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ON WEAKLY SYMMETRIC AND SPECIAL WEAKLY RICCI SYMMETRIC LP-SASAKIAN MANIFOLDS ADMITTING SEMI-SYMMETRIC SEMI-METRIC CONNECTION

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Abstract. The aim of this paper is to study the geometric properties of LP-Sasakian manifolds with respect to Levi-Civita connection when they are weakly symmetric, weakly Ricci symmetric and special weakly symmetric with respect to semi-symmetric semi-metric connection. An illustration of three dimensional LP-Sasakian manifold is given.
Keywords: LP-Sasakian manifolds, Levi-Civita connection, weakly Ricci symmetric LP-Sasakian manifolds.

1. Introduction

The concept of an LP-Sasakian manifold was first developed in 1989 by K. Matsumoto [9]. The identical idea was then independently suggested by I. Mihai and R. Rosca [11], who produced multiple results on this manifold. Additionally, Venkatesha and C.S. Bagewadi [19], I. Mihai, A.A. Shaikh and U.C. De [12], A.A. Shaikh [18], C. Ozgur [14] and others have explored the LP-Sasakian manifold.

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Subsequently, numerous geometers have published various works in this field ([8], [4], [15], [16], [6]).

A non-flat Riemannian manifold (M^n, g) $(n > 2)$ is called *weakly symmetric* if there exist 1-forms $\alpha, \beta, \gamma, \delta$ and σ such that

$$(1.1)\qquad \begin{aligned}(\nabla_X R)(Y,Z,V,U) &= \alpha(X)R(Y,Z,V,U) + \beta(Y)R(X,Z,V,U)\\ &+ \gamma(Z)R(Y,X,V,U) + \delta(V)R(Y,Z,X,U)\\ &+ \sigma(U)R(Y,Z,V,X),\end{aligned}$$

holds for all vector fields $X, Y, ..., V \in X(M)$, where R is the Riemannian curvature tensor of (M^n, g) of type $(0, 4)$ and ∇ is the covariant differentiation with respect to the Riemannian metric g. A weakly symmetric manifold is said to be *proper* if $\alpha = \beta = \gamma = \delta = \sigma = 0$ is not the case.

Let $\{e_i\}$, $(i = 1, 2, ..., n)$ be an orthonormal basis of the tangent space at point of the manifold. Then, putting $Y = U = e_i$ in (1.1) and taking summation for $1 \le i \le n$, we obtain

$$(1.2)\qquad \begin{aligned}(\nabla_X S)(Z,V) &= \alpha(X)S(Z,V) + \gamma(Z)S(X,V) + \delta(V)S(Z,X)\\ &+ \beta(R(X,Z)V) + \sigma(R(X,V)Z).\end{aligned}$$

A Riemannian manifold (M^n, g) $(n > 2)$ is called *weakly Ricci-symmetric* if there exist 1-forms ρ, μ, ν such that the relation

$$(1.3)\qquad (\nabla_X S)(Y,Z) = \rho(X)S(Y,Z) + \mu(Y)S(X,Z) + \nu(Z)S(X,Y),$$

holds for any vector fields X, Y, Z, where S is the Ricci tensor of type $(0, 2)$ of the manifold M^n. A weakly Ricci-symmetric manifold is said to be proper if $\rho = \mu = \nu = 0$ is not the case.

An n-dimensional Riemannian manifold (M^n, g) is called a special weakly Ricci-symmetric $(SWRS)_n$ manifold if

$$(1.4)\qquad (\nabla_X S)(Y,Z) = 2\alpha(X)S(Y,Z) + \alpha(Y)S(X,Z) + \alpha(Z)S(X,Y),$$

where α is a 1-form and is defined by

$$(1.5)\qquad \alpha(X) = g(X, \rho),$$

where ρ is the associated vector field.

We are the following known result.

Lemma 1.1. *[13] If $M : g = c$ is a surface in R^n, then the gradient vector field is a non-vanishing normal vector field on the entire surface M.*

2. LP-Sasakian manifold

A differentiable manifold of dimensional n(odd) is called LP-Sasakian manifold if it admits a $(1,1)$-tensor field ϕ, a contravariant vector field ξ, a covariant vector field η and a Lorentzian metric g which satisfy:

$$\phi^2 = I + \eta \otimes \xi, \quad \eta(\xi) = -1, \quad \phi(\xi) = 0, \quad \eta \circ \phi = 0, \tag{2.1}$$

$$g(\phi X, \phi Y) = g(X,Y) + \eta(X)\eta(Y), \quad g(X,\xi) = \eta(X), \tag{2.2}$$

for all $X, Y \in TM$.

Also LP-Sasakian manifold M^n satisfies

$$(\nabla_X \phi)Y = \{g(X,Y)\xi + 2\eta(Y)\eta(X)\xi\}, \tag{2.3}$$

$$\nabla_X \xi = \phi X, \tag{2.4}$$

where ∇ denotes the operator of covariant differentiation with respect to the Lorentzian metric g.

Example of LP-Sasakian manifold: Consider the 3-dimensional manifold $M = \{(x,y,z) \in R^3; z \neq 0\}$, where (x,y,z) are the standard co-ordinates in R^3. Let $\{E_1, E_2, E_3\}$ be linearly independent global frame field on M^n given by

$$E_1 = \frac{e^z}{x}\frac{\partial}{\partial x}, \quad E_2 = \frac{e^{z-ax}}{y}\frac{\partial}{\partial y}, \quad E_3 = \frac{\partial}{\partial z}. \tag{2.5}$$

Let g be the Riemannian metric defined by

$$\begin{aligned} g(E_1, E_2) &= g(E_1, E_3) = g(E_2, E_3) = 0, \\ g(E_1, E_1) &= g(E_2, E_2) = 1 \text{ and } g(E_3, E_3) = -1. \end{aligned}$$

The (ϕ, ξ, η) is given by

$$\begin{aligned} &\eta = -dz, \quad \xi = E_3 = \frac{\partial}{\partial z}, \\ &\phi E_1 = -E_1, \quad \phi E_2 = -E_2, \quad \phi E_3 = 0. \end{aligned}$$

The linearity property of ϕ and g yields that

$$\begin{aligned} \eta(E_3) &= -1, \quad \phi^2 U = U + \eta(U)E_3, \\ g(\phi U, \phi W) &= g(U,W) + \eta(U)\eta(W), \quad g(U,\xi) = \eta(U), \end{aligned}$$

for any vector fields U, W on M. By definition of Lie bracket, we have

$$[E_1, E_2] = -\frac{ae^z}{x}E_2, \quad [E_1, E_3] = -E_1, \quad [E_2, E_3] = -E_2. \tag{2.6}$$

The Levi-Civita connection with respect to above metric g is given by Koszula forumula

$$\begin{aligned} 2g(\nabla_X Y, Z) &= X(g(Y,Z)) + Y(g(Z,X)) - Z(g(X,Y)) \\ &\quad -g(X,[Y,Z]) - g(Y,[X,Z]) + g(Z,[X,Y]). \end{aligned}$$

Then we have,

$$\begin{array}{lll} \nabla_{E_1}E_1 = -E_3, & \nabla_{E_1}E_2 = 0, & \nabla_{E_1}E_3 = -E_1, \\ \nabla_{E_2}E_1 = \dfrac{ae^z}{x}E_2, & \nabla_{E_2}E_2 = -\dfrac{ae^z}{x}E_1 - E_3, & \nabla_{E_2}E_3 = -E_2, \\ \nabla_{E_3}E_1 = 0, & \nabla_{E_3}E_2 = 0, & \nabla_{E_3}E_3 = 0. \end{array}$$

The tangent vectors X and Y to M are expressed as linear combination of E_1, E_2, E_3, i.e., $X = a_1E_1 + a_2E_2 + a_3E_3$ and $Y = b_1E_1 + b_2E_2 + b_3E_3$, where a_i and b_j are scalars. Clearly (ϕ, ξ, η, g) and X, Y satisfy equations $(2.1), (2.2), (2.3)$ and (2.4). Thus M^n is LP-Sasakian manifold.

Also, in LP-Sasakian manifold M^n the following relations hold:

$$\eta(R(X,Y)Z) = \{g(Y,Z)\eta(X) - g(X,Z)\eta(Y)\}, \tag{2.7}$$

$$R(X,Y)\xi = \{\eta(Y)X - \eta(X)Y\}, \tag{2.8}$$

$$R(\xi,X)Y = \{g(X,Y)\xi - \eta(Y)X\}, \tag{2.9}$$

$$R(\xi,X)\xi = \{\eta(X)\xi + X\}, \tag{2.10}$$

$$S(X,\xi) = (n-1)\eta(X), \tag{2.11}$$

$$Q\xi = (n-1)\xi, \tag{2.12}$$

for any vector fields X, Y, Z, where $R(X,Y)Z$ is the curvature tensor and S is the Ricci tensor.

3. Semi-symmetric semi-metric connection

A. Friedmann and J.A. Schouten [5] introduced the idea of a semi-symmetric linear connection. A linear connection ∇ is said to be semi-symmetric connection if its torsion tensor T is of the form

$$T(X,Y) = \eta(Y)X - \eta(X)Y, \tag{3.1}$$

where η is a 1-form. Motivated by studies of author in [1], introduced the notion of semi-symmetric semi-metric connection $\widetilde{\nabla}$ on a contact metric manifold and it is defined as

$$\widetilde{\nabla}_X Y = \nabla_X Y - \eta(X)Y + g(X,Y)\xi, \tag{3.2}$$

where ∇ is Levi-Civita connection. A study on semi-symmetric connections and their properties can be found in [20, 3, 5, 7]. More recently, Mobin Ahmad and M. Danish Siddiqui [1] have studied a nearly Sasakian manifold with a semi-symmetric semi-metric connection, proving the results of integrability conditions of distribution of semi-invariant submanifolds of an approximately Sasakian manifold, inspired by research done by the author in [1]. Our focus is on LP-Sasakian manifolds that exhibit weakly symmetry.

A relation between the curvature tensor of M^n with respect to the semi-symmetric semi-metric connection $\widetilde{\nabla}$ and the Levi-Civita connection ∇ is given by

$$\begin{aligned}(3.3)\ \widetilde{R}(X,Y)Z &= R(X,Y)Z + 2[\eta(Y)g(X,Z) - \eta(X)g(Y,Z)]\xi + [g(X,\phi Y)\\ &- g(Y,\phi X)]Z + [g(Y,Z)\phi X - g(X,Z)\phi Y],\end{aligned}$$

where $\widetilde{R}$ and R are the Riemannian curvatures of the connections $\widetilde{\nabla}$ and ∇ respectively. From (3.3), it follows that

$$(3.4)\qquad \widetilde{S}(Y,Z) = S(Y,Z) + 2\eta(Y)\eta(Z) + 2g(Y,Z) - g(Z,\phi Y) + Tg(Y,Z),$$

where $T = trace\phi = g(\phi e_i, e_i)$, $\widetilde{S}$ and S are the Ricci tensors of the connections $\widetilde{\nabla}$ and ∇ respectively.

Taking Z instead of ξ, the above expression becomes

$$(3.5)\qquad \widetilde{S}(Y,\xi) = [(n-1) + T]\eta(Y).$$

4. Weakly symmetric LP-Sasakian manifold admitting semi-symmetric semi-metric connection

Let $\widetilde{M^n}$ denote LP-Sasakian manifold admitting semi-symmetric semi-metric connection. Let $\widetilde{M^n}$ be weakly symmetric. Then equation (1.2) may be written as

$$\begin{aligned}(4.1)\qquad (\widetilde{\nabla}_X\widetilde{S})(Z,V) &= \alpha(X)\widetilde{S}(Z,V) + \gamma(Z)\widetilde{S}(X,V) + \delta(V)\widetilde{S}(Z,X)\\ &+ \beta(\widetilde{R}(X,Z)V) + \sigma(\widetilde{R}(X,V)Z).\end{aligned}$$

Taking covariant differentiation of the Ricci tensor $\widetilde{S}$ with respect to X, we have

$$(4.2)\qquad (\widetilde{\nabla}_X\widetilde{S})(Z,V) = \widetilde{\nabla}_X\widetilde{S}(Z,V) - \widetilde{S}(\widetilde{\nabla}_X Z,V) - \widetilde{S}(Z,\widetilde{\nabla}_X V).$$

Putting $V = \xi$ in (4.2) and by virtue of (2.1), (2.4), (2.11), (3.2), (3.4), we find

$$\begin{aligned}(4.3)\qquad (\widetilde{\nabla}_X\widetilde{S})(Z,\xi) &= (n-1)\eta(\nabla_X Z) - (n-1)\eta(X)\eta(Z) - (n-1)g(X,Z)\\ &+(n-1)g(Z,\phi X) + X(T)\eta(Z) + T\eta(\nabla_X Z) - T\eta(X)\eta(Z)\\ &-Tg(X,Z) + Tg(Z,\phi X) + (n-1)g(Z,\phi X) - S(Z,\phi X)\\ &-2g(Z,\phi X) + g(Z,X) + \eta(X)\eta(Z).\end{aligned}$$

On the other hand replacing V with ξ in (4.1) and use (2.1), (2.11), (3.3), (3.4), (3.5), we immediately obtain

$$(4.4)\ (\widetilde{\nabla}_X\widetilde{S})(Z,\xi) = [(n-1)+T]\alpha(X)\eta(Z) + [(n-1)+T]\gamma(Z)\eta(X)$$

$$\begin{aligned}
&+ \quad \delta(\xi)S(Z,X) + 2\delta(\xi)\eta(Z)\eta(X) + 2\delta(\xi)g(Z,X) \\
&- \quad \delta(\xi)g(X,\phi Z) + T\delta(\xi)g(Z,X) + \eta(Z)\beta(X) \\
&- \quad \eta(X)\beta(Z) + g(X,\phi Z)\beta(\xi) - g(Z,\phi X)\beta(\xi) \\
&+ \quad \eta(Z)\beta(\phi X) - \eta(X)\beta(\phi Z) + \eta(Z)\sigma(X) - g(X,Z)\sigma(\xi) \\
&- \quad 2g(X,Z)\sigma(\xi) - 2\eta(X)\eta(Z)\sigma(\xi) + \eta(Z)\sigma(\phi X).
\end{aligned}$$

Hence, comparing the right hand side of the equations (4.3) and (4.4), we get

$$\begin{aligned}
&(n-1)\eta(\nabla_X Z) - (n-1)\eta(X)\eta(Z) - (n-1)g(X,Z) + (n-1)g(Z,\phi X) \\
&+X(T)\eta(Z) + T\eta(\nabla_X Z) - T\eta(X)\eta(Z) - Tg(X,Z) + Tg(Z,\phi X) \\
&+(n-1)g(Z,\phi X) - S(Z,\phi X) - 2g(Z,\phi X) + g(Z,X) + \eta(X)\eta(Z) \\
&= [(n-1)+T]\alpha(X)\eta(Z) + [(n-1)+T]\gamma(Z)\eta(X) + \delta(\xi)S(Z,X) \\
&+2\delta(\xi)\eta(Z)\eta(X) + 2\delta(\xi)g(Z,X) - \delta(\xi)g(X,\phi Z) + T\delta(\xi)g(Z,X) \\
&+\eta(Z)\beta(X) - \eta(X)\beta(Z) + g(X,\phi Z)\beta(\xi) - g(Z,\phi X)\beta(\xi) \\
&+\eta(Z)\beta(\phi X) - \eta(X)\beta(\phi Z) + \eta(Z)\sigma(X) - g(X,Z)\sigma(\xi) \\
&-2g(X,Z)\sigma(\xi) - 2\eta(X)\eta(Z)\sigma(\xi) + \eta(Z)\sigma(\phi X).
\end{aligned} \tag{4.5}$$

Plugging $Z = \xi$ in (4.5) and using these equations (2.1), (2.4), (2.11), we get the equation

$$\begin{aligned}
-X(T) &= -[(n-1)+T]\alpha(X) + [(n-1)+T]\gamma(\xi)\eta(X) \\
&+ [(n-1)+T]\delta(\xi)\eta(X) - \beta(X) - \eta(X)\beta(\xi) \\
&- \beta(\phi X) - \sigma(X) - \eta(X)\sigma(\xi) - \sigma(\phi X).
\end{aligned} \tag{4.6}$$

At this stage we can't give any geometric meaning to this equation. If we take $X = \xi$, then

$$\begin{aligned}
\xi(T) &= [(n-1)+T][\alpha(\xi) + \gamma(\xi) + \delta(\xi)], \\
i.e, \quad gradT.\xi &= [(n-1)+T][\alpha(\xi) + \gamma(\xi) + \delta(\xi)].
\end{aligned} \tag{4.7}$$

Since $[(n-1)+T] \neq 0$, we have $gradT$ is normal to ξ if and only if $[\alpha(\xi) + \gamma(\xi) + \delta(\xi)] = 0$.
Thus by Lemma 1.1 we can state the following:

Theorem 4.1. *Let $\widetilde{M}^n$ be weakly symmetric LP-Sasakian manifold with respect to semi-symmetric semi-metric connection. Then the sum of 1-forms α, γ and δ on vanish on the characteristic vector field ξ if and only if the gradient of trace of the endomorphism ϕ is normal to M^n along ξ.*

5. On special weakly Ricci-symmetric LP-Sasakian manifold admitting semi-symmetric semi-metric connection

Let $\widetilde{M}^n$ be special weakly Ricci-symmetric LP-Sasakian manifold. Then (1.4) may be written as

$$(\widetilde{\nabla}_X \widetilde{S})(Y,Z) = 2\alpha(X)\widetilde{S}(Y,Z) + \alpha(Y)\widetilde{S}(X,Z) + \alpha(Z)\widetilde{S}(X,Y). \tag{5.1}$$

Taking cyclic sum of (5.1). This implies that

$$(\widetilde{\nabla}_X\widetilde{S})(Y,Z)+(\widetilde{\nabla}_Y\widetilde{S})(Z,X)+(\widetilde{\nabla}_Z\widetilde{S})(X,Y) = 4[\alpha(X)\widetilde{S}(Y,Z)+\alpha(Y)\widetilde{S}(Z,X)+\alpha(Z)\widetilde{S}(X,Y)]. \tag{5.2}$$

Let $\widetilde{M}^n$ admit a cyclic Ricci tensor. Then (5.2) reduces to

$$0=\alpha(X)\widetilde{S}(Y,Z)+\alpha(Y)\widetilde{S}(Z,X)+\alpha(Z)\widetilde{S}(X,Y). \tag{5.3}$$

Now setting $Z=\xi$ in (5.3) and yield (2.1), (3.4), (3.5), we get

$$\begin{aligned} 0 &= [(n-1)+T]\alpha(X)\eta(Y)+[(n-1)+T]\alpha(Y)\eta(X)+\alpha(\xi)S(X,Y) \\ &+ 2\alpha(\xi)\eta(X)\eta(Y)+2\alpha(\xi)g(X,Y)-\alpha(\xi)g(Y,\phi X)+T\alpha(\xi)g(X,Y). \end{aligned} \tag{5.4}$$

Again setting $Y=\xi$ in (5.4) and employ (1.5) and (2.1), we obtain

$$2\eta(\rho)\eta(X)=\alpha(X). \tag{5.5}$$

Changing X to ξ in (5.5) and make use of (1.5) and (2.1), it follows that

$$\eta(\rho)=0. \tag{5.6}$$

By virtue of (5.6) in (5.5), we procure $\alpha(X)=0$, for all X.
This lead us to the following

Theorem 5.1. *Let $\widetilde{M}^n$ be special weakly Ricci-symmetric LP-Sasakian manifold M^n with respect to semi-symmetric semi-metric connection and admits a cyclic Ricci tensor. Then the 1-form α must vanish on M^n. However the converse holds trivially.*

Next setting $Z=\xi$ in (5.1), we have the following

$$(\widetilde{\nabla}_X\widetilde{S})(Y,\xi)=2\alpha(X)\widetilde{S}(Y,\xi)+\alpha(Y)\widetilde{S}(X,\xi)+\alpha(\xi)\widetilde{S}(X,Y). \tag{5.7}$$

The left hand side can be written in the form

$$(\widetilde{\nabla}_X\widetilde{S})(Y,\xi)=\widetilde{\nabla}_X\widetilde{S}(Y,\xi)-\widetilde{S}(\widetilde{\nabla}_XY,\xi)-\widetilde{S}(Y,\widetilde{\nabla}_X\xi). \tag{5.8}$$

By view of (1.5), (2.1), (2.11), (3.2), (3.4), (3.5), we infer that

$$\begin{aligned} &(n-1)\eta(\nabla_XY)-(n-1)\eta(X)\eta(Y)-(n-1)g(X,Y)+(n-1)g(Y,\phi X) \\ &+X(T)\eta(Y)+T\eta(\nabla_XY)-T\eta(X)\eta(Y)-Tg(X,Y)+Tg(Y,\phi X) \\ &+(n-1)g(Y,\phi X)-S(Y,\phi X)-2g(Y,\phi X)+g(Y,X)+\eta(X)\eta(Y) \\ &=2[(n-1)+T]\alpha(X)\eta(Y)+[(n-1)+T]\alpha(Y)\eta(X) \\ &+\eta(\rho)\{S(X,Y)+2\eta(X)\eta(Y)+2g(X,Y)-g(Y,\phi X)+Tg(X,Y)\}. \end{aligned} \tag{5.9}$$

Choosing $Y = \xi$ in (5.9) and utilize (1.5) and (2.1), (2.4), (2.11), gives

$$(5.10) \qquad -X(T) = -2[(n-1)+T]\alpha(X) + 2[(n-1)+T]\eta(\rho)\eta(X),$$
$$(5.11) \qquad i.e, \quad X(T) = 2[\eta(\rho)\eta(X) - \alpha(X)][(n-1)+T].$$

We know that $X(T) = gradT \cdot X$. Since $[(n-1)+T] \neq 0$. $gradT$ is normal to M^n if and only if $\eta(\rho)\eta(X) = \alpha(X)$.
Hence we state the following lemma 1.1.

Theorem 5.2. *Let $\widetilde{M}^n$ be special weakly Ricci-symmetric LP-Sasakian manifold M^n with respect to semi-symmetric semi-metric connection. Then the gradient of the trace of the endomorphism of T is normal to M^n if and only if $\eta(\rho)\eta(X) = \alpha(X)$.*

If we put $X = \xi$ in $\eta(\rho)\eta(X) = \alpha(X)$, then $\eta(\rho) = 0$. Thus $\alpha(X) = 0$.

Hence we can restate the Theorem 5.2 as follows:

Corollary 5.1. *Let $\widetilde{M}^n$ be special weakly symmetric LP-Sasakian manifold with respect to semi-symmetric semi-metric connection. Then the gradient of the trace of the endomorphism of T is normal to M^n along ξ if and only if 1-form vanish on the whole space M^n.*

We conclude from the above results:

Conclusion: If $\widetilde{M}^n$ is weakly symmetric LP-Sasakian manifold, then the sum of the 1-forms vanish along the characteristic vector field ξ if and only if the trace of endomorphism of ϕ is normal to M^n along ξ, whereas if $\widetilde{M}^n$ is special weakly Ricci-symmetric then the 1-form vanishes for every vector field if and only if trace of endomorphism ϕ is normal to M^n along ξ. If $\widetilde{M}^n$ admits cyclic Ricci tensor then the 1-form vanish the whole manifold M^n without any endomorphism.

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