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NUMERICAL MODELING OF JENKINS MODEL BASED FERROFLUID LUBRICATION SQUEEZE FILM PERFORMANCE IN ROUGH CURVED ANNULAR PLATES UNDER THE PRESENCE OF SLIP VELOCITY

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Abstract. An endeavour has been made to analyze the performance of a Jenkins model based magnetic squeeze film between curved transversely rough annular plates when the curved upper plate approaches the curved lower plate along surfaces governed by hyperbolic functions. Beavers and Joseph's slip model has been adopted to evaluate the effect of slip velocity. The random roughness of the bearing surfaces is characterised by a stochastic random variable with nonzero mean, variance, and skewness. The associated dimensionless stochastically averaged Reynolds equation is solved with suitable boundary conditions in non dimensional form to obtain the pressure distribution, leading to the expression for load carrying capacity. The graphical results establish that the bearing system registers an enhanced performance as compared to that of the bearing system dealing with a conventional lubricant. This investigation proves that albeit the bearing suffers due to transverse surface roughness, there exist sufficient scopes for obtaining a relatively better performance in the case of negatively skewed roughness by properly choosing curvature parameters and slip velocity. Further, Jenkins model based ferrofluid lubrication offers some measures in reducing the adverse effect of roughness when slip parameter is kept at minimum level. It is appealing to note that the variance(-ve) further enhances this positive effect.
Keywords: Jenkins model, magnetic squeeze film, ferrofluid lubrication, surface roughness.

1. Introduction

Nowadays, magnetic fluids are widely used in sealing of computer hard disk drives, rotating x-ray tubes, rotating shafts and rods. These are used as lubricants in bearing and dampers. Also, magnetic fluids are applied as heat controller in electric motors and hi-fi speaker systems without the need of change in their geometrical shapes. Ferrofluids are being greatly used in many magnetic fluid based scientific devices like sensors, accelerometer, pressure transducers, etc. and also, in actuating

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machines like electromechanical converters and energy converters etc. In the field of biomedicine also, these smart material have been found very useful. The use of magnetic fluids for cancer treatment by heating the tumour soaked in magnetic fluids by means of an alternating magnetic field, has drawn considerable attention, recently.

[14] proposed a simple flow model to describe the steady flow of magnetic fluids in the presence of slowly changing external magnetic fields. Quite a good number of papers are available in the literature for the study of different types of bearing using Neuringer and Rosensweig flow model for instance, in short bearing by [34], in the slider bearing by [46] and [40], the Journal bearing by [7] and [33], the circular plates by [41],[13], by[37] in conical plates. Later on, the simple flow model of Neuringer and Rosensweig was modified by [24] with Maugin's modification. It was found that Neuringer-Rosensweig model modified pressure while Jenkins flow model modified both the pressure and velocity of the ferrofluid. The steady-state effect of bearings with Jenkins model based magnetic fluids was discussed by [36],[39] and A [30]. It was concluded that the load carrying capacity of the bearing system increased with increasing magnetization of the magnetic fluid.

The use of squeezing flow between parallel walls expanded in many industrial and biological systems, such as machine elements, approaching gears, braking units, hydraulic dampers, skeletal bearings, synovial joints, moving pistons in engines and chocolate filler. To develop the equipment and machines better understanding of such flow models which describe the squeezing flow between parallel walls is always needed. The squeeze film performance between various geometrical configurations have considered in several investigations ([5],[4],[16],[2],[29]).

For performance of the bearing system, reduction of friction and wear is an enhanced necessary. It is noticed that slip velocity supports reducing the friction. [8] derived a slip boundary condition at the interface between a porous medium and fluid layer in an experimental study. Several researches concerned with slip velocity, have been presented; the circular disks by [25], the slider bearing by [43], [3],[30], [35], the radial sleeve bearing by [27] and infinitely long bearing by [22]. In all the above studies, it was observed that the slip effect played a crucial role for enhancing the bearing performance.

Surface roughness of machined mechanical parts and components has important influence on many performance characteristics of products such as wear resistance, corrosion resistance and fatigue strength. In recent years, surface roughness has been studied with much interest because all bearing surfaces tend to be rough through the manufacturing process, the wear and the impulsive damage. So, to enhance the performance of hydrodynamic lubrication in various bearings, it becomes important to evaluate the influence of surface roughness. [45] recognised the random characteristics of roughness and employed a stochastic method which was modified and developed by Christensen and Tonder([11],[10],[9]) to evaluate the effect of surface roughness. Christensen and Tonder's([11],[10],[9]) stochastic model assumes that the probability density function for the random variable characterizing the roughness is symmetric with the mean of the random variable equal to zero. According to this model, there are two types of roughness patterns which are of much interest in the roughness theory; one is transverse roughness and other one is longitudinal roughness. In the literature, quite a good number of authors ([26],[15],[44],[17],[12],[6],[32],[13],[28],[42],[31],[38]) have adopted this model to study the effect of surface roughness. All above authors noticed that roughness pattern played an important role to improve the performance of bearing system. [19] analyzed the effects of Shliomis model based ferrofluid lubrication of a squeeze film in rotating rough curved circular plates. It was observed that the adverse effect of transverse roughness could be reduced considerably at least in the case of negatively skewed roughness with a suitable choice of curvature parameters. [23] theoretically studied the effect of Shliomis model based ferrofluid lubrication of a squeeze film between curved rough annular plates with comparison between two different porous structures. It was seen that the effect of morphology parameter and volume concentration parameter increased the load carrying capacity. Recently, [18] analyzed the effect of slip velocity and surface roughness on the performance of Jenkins model based magnetic squeeze film in curved rough circular plates. It was manifest that for enhancing the performance characteristics of the bearing system the slip parameter was required to be reduced even if variance (-ve) occurs and suitable magnetic strength was in force.

Recently, it is observed that ([18]) Jenkins model modifies the Neuringer-Rosensweig model so far as magnetic fluid flow is concerned. Here, it was thought proper to investigate the behaviour of Jenkins model based ferrofluid lubrication of a curved rough annular squeeze film considering slip velocity.

2. Analysis

The geometry of the bearing configuration is exhibited in Figure 2.1 ([29]) which consists of two annular plates each of inside radius b and outside radius a , and the upper plate and lower plate are curved. Here r denotes the radial coordinates and h_0 is the central film thickness.

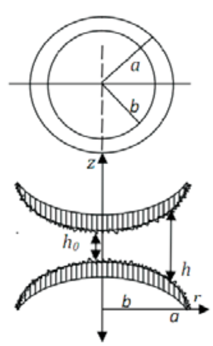

Fig. 2.1: Configuration of the bearing system

Jenkins proposed a simple model to describe the magnetic fluid flow, in 1972. However, the Jenkins model was a generalization of the Neuringer-Rosensweig model. With Maugin's modification, the governing equations of the model for steady state are ([24] and [36])

$$\rho(\bar{q}.\nabla)\bar{q} = -\nabla_p + \eta\nabla^2\bar{q} + \mu_0(\bar{M}.\nabla)\bar{H} + \frac{\rho.A^2}{2}\nabla\times[\frac{\bar{M}}{M}\times\{(\nabla\times\bar{q})\times\bar{M}\}] \tag{2.1}$$

together with

$$\nabla.\bar{q} = 0, \nabla\times\bar{H} = 0, \bar{M} = \bar{\mu}.\bar{H}, \nabla.(\bar{H}+\bar{M}) = 0$$

([29]). where ρ denotes the fluid density, $\bar{q}$ indicates the fluid velocity in the film region, $\bar{H}$ represents external magnetic field, $\bar{\mu}$ indicates magnetic susceptibility of the magnetic fluid, p denotes the film pressure, η denotes the fluid viscosity, μ_0 indicates the permeability of the free space, A being a material constant and $\bar{M}$ indicates magnetization vector. Equation (2.1) suggests that infact, Jenkins model is an improvement of Neuringer-Rosensweig model with an additional term

$$\frac{\rho.A^2}{2}\nabla\times[\frac{\bar{M}}{M}\times\{(\nabla\times\bar{q})\times\bar{M}\}] = \frac{\rho.A^2.\bar{\mu}}{2}\nabla\times[\frac{\bar{H}}{H}\times\{(\nabla\times\bar{q})\times\bar{H}\}] \tag{2.2}$$

which concerns the velocity of the fluid. At this point one wants to remember that Neuringer-Rosensweig model modifies the pressure while Jenkins model modifies both pressure and velocity of the ferrofluid.

Let (u, v, w) be the velocity of the fluid at any point (r, θ, z) between two solid surfaces, with OZ as axis. With the assumptions of hydrodynamic lubrication and remembering that the flow is steady and axially symmetric, the equations of motion turn out to be

$$(1 - \frac{\rho.A^2.\bar{\mu}.H}{2\eta})\frac{\partial^2 u}{\partial z^2} = \frac{1}{\eta}\frac{d}{dr}(p - \frac{\mu_0\bar{\mu}}{2}H^2) \tag{2.3}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{\partial w}{\partial z} = 0 \tag{2.4}$$

Solving the above equation (2.3) with the boundary conditions, $u = 0$ when $z = 0, h$, one obtains

$$u = \frac{z(z-h)}{2\eta(1 - \frac{\rho A^2\bar{\mu}H}{2\eta})}\frac{d}{dr}(p - \frac{\mu_0\bar{\mu}}{2}H^2) \tag{2.5}$$

Replacing the value of u in equation (2.4) and integrating it with respect to z over the interval $(0, h)$ one gets the Reynolds type equation for the film pressure as

$$\frac{1}{r}\frac{d}{dr}(\frac{h^3}{(1 - \frac{\rho A^2\bar{\mu}H}{2\eta})}r\frac{d}{dr}(p - \frac{\mu_0\bar{\mu}}{2}H^2)) = 12\eta\dot{h_0} \tag{2.6}$$

The bearing surfaces are taken to be transversely rough. In view of the stochastic theory of Christensen and Tonder ([11],[10],[9]), the thickness h of the lubricant film is assumed as

$$h = \bar{h} + h_s \tag{2.7}$$

where $\bar{h}$ indicates the mean film thickness and h_s denotes the deviation from the mean film thickness characterizing the random roughness of the bearing surfaces. h_s is directed by the probability density function

$$f(h_s) = \begin{cases} \frac{35}{32c^7}(c^2 - h_s^2)^3 & ; \quad -c \leq h_s \leq c \\ 0 & , \quad elsewhere, \end{cases} \tag{2.8}$$

wherein c stands for the maximum deviation from the mean film thickness. The mean α, the standard deviation σ and the parameter ϵ which is the measure of symmetry of the random variable h_s, are considered as in the study of Christensen and Tonder ([11],[10],[9]). It is assumed that the upper plate lying along the surface governed by ([29],[31] ,[20])

$$z_u = h_0 \frac{1}{(1 + \beta r)}; b \leq r \leq a$$

approaches with normal velocity $\dot{h_0}$ to the lower plate lying along the surface determined by

$$z_l = h_0[\frac{1}{(1 + \gamma r)} - 1]; b \leq r \leq a$$

where β and γ represent the curvature parameter of the respective plates. The film thickness $h(r)$ then, is defined by ([29],[31], [21])

$$h(r) = h_0[\frac{1}{(1 + \beta r)} - \frac{1}{(1 + \gamma r)} + 1]; b \leq r \leq a \tag{2.9}$$

Modification of the method of Christensen and Tonder ([11],[10],[9]) on certain simplifications yields, under usual assumptions of hydro-magnetic lubrication ([29],[1], [6]) the Reynolds type equation;

$$\frac{1}{r}\frac{d}{dr}(\frac{g(h)}{(1 - \frac{\rho A^2 \bar{\mu} H}{2\eta})} r \frac{d}{dr}(p - \frac{\mu_0 \bar{\mu}}{2} H^2)) = 12\eta \dot{h_0} \tag{2.10}$$

where

$$H^2 = K(r - b)(a - r),$$
$$g(h) = (h^3 + 3h^2\alpha + 3(\sigma^2 + \alpha^2)h + 3\sigma^2\alpha + \alpha^3 + \epsilon)(\frac{4 + sh}{2 + sh})$$

Where K is a suitably chosen constant so as to produce a required magnetic strength ([29]). Introducing the non dimensional quantities

$$\bar{h} = \frac{h}{h_0} = [\frac{1}{(1 + BR)} - \frac{1}{(1 + CR)} + 1], R = \frac{r}{b}, P = -\frac{h_0^3 p}{\eta b^2 \dot{h_0}}, B = \beta b, C = \gamma b, \bar{s} = sh_0$$

$$\mu^* = -\frac{K\mu_0\bar{\mu}h_0^3}{\eta\dot{h_0}}, k = \frac{a}{b}, \bar{A}^2 = \frac{\rho A^2\bar{\mu}b\sqrt{K}}{2\eta}, \bar{\sigma} = \frac{\sigma}{h_0}, \bar{\alpha} = \frac{\alpha}{h_0}, \bar{\epsilon} = \frac{\epsilon}{h_0^3} \tag{2.11}$$

and using the equation (2.11), equation (2.10) assumes the form

$$\frac{1}{R}\frac{d}{dR}\left(\frac{g(\bar{h})}{(1-\bar{A}^2\sqrt{(R-1)(k-R)})}R\frac{d}{dR}(p-\frac{1}{2}\mu^*(R-1)(k-R))\right) = -12 \tag{2.12}$$

where

$$g(\bar{h}) = (\bar{h}^3 + 3\bar{h}^2\bar{\alpha} + 3(\bar{\sigma}^2+\bar{\alpha}^2)\bar{h} + 3\bar{\sigma}^2\bar{\alpha} + \bar{\alpha}^3 + \bar{\epsilon})(\frac{4+\bar{s}\bar{h}}{2+\bar{s}\bar{h}})$$

Solving equation (2.12) under the non dimensional boundary conditions

$$P(1) = P(k) = 0 \tag{2.13}$$

one obtains the expression for the non dimension pressure distribution as

$$P = \frac{\mu^*}{2}(R-1)(k-R) - 6\int_1^R \frac{R}{g(\bar{h})}(1-\bar{A}^2\sqrt{(R-1)(k-R)})dR$$

$$+6\left[\frac{\int_1^k \frac{R}{g(\bar{h})}(1-\bar{A}^2\sqrt{(R-1)(k-R)})dR}{\int_1^k \frac{1}{Rg(\bar{h})}(1-\bar{A}^2\sqrt{(R-1)(k-R)})dR}\right]\int_1^R \frac{1}{Rg(\bar{h})}(1-\bar{A}^2\sqrt{(R-1)(k-R)})dR \tag{2.14}$$

The dimensionless load carrying capacity of the bearing system then, is derived as

$$W = -\frac{h_0^3 w}{2\pi\eta b^4\dot{h_0}} = \frac{\mu^*}{24}(k^2-1)(k-1)^2 + 3\int_1^k \frac{R^3}{g(\bar{h})}(1-\bar{A}^2\sqrt{(R-1)(k-R)})dR$$

$$-3\frac{[\int_1^k \frac{R}{g(\bar{h})}(1-\bar{A}^2\sqrt{(R-1)(k-R)})dR]^2}{\int_1^k \frac{1}{Rg(\bar{h})}(1-\bar{A}^2\sqrt{(R-1)(k-R)})dR} \tag{2.15}$$

3. Results and Discussions

It is clearly seen that the dimensionless pressure distribution and load carrying capacity are determined from equation (2.14) and equation (2.15), respectively. The expression for the load carrying capacity is linear with respect to the magnetization parameter which makes it clear that the load carrying capacity will be increased for increasing values of magnetization parameter. Probably, this may be owing to the fact that the magnetization induces an increase in the viscosity of the lubricant leading to increased pressure and consequently the load carrying capacity. Further, it is observed from equation (2.15) that the load carrying capacity increases by $(\mu^*/24)(k^2-1)(k-1)^2$ as compared to the case of conventional lubricant based bearing system.

Taking the roughness parameters to be zero this study reduces to the corresponding Jenkins model based magnetic squeeze film performance in curved smooth annular plates. Further, removing magnetization this investigation reduces to squeeze film performance in curved smooth annular plates.

The variation of dimensionless load carrying capacity with respect to the material constant parameter is presented in Figures 3.1-3.7. These figures tend to indicate that the load carrying capacity decreases with increasing values of the material constant parameter. This is in accordance with the results obtained experimentally ([1],[29]).

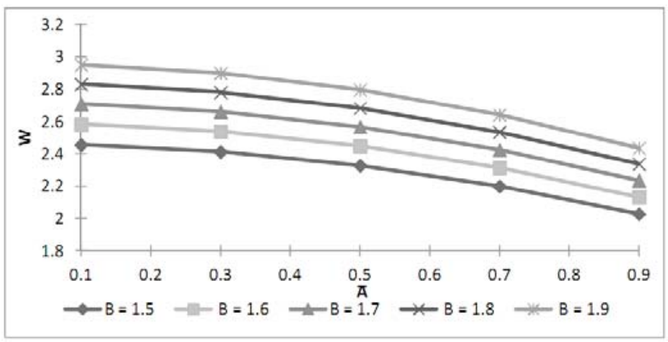

FIG. 3.1: Variation of Load carrying capacity with respect to $\bar{A}$ and B

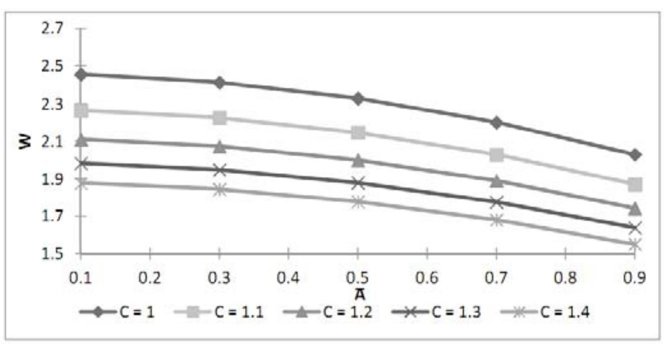

FIG. 3.2: Variation of Load carrying capacity with respect to $\bar{A}$ and C.

Figures 3.8-3.13 depict the effect of upper plate's curvature parameter on the distribution of non dimensional load carrying capacity. It can be easily seen that the

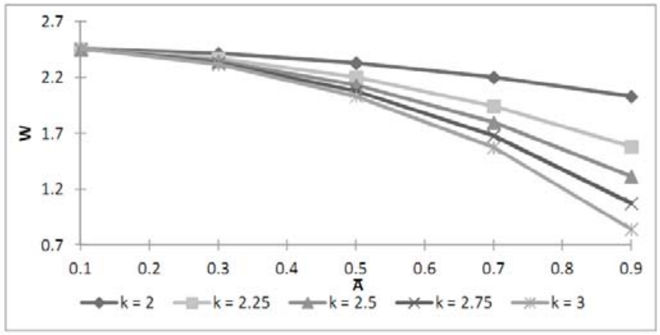

FIG. 3.3: Variation of Load carrying capacity with respect to $\bar{A}$ and k.

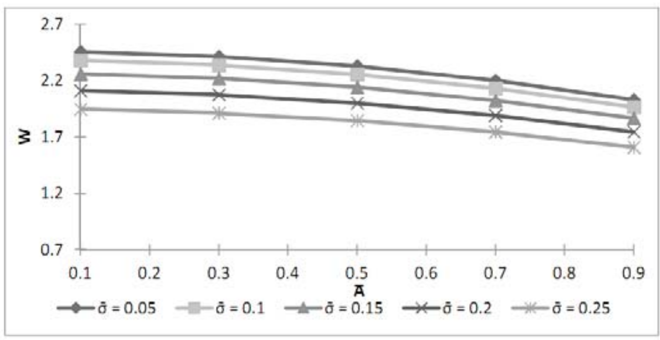

FIG. 3.4: Variation of Load carrying capacity with respect to $\bar{A}$ and $\bar{\sigma}$.

upper plate's curvature parameter significantly increases the load carrying capacity with respect to other parameters, which is in conformity with the observations of [18].

In Figures 3.14-3.18, one can visualize the effect of lower plate's curvature parameter on the variation of load carrying capacity. These figures suggest that the trends of the load carrying capacity with respect to lower plate's curvature parameter are almost opposite to that of upper plate's curvature parameter.

The effect of aspect ratio on the distribution of load carrying capacity is given in Figures 3.19-3.22. In this type of bearing system the aspect ratio plays an important role in improving the performance of the bearing system.

The effect of standard deviation associated with roughness is presented in Figures 3.23-3.25. It is manifest that the standard deviation causes considerably load reduction. This is because the motion of the lubricant gets retarded by the composite roughness of the surfaces. The combined effect of slip and standard deviation is relatively adverse (Figure 3.25) because the decreased load carrying capacity due to the standard deviation gets further decreased due to slip velocity.

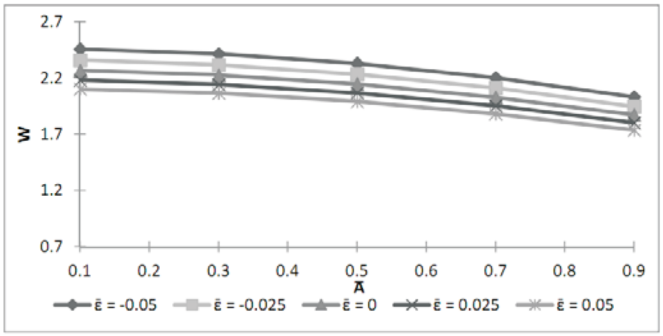

FIG. 3.5: Variation of Load carrying capacity with respect to $\bar{A}$ and $\bar{\epsilon}$.

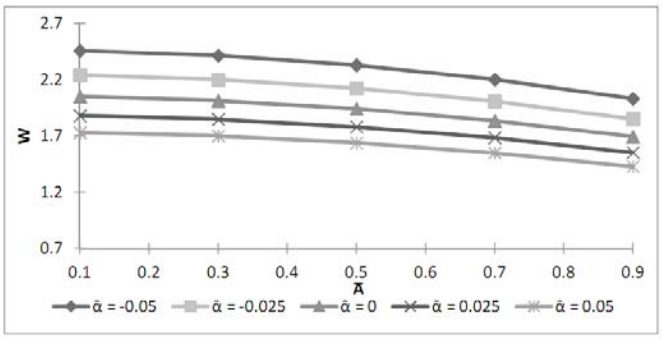

FIG. 3.6: Variation of Load carrying capacity with respect to $\bar{A}$ and $\bar{\alpha}$.

Figures 3.26 and 3.27 dealing with the effect of skewness indicate that the positive skewness decreases the load carrying capacity while the load carrying capacity gets increased due to negative skewed roughness. Further, it is appealing to note that the trends of load carrying capacity with respect to variance almost follow the path of skewness (Figure 3.28). This means the negative skewed roughness may play a good role in enhancing the performance of the bearing system when variance (-ve) occurs.

Some of the points from the graphical representations are as under

- The adverse effect of roughness can be reduced to a considerable extent by the positive effect of magnetization, when the slip parameter is minimized.
- With a suitable ratio of curvature parameters the combined effect of variance (-ve) and negatively skewed roughness may provide some measures in lowering the adverse effect of slip parameter when material constant is at the reduced level.
- A close scrutiny of the results presented here and the results found in [31]

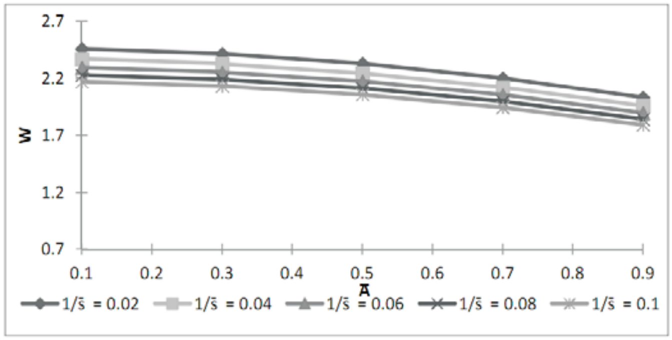

FIG. 3.7: Variation of Load carrying capacity with respect to $\bar{A}$ and $1/\bar{s}$.

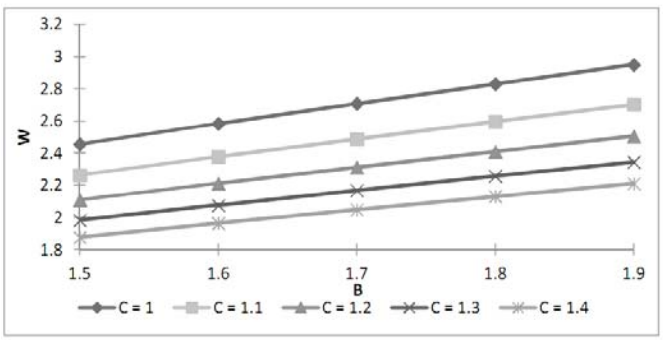

FIG. 3.8: Variation of Load carrying capacity with respect to B and C.

directly indicates that Jenkins model raises the load carrying capacity at least by 3 percent as compared to Neuringer-Rosensweig model.

- For boosting the performance the bearing system, however, the slip must be kept at minimum even if Jenkins model is deployed and suitable magnetic strength is in place.

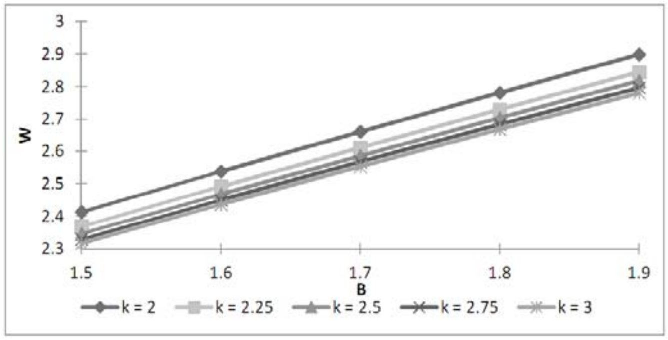

FIG. 3.9: Variation of Load carrying capacity with respect to B and k.

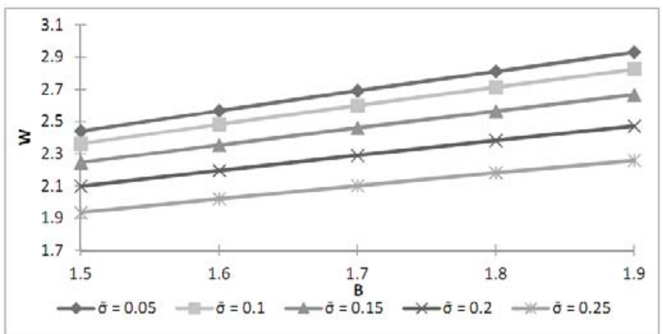

FIG. 3.10: Variation of Load carrying capacity with respect to B and $\bar{\sigma}$.

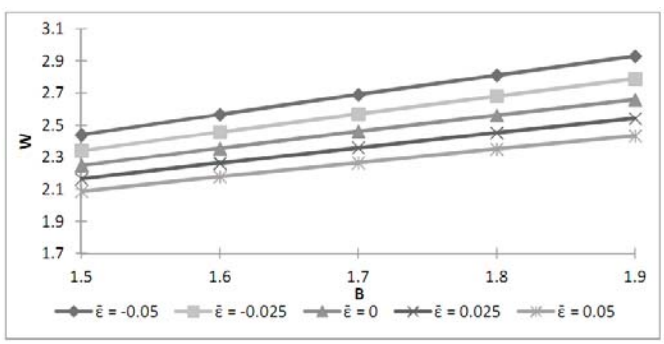

FIG. 3.11: Variation of Load carrying capacity with respect to B and $\bar{\varepsilon}$.

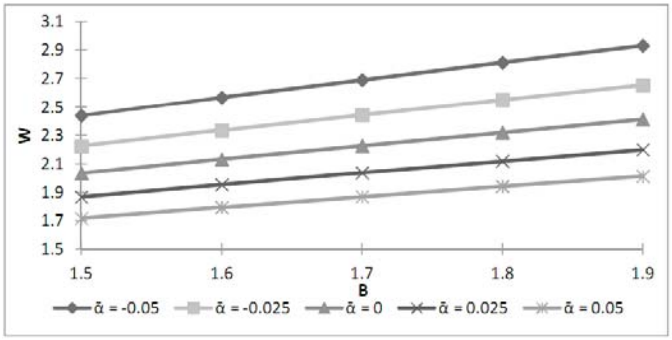

FIG. 3.12: Variation of Load carrying capacity with respect to B and $\bar{\alpha}$.

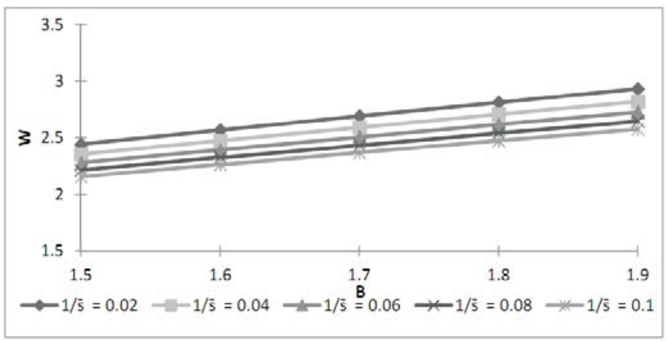

FIG. 3.13: Variation of Load carrying capacity with respect to B and $1/\bar{s}$.

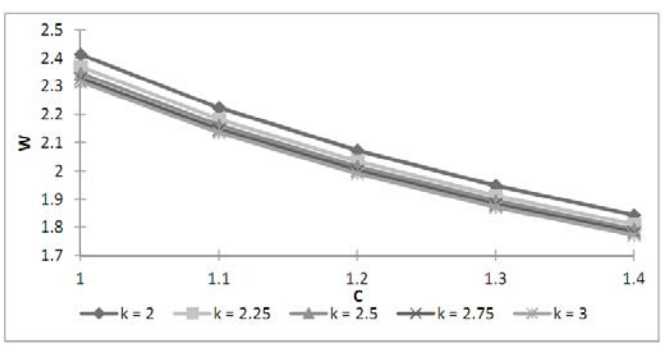

FIG. 3.14: Variation of Load carrying capacity with respect to C and k.

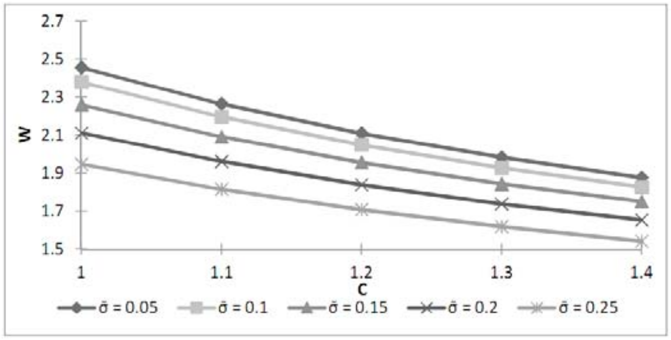

Fig. 3.15: Variation of Load carrying capacity with respect to C and $\bar{\sigma}$.

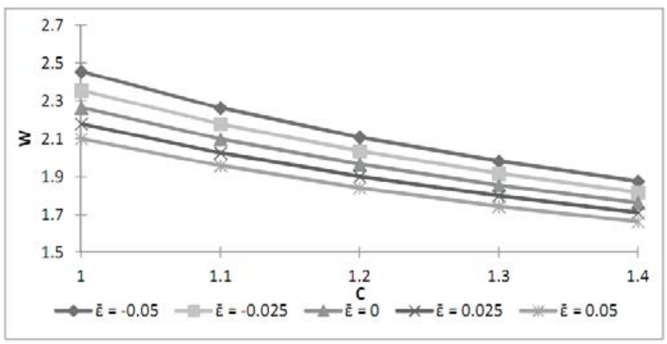

Fig. 3.16: Variation of Load carrying capacity with respect to C and $\bar{\epsilon}$.

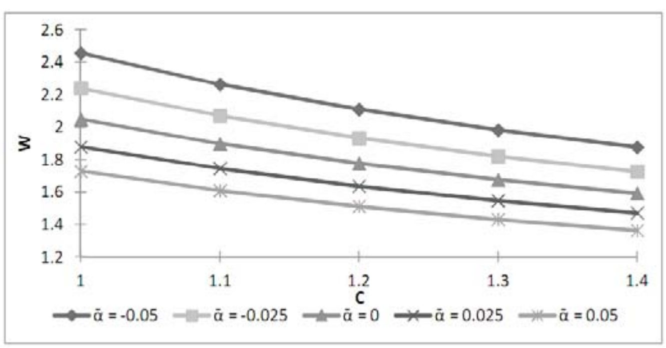

Fig. 3.17: Variation of Load carrying capacity with respect to C and $\bar{\alpha}$.

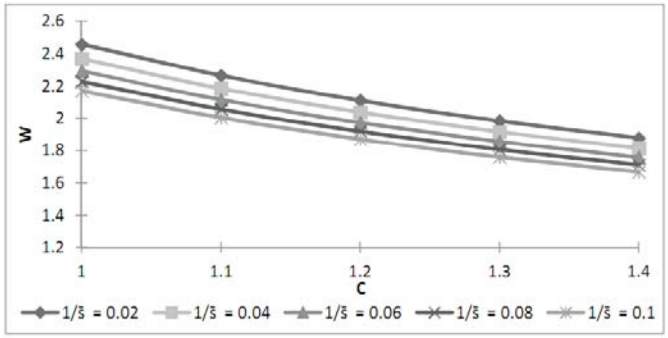

FIG. 3.18: Variation of Load carrying capacity with respect to C and $1/\bar{s}$.

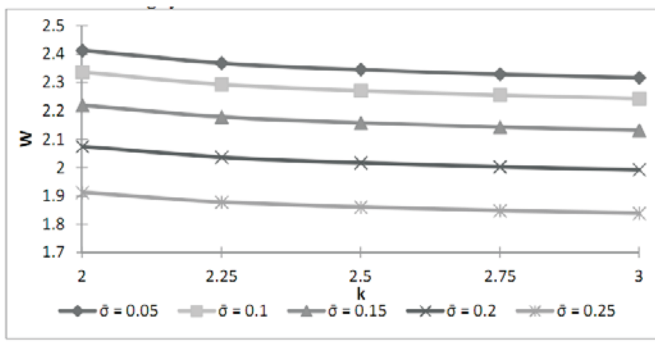

FIG. 3.19: Variation of Load carrying capacity with respect to k and $\bar{\sigma}$.

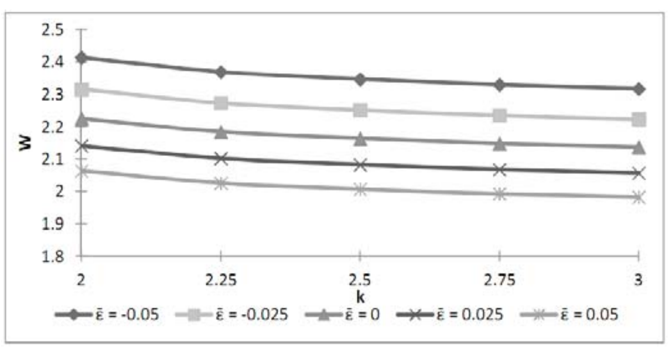

FIG. 3.20: Variation of Load carrying capacity with respect to k and $\bar{\varepsilon}$.

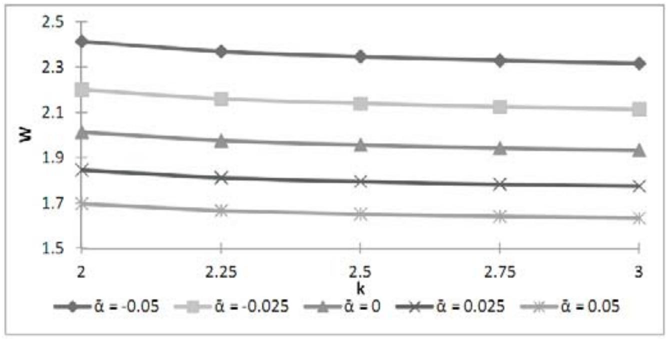

FIG. 3.21: Variation of Load carrying capacity with respect to k and $\bar{\alpha}$.

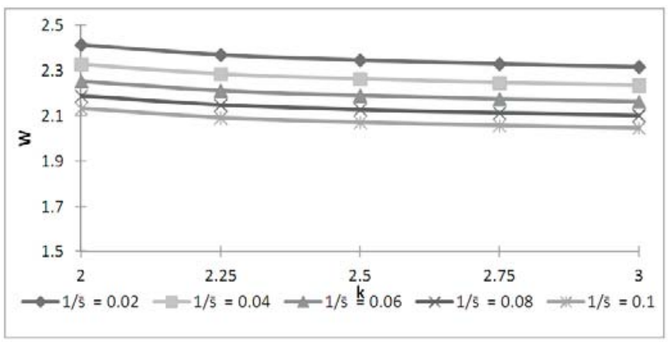

FIG. 3.22: Variation of Load carrying capacity with respect to k and $1/\bar{s}$.

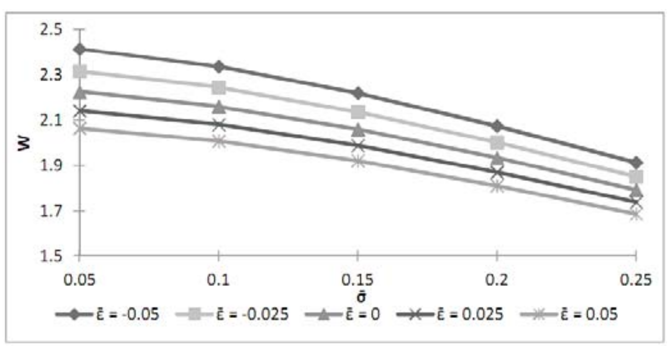

FIG. 3.23: Variation of Load carrying capacity with respect to $\bar{\sigma}$and $\bar{\epsilon}$.

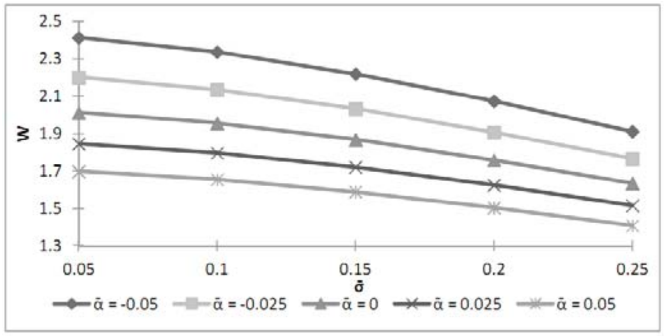

Fig. 3.24: Variation of Load carrying capacity with respect to $\bar{\sigma}$ and $\bar{\alpha}$.

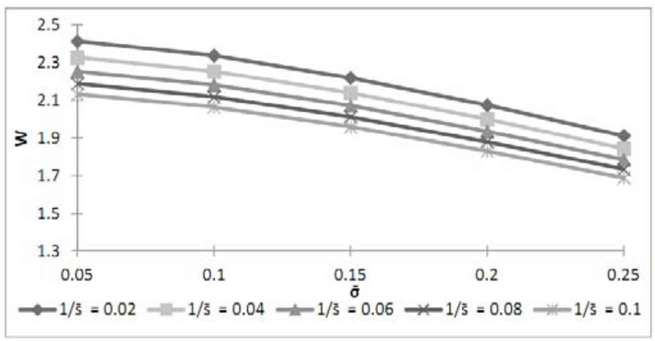

Fig. 3.25: Variation of Load carrying capacity with respect to $\bar{\sigma}$and $1/\bar{s}$.

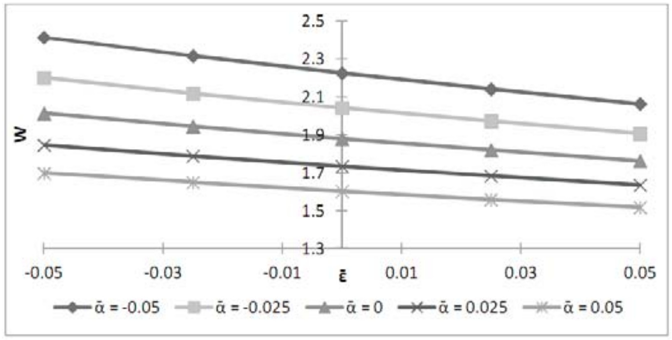

Fig. 3.26: Variation of Load carrying capacity with respect to $\bar{\epsilon}$ and $\bar{\alpha}$.

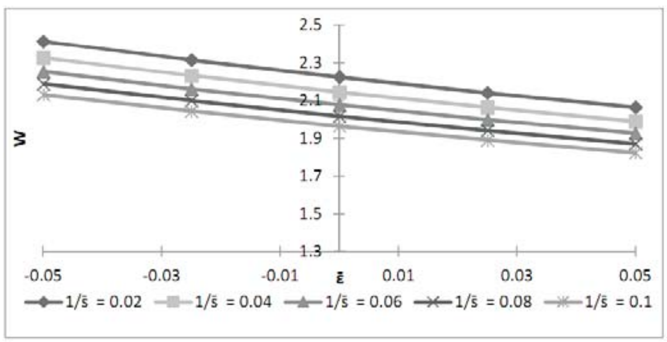

Fig. 3.27: Variation of Load carrying capacity with respect to $\bar{\epsilon}$ and $1/\bar{s}$.

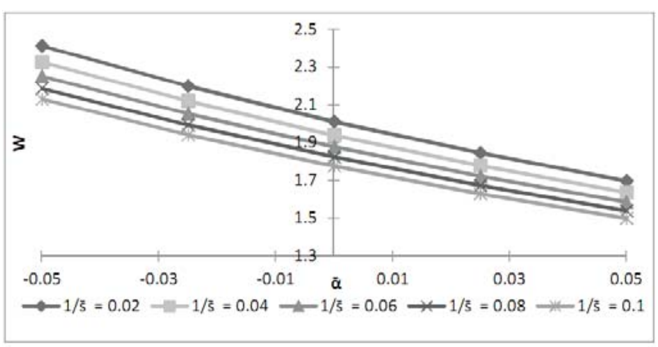

Fig. 3.28: Variation of Load carrying capacity with respect to $\bar{\alpha}$ and $1/\bar{s}$.

4. Conclusion

This investigation makes it clear that the roughness merits a serious attention while designing this Jenkins model based magnetic fluid lubricated bearing system even if suitable values of curvature parameter, slip velocity and material constant are chosen. This is all the more significant from bearing's life period point of view. In addition, the bearing can support a load even in the absence of flow unlike the case of a conventional lubricant based bearing system. Also, this study suggests that the Jenkins model scores over the Neuringer-Rosensweig model for augmenting the performance characteristics of the bearing system.

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