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LINEAR TO NON-LINEAR TOPOLOGY VIA γ-OPEN SETS IN THE ENVIRONMENT OF BITOPOLOGICAL SPACES

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Abstract. Generalizations of open sets always gives a linear structure in an ordinary topological space. This paper proposes that there exists a non-linear structure in a given bitopological space via γ-open sets of the context. The new structure is also studied in the light of hyperconnectedness to show that it is completely independent from the original one. Also, the relationships between extremally disconnectedness, connectedness and hyperconnectedness are presented in the same environment by means of γ-open set. Moreover, the idea of maximal γ-hyperconnectedness is initiated in this work and some important results related to filter, ultrafilter, door space are established. Finally, some functions concerned with $(1, 2)$ γ-open sets are introduced and interrelationships among them are produced. Some suitable examples and counter examples are properly placed to make the paper self sufficient.
Key words: bitopological space, gamma-open set, non-linear structures.

1. Introduction

In 1987, D. Andrijevic first introduced the concept of γ-open set in a topological space [10]. It is seen that γ-open set is a generalization of open set and the family of all these γ-open sets constitutes a topological structure on a given topological space. Again in 1997, A. A. El Atik, introduced γ-open set [1] in a different way and in his study also, it is observed that γ-open set is a generalized form of open set, where the collection of all these γ-open sets forms a supra topology. Then, I. M. Hanafy extended the theory of γ-open set (in the sense of Atik et al.) in fuzzy topological

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space [13] in the year 1999. In that environment also the relation between fuzzy open set and fuzzy γ-open set becomes linear in nature, that is every fuzzy open set is a fuzzy γ-open set. Very recently in 2017, B. Bhattacharya initiated the notion of fuzzy γ^*-open set [3] in a fuzzy topological space, which is in the direction of D. Andrijevic. Surprisingly, it has been established that fuzzy γ^*-open sets and fuzzy open sets are both independent concepts, that is, the relation between them is not linear.

In the same work, the following question was raised :
Is there any other collection of sets or any other form of topology which structures an independent topology for a given topological space (X, τ)?

In the present paper we are going to find an answer to the above question in the environment of bitopological space.

Bitopological space was first introduced by J C. Kelly [14] in 1963. Maheswari and Prasad [20] extended the idea of semi open set and semi continuity to the bitopological structure in 1977. Also the initiative for studying pre-open set [15] and α-open set [16] was taken by M. Jelic in 1990, in this environment.

The concept of $(1,2)$ open set and $(1,2)^*$ open set [18] is introduced by M. L. Thivagar in 1991. Recently, A. Paul and B. Bhattacharya extended γ-open set in bitopological space, namely $(1,2)$ γ-open set and $(1,2)^*\gamma$-open set [2]. They demonstrated that $(1,2)$ γ-open set and $(1,2)$ open set in a bitopological spaces are completely independent of each other. Similarly, $(1,2)^*$ γ-open set and $(1,2)^*$ open set are also independent. The concept of such types of γ-open sets in bitopological spaces are extended in fuzzy environment too [4, 5, 6, 7, 8] .

The concept of hyperconnectedness in a topological space was first introduced by P. M. Mathew in 1988 [19]. Later in 2001, hyperconnectedness and maximal hyperconnectedness were studied by B. Garai and C. Bandyopadhyay in bitopological structure [9]. They also characterized hyperconnected bitopological space and maximal hyperconnected bitopological space using filter. Moreover, the concept of almost b-continuous function in the same space was introduced and investigated by D. J. Sarma and S. Acharjee [12].

In this present treatise, we introduce the notion of $(1,2)$ γ-hyperconnected bitopological space and show that $(1,2)$ γ-hyperconnected bitopological space and general hyperconnected bitopological space are not linearly related, that is they are two completely independent structures. Also we define the concept of $(1,2)$ extremally γ-disconnected space and $(1,2)$ γ-connected space. We demonstrated that their relation with the usual extremally disconnected sapce and connected space are also non-linear. Finally, we introduce some functions relating to $(1,2)$ γ-open set. We establish important interrelationship among them.

2. Preliminaries

Before going to the main section we need some basic and preliminary ideas about the existing definitions and results which will play a major role in this study.

Definition 2.1. [14] Let X be a non-empty set and τ_1 and τ_2 be two topologies defined on X. Then the structure (X, τ_1, τ_2) is known to be a bitopological space.

Definition 2.2. [18] Let (X, τ_1, τ_2) be a bitopological space. Then a set $A \in \tau_1 \cup \tau_2$ is said to be a $(1,2)$ open set or T_1T_2-open set.
Let us consider the collection $S = \{B : B = A_1 \cup A_2, A_1 \in \tau_1, A \in \tau_2\}$. Then the members of S are known as $(1,2)^*$ open sets or $T_{1,2}$-open sets. The collection of all $(1,2)$ open sets (resp. $(1,2)^*$ open sets) are denoted by $(1,2)$ $O(X)$ (resp.$(1,2)^*$ $O(X)$) and the family of all $(1,2)$ closed sets (resp. $(1,2)^*$ closed sets) are denoted by $(1,2)$ $cl(X)$ (resp.$(1,2)^*$ $cl(X)$)
The complement of a $(1,2)$ open set (T_1T_2-*open* set) is called a $(1,2)$ closed set (T_1T_2-*closed* set) and the complement of a $(1,2)^*$-open set ($T_{1,2}$-*open* set) is called a $(1,2)^*$ closed set ($T_{1,2}$-*closed* set).

Definition 2.3. [18] Let S be a subset of (X, T_1, T_2). Then

(i) The $T_{1,2}$-*interior* of S, denoted by $T_{1,2}$-$int(S)$, is defined by $T_{1,2}\text{-}int(S) = \bigcup\{G : G \subseteq S, G \text{ is } T_{1,2}\text{-open}\}$. Similarly, the T_1T_2-*interior* of S, denoted by T_1T_2-$int(S)$, is defined by $T_1T_2\text{-}int(S) = \bigcup\{G : G \subseteq S, G \text{ is } T_1T_2\text{-open}\}$.

(ii) The $T_{1,2}$-*closure* of S, denoted by $T_{1,2}$-$cl(S)$, is defined by $T_{1,2}\text{-}cl(S) = \bigcap\{G : S \subseteq G, G \text{ is } T_{1,2}\text{-closed}\}$. Similarly, the T_1T_2-*closure* of S, denoted by T_1T_2-$cl(S)$, is defined by $T_1T_2\text{-}cl(S) = \bigcap\{G : S \subseteq G, G \text{ is } T_1T_2\text{-closed}\}$

Definition 2.4. Any subset A of a bitopological space (X, T_1, T_2) is said to be

(i) [20] $(1,2)$ semi open set if $A \subseteq T_2\text{-}cl(T_1\text{-}int(A))$.

(ii) [15] $(1,2)$ pre open set if $A \subseteq T_1\text{-}int(T_2\text{-}cl(A))$.

(iii) [21] $(1,2)$ regular open set if $A = T_1\text{-}int(T_2\text{-}cl(A))$.

Here by T_2-*cl* and T_1-*int* we mean the *closure* operator with respect to T_2 and *interior* operator with respect to T_1 respectively.

The complement of $(1,2)$ semi open, $(1,2)$ pre open, $(1,2)$ regular open sets are called $(1,2)$ semi closed, $(1,2)$ pre closed, $(1,2)$ regular closed sets respectively. The collection of all $(1,2)$ semi open, $(1,2)$ pre open, $(1,2)$ regular open sets are denoted by $(1,2)$ $PO(X)$, $(1,2)$ $RO(X)$ and $(1,2)$ $SO(X)$ respectively.

Definition 2.5. [2] A subset A of a bitopological space (X, T_1, T_2) is said to be $(1,2)$ γ-open set if for any non-empty $(1,2)$ pre open set B in X, $A \cap B \subseteq T_1\text{-}int(T_2\text{-}cl(A \cap B))$. The complement of a $(1,2)$ γ-open set is known as $(1,2)$ γ-closed set. The collection of all $(1,2)$ γ-open sets and $(1,2)$ γ-closed sets are denoted by $(1,2)$ γ-$O(X)$ and $(1,2)$ γ-$cl(X)$ respectively.
The $(1,2)$ γ-interior and $(1,2)$ γ-closure of a set A is denoted by $(1,2)$ $int_\gamma(A)$ and $(1,2)$ $cl_\gamma(A)$ respectively.

3. $(1, 2)$ γ-Hyperconnectedness in Bitopological Space

In this section, we introduce the concept of $(1, 2)$ γ-hyperconnectedness, $(1, 2)$ γ-connectedness and $(1, 2)$ extremally γ-disconnectedness in a bitopological space. We establish the interrelationship among them. Also, we find that hyperconnectedness and $(1, 2)$ γ-hyperconnectedness are two completely independent notions. Similarly the relation between the concepts $(1, 2)$ extremally γ-disconnectedness and extremally disconnectedness; $(1, 2)$ γ-connectedness and connectedness are non-linear in nature. Furthermore, we define $(1, 2)$ γ-regular open set and $(1, 2)$ γ-semi open set and characterized them in a bitopological space which is $(1, 2)$ γ-hyperconnected.

Definition 3.1. A subset A of (X, T_1, T_2) is said to be $(1, 2)$ dense if T_2-$cl(A) = X$ and $(1, 2)$ nowhere dense if T_1-$int(T_2$-$cl(A)) = \phi$.
A bitopological space X is said to be $(1, 2)$ hyperconnected if for every $(1, 2)$ open set A in X, A is $(1, 2)$ dense in X.

Definition 3.2. A subset A of (X, T_1, T_2) is said to be $(1, 2)$ γ-dense if $(1, 2)$ $cl_\gamma(A)$ $= X$ and $(1, 2)$ nowhere γ-dense if $(1, 2)$ $int_\gamma((1, 2)$ $cl_\gamma(A)) = \phi$.
A bitopological space is said to be $(1, 2)$ γ-hyperconnected space if every $(1, 2)$ γ-open set is $(1, 2)$ γ-dense therein.

Remark 3.1. Both the concepts of $(1, 2)$ hyperconnectedness and $(1, 2)$ γ-hyperconnectedness are independent from each other in a bitopological space.

Example 3.1. A bitopological space which is $(1, 2)$ hyperconnected may not be $(1, 2)$ γ-hyperconnected.
Let us consider a bitopological space (X, T_1, T_2) with $X = \{a, b, c\}$, $T_1 = \{\phi, X, \{a\}, \{a, b\}\}$ and $T_2 = \{\phi, X, \{a, b\}\}$. Then since, T_2-$cl\{a\} = X$, T_2-$cl\{a, b\} = X$, therefore X is $(1, 2)$ hyperconnected. Again, $(1, 2)$ $PO(X) = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ and $(1, 2)$ γ-$O(X) = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$. So $(1, 2)$ γ-$cl(X) = \{\phi, X, \{b, c\}, \{a, c\}, \{c\}\}$. Now $(1, 2)$ $cl_\gamma\{a\} = \{a, c\} \neq X$. Hence, X is not $(1, 2)$ γ-hyperconnected.

Example 3.2. A $(1, 2)$ γ-hyperconnected space may not be $(1, 2)$ hyperconnected in bitopological structure.
Let $X = \{a, b, c\}$ be any non-empty set and T_1, T_2 be two topologies defined on the set X, such that $T_1 = \{\phi, X, \{a, c\}\}$ and $T_2 = \{\phi, X, \{b\}, \{a, b\}, \{b, c\}\}$. Now $(1, 2)$ $PO(X) = \{\phi, X, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ and $(1, 2)$ γ-$O(X) = \{\phi, X, \{b\}, \{a, b\}\}$. Therefore $(1, 2)$ γ-$cl(X) = \{\phi, X, \{a, c\}, \{c\}\}$. Since $(1, 2)$ $cl_\gamma\{b\} = (1, 2)$ $cl_\gamma\{a, b\} = X$, X is a $(1, 2)$ γ-hyperconnected space. Again, since T_2-$cl\{a, c\} = \{a, c\} \neq X$, so X is not a $(1, 2)$ hyperconnected space.

Definition 3.3. A bitopological space (X, T_1, T_2) is said to be a $(1, 2)$ extremally disconnected space if the T_1T_2-*closure* of every T_1T_2-open subset of X is again a T_1T_2-open set in X.

Definition 3.4. A bitopological space (X, T_1, T_2) is said to be $(1, 2)$ extremelly γ-disconnected if $(1, 2)$ γ-closure of every $(1, 2)$ γ-open set is $(1, 2)$ γ-open.

Definition 3.5. A bitopological space (X, T_1, T_2) is said to be a $(1,2)$ connected space if X cannot be expressed as the union of two non-empty disjoint $(1,2)$ open sets in X.
It is said to be $(1,2)$ γ-connected if X cannot be expressed as the union of two non-empty disjoint $(1,2)$ γ-open sets of X.

Remark 3.2. The relation between the notions of $(1,2)$ extremally disconnectedness and $(1,2)$ extremally γ-disconnectedness is non-linear, that is these notions are independent of each other.

Example 3.3. In bitopological environment, a $(1,2)$ extremally γ-disconnected space may not be a $(1,2)$ extremally disconnected space.
Let us consider a bitopological space (X, T_1, T_2), where $X = \{a, b, c\}, T_1 = \{\phi, X, \{a\}\}$ and $T_2 = \{\phi, X, \{c\}, \{a, c\}\}$. Then $(1,2)$ $O(X) = \{\phi, X, \{a\}, \{c\}, \{a, c\}\}$ and so $(1,2)$ $cl(X) = \{\phi, X, \{b\}, \{a, b\}, \{b, c\}\}$. Now $T_1T_2\text{-}cl\{a\} = \{a, b\} \notin (1,2)$ $O(X)$. Thus X is not a $(1,2)$ extremally disconnected space. Again, on the other hand $(1,2)$ $\gamma\text{-}O(X) = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$ and so $(1,2)$ $\gamma\text{-}cl(X) = \{\phi, X, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$. Clearly it can be observed that $(1,2)$ γ-*closure* of every $(1,2)$ γ-open set in X is a $(1,2)$ γ-open set in X. Hence X is a $(1,2)$ extremally γ-disconnected space.

Example 3.4. A $(1,2)$ extremally disconnected space may not be a $(1,2)$ extremally γ-disconnected space.
We consider the space (X, T_1, T_2) with $X = \{a, b, c\}, T_1 = \{\phi, X, \{b, c\}\}$ and $T_2 = \{\phi, X, \{a\}\}$. Then $(1,2)$ $O(X) = (1,2)$ $cl(X) = \{\phi, X, \{a\}, \{b, c\}\}$ and so obviously X is a $(1,2)$ extremally disconnected space. Also $(1,2)$ $PO(X) = \{\phi, X, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\}$ and $(1,2)\gamma\text{-}O(X) = \{\phi, X, \{b\}, \{c\}, \{b, c\}\}$. Thus $(1,2)\gamma\text{-}cl(X) = \{\phi, X, \{a, c\}, \{a, b\}, \{a\}\}$. Now $\{b\} \in (1,2)$ $\gamma\text{-}O(X)$ but $(1,2)cl_\gamma\{b\} = \{a, b\} \notin (1,2)\gamma\text{-}O(X)$. Therefore X is not a $(1,2)$ extremally γ-disconnected space.

Theorem 3.1. *Let (X, T_1, T_2) be a bitopological space which is $(1,2)$ γ-hyperconnected. Then, it is a $(1,2)$ extremelly γ-disconnected space.*

Proof. Suppose that, X is a $(1,2)$ γ-hyperconnected space. Then, for every $(1,2)$ γ-open set A of X, $(1,2)$ $cl_\gamma(A) = X$, which is itself a $(1,2)$ γ-open set. Hence the theorem. □

Remark 3.3. A bitopological space which is $(1,2)$ extremelly γ-disconnected may not be a $(1,2)$ γ-hyperconnected space.

Example 3.5. Let $X = \{a, b, c\}$ be any non-empty set and T_1, T_2 be two topologies defined on the set X, such that, $T_1 = \{\phi, X, \{a\}, \{b, c\}\}$ and $T_2 = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$. Then, $(1,2)$ $PO(X) = \{\phi, X, \{a\}, \{b, c\}\}$ and $(1,2)$ $\gamma\text{-}O(X) = \{\phi, X, \{a\}, \{b, c\}\}$. Thus it is observed that $(1,2)$ γ-closure of all the $(1,2)$ γ-open sets is again a $(1,2)$ γ-open set. Therefore X is a $(1,2)$ extremally γ-disconnected space. But $(1,2)$ $cl_\gamma\{a\} = \{a\} \neq X$. Hence X is not a $(1,2)$ γ-hyperconnected space.

Theorem 3.2. *Every bitopological space which is $(1,2)$ γ-hyperconnected, is necessarily a $(1,2)$ γ-connected space.*

Proof. The proof is obvious from the definitions of $(1,2)$ γ-hyperconnected space and $(1,2)$ γ-connected space and hence ignored. $\square$

Remark 3.4. A bitopological $(1,2)$ γ-connected space may not be $(1,2)$ γ-hyperconnected. It can be verified from the following example.

Example 3.6. We consider the bitopological space given in example 3.1. It can be easily observed that, $\{a\}$ and $\{b\}$ are the only non-empty disjoint $(1,2)$ γ-open sets, but $\{a\} \cup \{b\} = \{a,b\} \neq X$. Therefore, X is $(1,2)$ γ-connected. Whereas, it is demonstrated in example 1 that X is not $(1,2)$ γ-hyperconnected.

Remark 3.5. In a bitopological space the idea of $(1,2)$ connectedness and $(1,2)$ γ-connectedness are both independent.

Example 3.7. A $(1,2)$ γ-connected space may not be a $(1,2)$ connected space.
Let us consider the bitopological space considered in example 3.2. In that case $(1,2)$ $O(X) = \{\phi, X, \{b\}, \{a,b\}, \{b,c\}, \{a,c\}\}$ and $(1,2)$ γ-$O(X) = \{\phi, X, \{b\}, \{a,b\}\}$. Now $\{b\}$ and $\{a,c\}$ are the only disjoint non-empty $(1,2)$ open sets and $\{b\} \cup \{a,c\} = X$. Thus X is not a $(1,2)$ connected space. Again there are no disjoint non-empty $(1,2)$ γ-open sets in X and so X is automatically a $(1,2)$ γ-connected space.

Example 3.8. A $(1,2)$ connected space may not be a $(1,2)$ γ-connected space.
Let us consider the bitopological space (X, T_1, T_2), where $X = \{a,b,c\}, T_1 = \{\phi, X, \{a\}, \{b,c\}\}$ and $T_2 = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}\}$. Then $(1,2)$ $O(X) = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}$ and $(1,2)$ γ-$O(X) = \{\phi, X, \{a\}, \{b,c\}\}$. Here $\{a\}$ and $\{b,c\}$ are the only non-empty disjoint $(1,2)$ γ-open sets in X and $\{a\} \cup \{b,c\} = X$, therefore X is not a $(1,2)$ γ-connected space. Again, $\{a\}$ and $\{b\}$ are two non-empty disjoint $(1,2)$ open sets in X, but $\{a\} \cup \{b\} = \{a,b\} \neq X$. Thus X is a $(1,2)$ connected space.

Theorem 3.3. *Let (X, T_1, T_2) be a bitopological space which is $(1,2)$ γ-hyperconnected and $A \subseteq X$ be a non-empty subset. Then A is either a $(1,2)$ γ-open set or $(1,2)$ nowhere γ-dense set.*

Proof. We suppose $A \subseteq X$ is not nowhere γ-dense set. Then, $(1,2)$ $int_\gamma((1,2)$ $cl_\gamma(A)) \neq \phi$. Then $(1,2)$ $cl_\gamma(X - (1,2)$ $cl_\gamma(A)) = X - (1,2)$ $int_\gamma((1,2)$ $cl_\gamma(A)) \neq X$. So, $(1,2)$ $cl_\gamma((1,2)$ $int_\gamma((1,2)$ $cl_\gamma(A))) = X$. Again, $(1,2)$ $cl_\gamma((1,2)$ $int_\gamma((1,2)$ $cl_\gamma(A))) \subseteq (1,2)$ $cl_\gamma(A)$, which implies $X \subseteq (1,2)$ $cl_\gamma(A)$. Therefore, $(1,2)$ $cl_\gamma(A) = X$. Thus, A is a $(1,2)$ γ-dense set. Hence the theorem. $\square$

To characterize bitopological γ-hyperconnected space, we now introduce $(1,2)$ γ-regular open set.

Definition 3.6. A subset A of a bitopolgical space (X, T_1, T_2) is said to be $(1,2)$ γ-regular open if $A = (1,2)$ $int_\gamma((1,2)$ $cl_\gamma(A))$.

Theorem 3.4. *Any bitopological space (X, T_1, T_2) is a $(1,2)$ γ-hyperconnected space iff there exists no $(1,2)$ γ-regular open set in X other than ϕ and X itself.*

Proof. Let (X, T_1, T_2) be a $(1,2)$ γ-hyperconnected space. Suppose $A \subset X$ be a $(1,2)$ γ-regular open set. Then $A = (1,2)\ int_\gamma((1,2)\ cl_\gamma(A))$, which implies, $A^c = ((1,2)\ int_\gamma((1,2)\ cl_\gamma(A)))^c = (1,2)\ cl_\gamma(X - (1,2)\ cl_\gamma(A))$. Now, since $A \neq \phi$, so $A^c = (1,2)\ cl_\gamma(X - ((1,2)\ cl_\gamma(A))) \neq X$. But this is a contradiction to the fact that $X - ((1,2)\ cl_\gamma(A))$ is a $(1,2)$ γ-open set and so $(1,2)\ cl_\gamma(X - ((1,2)\ cl_\gamma(A))) = X$. Thus ϕ and X are the only $(1,2)$ γ-regular open subsets in X.
Conversely, suppose ϕ and X are the only $(1,2)$ γ-regular open set in X. If possible let us suppose that X is not a $(1,2)$ γ-hyperconnected space. This implies that, there exists a $(1,2)$ γ-open set A in X such that $(1,2)\ cl_\gamma(A) \neq X$. Then $(1,2)\ cl_\gamma((1,2)\ int_\gamma(A)) = \phi$, which is not possible. Thus X is a $(1,2)$ γ-hyperconnected space. $\square$

Definition 3.7. A subset A of a bitopological space X is said to be a $(1,2)$ γ-semi open set if $A \subseteq (1,2)\ cl_\gamma((1,2)\ int_\gamma(A))$. The collection of all $(1,2)$ γ-semi open sets in a bitopological space (X, T_1, T_2) is denoted by $(1,2)$ γ-$SO(X)$.

Theorem 3.5. *In a bitopological $(1,2)$ γ-hyperconnected space any subset A is $(1,2)$ γ-semi open set if $(1,2)\ int_\gamma(A) \neq \phi$.*

Proof. Let X be a bitopological space which is $(1,2)$ γ-hyperconnected and $A\ (\neq \phi) \subseteq X$, where $(1,2)\ int_\gamma(A) \neq \phi$. Then, $(1,2)\ cl_\gamma((1,2)\ int_\gamma(A)) = X$ and so $A \subseteq (1,2)\ cl_\gamma((1,2)\ int_\gamma(A))$. Thus every $A \subseteq X$ is a $(1,2)$ γ-semi open set if $(1,2)\ int_\gamma(A) \neq \phi$. $\square$

Remark 3.6. Let X be a bitopological space which is $(1,2)$ γ-hyperconnected. Then the collection of all $(1,2)$ γ-semi open sets in X forms a topology. We prove the finite intersection condition to form a topology in the following proposition.

Proposition 3.1. *Finite intersection of $(1,2)$ γ-semi open sets is a $(1,2)$ γ-semi open set in a bitopological $(1,2)$ γ-hyperconnected space.*

Proof. Let us suppose that A and B be two non-empty $(1,2)$ γ-semi open sets in X. So, $A \subseteq (1,2)\ cl_\gamma((1,2)\ int_\gamma(A))$, $B \subseteq (1,2)\ cl_\gamma((1,2)\ int_\gamma(B))$. Consequently, $(1,2)\ cl_\gamma(A) = (1,2)\ cl_\gamma((1,2)\ int_\gamma(A)) = X$ and $(1,2)\ cl_\gamma(B) = (1,2)\ cl_\gamma((1,2)\ int_\gamma(B)) = X$. Now, $(1,2)\ cl_\gamma((1,2)\ int_\gamma(A \cap B)) = (1,2)\ cl_\gamma((1,2)\ int_\gamma(A)) \cap (1,2)\ cl_\gamma((1,2)\ int_\gamma(B)) = X$. Thus $A \cap B \subseteq (1,2)\ cl_\gamma((1,2)\ int_\gamma(A)) \cap (1,2)\ cl_\gamma((1,2)\ int_\gamma(B)) = (1,2)\ cl_\gamma((1,2)\ int_\gamma(A \cap B))$. Hence $A \cap B$ is a $(1,2)$ γ-semi open set. $\square$

4. $(1,2)$ Maximal γ-Hyperconnected Space and $(1,2)$ γ-Door Space

In this particular section, we introduce $(1,2)$ maximal γ-hyperconnectedness in a bitopological space. We characterize $(1,2)$ γ-hyperconnected space and $(1,2)$ maximal γ-hyperconnected space using filter and ultra filter. We also define $(1,2)$ γ-door space and establish the relationship of this space with $(1,2)$ γ-hyperconnected space and $(1,2)$ maximal γ-hyperconnected space.
Before going to the main results, here we put the definitions of filter and ultrafilter from the available literature.

- Let S be a non-empty set. A filter on S is a subset P of the power set $P(S)$ of S with the following properties:
  (i) $S \in P$ and $\phi \notin P$
  (ii) $A \in P$ and $A \subseteq B \subseteq S \Rightarrow B \in P$
  (iii) $A, B \in P \Rightarrow A \cap B \in P$.
- Let P be filter on a non-empty set S. Then P is an ultrafilter if for any $A \subseteq S$, either A or $S - A$ is in P.

Theorem 4.1. *A bitpological space (X, T_1, T_2) is $(1,2)$ γ-hyperconnected iff $(1,2)$ γ-$SO(X) - \{\phi\}$ is a filter on X.*

Proof. Let X be a bitopological space which is $(1,2)$ γ-hyperconnected. If $A, B \in (1,2)$ γ-$SO(X) - \{\phi\}$ then, there exists $(1,2)$ γ-open sets U and V such that, $U \subseteq A$ and $V \subseteq B$. But since X is a $(1,2)$ γ-hyperconnected space, $U \cap B \neq \phi$ and thus $A \cap B \neq \phi$. Therefore, $A \cap B \in (1,2)$ γ-$SO(X) - \{\phi\}$.
On the other hand, let $A \in (1,2)$ γ-$SO(X) - \{\phi\}$ and $A \subseteq B$. Then, there exists a non-empty $(1,2)$ γ-open set U such that $U \subseteq A \subseteq B$ and so, $B \in (1,2)$ γ-$SO(X) - \{\phi\}$. Hence $(1,2)$ γ-$SO(X) - \{\phi\}$ is a filter in X.
Since every $(1,2)$ γ-open set is $(1,2)$ γ-semi open, then the sufficient part is obvious. □

Remark 4.1. The pair $(X, (1,2)$ γ-$SO(X))$ is $(1,2)$ γ-hyperconnected, since $(1,2)$ γ-$SO(X) - \{\phi\}$ is a filter in X.

Remark 4.2. From the previous result it is clear that, if (X, T_1, T_2) is a $(1,2)$ γ-hyperconnected space then, (X, S_1, S_2) is also a $(1,2)$ γ-hyperconnected space, where S_1, S_2 are the collections of $(1,2)$ γ-semi open sets in T_1, T_2 respectively.

Now we will characterize $(1,2)$-hyperconnectedness via maximality in a bts. For that we recall the definition of maximality in a bts.

Definition 4.1. [9] A bts (X, τ, σ) is called a pairwise maximal P-space with a property P if (X, τ', σ') has the property P with $\tau \subseteq \tau'$ and $\sigma \subseteq \sigma'$, then $\tau = \tau'$ and $\sigma = \sigma'$.

Theorem 4.2. *If a bitopological space is $(1,2)$ maximal γ-hyperconnected then, $(1,2)$ γ-$SO(X) - \{\phi\}$ is an ultrafilter on X.*

Proof. Let us suppose (X, T_1, T_2) is a $(1,2)$ maximal γ-hyperconnected space. For any $E \subseteq X$, suppose $E \notin (1,2)$ γ-$SO(X) - \{\phi\}$. Then, E is not a $(1,2)$ γ-open set. We consider the simple extensions $T_1(E)$, $T_2(E)$ of T_1, T_2 respectively. Since (X, T_1, T_2) is $(1,2)$ maximal γ-hyperconnected, $(X, T_1(E), T_2(E))$ is not $(1,2)$ maximal γ-hyperconnected. Thus, there exists non-empty subsets $G, H \in (X, T_1(E), T_2(E))$ such that $G \cap H \neq \phi$. Let, $G = G_1 \cup (G_2 \cap E), H = H_1 \cup (H_2 \cap E)$, where G_1, G_2, H_1, H_2 are $(1,2)$ γ-open sets. Then, $G_1 \cap H_1 = \phi$. Since (X, T_1, T_2)

is a $(1,2)$ γ-hyperconnected space, either $G_1 = \phi$ or $H_1 = \phi$. Suppose $G_1 = \phi$. If $H_1 = \phi$, then $G_2 \neq \phi, H_2 \neq \phi$, since $G \neq \phi, H \neq \phi$. Thus by $(1,2)$ γ-hyperconnectivity, $G_2 \cap H_2 \neq \phi$. Again, since, $G \cap H = \phi$, we have $G_2 \cap H_2 \cap E = \phi$. Hence $G_2 \cap H_2 \subseteq E^c$. Therefore, $E^c \in (1,2)$ γ-$SO(X) - \{\phi\}$.
Now, we consider $H_1 \neq \phi$. Since $G \neq \phi$, we have $G_2 \neq \phi$. Then, $G_2 \cap H_1 \neq \phi$. From the relation $G \cap H = \phi$, it follows that, $(G_2 \cap E) \cap H_1 = \phi$. Hence, $G_2 \cap H_1 \subseteq E^c$ and so $E^c \in (1,2)$ γ-$SO(X) - \{\phi\}$. Therefore, $(1,2)$ γ-$SO(X) - \{\phi\}$ is an ultrafilter. □

Corollary 4.1. *If (X, T_1, T_2) is a $(1,2)$ maximal γ-hyperconnected space then, the collection of all non-empty $(1,2)$ γ-open sets is an ultrafilter.*

Theorem 4.3. *Let (X, T_1, T_2) be a bitopological space such that $(1,2)$ γ-$SO(X) - \{\phi\}$ is an ultrafilter. Then, $(X, (1,2)$ γ-$SO(X))$ is $(1,2)$ maximal γ-hyperconnected.*

Proof. It is obvious that, $(X, (1,2)$ γ-$SO(X))$ is $(1,2)$ γ-hyperconnected, because $(1,2)$ γ-$SO(X))$ is a filter. Suppose that, it is not a $(1,2)$ maximal γ-hyperconnected space. Then, there exists another $(1,2)$ γ-hyperconnected space (X, T_1^*, T_2^*) such that $(1,2)$ γ-$SO(X, T_1^*, T_2^*) \subset (1,2)$ γ-$O(X, T_1^*, T_2^*)$. Thus, $(1,2)$ γ-$SO(X, T_1, T_2) \subset (1,2)$ γ-$SO(X, T_1^*, T_2^*)$, which leads to a contradiction, since $(1,2)$ γ-$SO(X, T_1^*, T_2^*) - \{\phi\}$ is a filter which is greater than $(1,2)$ γ-$SO(X, T_1, T_2)$, but $(1,2)$ γ-$SO(X, T_1, T_2)$ is ultrafilter. Hence the result. □

Definition 4.2. A bitopological space (X, T_1, T_2) is said to be a $(1,2)$ γ-door space if for any $E \subseteq X$, $E \in (1,2)$ γ-$O(X)$ or $E^c \in (1,2)$ γ-$O(X)$.

Remark 4.3. If a bitopological space is $(1,2)$ γ-door then it is not necessarily true that the topological spaces $(X, T_1), (X, T_2)$ are also $(1,2)$ γ-door.

Example 4.1. Let (X, T_1, T_2) be a bitopological space such that $X = \{a, b, c\}$, $T_1 = \{\phi, X, \{a\}, \{b\},$
$\{a, b\}\}, T_2 = \{\phi, X, \{a\}, \{a, c\}\}$. Then $(1,2)$ $PO(X) = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$ and $(1,2)$ γ-$O(X) = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$. Clearly (X, T_1, T_2) is a $(1,2)$ γ-door space. Now $\{c\}, \{a, b\} \notin T_1$. Therefore, (X, T_1) is not a $(1,2)$ γ-door space. Again, since $\{c\}, \{a, b\} \notin T_2$, then (X, T_2) is also not a $(1,2)$ γ-door space.

Theorem 4.4. *If (X, T_1, T_2) is $(1,2)$ γ-hyperconnected and $(1,2)$ γ-door, then it is $(1,2)$ maximal γ-hyperconnected and $(1,2)$ minimal γ-door in X.*

Proof. Suppose (X, T_1, T_2) and (X, T_1^*, T_2^*) be two bitopological spaces such that (X, T_1^*, T_2^*) is stronger than (X, T_1, T_2). We suppose (X, T_1, T_2) is $(1,2)$ γ-hyperconnected and $(1,2)$ γ-door. If possible let us suppose that (X, T_1^*, T_2^*) is also $(1,2)$ γ-hyperconnected. We denote the collection of all $(1,2)$ γ-open sets of (X, T_1^*, T_2^*) by $(1,2)$ γ-$O^*(X)$. Then obviously, $(1,2)$ γ-$O(X) \subset (1,2)$ γ-$O^*(X)$. Let G be a non-empty set such that $G \in (1,2)$ γ-$O^*(X)$, $G \notin (1,2)$ γ-$O(X)$. Now, since (X, T_1, T_2) is $(1,2)$ γ-door, therefore $X - G \in (1,2)$ γ-$O(X)$ and thus $X - G \in (1,2)$ γ-$O^*(X)$. Therefore $G, X - G$ are non-empty $(1,2)$ γ-open sets in (X, T_1^*, T_2^*)

with $G \cap (X - G) \neq \phi$, which contradicts to our assumption that (X, T_1^*, T_2^*) is $(1,2)$ γ-hyperconnected. Thus we can say that, there does not exist any $(1,2)$ γ-hyperconnected space stronger than (X, T_1, T_2). Hence, (X, T_1, T_2) is $(1,2)$ maximal γ-hyperconnected.
Again, suppose that (X, T_1, T_2) is not a minimal $(1,2)$ γ-door space. Then there exists another bitopology (X, T_1^*, T_2^*) on X, weaker than (X, T_1, T_2), which is also $(1,2)$ γ-door. Let G be a non-empty subset of X such that $G \in (1,2)$ γ-$O(X), G \notin (1,2)$ γ-$O^*(X)$. This implies $X - G \in (1,2)$ γ-$O^*(X)$. Thus, $G, X - G$ are non-empty $(1,2)$ γ-open sets in (X, T_1, T_2). This is a contradiction, since $G \cap X - G = \phi$. Hence, (X, T_1, T_2) is $(1,2)$ minimal γ-door. $\square$

Definition 4.3. A bitopological space (X, T_1, T_2) is said to be $(1,2)$ submaximal if every $(1,2)$ γ-dense subset of X is a $(1,2)$ γ-open set.

Theorem 4.5. *A bitopological space (X, T_1, T_2) is a $(1,2)$ maximal γ-hyperconnected space iff it is $(1,2)$ submaximal and $(1,2)$ γ-hyperconnected.*

Proof. Suppose (X, T_1, T_2) is a $(1,2)$ maximal γ-hyperconnected space. Let $E \subset X$ be a $(1,2)$ γ-dense set. Then by corollary 4.5, we have, $(1,2)$ γ-$O(X) - \{\phi\}$ is an ultrafilter. So E must be a $(1,2)$ γ-open set. For if E is not a $(1,2)$ γ-open set, then E^c must be $(1,2)$ γ-open since $(1,2)$ γ-$O(X) - \{\phi\}$ is an ultrafilter. In that case, E is a $(1,2)$ γ-closed set and hence $(1,2)$ $cl_\gamma(E) = E$. Again, since E is a $(1,2)$ γ-dense set, $(1,2)$ $cl_\gamma(E) = X$. Therefore, $E = X$. Thus X is a $(1,2)$ submaximal space.
Conversely suppose (X, T_1, T_2) is $(1,2)$ γ-submaximal and $(1,2)$ γ-hyperconnected. Let (X, T_1^*, T_2^*) be a $(1,2)$ γ-hyperconnected space stronger than (X, T_1, T_2). Then, $(1,2)$ γ-$O(X) \subseteq (1,2)$ γ-$O^*(X)$. If G be any non-empty $(1,2)$ γ-open set in (X, T_1^*, T_2^*), then $(1,2)$ $cl_\gamma(G) = X$ in (X, T_1^*, T_2^*). This implies, $(1,2)$ $cl_\gamma(G) = X$ in (X, T_1, T_2). Thus G is $(1,2)$ γ-dense in (X, T_1, T_2). Thus G is a $(1,2)$ γ-open set, that is $G \in (1,2)$ γ-$O(X)$. Therefore, $(1,2)$ γ-$O(X) = (1,2)$ γ-$O^*(X)$. Hence (X, T_1, T_2) is a $(1,2)$ maximal γ-hyperconnected space. $\square$

5. Results on Some Related Functions

In this section we define the notions of $(1,2)$ γ-feebly continuous function, $(1,2)$ γ-semi continuous function, $(1,2)$ γ-almost continuous function and $(1,2)\gamma$-contra continuous function in a bitopological space. We establish some properties related to these functions and also the interrelationship between these functions.

Definition 5.1. A function $f : (X, T_1, T_2) \to (Y, \sigma_1, \sigma_2)$ from a bitopological space X to another bitopological space Y is said to be a $(1,2)$ γ-feebly continuous function if for every $(1,2)$ γ-open set B of Y, $f^{-1}(B) \neq \phi$.

Definition 5.2. A function $f : (X, T_1, T_2) \to (Y, \sigma_1, \sigma_2)$ from a bitopological space X to another bitopological space Y is said to be a $(1,2)$ γ-semi continuous function if for every σ_i-open set B of Y, $f^{-1}(B)$ is $(1,2)$ γ-semi open set in Y.

Theorem 5.1. *Every* $(1,2)$ γ*-semi continuous function is a* $(1,2)$ γ*-feebly continuous function.*

Proof. It is obvious. $\square$

Remark 5.1. Converse of the above theorem is not true, that is, a $(1,2)$ γ-feebly continuous function may not be a $(1,2)$ γ-semi continuous function.

Example 5.1. Let us consider two bitopological spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2) with $X = \{a, b, c\}, Y = \{1, 2, 3\}, \tau_1 = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}, \tau_2 = \{\phi, X, \{b\}, \{c\}, \{b ,c\}\}, \sigma_1 = \{\phi, Y, \{2\}, \{1, 2\}\}, \sigma_2 = \{\phi, Y, \{1\}, \{2, 3\}\}$. We define a function $f : X \to Y$ in such a way that $f(a) = 3, f(b) = f(c) = 2$. Now, $(1,2)$ γ-$O(Y) = \{\phi, Y, \{2\}\}$ and $f^{-1}(\{2\}) = \{b, c\} \neq \phi$. Also, $(1,2)$ $int_\gamma(f^{-1}(\{2\}) = (1,2)$ $int_\gamma\{b, c\} = \{b\} \neq \phi$. Therefore, f is a $(1,2)$ γ-feebly continuous function. But since $\{b, c\}$ is not $(1,2)$ γ-semi open in X, thus f is not a $(1,2)$ γ-semi continuous function.

Nonetheless the converse of the theorem 5.1 is true if we add the condition of $(1,2)$ γ-hyperconnectedness. It is explained in the theorem below.

Theorem 5.2. *Every* $(1,2)$ γ*-feebly continuous function is a* $(1,2)$ γ*-semi continuous function in a* $(1,2)$ γ*-hyperconnected space.*

Proof. Let us consider two bitopological space (X, τ_1, τ_2) and (Y, σ_1, σ_2) and $f : X \to Y$ be any function which is $(1,2)$ γ-feebly continuous. We suppose λ be a $(1,2)$ γ-open set in Y. Then, $f^{-1}(\lambda) \neq \phi$ and therefore $(1,2)$ $int_\gamma(f^{-1}(\lambda)) \neq \phi$. Now, since X is $(1,2)$ γ-hyperconnected, so $f^{-1}(\lambda)$ is a $(1,2)$ γ-semi open set in X. Hence f is a $(1,2)$ γ-semi continuous function. $\square$

Definition 5.3. A function $f : (X, T_1, T_2) \to (Y, \sigma_1, \sigma_2)$ from a bitopological space X to another bitopological space Y is said to be a $(1,2)$ γ-almost continuous function if the inverse of each $(1,2)$-regular open set of Y is $(1,2)$ γ-open set in X.
f is said to be a $(1,2)$ γ-contra continuous function if the inverse of each σ_2-open set is $(1,2)$ γ-closed set in X.

Theorem 5.3. *If* $f : (X, T_1, T_2) \to (Y, \sigma_1, \sigma_2)$ *be a* $(1,2)$ γ*-almost continuous function from a bitopological space* X *to another bitopological space* Y*. Then* f *is* $(1,2)$ γ*-contra continuous.*

Proof. Let us suppose that f is $(1,2)$ γ-almost continuous. Then by definition, $f(A) \subseteq \sigma_1$-$int(\sigma_2$-$cl(B))$, where A is a $(1,2)$ γ-open set in X and B is a σ_1-open set in Y. Then, $f(A) \subseteq \sigma_2$-$cl(B)$. Hence f is $(1,2)$ γ-contra continuous. $\square$

Remark 5.2. A $(1,2)$ γ-contra continuous function between two bitopological spaces may not be a $(1,2\ )\gamma$-almost continuous function therein. It is verified in the next example.

Example 5.2. Let us consider two bitopological spaces (X, T_1, T_2) and (Y, σ_1, σ_2) with $X = \{a, b, c\}$, $Y = \{1, 2, 3\}$, $T_1 = \{\phi, X, \{a\}, \{a, b\}\}$, $T_2 = \{\phi, X, \{b\}, \{a, b\}\}, \sigma_1 = \{\phi, Y, \{1\}, \{2\}, \{1, 2\}\}$ and $\sigma_2 = \{\phi, Y, \{1\}, \{1, 2\}\}$. Then, $(1,2)$ $PO(X) = \{\phi, X, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$, $(1,2)$ γ-$O(X) = \{\phi, X, \{a\}, \{c\}, \{a, c\}\}$ and $(1,2)$ γ-$cl(X) = \{\phi, X, \{b\}, \{b, c\}, \{a, b\}\}$. Also, $(1,2)$ $PO(Y) = \{\phi, Y, \{1\}, \{2\}, \{1, 2\}, \{2, 3\}\}$, $(1,2)$ γ-$O(Y) = \{\phi, Y, \{1\}, \{2\}, \{1, 2\}, \{2, 3\}\}$. Now, we consider a function $f : X \to Y$ defined as $f(a) = 2, f(b) = 1, f(c) = 3$. Then, $f^{-1}(\{1\}) = \{b\}, f^{-1}(\{1, 2\}) = \{b, c\}$ and both are $(1,2)$ γ-closed sets in X. Then f is $(1,2)$ γ-contra continuous function. Again $\{1\}$ is $(1,2)$ regular open in Y and $f^{-1}(\{1\}) = \{b\}$, which is not a $(1,2)$ γ-open set in X. Thus f is not a $(1,2)$ γ-almost continuous function.

6. Conclusion

In this paper we presented a non-linear structure in the environment of bitopological space passing through γ-open sets. We established that the notions of $(1,2)$ hyperconnectedness and $(1,2)$ γ-hyperconnectedness are completely independent of each other in the same structure. In the same space, the collection of all $(1,2)$ γ-semi open sets does not form topology, rather it forms a supratopology. But in our study we proved that the collection of all $(1,2)$ γ-semi open sets constitutes a topology if the space is a $(1,2)$ γ-hyperconnected space. Also, it has been shown that in a $(1,2)$ γ-hyperconnected space (X, T_1, T_2), the collections of all $(1,2)$ γ-semi open sets with respect to both the topologies T_1 and T_2 together with the set X form another $(1,2)$ γ-hyperconnected space. We characterized $(1,2)$ γ-hyperconnected space with the help of filters and ultrafilters. Finally, different types of functions viz. $(1,2)$ γ-semi continuous functions, $(1,2)$ γ-feebly continuous functions, $(1,2)$ γ-almost continuous functions and $(1,2)$ γ-contra continuous functions are introduced and the interrelationship among these functions are discussed.

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