, $A_1, A_2, \ldots, A_m$ be nonempty closed subsets of X and $Y = \cup_{i=1}^m A_i$. Suppose that $g\colon Y \to Y$ is a cyclic generalized g_ψ-contraction mapping, for some $\psi \in \Psi$. Then g has a unique fixed point. Moreover, the fixed point of g belongs to $\cap_{i=1}^m A_i$.*

Proof. Let $x_0 \in A_1$ (such a point exists since $A_1 \neq \emptyset$). Define the sequence $\{x_n\}$ in X by $x_{n+1} = gx_n$, $n = 0,1,2,\ldots$. We shall prove that

$$\lim_{n\to\infty} S(x_{n+1},x_{n+1},x_{n+2}) = 0. \tag{3.2}$$

If for some k, we have $\lim_{k\to\infty} S(x_{k+1},x_{k+1},x_{k+2}) = 0$, then equation (3.2) follows immediately. So, we can assume that $S(x_{n+1},x_{n+1},x_{n+2}) > 0$ for all n. From the condition (**I**), we observe that for all n, there exists $i = i_n \in \{1,2,\ldots,m\}$ such that $(x_{n+1},x_{n+1},x_{n+2}) \in A_i \times A_i \times A_{i+1}$. Then from condition (**II**) and using Lemma 2.1, we have

$$\begin{aligned}S(x_{n+1},x_{n+1},x_{n+2}) \quad &= \quad S(gx_n,gx_n,gx_{n+1})\\ &\leq \quad b_1\, \mathbf{n}(x_n,x_n,x_{n+1}) + b_2\, \mathbf{N}(x_n,x_n,x_{n+1}), \quad n = 1,2,\ldots.\end{aligned} \tag{3.3}$$

On the other hand, we have

$$\begin{aligned}\mathbf{n}(x_n,x_n,x_{n+1}) \quad &= \quad \psi\Big(S(x_{n+1},x_{n+1},x_{n+2})\frac{1+S(x_n,x_n,x_{n+1})}{1+S(x_n,x_n,x_{n+1})}\Big)\\ &= \quad \psi(S(x_{n+1},x_{n+1},x_{n+2})),\end{aligned}$$

and

$$\mathbf{N}(x_n,x_n,x_{n+1}) = \max\Big\{\psi(S(x_n,x_n,x_{n+1}), \psi\big(\frac{1}{2}S(x_n,x_n,x_{n+2})\big)\Big\}.$$

• If $\mathbf{N}(x_n, x_n, x_{n+1}) = \psi(S(x_n, x_n, x_{n+1})$, we obtain from (3.3) and the property of ψ that

$$\begin{aligned} S(x_{n+1}, x_{n+1}, x_{n+2}) &\leq b_1\, \psi(S(x_{n+1}, x_{n+1}, x_{n+2})) + b_2\, \psi(S(x_n, x_n, x_{n+1}) \\ &< b_1\, S(x_{n+1}, x_{n+1}, x_{n+2}) + b_2\, S(x_n, x_n, x_{n+1}), \end{aligned}$$

that is,

$$S(x_{n+1}, x_{n+1}, x_{n+2}) \leq \Big(\frac{b_2}{1-b_1}\Big)\, \psi(S(x_n, x_n, x_{n+1}). \tag{3.4}$$

• If $\mathbf{N}(x_n, x_n, x_{n+1}) = \psi\big(\frac{1}{2}S(x_n, x_n, x_{n+2})\big)$, we obtain from (3.3) and the property of ψ that

$$\begin{aligned} S(x_{n+1}, x_{n+1}, x_{n+2}) &\leq b_1\, \psi(S(x_{n+1}, x_{n+1}, x_{n+2})) + b_2\, \psi\big(\frac{1}{2}S(x_n, x_n, x_{n+2})\big) \\ &< b_1\, S(x_{n+1}, x_{n+1}, x_{n+2}) + b_2\, \frac{1}{2}S(x_n, x_n, x_{n+2}). \end{aligned} \tag{3.5}$$

By $(S2)$ and Lemma 2.1, we have

$$\begin{aligned} S(x_n, x_n, x_{n+2}) &\leq 2S(x_n, x_n, x_{n+1}) + S(x_{n+2}, x_{n+2}, x_{n+1}) \\ &= 2S(x_n, x_n, x_{n+1}) + S(x_{n+1}, x_{n+1}, x_{n+2}). \end{aligned}$$

Therefore, we have

$$\frac{1}{2}S(x_n, x_n, x_{n+2}) \leq S(x_n, x_n, x_{n+1}) + \frac{1}{2}S(x_{n+1}, x_{n+1}, x_{n+2}). \tag{3.6}$$

Combining (3.5) with (3.6), we obtain

$$\begin{aligned} S(x_{n+1}, x_{n+1}, x_{n+2}) &\leq b_1\, S(x_{n+1}, x_{n+1}, x_{n+2}) + b_2\, [S(x_n, x_n, x_{n+1}) \\ &\quad + \frac{1}{2}S(x_{n+1}, x_{n+1}, x_{n+2})], \end{aligned}$$

that is,

$$S(x_{n+1}, x_{n+1}, x_{n+2}) \leq \Big(\frac{2b_2}{2-2b_1-b_2}\Big)\, [S(x_n, x_n, x_{n+1}). \tag{3.7}$$

Define $\mu = \max\Big\{\frac{b_2}{1-b_1}, \frac{2b_2}{2-2b_1-b_2}\Big\} < 1$ and let $Q_{n+1} = S(x_{n+1}, x_{n+1}, x_{n+2})$ and $Q_n = S(x_n, x_n, x_{n+1})$. Consequently, it can be concluded that

$$Q_{n+1} \leq \mu\, Q_n \leq \mu^2\, Q_{n-1} \leq \ldots \leq \mu^{n+1}\, Q_0. \tag{3.8}$$

Therefore, since $0 \leq \mu < 1$, taking the limit as $n \to \infty$, we have $S(x_{n+1}, x_{n+1}, x_{n+2}) \to 0$, which is (3.2).

Thus for all $n < m$, by using ($S2$), Lemma 2.1 and equation (3.8), we have

$$\begin{aligned} S(x_n, x_n, x_m) &\leq 2S(x_n, x_n, x_{n+1}) + S(x_m, x_m, x_{n+1}) \\ &= 2S(x_n, x_n, x_{n+1}) + S(x_{n+1}, x_{n+1}, x_m) \\ &\dots \\ &\leq 2[\mu^n + \ldots + \mu^{m-1}]S(x_0, x_0, x_1) \\ &\leq \Big(\frac{2\mu^n}{1-\mu}\Big)S(x_0, x_0, x_1). \end{aligned}$$

Taking the limit as $n, m \to \infty$, we get

$$S(x_n, x_n, x_m) \to 0,$$

since $0 < \mu < 1$. Thus, we have $S(x_n, x_n, x_m) \to 0$ as $n, m \to \infty$.

This shows that the sequence $\{x_n\}$ is a Cauchy sequence in the complete S-metric space (X, S). Since Y is closed in (X, S), then (Y, S) is also complete and there exists $u \in Y = \cup_{i=1}^m A_i$. Notice that the iterative sequence $\{x_n\}$ has an infinite number of terms in A_i for each $i = 1, 2, \ldots, m$. Hence in each A_i, $i = 1, 2, \ldots, m$, we can construct a subsequence of $\{x_n\}$ that converges to u. Using that each A_i, $i = 1, 2, \ldots, m$, is closed, we conclude that $u \in \cup_{i=1}^m A_i$ and thus $\cup_{i=1}^m A_i \neq \emptyset$.

Now, we shall prove that u is a fixed point of g (which is possible since u belongs to each A_i). Indeed, since $u \in \cup_{i=1}^m A_i$, so for all n, there exists $i(n) \in \{1, 2, \ldots, m\}$ such that $x_n \in A_{i(n)}$, using (**II**) and Lemma 2.1, we obtain

$$\begin{aligned} S(x_{n+1}, x_{n+1}, gu) &= S(gx_n, gx_n, gu) \\ &\leq b_1\, \mathbf{n}(x_n, x_n, u) + b_2\, \mathbf{N}(x_n, x_n, u), \end{aligned} \tag{3.9}$$

for all n. On the other hand, we have

$$\mathbf{n}(x_n, x_n, u) = \psi\Big(S(gu, gu, u)\frac{1 + S(x_{n+1}, x_{n+1}, x_n)}{1 + S(x_n, x_n, u)}\Big),$$

on letting $n \to +\infty$ and using the continuity of ψ, condition ($S1$) and Lemma 2.1, we obtain that

$$\mathbf{n}(x_n, x_n, u) \to \psi(S(u, u, gu)),$$

and

$$\begin{aligned} \mathbf{N}(x_n, x_n, u) &= \max\Big\{\psi(S(x_n, x_n, u)), \psi(S(gx_n, gx_n, x_n)), \psi(S(gx_n, gx_n, x_n)), \\ &\qquad \psi\big(\frac{1}{2}[S(gx_n, gx_n, u) + S(gu, gu, x_n)]\big)\Big\} \\ &= \max\Big\{\psi(S(x_n, x_n, u)), \psi(S(x_{n+1}, x_{n+1}, x_n)), \psi(S(x_{n+1}, x_{n+1}, x_n)), \\ &\qquad \psi\big(\frac{1}{2}[S(x_{n+1}, x_{n+1}, u) + S(gu, gu, x_n)]\big)\Big\}. \end{aligned}$$

On letting $n \to +\infty$ and using the continuity of ψ, condition $(S1)$ and Lemma 2.1, we obtain that

$$\mathbf{N}(x_n, x_n, u) \to \psi\Big(\frac{S(u,u,gu)}{2}\Big).$$

On letting $n \to +\infty$ in (3.9) and using (3.10) and (3.10), we obtain

$$\begin{aligned} S(u,u,gu) &\leq b_1\,\psi(S(u,u,gu)) + b_2\,\psi\Big(\frac{S(u,u,gu)}{2}\Big) \\ &\leq b_1\,\psi(S(u,u,gu)) + b_2\,\psi(S(u,u,gu)). \end{aligned}$$

Suppose that $S(u,u,gu) > 0$. In this case, using condition (Ψ_2), we get

$$\begin{aligned} S(u,u,gu) &< b_1\,S(u,u,gu) + b_2\,S(u,u,gu) \\ &= (b_1+b_2)S(u,u,gu) < S(u,u,gu), \quad \text{since } b_1+b_2<1, \end{aligned}$$

which is a contradiction. Hence $S(u,u,gu) = 0$. Thus, $gu = u$. This shows that u is a fixed point of g.

Finally, we prove that u is the unique fixed point of g. Assume that v is another fixed point of g, that is, $gv = v$ with $v \neq u$. From condition (**I**), this implies that $v \in \cup_{i=1}^m A_i$. Now, we apply condition (**II**) for $x = y = u$ and $z = v$, we obtain

$$\begin{aligned} S(u,u,v) &= S(gu,gu,gv) \\ &\leq b_1\,\mathbf{n}(u,u,v) + b_2\,\mathbf{N}(u,u,v), \end{aligned} \tag{3.10}$$

where

$$\begin{aligned} \mathbf{n}(u,u,v) &= \psi\Big(S(gv,gv,v)\frac{1+S(gu,gu,u)}{1+S(u,u,v)}\Big) \\ &= \psi\Big(S(v,v,v)\frac{1+S(u,u,u)}{1+S(u,u,v)}\Big). \end{aligned}$$

Using the property of ψ and condition $(S1)$, we get

$$\mathbf{n}(u,u,v) \to 0, \tag{3.11}$$

and

$$\begin{aligned} \mathbf{N}(u,u,v) &= \max\Big\{\psi(S(u,u,v)), \psi(S(gu,gu,u)), \psi(S(gu,gu,u)), \\ &\qquad \psi\Big(\frac{1}{2}[S(gu,gu,v) + S(gv,gv,u)]\Big)\Big\} \\ &= \max\Big\{\psi(S(u,u,v)), \psi(S(u,u,u)), \psi(S(u,u,u)), \\ &\qquad \psi\Big(\frac{1}{2}[S(u,u,v) + S(v,v,u)]\Big)\Big\}. \end{aligned}$$

Using Lemma 2.1, condition $(S1)$ and the property of ψ, we get

$$\mathbf{N}(u,u,v) \to S(u,u,v). \tag{3.12}$$

If $S(u,u,v) > 0$, from equations (3.10), (3.11) and (3.12), we get

$$S(u,u,v) \leq b_2\, S(u,u,v) < S(u,u,v), \tag{3.13}$$

which is a contradiction. Hence, $S(u,u,v) = 0$, that is, $u = v$. Thus we have proved the uniqueness of the fixed point. This completes the proof. □

Next, we derive some fixed point theorems from Theorem 3.1.

If we take $m = 1$ and $A_1 = X$ in Theorem 3.1, then we obtain immediately the following result.

Corollary 3.1. *Let (X,S) be a complete S-metric space and $g\colon X \to X$ satisfies the following condition: there exists $b_1, b_2 \in [0,1)$ with $b_1 + b_2 < 1$ and some $\psi \in \Psi$ such that*

$$\begin{aligned} S(gx,gy,gz) \quad \leq \quad & b_1\, \psi\Big(S(gz,gz,z)\frac{1+S(gy,gy,y)}{1+S(x,y,z)}\Big) \\ & + b_2 \max\Big\{\psi(S(x,y,z)), \psi(S(gx,gx,x)), \psi(S(gy,gy,y)), \\ & \qquad \psi\big(\frac{1}{2}[S(gx,gx,z)+S(gz,gz,x)]\big)\Big\}, \end{aligned}$$

for all $x,y,z \in X$. Then g has a unique fixed point.

Remark 3.1. Corollary 3.1 extends and generalizes many existing fixed point theorems in the literature to the setting of complete S-metric spaces (see, [7, 12]).

Corollary 3.2. *Let (X,S) be a complete S-metric space, $m \in \mathbb{N}$, $A_1, A_2, \ldots, A_m$ be nonempty closed subsets of X, $Y = \cup_{i=1}^m A_i$ and $g\colon Y \to Y$. Suppose that there exists a nondecreasing function $\psi \in \Psi$ such that:*

$(h1)$ $Y = \cup_{i=1}^m A_i$ is a cyclic representation of Y with respect to g;

$(h2)$ there exist $b_1, b_2 \in [0,1)$ with $b_1 + b_2 < 1$ such that for all $(x,y,z) \in A_i \times A_i \times A_{i+1}$, $i = 1,2,\ldots,m$ (with $A_{m+1} = A_1$),

$$\begin{aligned} S(gx,gy,gz) \quad \leq \quad & b_1\, \psi\Big(S(gz,gz,z)\frac{1+S(gy,gy,y)}{1+S(x,y,z)}\Big) \\ & + b_2\, \psi\Big(\max\Big\{S(x,y,z), S(gx,gx,x), S(gy,gy,y), \\ & \qquad \frac{1}{2}[S(gx,gx,z)+S(gz,gz,x)]\Big\}\Big), \end{aligned} \tag{3.14}$$

for all $x,y,z \in X$. Then g has a unique fixed point. Moreover, the fixed point of g belongs to $\cap_{i=1}^m A_i$.

Proof. It follows from Theorem 3.1 by taking that if $\psi \in \Psi$ is a nondecreasing function, we have

$$\begin{aligned} \mathbf{N}(x,y,z) \quad = \quad & \psi\Big(\max\Big\{S(x,y,z), S(gx,gx,x), S(gy,gy,y), \\ & \qquad \frac{1}{2}[S(gx,gx,z)+S(gz,gz,x)]\Big\}\Big). \end{aligned}$$

□

Remark 3.2. It is clear that the conclusions of the Corollary 3.2 remain valid if in condition (3.14), the second term of the right-hand side is replaced by one of the following terms:

$$b_2\,\psi(S(x,y,z));\;\; b_2\,\psi\bigg(\frac{1}{2}[S(gx,gx,z)+S(gz,gz,x)]\bigg);$$

$$b_2\,\max\Big\{\psi(S(gx,gx,x)),\psi(S(gy,gy,y))\Big\};$$

$$\text{or}\;\; b_2\,\max\Big\{\psi(S(x,y,z)),\psi(S(gx,gx,x)),\psi(S(gy,gy,y))\Big\}.$$

Corollary 3.3. *Let (X,S) be a complete S-metric space, $m\in\mathbb{N}$, $A_1,A_2,\dots,A_m$ be nonempty closed subsets of X, $Y=\cup_{i=1}^m A_i$ and $g\colon Y\to Y$. Suppose that there exist five positive constants d_j, $j=1,2,3,4,5$ with $\sum_{j=1}^5 d_j<1$ such that:*

$(h1)$ $Y=\cup_{i=1}^m A_i$ is a cyclic representation of Y with respect to g;

$(h2)$ for all $(x,y,z)\in A_i\times A_i\times A_{i+1}$, $i=1,2,\dots,m$ (with $A_{m+1}=A_1$),

$$\begin{aligned} S(gx,gy,gz) \;\leq\;& d_1\left(S(gz,gz,z)\frac{1+S(gy,gy,y)}{1+S(x,y,z)}\right)+d_2\,S(x,y,z)\\ &+d_3\,S(gx,gx,x)+d_4\,S(gy,gy,y)\\ &+d_5\,\frac{1}{2}[S(gx,gx,z)+S(gz,gz,x)], \end{aligned}\tag{3.15}$$

for all $x,y,z\in X$. Then g has a unique fixed point. Moreover, the fixed point of g belongs to $\cap_{i=1}^m A_i$.

Proof. It follows from Theorem 3.1 with $\psi(t)=(d_1+d_2+d_3+d_4+d_5)t$. □

As special case we obtain S-metric space versions of Banach ([4]), Kannan ([10]) and Chatterjea ([5]) fixed point results (relation (1), (4) and (11) in [23]) in the cyclic variant from Corollary 3.3.

Corollary 3.4. *Let (X,S) be a complete S-metric space, $m\in\mathbb{N}$, $A_1,A_2,\dots,A_m$ be nonempty closed subsets of X and $Y=\cup_{i=1}^m A_i$. Let $g\colon Y\to Y$ be such that:*

(1) $Y=\cup_{i=1}^m A_i$ is a cyclic representation of Y with respect to g;

(2) there exists $\delta\in[0,1)$ such that one of the following conditions hold for all $(x,y,z)\in A_i\times A_i\times A_{i+1}$, $i=1,2,\dots,m$ (with $A_{m+1}=A_1$),

$$S(gx,gy,gz)\leq\delta\,S(x,y,z),$$

$$S(gx,gy,gz)\leq\frac{\delta}{2}\,[S(x,x,gx)+S(y,y,gy)],$$

$$S(gx,gy,gz)\leq\frac{\delta}{2}\,[S(x,x,gy)+S(y,y,gx)],$$

for all $x,y,z\in X$. Then g has a unique fixed point $u\in Y$.

Proof. It follows from Corollary 3.3 by taking (1) $d_2 = \delta$ and $d_1 = d_3 = d_4 = d_5 = 0$, (2) $d_3 = d_4 = \frac{\delta}{2}$ and $d_1 = d_2 = d_5 = 0$, and (3) $d_5 = \delta$ and $d_1 = d_2 = d_3 = d_4 = 0$. □

If we take $b_1 = 0$, $b_2 = 1$ and $\max\Big\{\psi(S(x,y,z)), \psi(S(gx,gx,x)), \psi(S(gy,gy,y)), \psi\big(\frac{1}{2}[S(gx,gx,z)+S(gz,gz,x)]\big)\Big\} = \psi(S(x,y,z))$ in the Theorem 3.1, then we obtain the following result as corollary.

Corollary 3.5. *Let (X,S) be a complete S-metric space, $m \in \mathbb{N}$, $A_1, A_2, \ldots, A_m$ be nonempty closed subsets of X, $Y = \cup_{i=1}^m A_i$, $g\colon Y \to Y$ an operator and $Y = \cup_{i=1}^m A_i$ is a cyclic representation of Y with respect to g. Suppose that g satisfies the following condition: for any $(x,y,z) \in A_i \times A_i \times A_{i+1}$, $i = 1,2,\ldots,m$ with $A_{m+1} = A_1$,*

$$S(gx,gy,gz) \le \psi(S(x,y,z)).$$

Then g has a unique fixed point. Moreover, the fixed point of g belongs to $\cap_{i=1}^m A_i$.

Remark 3.3. Corollary 3.4 extends the corresponding result of Kirk et al. [14] to the setting of S-metric space.

If we take $A_1 = A_2 = \ldots = A_m = X$ and $\psi(t) = k\,t$, where $0 < k < 1$ in the Corollary 3.4, then we obtain the following result.

Corollary 3.6. ([24]) *Let (X,S) be a complete S-metric space and $g\colon X \to X$ be a mapping such that for any $x,y,z \in X$,*

$$S(gx,gy,gz) \le k\,S(x,y,z),$$

where $0 < k < 1$. Then g has a unique fixed point in X.

Remark 3.4. Corollary 3.5 also extends the well-known Banach fixed point theorem [4] form complete metric space to the setting of complete S-metric space.

Now, we give some examples in support of our results.

Example 3.1. Let $X = [0,1]$ and $g\colon X \to X$ be given by $g(x) = \frac{x}{8}$. Let $A_1 = [0, \frac{1}{2}]$ and $A_1 = [\frac{1}{2}, 1]$. Define the function $S\colon X^3 \to [0,\infty)$ by $S(x,y,z) = \max\{x,y,z\}$ for all for all $x,y,z \in X$, then S is an S-metric on X. Now, define the function $\psi\colon [0,+\infty) \to [0,+\infty)$ by $\psi(t) = \frac{t}{2}$, $t \in [0,1]$. Then ψ has the properties mentioned in Corollary 3.5. Let $x \ge y \ge z$ for all $x,y,z \in X$. It is clear that $X = \cup_{i=1}^2 A_i$ is a cyclic representation of X with respect to g.

(1) Now, consider the inequality of Corollary 3.5, we have

$$\begin{aligned} S(gx, gy, gz) &= S\Big(\frac{x}{8}, \frac{y}{8}, \frac{z}{8}\Big) \\ &= \max\Big\{\frac{x}{8}, \frac{y}{8}, \frac{z}{8}\Big\} \\ &= \frac{x}{8} \leq \psi(S(x, y, z)) = \psi(\max\{x, y, z\}) \\ &= \psi(x) = \frac{x}{2} \end{aligned}$$

or

$$\frac{1}{8} \leq \frac{1}{2},$$

which is true. Thus, all the conditions of Corollary 3.5 are satisfied and $u = \frac{1}{2} \in \cup_{i=1}^{2} A_i$ is a unique fixed point of g.

(2) Again, consider the inequality of Corollary 3.6, we have

$$\begin{aligned} S(gx, gy, gz) &= S\Big(\frac{x}{8}, \frac{y}{8}, \frac{z}{8}\Big) \\ &= \max\Big\{\frac{x}{8}, \frac{y}{8}, \frac{z}{8}\Big\} \\ &= \frac{x}{8} \leq k\, S(x, y, z) = k\, max\{x, y, z\} \\ &= k\, x, \end{aligned}$$

or

$$k \geq \frac{1}{8}.$$

If we take $0 < k < 1$, then all the conditions of Corollary 3.6 are satisfied and $u = 0 \in X$ is a unique fixed point of g.

Example 3.2. Let $X = [0, 1]$. We define $S\colon X^3 \to \mathbb{R}_+$ by

$$S(x, y, z) = \begin{cases} 0, & \text{if } x = y = z, \\ \max\{x, y, z\}, & \text{if otherwise}, \end{cases}$$

for all $x, y, z \in X$. Then (X, S) is a complete S-metric space. Suppose $A_1 = [0, 1]$, $A_2 = [0, \frac{1}{2}]$ and $Y = \cup_{i=1}^{2} A_i$. Consider the mapping $g\colon Y \to Y$ such that $g(x) = \frac{x^2}{2(1+x)}$ for all $x \in Y$. It is clear that $Y = \cup_{i=1}^{2} A_i$ is a cyclic representation of X with respect to g. Let us suppose that $\psi\colon [0, +\infty) \to [0, +\infty)$ be such that $\psi(t) = \frac{t^2}{1+t}$, $t \in [0, 1]$. Then ψ has the properties mentioned in Theorem 3.1. Moreover, the mapping g is a cyclic representation of Y with respect to g. Without loss of generality, we assume that $x \geq y \geq z$ for all $x, y, z \in Y$. Then

$$\begin{aligned} S(gx, gy, gz) &= \max\{gx, gy, gz\} \\ &= \max\Big\{\frac{x^2}{2(1+x)}, \frac{y^2}{2(1+y)}, \frac{z^2}{2(1+z)}\Big\} \\ &= \frac{x^2}{2(1+x)}, \end{aligned}$$

$$S(x,y,z)=\max\{x,y,z\}=x.$$

On the other hand,

$$\begin{aligned}\mathbf{N}(x,y,z) &= \max\Big\{\psi(S(x,y,z)),\psi\Big(S\Big(\frac{x^2}{2(1+x)},\frac{x^2}{2(1+x)},x\Big)\Big),\\ &\qquad\psi\Big(S\Big(\frac{y^2}{2(1+y)},\frac{y^2}{2(1+y)},y\Big)\Big),\\ &\qquad\psi\Big(\frac{1}{2}\Big[S\Big(\frac{x^2}{2(1+x)},\frac{x^2}{2(1+x)},z\Big)+S\Big(\frac{z^2}{2(1+z)},\frac{z^2}{2(1+z)},x\Big)\Big]\Big)\Big\}\\ &= \max\Big\{\psi(x),\psi(x),\psi(y),\psi\Big(\frac{1}{2}\Big[\max\Big\{\frac{x^2}{2(1+x)},z\Big\}+x\Big]\Big)\Big\}\\ &= \psi(x).\end{aligned}$$

(Since it was used that the function ψ is increasing and since $x\geq z$, $x\geq\frac{x^2}{2(1+x)}$, that $\frac{1}{2}\Big[\max\Big\{\frac{x^2}{2(1+x)},z\Big\}+x\Big]\leq x$.)

Hence in this case

$$S(gx,gy,gz)\leq\frac{1}{2}\,\mathbf{N}(x,y,z)$$

is satisfied for $b_1=0$. Thus, the condition (**II**) holds for $b_1=0$ and $b_2=\frac{1}{2}$.

Hence, all conditions of Theorem 3.1 are satisfied (with $m=2$) and so g has a unique fixed point which is in this case is $u=0\in\cap_{i=1}^{2}A_i$.

Example 3.3. Let $X=[0,1]$ and $S\colon X^3\to\mathbb{R}_+$ be given by

$$S(x,y,z)=\begin{cases}|x-z|+|y-z|, & \text{if } x,y,z\in[0,1)\\ 1, & \text{if } x=1 \text{ or } y=1 \text{ or } z=1,\end{cases}$$

for all $x,y,z\in X$. Then (X,S) is a complete S-metric space.

If a mapping $g\colon X\to X$ is given by

$$g(x)=\begin{cases}1/2, & \text{if } x,y,z\in[0,1)\\ 1/6, & \text{if } x=y=z=1,\end{cases}$$

and $A_1=[0,\frac{1}{2}]$, $A_2=[\frac{1}{2},1]$, then $A_1\cup A_2=X$ is a cyclic representation of X with respect to g. Now, define the function $\psi\colon[0,\infty)\to[0,1)$ and $\psi(t)=\frac{3t}{4}$, $t\in[0,1]$. Then ψ has the properties mentioned in Corollary 3.5. Moreover, the mapping g is a cyclic representation of Y with respect to g. Without loss of generality, we assume that $x\geq y\geq z$ for all $x,y,z\in X$. Indeed, consider the following cases.

Case I: If $x,y\in[0,\frac{1}{2}]$, $z\in[\frac{1}{2},1)$ or $z\in[0,\frac{1}{2}]$, $x,y\in[\frac{1}{2},1)$. Then

$$\begin{aligned}S(gx,gy,gz) &= S\Big(\frac{1}{2},\frac{1}{2},\frac{1}{2}\Big)=0\\ &\leq \psi(S(x,y,z)).\end{aligned}$$

Thus, the inequality of Corollary 3.5 is trivially satisfied.

Case II: If $x, y \in [0, \frac{1}{2}]$ and $z = 1$. Then

$$S(gx, gy, gz) = S\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{6}\right) = \frac{2}{3},$$

$$S(x, y, z) = 1,$$

and

$$\psi(S(x, y, z)) = \frac{3}{4}.$$

Consequently,

$$\begin{aligned} S(gx, gy, gz) &= \frac{2}{3} \leq \psi(S(x, y, z)) \\ &= \frac{3}{4}, \end{aligned}$$

which is true. Thus, all the conditions of Corollary 3.5 are satisfied.

Case III: If $x, z \in [0, \frac{1}{2}]$ and $y = 1$. Then

$$S(gx, gy, gz) = S\left(\frac{1}{2}, \frac{1}{6}, \frac{1}{2}\right) = \frac{1}{3},$$

$$S(x, y, z) = 1,$$

and

$$\psi(S(x, y, z)) = \frac{3}{4}.$$

Consequently,

$$S(gx, gy, gz) = \frac{1}{3} \leq \psi(S(x, y, z)) = \frac{3}{4},$$

which is true. Thus, all the conditions of Corollary 3.5 are satisfied.

Case IV: If $y, z \in [0, \frac{1}{2}]$ and $x = 1$. Then

$$S(gx, gy, gz) = S\left(\frac{1}{6}, \frac{1}{2}, \frac{1}{2}\right) = \frac{1}{3},$$

$$S(x, y, z) = 1,$$

and

$$\psi(S(x, y, z)) = \frac{3}{4}.$$

Consequently,

$$S(gx, gy, gz) \quad = \quad \frac{1}{3} \leq \psi(S(x, y, z)) = \frac{3}{4},$$

which is true. Thus, all the conditions of Corollary 3.5 are satisfied.

Considering all the above cases, we conclude that the inequality used in Corollary 3.5 remains valid for ψ and g constructed in the above example and consequently by applying Corollary 3.5, g has a unique fixed point (which is $u = \frac{1}{2} \in A_1 \cap A_2$).

4. Application to well posedness fixed point problem

The notion of well posedness of a fixed point problem has generated much interest to several mathematicians, for example [1, 2, 8, 15, 20, 21, 22]. Here, we study well posedness of a fixed point problem of mappings in Theorem 3.1.

Definition 4.1. ([8]) Let (X, d) be a metric space and $g\colon X \to X$ be a mapping. The fixed point problem of g is said to be well-posed if

(i) g has a unique fixed point u in X;

(ii) for any sequence $\{x_n\}$ of points in X such that $\lim_{n\to\infty} d(gx_n, x_n) = 0$, we have $\lim_{n\to\infty} d(x_n, u) = 0$.

Now, we generalize the above notion in S-metric space.

Definition 4.2. Let (X, S) be a S-metric space and $g\colon X \to X$ be a mapping. The fixed point problem of g is said to be well-posed if

(i) g has a unique fixed point u in X;

(ii) for any sequence $\{x_n\}$ of points in X such that $\lim_{n\to\infty} S(gx_n, gx_n, x_n) = 0 = \lim_{n\to\infty} S(x_n, x_n, gx_n)$, we have $\lim_{n\to\infty} S(x_n, x_n, u) = 0 = \lim_{n\to\infty} S(u, u, x_n)$.

Concerning the well-posedness of the fixed point problem in a S-metric space satisfying the conditions of Theorem 3.1, we have the following result.

Theorem 4.1. *Let $g\colon Y \to Y$ be a self mapping as in Theorem 3.1. Then the fixed point problem for g is well posed.*

Proof. From Theorem 3.1, we know that g has a unique fixed point, say, $u \in Y$. Let $\{x_n\} \subset Y$ be a sequence in Y such that $\lim_{n\to\infty} S(x_n, x_n, gx_n) = 0 = \lim_{n\to\infty} S(gx_n, gx_n, x_n)$. Then using $(S1)$, Lemma 2.1, condition (**II**) and the property of ψ, we have

$$\begin{aligned} S(x_n, x_n, u) &\leq 2S(x_n, x_n, gx_n) + S(u, u, gx_n) \\ &= 2S(x_n, x_n, gx_n) + S(gx_n, gx_n, gu) \\ &\leq 2S(x_n, x_n, gx_n) + b_1\, \mathbf{n}(x_n, x_n, u) \\ &\quad + b_2\, \mathbf{N}(x_n, x_n, u), \end{aligned} \tag{4.1}$$

where

$$\mathbf{n}(x_n, x_n, u) = \psi\Big(S(gu, gu, u)\frac{1 + S(gx_n, gx_n, x_n)}{1 + S(x_n, x_n, u)}\Big)$$

$$(4.2) \qquad = \psi\Big(S(u, u, u)\frac{1 + S(gx_n, gx_n, x_n)}{1 + S(x_n, x_n, u)}\Big) = 0,$$

and

$$\begin{aligned}
\mathbf{N}(x_n, x_n, u) &= \max\Big\{\psi(S(x_n, x_n, u)), \psi(S(gx_n, gx_n, x_n)), \psi(S(gx_n, gx_n, x_n)), \\
&\qquad \psi\Big(\frac{1}{2}[S(gx_n, gx_n, u) + S(gu, gu, x_n)]\Big)\Big\} \\
&= \max\Big\{\psi(S(x_n, x_n, u)), \psi(S(gx_n, gx_n, x_n)), \psi(S(gx_n, gx_n, x_n)), \\
&\qquad \psi\Big(\frac{1}{2}[S(gx_n, gx_n, u) + S(u, u, x_n)]\Big)\Big\} \\
&= \max\Big\{\psi(S(x_n, x_n, u)), \psi(S(gx_n, gx_n, x_n)), \psi(S(gx_n, gx_n, x_n)), \\
&\qquad \psi\Big(\frac{1}{2}[2S(gx_n, gx_n, x_n) + 2S(x_n, x_n, u)]\Big)\Big\} \\
&= \max\Big\{\psi(S(x_n, x_n, u)), \psi(S(gx_n, gx_n, x_n)), \psi(S(gx_n, gx_n, x_n)), \\
&\qquad \psi\big(S(gx_n, gx_n, x_n) + S(x_n, x_n, u)\big)\Big\} \\
(4.3) \qquad &= \psi(S(gx_n, gx_n, x_n)).
\end{aligned}$$

From equations (4.1)-(4.3), we obtain

$$(4.4) \qquad S(x_n, x_n, u) \le 2S(x_n, x_n, gx_n) + b_2\, \psi(S(x_n, x_n, u)).$$

Using the property of ψ in equation (4.4), we obtain

$$S(x_n, x_n, u) < 2S(x_n, x_n, gx_n) + b_2\, S(x_n, x_n, u),$$

taking the limit as $n \to \infty$ in the above inequality, we get $S(x_n, x_n, u) \to 0$ as $n \to \infty$ since $b_2 < 1$, which is equivalent to saying that $x_n \to u$ as $n \to \infty$. This completes the proof. □

5. Conclusion

In this paper, we prove some fixed point theorems for generalized g_ψ-cyclic contractions in the setting of complete S-metric spaces. Also we give some examples in support of our results. The results presented in this paper extend, generalize and improve several fixed point results in the literature (see, e.g., [11, 12, 16, 17, 24] and many others) to the setting of complete S-metric spaces.

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REFERENCES

1. M. Akkouchi: *Well-posedness of the fixed point problem for certain asymptotically regular mappings*, Ann. Math. Sil. **23** (2009), 43–52.
2. M. Akkouchi and V. Popa: *Well-posedness of the fixed point problem for mappings satisfying an implicit relations*, Demonstr. Math. **43(4)** (2010), 923–929.
3. A. H. Ansari and K. Zoto: *Some fixed point theorems and cyclic contractions in dislocated quasi-metric spaces*, Facta Universitatis, Ser. Math. Inf. **33(1)** (2018), 91–106.
4. S. Banach: *Sur les opération dans les ensembles abstraits et leur application aux équation integrals*, Fund. Math. **3**(1922), 133–181.
5. S. K. Chatterjea: *Fixed point theorems compactes*, Rend. Acad. Bulgare Sci. **25**(1972), 727–730.
6. C. M. Chen: *Fixed point theorem for the cyclic weaker Meir-Keeler function in complete metric spaces*, Fixed Point Theory Appl. **2012:17** (2012). doi:10.1186/1687-1812-2012-17.
7. C. Di Bari and V. Popa: *Fixed points for weak φ-contractions on partial metric spaces*, Int. J. Engin. Contemp. Math. Sci. **1** (2011), 5–13.
8. F. S. De Blasi and J. Myjak: *Sur la porosité des contractions sans point fixed*, C. R. Acad. Sci. Paris **308** (1989), 51–56.
9. A. Gupta: *Cyclic contraction on S-metric space*, Int. J. Anal. Appl. **3(2)** (2013), 119–130.
10. R. Kannan: *Some results on fixed point theorems*, Bull. Calcutta Math. Soc. **60**(1969), 71–78.
11. E. Karapinar: *Fixed point theorem for cyclic weak ϕ-contraction*, Appl. Math. Lett. **24** (2011), 822–825.
12. E. Karapinar, I. M. Erhan and A. Yildiz-Ulus: *Fixed point theorem for cyclic maps on partial metric spaces*, Appl. Math. Inf. Sci. **6** (2012), 239–244.
13. E. Karapinar and H. K. Nashine: *Fixed point theorems for Kannan type cyclic weakly contractions*, J. Nonlinear Anal. Optim. **4(1)** (2013), 29–35.
14. W. A. Kirk, P. S. Shrinavasan and P. Veeramani: *Fixed points for mappings satisfying cyclical contractive conditions*, Fixed Point Theory **4** (2003), 79–89.
15. B. K. Lahiri and P. Das: *Well-posedness and porosity of a certain class of operators*, Demonstr. Math. **38(1)** (2005), 169–176.
16. H. K. Nashine, Z. Kadelburg and S. Radenović: *Fixed point theorems via various cyclic contractive conditions in partial metric spaces*, Publ. de L'Institut Mathematique, Nouvelle Serie, tome **93(107)** (2013), 69–93.
17. M. Pacurar and I. A. Rus: *Fixed point theorem for cyclic ϕ-contractions*, Nonlinear Anal. **72** (2010), 1181–1187.
18. M. A. Petric: *Some results concerning cyclical contractive mappings*, General Math. **18** (2010), 213–226.
19. M. A. Petric: *Best proximity point theorems for weak cyclic Kannan contractions*, Filomat **25(1)** (2011), 145–154.

20. V. Popa: *Well-posedness of fixed point problems in orbitally complete metric spaces*, Stud. Cercet. Ştiinţ., Ser. Mat. Univ. Bacău **16** (2006), Supplement. Proceedings of ICMI 45, Bacău, Sept. 18-20 (2006), 209–214.

21. V. Popa: *Well-posedness of fixed point problems in compact metric spaces*, Bul., Univ. Petrol-Gaze Ploie ş ti, Ser. Mat. Inform. Fiz. **60(1)** (2008), 1–4.

22. S. Reich and A. T. Zaslawski: *Well-posedness of fixed point problems*, Far East J. Math. Sci., Special Volume part III (2001), 393–401.

23. B. E. Rhoades: *A comparison of various definitions of contractive mappings*, Trans. Amer. Math. Soc. **333** (1977), 257–290.

24. S. Sedghi, N. Shobe and A. Aliouche: *A generalization of fixed point theorems in S-metric spaces*, Mat. Vesn. **64(3)** (2012), 258–266.

25. S. Sedghi and N. V. Dung: *Fixed point theorems on S-metric spaces*, Mat. Vesn. **66(1)** (2014), 113–124.