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FIXED POINT RESULTS FOR $(\alpha - \beta)$-ADMISSIBLE ALMOST Z-CONTRACTIONS IN METRIC-LIKE SPACE VIA SIMULATION FUNCTION

Archana Dewangan[1], Anil Kumar Dubey[2], Urmila Mishra[3] and R. P. Dubey[1]

[1] **Department of Mathematics, Dr. C. V. Raman University**
Kargi Road, Kota, Bilaspur-495113 (Chhattisgarh), India
[2] **Department of Mathematics, Bhilai Institute of Technology**
Bhilai House, Durg (Chhattisgarh), India
[3] **Department of Mathematics, Vishwavidyalaya Engineering College**
(A constituent college of CSVTU, Bhilai)
Ambikapur-497116 (Chhattisgarh), India

Abstract. In this paper, we establish the existence and uniqueness of a fixed point of (α, β)-admissible almost Z-contractions via simulation functions in metric-like spaces. Our results generalize and unify several fixed point theorem in literature.
Key words: fixed point, metric-like space, simulation function.

1. Introduction

The well known Banach contraction principle [8] established the existence and uniqueness of fixed point of a contraction on a complete metric space. Since then, several authors generalized this principle by introducing the various contractions on usual metric spaces such that as b-metric space, partial metric space, metric-like space etc. As generalizations if standard metric spaces, metric-like spaces were considered first by Hitzler and Seda [13] under the name of dislocated metric spaces.

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Corresponding Author: Urmila Mishra, Department of Mathematics, Vishwavidyalaya Engineering College, (A constituent college of CSVTU, Bhilai), Ambikapur-497116 (Chhattisgarh), India | E-mail: mishra.urmila@gmail.com
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Afterwards Amini-harandi [2] proved some fixed point results in the class of metric-like space. Very recently many authors have obtained fixed point results in the setting of metric-like spaces, for example see [1, 4, 5, 6, 19, 24]. Let us recall some notations and definitions we will need in the sequel.

2. Preliminaries

Definition 2.1. ([2, 5]) Let X be a non empty set. A function $\sigma : X \times X \to R^+$ is said to be a metric-like (or a dislocated metric) on X, if for any $x, y, z \in X$ the following conditions hold true:

(σ_1) $\sigma(x, y) = 0 \quad \text{implies} \quad x = y$;

(σ_2) $\sigma(x, y) = \sigma(y, x)$;

(σ_3) $\sigma(x, z) \leq \sigma(x, y) + \sigma(y, z)$.

The pair (X, σ) is then called a metric-like space.

Then a metric-like on X satisfies all conditions of a metric except that $\sigma(x, x)$ may be positive for $x \in X$. Each metric-like σ on X generates a topology τ_σ on X, whose base is the family of open σ-balls, then for all $x \in X$ and $\epsilon > 0$

$$B_\sigma(X, \epsilon) = \{y \in X : \sigma(x, y) - \sigma(x, x) < \epsilon\}.$$

Now, let (X, σ) be a metric-like space. A sequence $\{x_n\}$ in the metric-like space (X, σ) converges to a point $x \in X$ if and only if $\lim_{n\to+\infty} \sigma(x_n, x) = \sigma(x, x)$.

Let (X, σ) be metric-like space, and let $T : X \to X$ be a continuous mapping. Then $\lim_{n\to+\infty} x_n = x$ implies $\lim_{n\to+\infty} T(x_n) = T(x)$.

A sequence $\{x_n\}$ is Cauchy in (X, σ), if and only if $\lim_{n,m\to+\infty} \sigma(x_m, x_n)$ exists and is finite. Moreover, the metric-like space (X, σ) is called complete, if and only if for every Cauchy sequence $\{x_n\}$ in X, there exists $x \in X$ such that $\lim_{n\to+\infty} \sigma(x_n, x) = \sigma(x, x) = \lim_{n,m\to+\infty} \sigma(x_n, x_m)$.

Every partial metric space and metric space is a metric-like space.

Example 2.1. ([24]) Let $X = \{1, 2, 3\}$ and $\sigma(x, y) = \begin{cases} 3 : & \text{if} \quad x = y \\ 2 : & \text{otherwise.} \end{cases}$

Then (X, σ) is a metric-like space. It is neither a partial metric space ($\sigma(1, 1) = 3$) and ($\sigma(1, 2) = 2$) nor a metric space ($\sigma(1, 1) = 3 \neq 0$).

Remark 2.1. ([1]) A subset A of a metric-like space (X, σ) is bounded if there is a point $b \in X$ and a positive constant k such that $\sigma(a, b) \leq k$ for all $a \in A$.

Remark 2.2. ([1, 2]) Let $X = \{0, 1\}$ such that $\sigma(x, y) = 1$ for each $x, y \in X$ and let $x_n = 1$ for each $n \in N$. Then it is easy to see that $x_n \to 0$ and $x_n \to 1$ and so in metric like space, the limit of a convergence sequence is not necessarily unique.

The following Lemma is useful to prove our results.

Lemma 2.1. ([2, 5, 12]) *Let (X,σ) be a metric-like space. Let $\{x_n\}$ be a sequence in X such that $x_n \to x$, where $x \in X$ and $\sigma(x,y)=0$. Then for all $y \in X$ we have* $\lim_{n\to+\infty}\sigma(x_n,y)=\sigma(x,y)$.

Definition 2.2. ([23]) For a non empty set X, let $T: X \to X$ and $\alpha : X \times X \to [0,+\infty)$ be given mappings. We say that T is α-admissible, if for all $x,y \in X$, we have $\alpha(x,y) \geq 1 \quad \text{implies} \quad \alpha(Tx,Ty) \geq 1$.

The concept of α-admissible mappings has been used in many works, see for example [6, 14, 17, 20, 22]. Later, Karapinar et al. [16] introduced the notion of triangular α-admissible mappings.

Definition 2.3. ([16]) Let $T: X \to X$ and $\alpha : X \times X \to [0,+\infty)$ be given mappings. A mapping $T: X \to X$ is called a triangular α-admissible if:

(T_1) T is α-admissible;

(T_2) $\alpha(x,y) \geq 1$ and $\alpha(y,z) \geq 1 \quad \text{implies} \quad \alpha(x,z) \geq 1$ for all $x,y,z \in X$.

Chandok [11] introduced the concept of (α,β)-admissible Geraghty type contractive mapping, with sufficient condition for the existence of a fixed point for such class of generalized non-linear contractive mapping in metric space proved some fixed point results.

Definition 2.4. ([11]) Let X be a non empty set, $T: X \times X$ and $\alpha,\beta : X \times X \to R^+$, we say that T is an (α,β)-admissible mapping if $\alpha(x,y) \geq 1$ and $\beta(x,y) \geq 1$ implies $\alpha(Tx,Ty) \geq 1$ and $\beta(Tx,Ty) \geq 1$ for all $x,y \in X$.

Berinde [9, 10] extended the class of contractive mappings, introducing the notion of almost contractions as follows.

Definition 2.5. Let (X,d) be a metric space. A self mapping T on X is called an almost contraction if there are constants $\lambda \in (0,1)$ and $\theta \geq 0$ such that

$$d(Tx,Ty) \quad \leq \quad \lambda d(x,y) + \theta d(y,Tx), \ \textit{for all } x,y \in X.$$

Berinde [9] proved that every almost contraction mapping defined in termsw of a complete metric sapce has at least one fixed point. Subsequently, Babu et al. [7] demonstrated that almost contractions type mappings have a unique fixed point under conditions that present the notion of B-almost contraction.

Definition 2.6. Let (X,d) be a metric space. A self mapping T on X is called an B-almost contraction if there are constants $\lambda \in (0,1)$ and $\theta \geq 0$ such that

$$d(Tx,Ty) \quad \leq \quad \lambda d(x,y) + \theta N(x,y) \ \textit{for all } x,y \in X,$$

where $N(x,y) = min\{d(x,Tx), d(y,Ty), d(x,Ty), d(y,Tx)\}$.

Khojasteh et al. [18] presented the notion of Z-contraction involving a new class of mappings, namely simulation function to prove the following Theorem.

Theorem 2.1. *Let (X, d) be a complete metric space and $T : X \to X$ be a Z-contraction with respect to a function ς satisfying certain conditions, that is,*

$$\zeta(d(Tx, Ty), d(x, y)) \geq 0$$

for all $x, y \in X$. Then, T has a unique fixed point, and for every initial point $x_0 \in X$, the Picard sequence $\{T^n x_0\}$ converges to this fixed point.

A simple example of Z-contraction is the Banach contraction, which can be obtained by taking $\lambda \in [0, 1)$ and $\zeta(t, s) = \lambda s - t$ for all $t, s \in [0, +\infty)$ in above result.

Definition 2.7. [18] Let $\zeta : [0, +\infty) \times [0, +\infty) \to R$ be a function, then ζ is called a simulation function if it satisfies the following conditions:

(ζ_1) $\zeta(0, 0) = 0$.

(ζ_2) $\zeta(t, s) < s - t$, for all $t, s > 0$.

(ζ_3) If $(t_n), (s_n)$ are sequences in $(0, +\infty)$ such that $\lim_{n\to+\infty} t_n = lim_{n\to+\infty} s_n > 0$, then $\lim_{n\to+\infty} \sup \zeta(t_n, s_n) < 0$.

(ζ_4) If $(t_n), (s_n)$ are sequences in $(0, +\infty)$ such that $\lim_{n\to+\infty} t_n = lim_{n\to+\infty} s_n > 0$ and $t_n < s_n$ for all $n \in N$, then $\lim_{n\to+\infty} \sup \zeta(t_n, s_n) < 0$.

If the function ζ satisfies the conditions $(\zeta_1) - (\zeta_3)$, we say that ζ is a simulation function according to the sense of Khojasteh et al.[18]. If it satisfies (ζ_2) and (ζ_3), it is a simulation function according to the sense of Argoubi et al.[3] and if it satisfies (ζ_1), (ζ_2) and (ζ_4), then it is a simulation function according to the sense of Roldan-Lopez-de-Hierro et al.[21].

Remark 2.3. ([18]) It is clear from the definition of simulation function that $\zeta(t, s) < 0$ for all $t \geq s > 0$. Therefore if T is a Z-contraction with respect to $\zeta \in z$ then, for all distinct $x, y \in X$ such that $d(Tx, Ty) < d(x, y)$. This shows that every Z-contraction mapping is contraction, therefore it is continuous.

In this study, by combining the ideas in [15] and [24], we prove some fixed point results for (α, β)-admissible almost Z-contraction with respect to ζ. Moreover, one example is given to support the obtained result.

3. Main Results

Firstly, we give the following definition which will be used in our main results.

Definition 3.1. ([15]) Let (X, d) be a metric space and $\zeta \in z$. We say that $T : X \to X$ is an almost Z-contraction if there is a constant $\theta \geq 0$ such that

$$\zeta(\alpha(Tx, Ty), d(x, y) + \theta N(x, y)) \geq 0 \tag{3.1}$$

for all $x, y \in X$, where $N(x, y) = \min\{d(x, Tx), d(y, Ty), d(x, Ty), d(y, Tx)\}$

Remark 3.1. If T is an almost Z-contraction with respect to $\zeta \in Z$, then

$$d(Tx, Ty) < d(x, y) + \theta N(x, y) \tag{3.2}$$

for all $x, y \in X$.

Our main result is as follows.

Theorem 3.1. *Let (X, σ) be a complete metric-like space and a continuous mapping $T : X \to X$ be a (α, β)-admissible almost Z-contraction with respect to a ζ simulation function satisfying as*

$$\zeta(\alpha(Tx, Ty)\beta(Tx, Ty)\sigma(Tx, Ty), \sigma(x, y) + \theta N(x, y)) \geq 0 \tag{3.3}$$

for all $x, y \in X$ and there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1, \beta(x_0, Tx_0) \geq 1$. Then, T has a unique fixed point $u \in X$.

Proof. Let x_n be a sequence in X such that $x_{n+1} = Tx_n$ for all $n = 0, 1, 2, 3....$ If $x_n = x_{n+1}$ then $Tx_n = x_{n+1} = x_n$ i.e. x_n is a fixed point of T. So proof is trivial. Now, we consider $x_n \neq x_{n+1}$ for all $n \in N \cup \{0\}$.

Since $\alpha(x_0, Tx_0) \geq 1$ implies $\alpha(x_0, x_1) \geq 1$ and T is an (α, β)-admissible, so

$$\alpha(Tx_0, Tx_1) \geq 1 \quad \text{implies} \quad \alpha(x_1, x_2) \geq 1.$$

Continuing, we have for all $n \geq 0$

$$\alpha(x_n, x_{n+1}) \geq 1. \tag{3.4}$$

Similarly for all $n \geq 0$, we obtain

$$\beta(x_n, x_{n+1}) \geq 1. \tag{3.5}$$

From (3.3), we have

$$\begin{aligned} 0 \leq\ & \zeta(\alpha(Tx_{n-1}, Tx_n)\beta(Tx_{n-1}, Tx_n)\sigma(Tx_{n-1}, Tx_n), \sigma(x_{n-1}, x_n) \\ & +\theta N(x_{n-1}, x_n)) \end{aligned}$$

i.e.

$$0 \leq \zeta(\alpha(x_n, x_{n+1})\beta(x_n, x_{n+1})\sigma(x_n, x_{n+1}), \sigma(x_{n-1}, x_n) + \theta N(x_{n-1}, x_n)) \tag{3.6}$$

Since

$$\begin{aligned} N(x_{n-1}, x_n) &= min\{\sigma(x_{n-1}, Tx_{n-1}), \sigma(x_n, Tx_n), \sigma(x_{n-1}, Tx_n), \sigma(x_n, Tx_{n-1})\} \\ &= min\{\sigma(x_{n-1}, x_n), \sigma(x_n, x_{n+1}), \sigma(x_{n-1}, x_{n+1}), \sigma(x_n, x_n)\} = 0. \end{aligned}$$

Therefore, from (3.6) and by ζ_2, we have

$$\begin{aligned} 0 &\leq \zeta(\alpha(x_n, x_{n+1})\beta(x_n, x_{n+1})\sigma(x_n, x_{n+1}), \sigma(x_{n-1}, x_n)) \\ &< \sigma(x_{n-1}, x_n) - \alpha(x_n, x_{n+1})\beta(x_n, x_{n+1})\sigma(x_n, x_{n+1}) \end{aligned}$$

$$\alpha(x_n, x_{n+1})\beta(x_n, x_{n+1})\sigma(x_n, x_{n+1}) < \sigma(x_{n-1}, x_n). \tag{3.7}$$

We know,

$$\sigma(x_n, x_{n+1}) \leq \alpha(x_n, x_{n+1})\beta(x_n, x_{n+1})\sigma(x_n, x_{n+1}). \tag{3.8}$$

Since $\alpha(x_n, x_{n+1}) \geq 1$ and $\beta(x_n, x_{n+1}) \geq 1$. From (3.7) and (3.8) for all $n \geq 0$, we have

$$\sigma(x_n, x_{n+1}) \leq \alpha(x_n, x_{n+1})\beta(x_n, x_{n+1})\sigma(x_n, x_{n+1}) < \sigma(x_{n-1}, x_n) \tag{3.9}$$

i.e.

$$\sigma(x_n, x_{n+1}) < \sigma(x_{n-1}, x_n). \tag{3.10}$$

The sequence $\{\sigma(x_n, x_{n+1})\}$ is non increasing. So there exist $r \geq 0$ such that $lim_{n\to+\infty}\sigma(x_{n-1}, x_n) = r$. We prove that

$$\lim_{n\to+\infty} \sigma(x_{n-1}, x_n) = 0. \tag{3.11}$$

Now, we assume on the contrary such that $r > 0$. By (3.9), we have

$$\lim_{n\to+\infty} \{\alpha(x_n, x_{n+1})\beta(x_n, x_{n+1})\sigma(x_n, x_{n+1})\} = r.$$

Since $r > 0$ and letting $s_n = \alpha(x_n, x_{n+1})\beta(x_n, x_{n+1})\sigma(x_n, x_{n+1})$ and $t_n = \sigma(x_n, x_{n+1})$ such that $\lim_{n\to+\infty} s_n = \lim_{n\to+\infty} t_n = r$, then by (ζ_3) $\lim_{n\to+\infty} \sup \zeta(s_n, t_n) < 0$.

Since $\zeta(s_n,t_n)\geq 0$, so $0\leq \lim_{n\to+\infty}\sup\zeta(s_n,t_n)<0$, which is a contradiction. So, our assumption is false. Hence $r=0$. Again we show that $\{x_n\}$ is a Cauchy sequence in (X,σ), i.e.

$$\lim_{n,m\to+\infty}\sigma(x_n,x_m)=0. \tag{3.12}$$

Suppose on the contrary that is $\{x_n\}$ is not a Cauchy sequence. Then there exist $\epsilon>0$ for which we can assume subsequences $x_{n_{(k)}}$ and $x_{m_{(k)}}$ of x_n with $n(k)>m(k)>k$ such that for every k

$$\sigma(x_{n_{(k)}},x_{m_{(k)}})\geq\epsilon \tag{3.13}$$

and $n_{(k)}$ is the smallest number such that (3.13) holds.

From (3.13), we get

$$\sigma(x_{n_{(k)}-1},x_{m_{(k)}})<\epsilon. \tag{3.14}$$

Then by triangular inequality and (3.12), we have

$$\begin{aligned}\epsilon\leq\sigma(x_{n_{(k)}},x_{m_{(k)}}) &\leq \sigma(x_{n_{(k)}},x_{n_{(k)}-1})+\sigma(x_{n_{(k)}-1},x_{m_{(k)}})\\ &< \sigma(x_{n_{(k)}},x_{n_{(k)}-1})+\epsilon.\end{aligned}$$

Taking $n\to+\infty$ in above equation and applying (3.11), we get

$$lim_{n\to+\infty}\sigma(x_{n_{(k)}},x_{m_{(k)}})=\epsilon. \tag{3.15}$$

From the triangular inequality, we have

$$\sigma(x_{n_{(k)}+1},x_{m_{(k)}})\leq\sigma(x_{n_{(k)}+1},x_{n_{(k)}})+\sigma(x_{n_{(k)}},x_{m_{(k)}})$$

taking limit $n\to+\infty$ and using (3.11), (3.13) and (3.15), we have

$$\lim_{n\to+\infty}\sigma(x_{n_{(k)}+1},x_{m_{(k)}})=\epsilon. \tag{3.16}$$

Similarly, it is easy to show that

$$\lim_{n\to+\infty}\sigma(x_{n_{(k)}+1},x_{m_{(k)}+1})=\epsilon. \tag{3.17}$$

Since T is an (α,β)-admissible almost Z-contraction with respect to ζ and using (ζ_3)

$$\begin{aligned} 0 &\leq \lim_{n\to+\infty} Sup\zeta(\alpha(Tx_{n_{(k)}}, Tx_{m_{(k)}})\beta(Tx_{n_{(k)}}, Tx_{m_{(k)}})\sigma(Tx_{n_{(k)}}, Tx_{m_{(k)}}),\\ &\sigma(x_{n_{(k)}}, x_{m_{(k)}}) + \theta N(x_{n_{(k)}}, x_{m_{(k)}})) \end{aligned}$$

$$0 \leq \lim_{n\to+\infty} Sup\zeta(\alpha(x_{n_{(k)}+1}, x_{m_{(k)}+1})\beta(x_{n_{(k)}+1}, x_{m_{(k)}+1}) \quad (3.18)$$

$$\sigma(x_{n_{(k)}+1}, x_{m_{(k)}+1}), \sigma(x_{n_{(k)}}, x_{m_{(k)}}) + \theta N(x_{n_{(k)}}, x_{m_{(k)}})) < 0$$

Since

$$\begin{aligned} N(x_{n_{(k)}}, x_{m_{(k)}}) &= \min\{\sigma(x_{n_{(k)}}, Tx_{n_{(k)}}), \sigma(x_{m_{(k)}}, Tx_{m_{(k)}}), \sigma(x_{n_{(k)}}, Tx_{m_{(k)}}),\\ &\sigma(x_{m_{(k)}}, Tx_{n_{(k)}})\}\\ &= \min\{\sigma(x_{n_{(k)}}, x_{n_{(k)}+1}), \sigma(x_{m_{(k)}}, x_{m_{(k)}+1}), \sigma(x_{n_{(k)}}, x_{m_{(k)}+1}),\\ &\sigma(x_{m_{(k)}}, x_{n_{(k)}+1})\} \end{aligned}$$

taking $n \to +\infty$ and using (3.11), we obtain

$$\lim_{n\to+\infty} N(x_{n_{(k)}}, x_{m_{(k)}}) = 0, \quad (3.19)$$

from (3.18) and (3.19), we have

$$\begin{aligned} 0 &\leq \lim_{n\to+\infty} \sup \zeta(\alpha(x_{n_{(k)}+1}, x_{m_{(k)}+1})\beta(x_{n_{(k)}+1}, x_{m_{(k)}+1})\sigma(x_{n_{(k)}+1}, x_{m_{(k)}+1}),\\ &\sigma(x_{n_{(k)}}, x_{m_{(k)}})) < 0, \end{aligned}$$

which is a contradiction due to our assumption. So, $\{x_n\}$ is a Cauchy sequence. Since X is complete metric-like space, then there exists $x \in X$ and using (3.12) such that

$$\lim_{n\to+\infty} \sigma(x_n, x) = \sigma(x, x) = \lim_{n,m\to+\infty} \sigma(x_n, x_m) = 0. \quad (3.20)$$

Now, we show that x is a fixed point of T. Since T is continuous and $x_n \to x$ as $n \to +\infty$. So from (3.20)

$$\lim_{n\to+\infty} \sigma(x_{n+1}, Tx) = \lim_{n\to+\infty} \sigma(Tx_n, Tx) = \sigma(Tx, Tx) = 0. \quad (3.21)$$

Using Lemma (2.1) and (3.21), we have

$$\lim_{n\to+\infty} \sigma(x_{n+1}, Tx) = \sigma(x, Tx), \quad (3.22)$$

from (3.21) and (3.22),

$$\sigma(x,Tx)=\sigma(Tx,Tx)=0. \tag{3.23}$$

Hence, $Tx=x$, that is x is a fixed point of T. Now, we shall show that the uniqueness of fixed point of x. We argue by contrary. Assume that there exists $u\in X$ such that $Tu=u$ and $x\neq u$. Now,

$$0\leq \zeta(\alpha(Tx,Tu)\beta(Tx,Tu)\sigma(Tx,Tu),\sigma(x,u)+\theta N(x,u)), \tag{3.24}$$

where $N(x,u)=\min\{\sigma(x,Tx),\sigma(u,Tu),\sigma(x,Tu),\sigma(u,Tx)\}$,

$$i.e.\quad N(x,u)=0. \tag{3.25}$$

From (3.24) and (3.25), we have

$$\begin{array}{rcl} 0 & \leq & \zeta(\alpha(Tx,Tu)\beta(Tx,Tu)\sigma(Tx,Tu),\sigma(x,u)) \\ & \leq & \sigma(x,u)-\alpha(Tx,Tu)\beta(Tx,Tu)\sigma(Tx,Tu) \\ & = & \sigma(x,u)-\alpha(x,u)\beta(x,u)\sigma(x,u) \\ & = & \sigma(x,u)[1-\alpha(x,u)\beta(x,u)]<0, \end{array}$$

since $\alpha(x,u)\geq 1,\beta(x,u)\geq 1$, which is a contradiction, so $x=u$. Hence T has a unique fixed point. □

Corollary 3.1. *In Theorem* 3.1*, if we have choose any one of the ζ simulation given below, we have the same result and proof are similar to these corollary.*

$$\zeta(\alpha(x,Tx)\beta(y,Ty)\sigma(Tx,Ty),\sigma(x,y)+\theta N(x,y))\geq 0 \tag{3.26}$$

$$\zeta(\alpha(x,y)\beta(Tx,Ty)\sigma(Tx,Ty),\sigma(x,y)+\theta N(x,y))\geq 0 \tag{3.27}$$

$$\zeta(\alpha(x,y)\beta(x,y)\sigma(Tx,Ty),\sigma(x,y)+\theta N(x,y))\geq 0 \tag{3.28}$$

$$\zeta(\alpha(Tx,Ty)\beta(x,y)\sigma(Tx,Ty),\sigma(x,y)+\theta N(x,y))\geq 0. \tag{3.29}$$

The following example shows that our main result i.e. Theorem 3.1 is a proper generalization of [15] and [24].

Example 3.1. [12] Take $X = [0, +\infty)$ endowed with the metric-like $\sigma(x, y) = x^2 + y^2$. Consider the mapping $T : X \to X$ given by

$$T(x) = \begin{cases} \frac{x^2}{x+1}, & \text{if} \quad x \in [0, 1], \\ x^2, & \text{if} \quad x > 1. \end{cases}$$

Note that (X, σ) is a complete metric-like space. Define mappings $\alpha, \beta : X \times X \to R^+$ by

$$\alpha(x, y) = \beta(x, y) = \begin{cases} 1 & \text{if} \quad x, y \in [0, 1], \\ 0 & \text{if} \quad \textit{otherwise}. \end{cases}$$

Note that T is an (α, β)-admissible mapping if $\alpha(x, y) \geq 1$ and $\beta(x, y) \geq 1$ implies $\alpha(Tx, Ty) \geq 1$ and $\beta(Tx, ty) \geq 1$ for all $x, y \in X$. By definition of α and β this implies that $x, y \in [0, 1]$. Thus

$$\alpha(Tx, Ty) = \alpha\left(\frac{x^2}{x+1}, \frac{y^2}{y+1}\right) = 1.$$

Similarly $\beta(Tx, Ty) = 1$.
From above, it is clear that T is an (α, β)-admissible mapping. Let $\zeta(t, s) = \lambda s - t$, $\lambda \in [0, 1]$ for all $s, t > 0$. Also for $x, y \in X$ such that $\alpha(x, y) \geq 1$ and $\beta(x, y) \geq 1$. So, $x, y \in [0, 1]$. In this case, we have

$$(\alpha(Tx, Ty)\beta(Tx, Ty)\sigma(Tx, Ty), \sigma(x, y) + \theta N(x, y))$$

$$= \left((\frac{x^2}{x+1})^2 + (\frac{y^2}{y+1})^2, x^2 + y^2 + \theta N(x, y) \right). \tag{3.30}$$

Here $\theta \geq 0$ and

$$\begin{aligned} N(x, y) &= \min\{\sigma(x, Tx), \sigma(y, Ty), \sigma(x, Ty), \sigma(y, Tx)\} \\ &= \min\left\{ x^2 + \left(\frac{x^2}{x+1}\right)^2, y^2 + \left(\frac{y^2}{y+1}\right)^2, x^2 + (\frac{y^2}{y+1}\Big)^2, y^2 + \left(\frac{x^2}{x+1}\right)^2 \right\}. \end{aligned}$$

Since $x, y \in [0, 1]$

$$N(x, y) = 0 \tag{3.31}$$

from (3.30) and (3.31), we have

$$(\alpha(Tx, Ty)\beta(Tx, Ty)\sigma(Tx, Ty), \sigma(x, y) + \theta N(x, y)) = \left(\left(\frac{x^2}{x+1}\right)^2 + \left(\frac{y^2}{y+1}\right)^2, x^2 + y^2 \right).$$

It follows that

$$\begin{aligned} &\zeta(\alpha(Tx, Ty)\beta(Tx, Ty)\sigma(Tx, Ty), \sigma(x, y) + \theta N(x, y)) \\ &\qquad = \zeta\left(\left(\tfrac{x^2}{x+1}\right)^2 + \left(\tfrac{y^2}{y+1}\right)^2, x^2 + y^2 \right) \\ &\qquad = \lambda(x^2 + y^2) - \left(\left(\tfrac{x^2}{x+1}\right)^2 + \left(\tfrac{y^2}{y+1}\right)^2 \right). \end{aligned}$$

If we take $\lambda = \frac{1}{2}$, we get

$$\begin{aligned}\zeta(\alpha(Tx,Ty)\beta(Tx,Ty)\sigma(Tx,Ty),\sigma(x,y)+\theta N(x,y))\\ = \tfrac{1}{2}(x^2+y^2) - \left(\left(\tfrac{x^2}{x+1}\right)^2 + \left(\tfrac{y^2}{y+1}\right)^2\right) \geq 0\end{aligned}$$

i.e. $\zeta(\alpha(Tx,Ty)\beta(Tx,Ty)\sigma(Tx,Ty),\sigma(x,y)+\theta N(x,y)) \geq 0$. Also let $\{x_n\}$ be a sequence in X such that $\alpha(x_n,x_{n+1}) \geq 1$, $\beta(x_n,x_{n+1}) \geq 1$ for all n and $x_n \to x \in X$. Then, $\{x_n\} \subset [0,1]$ and $x_n^2 + x^2 \to 2x^2$ as $n \to +\infty$. Thus, $x_n \to x$ as $n \to +\infty$ in $(X, |.|)$. This implies that $x \in [0,1]$ and so $\alpha(x_n,x) = 1$, $\beta(x_n,x) = 1$ for all n. Moreover, there exists $x_0 \in X$ such that $\alpha(x_0,Tx_0) \geq 1$, $\beta(x_0,Tx_0) \geq 1$. In fact, for $x_0 = 1$, we have $\alpha(1,T1) = \alpha(1,1/2) = 1 = \beta(1,T1)$. Thus, all the conditions of Theorem 3.1 are verified. Here $x = 0$ is the unique fixed point of T.

4. Conclusions

In this paper, we studied (α,β)-admissible almost Z-contraction for a mapping T over a nonempty set X endowed with a complete metric-like space. Based on this a new contraction, some fixed point results are obtained. Our results are generalization for many existing results in the literature. Finally, we show the usability of our result by setting up one example.

Competing Interests

The authors declare that they have no competing interests.

Author's contribution

All authors contributed equally and significantly in writing this article. All authors read and approved final manuscript.

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