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$BF-$OSTROWSKI TYPE INEQUALITIES FOR $(A, B, G, D)-$CONVEX

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Abstract. In this paper, for the first time we present the generalized notion of $(A, B, G, D)-$ convex (concave) function in mixed kind, which is the generalization of functions given in [15], [2], [4], [14], [16] and [3]. We would like to state well-known Ostrowski inequality via Fuzzy Riemann Integrals for $(A, B, G, D)-$ convex (concave) function in mixed kind. Moreover we establish some Fuzzy Ostrowski type inequalities for the class of functions whose derivatives in absolute values at certain powers are (A, B, G, D)-convex (concave) functions in mixed kind by using different techniques including Hölder's inequality [27] and power mean inequality [26]. Also, various established results would be captured as special cases with respect to the convexity of function.
Keywords: fuzzy Riemann Integral, convex (concave) function, Ostrowski inequality.

1. Introduction

From the literature, we recall and introduce some definitions for various convex (concave) functions.

Definition 1.1. [3] A function $\phi : I \subset \mathbb{R} \to \mathbb{R}$ is said to be convex (concave) function, if

$$\phi\left(tx + (1-t)y\right) \leq (\geq) t\phi(x) + (1-t)\phi(y),$$

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$\forall x, y \in I, t \in [0,1]$.

We recall here definition of $P-$convex(concave) function from [14]:

Definition 1.2. We say that $\phi : I \subset \mathbb{R} \to \mathbb{R}$ is a $P-$convex(concave) function, if ϕ is a non-negative and $\forall x, y \in I$ and $t \in [0,1]$, we have

$$\phi\left(tx + (1-t)y\right) \leq (\geq) \phi(x) + \phi(y).$$

Here we also have definition of quasi$-$convex(concave) function (for detailed discussion see [16].

Definition 1.3. A function $\phi : I \subset \mathbb{R} \to \mathbb{R}$ is known as quasi$-$convex(concave), if

$$\phi(tx + (1-t)y) \leq (\geq) \max\{\phi(x), \phi(y)\}$$

$\forall x, y \in I$, $t \in [0,1]$.

Now we present the definition of $s-$convex functions in the first kind as follows which are extracted from [22]:

Definition 1.4. [4] Let $s \in [0,1]$. A function $\phi : I \subset [0,\infty) \to [0,\infty)$ is said to be $s-$convex (concave) function in the 1^{st} kind, if

$$\phi\left(tx + (1-t)y\right) \leq (\geq) t^s \phi(x) + (1-t^s)\phi(y),$$

$\forall x, y \in I, t \in [0,1]$.

Remark 1.1. Note that in this definition, we also included $s = 0$. Further, if we put $s = 0$, we get quasi$-$convexity (see Definition 1.3).

For the second kind of convexity, we recall the definition from [22].

Definition 1.5. Let $s \in [0,1]$. A function $\phi : I \subset [0,\infty) \to [0,\infty)$ is said to be $s-$convex (concave) function in the 2^{nd} kind, if

$$\phi\left(tx + (1-t)y\right) \leq (\geq) t^s \phi(x) + (1-t)^s \phi(y),$$

$\forall x, y \in I, t \in [0,1]$.

Remark 1.2. In a similar manner, we have slightly improved the definition of a second-kind convexity by including $s = 0$. Further if we put $s = 0$, we easily get $P-$convexity (see Definition 1.2).

Now we introduce a new class of functions which would be called the class of $(s,r)-$ convex (concave) functions in the mixed kind:

Definition 1.6. Let $(s, r) \in [0, 1]^2$. A function $\phi : I \subset [0, \infty) \to [0, \infty)$ is said to be $(s, r)-$convex (concave) function in mixed kind, if

$$\phi\,(tx + (1-t)y) \leq (\geq) t^{rs}\phi(x) + (1-t^r)^s\phi(y),$$

$\forall x, y \in I, t \in [0, 1]$.

Definition 1.7. [15] Let $(A, B) \in [0, 1]^2$. A function $\phi : I \subset [0, \infty) \to [0, \infty)$ is said to be $(A, B)-$convex(concave) in the 1^{st} kind, if

$$\phi\,(tx + (1-t)y) \leq (\geq) t^{A}\phi(x) + (1-t^B)\phi(y),$$

$\forall x, y \in I, t \in [0, 1]$.

Definition 1.8. [15] Let $(A, B) \in [0, 1]^2$. A function $\phi : I \subset [0, \infty) \to [0, \infty)$ is said to be $(A, B)-$convex(concave) function in the 2^{nd} kind, if

$$\phi\,(tx + (1-t)y) \leq (\geq) t^{A}\phi(x) + (1-t)^B\phi(y),$$

$\forall x, y \in I, t \in [0, 1]$.

Next, we introduce $(A, B, G, D)-$convex(concave) in mixed kind.

Definition 1.9. Let $(A, B, G, D) \in [0, 1]^4$. A function $\phi : I \subset [0, \infty) \to [0, \infty)$ is said to be $(A, B, G, D)-$convex(concave) function in mixed kind, if

$$\phi\,(tx + (1-t)y) \leq (\geq) t^{AG}\phi(x) + (1-t^{BG})^D\phi(y), \tag{1.1}$$

$\forall x, y \in I, t \in [0, 1]$.

Remark 1.3. In Definition 1.9, we have the following cases.

1. If we choose $G = D = 1$ in (1.1), we get $(A, B)-$convex (concave) in 1^{st} kind function.
2. If we choose $B = G = 1$ in (1.1), we get $(A, B)-$convex (concave) in 2^{nd} kind function.
3. If we choose $A = D = s$, $B = 1, G = r$, where $s, r \in [0, 1]$ in (1.1), we get $(s, r)-$convex (concave) in mixed kind function.
4. If we choose $A = B = s$ and $G = D = 1$ where $s \in [0, 1]$ in (1.1), we get $s-$convex (concave) in 1^{st} kind function.
5. If we choose $A = B = 0$, and $G = D = 1$, in (1.1), we get quasi$-$convex (concave) function.
6. If we choose $A = D = s$, $B = G = 1$ where $s \in [0, 1]$ in (1.1), we get $s-$convex (concave) in 2^{nd} kind function.
7. If we choose $A = D = 0$, and $B = G = 1$, in (1.1), we get $P-$convex (concave) function.

8. If we choose $A = B = G = D = 1$ in (1.1), gives us ordinary convex (concave) function.

In almost every field of science, inequalities play an important role. Although it is very vast discipline but our focus is mainly on the Ostrowski type inequalities. In 1938, the Ostrowski established the following interesting integral inequality for differentiable mappings with bounded derivatives. This inequality is well known in the literature as the Ostrowski inequality.

Theorem 1.1. *[23] Let $\varphi : [\rho_a, \rho_b] \to \mathbb{R}$ be a differentiable function on (ρ_a, ρ_b) with the property that $|\varphi'(t)| \le M$ for all $t \in (\rho_a, \rho_b)$. Then*

$$\left|\varphi(x) - \frac{1}{\rho_b - \rho_a}\int_{\rho_a}^{\rho_b} \varphi(t)dt\right| \le M(\rho_b - \rho_a)\left[\frac{1}{4} + \left(\frac{x - \frac{\rho_a+\rho_b}{2}}{\rho_b - \rho_a}\right)^2\right], \tag{1.2}$$

for all $x \in (\rho_a, \rho_b)$. The constant $\frac{1}{4}$ is the best possible in the kind that it cannot be replaced by a smaller quantity.

Definition 1.10. [29] A fuzzy number is $\phi : \mathbb{R} \to [0, 1]$ can be defined as

1. $[\phi]^0 = Closure(\{r \in \mathbb{R} : \phi(r) > 0\})$ is compact.
2. ϕ is Normal.(i.e, $\exists\ r_0 \in \mathbb{R}$ such that $\phi(r_0) = 1$).
3. ϕ is fuzzy convex, i.e, $\phi(\eta r_1 + (1 - \eta)r_2) \ge \min\{\phi(r_1), \phi(r_2)\}$, $\forall r_1, r_2 \in \mathbb{R}$, $\eta \in [0, 1]$.
4. $\forall r_0 \in R$ and $\epsilon > 0$, $\exists$ Neighborhood $V(r_0)$, such that $\phi(r) \le \phi(r_0)+\epsilon$, $\forall r \in \mathbb{R}$.

Definition 1.11. [30] For any $\zeta \in [0, 1]$, and ϕ be any fuzzy number, then $\zeta-$level set $[\phi]^\zeta = \{r \in \mathbb{R} : \phi(r) \ge \zeta\}$. Moreover $[\phi]^\zeta = \left[\phi_-^{(\zeta)}, \phi_+^{(\zeta)}\right], \forall\zeta \in [0, 1]$.

Proposition 1.1. *[31] Let $\phi, \varphi \in F_{\mathbb{R}}$(Set of all Fuzzy numbers) and $\eta \in \mathbb{R}$, then the following properties holds:*

1. $[\phi]^{\zeta_1} \subseteq [\varphi]^{\zeta_2}$ *whenever* $0 \le \zeta_2 \le \zeta_1 \le 1$.
2. $[\phi + \varphi]^\zeta = [\phi]^\zeta + [\varphi]^\zeta$.
3. $[\eta \odot \phi]^\zeta = \eta\,[\phi]^\zeta$.
4. $\phi \oplus \varphi = \varphi \oplus \phi$.
5. $\eta \odot \phi = \phi \odot \eta$.
6. $\widetilde{1} \odot \phi = \phi$.

$\forall\zeta\in[0,1]$, *where* $\widetilde{1}\in F_{\mathbb{R}}$, *defined by* $\forall r\in\mathbb{R},\widetilde{1}(r)=1$.

Definition 1.12. [29] Let $D:F_{\mathbb{R}}\times F_{\mathbb{R}}\to\mathbb{R}_{+}\cup\{0\}$, defined as

$$D(\phi,\varphi)=\sup_{\zeta\in[0,1]}\max\left\{\left|\phi_{-}^{(\zeta)},\phi_{+}^{(\zeta)}\right|,\left|\varphi_{-}^{(\zeta)},\varphi_{+}^{(\zeta)}\right|\right\}$$

$\forall\phi,\varphi\in F_{\mathbb{R}}$. Then D is metric on $F_{\mathbb{R}}$.

Proposition 1.2. *[29] Let* $\phi_1,\phi_2,\phi_3,\phi_4\in F_{\mathbb{R}}$ *and* $\eta\in F_{\mathbb{R}}$, *we have*

1. $(F_{\mathbb{R}},D)$ *is complete.*
2. $D(\phi_1\oplus\phi_3,\phi_2\oplus\phi_3)=D(\phi_1,\phi_2)$.
3. $D(\eta\odot\phi_1,\eta\odot\phi_2)=|\eta|D(\phi_1,\phi_2)$.
4. $D(\phi_1\oplus\phi_2,\phi_3\oplus\phi_4)=D(\phi_1,\phi_3)+D(\phi_2,\phi_4)$.
5. $D(\phi_1\oplus\phi_2,\widetilde{0})\leq D(\phi_1,\widetilde{0})+D(\phi_2,\widetilde{0})$.
6. $D(\phi_1\oplus\phi_2,\phi_3)\leq D(\phi_1,\phi_3)+D(\phi_2,\widetilde{0})$,

where $\widetilde{0}\in F_{\mathbb{R}}$, *defined by* $\forall r\in\mathbb{R},\widetilde{0}(r)=0$.

Definition 1.13. [30] Let $\phi,\varphi\in F_{\mathbb{R}}$, if $\exists\ \theta\in F_{\mathbb{R}}$, such that $\phi=\varphi\oplus\theta$, then θ is $H-$difference of ϕ and φ, denoted by $\theta=\phi\ominus\varphi$.

Definition 1.14. [30] A function $\phi:[r_0,r_0+\epsilon]\to F_{\mathbb{R}}$ is $H-$differentiable at r, if $\exists\ \phi'(r)\in F_{\mathbb{R}}$, i.e both limits

$$\lim_{h\to0^+}\frac{\phi(r+h)\ominus\phi(r)}{h},\ \lim_{h\to0^+}\frac{\phi(r)\ominus\phi(r-h)}{h}$$

exists and are equal to $\phi'(r)$.

Definition 1.15. [28] Let $\phi:[\rho_a,\rho_b]\to F_{\mathbb{R}}$, if $\forall\zeta>0,\exists\eta>0$, for any partition $P=\{[u,v]:D\}$ of $[\rho_a,\rho_b]$ with norm $D(P)<\eta$, we have

$$D\left(\sum_{P}^{*}(v-u)\phi(D),\varphi\right)<\zeta,$$

then we say that ϕ is Fuzzy–Riemann integrable to $\varphi\in F_{\mathbb{R}}$, we write it as

$$\varphi=(FR)\int_{\rho_a}^{\rho_b}\phi(x)dx.$$

In order to prove our main results, we need the following Lemma that has been obtained in [5].

Lemma 1.1. *Let $\varphi : [\rho_a, \rho_b] \to F_{\mathbb{R}}$ be an absolutely continuous mapping on (ρ_a, ρ_b) with $\rho_a < \rho_b$. If $\varphi' \in C_F[\rho_a, \rho_b] \cap L_F[\rho_a, \rho_b]$, then for $x \in (\rho_a, \rho_b)$ the following identity holds:*

$$\frac{1}{\rho_b - \rho_a} \odot (FR) \int_{\rho_a}^{\rho_b} \varphi(t)dt \oplus \frac{(x-\rho_a)^2}{\rho_b - \rho_a} \odot (FR) \int_0^1 t \odot \varphi'(tx + (1-t)\rho_a)dt$$

$$= \varphi(x) \oplus \frac{(\rho_b - x)^2}{\rho_b - \rho_a} \odot (FR) \int_0^1 t \odot \varphi'(tx + (1-t)\rho_b)dt. \tag{1.3}$$

We make use of the beta function of Euler type, which is for $x, y > 0$ defined as

$$B(x, y) = \int_0^1 t^{x-1}(1-t)^{y-1}dt = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)},$$

where $\Gamma(x) = \int_0^\infty e^{-u}u^{x-1}du$.

2. Generalized Fuzzy Ostrowski type inequalities via $(A, B, G, D)-$convex functions in mixed kind

Theorem 2.1. *Suppose all the assumptions of Lemma 1.1 hold. Additionally, assume that $D(\varphi', \widetilde{0})$ is $(A, B, G, D)-$convex function on $[\rho_a, \rho_b]$ and $D(\varphi'(x), \widetilde{0}) \leq M$. Then $\forall x \in (\rho_a, \rho_b)$, the following inequality holds:*

$$D\left(\varphi(x), \frac{1}{\rho_b - \rho_a} \odot (FR) \int_{\rho_a}^{\rho_b} \varphi(t)dt\right)$$

$$\leq M \left(\frac{1}{AG+2} + \frac{B\left(\frac{2}{BG}, D+1\right)}{BG}\right) I(x), \tag{2.1}$$

where $I(x) = \frac{(x-\rho_a)^2 + (\rho_b - x)^2}{\rho_b - \rho_a}$.

Proof. From the Lemma 1.1

$$D\left(\varphi(x), \frac{1}{\rho_b - \rho_a} \odot (FR) \int_{\rho_a}^{\rho_b} \varphi(t)dt\right)$$

$$\leq D\left(\frac{(x-\rho_a)^2}{\rho_b - \rho_a} \odot (FR) \int_0^1 t \odot \varphi'(tx + (1-t)\rho_a)dt,\right.$$

$$\left.\frac{(\rho_b - x)^2}{\rho_b - \rho_a} \odot (FR) \int_0^1 t \odot \varphi'(tx + (1-t)\rho_b)dt\right),$$

$$\leq D\left(\frac{(x-\rho_a)^2}{\rho_b - \rho_a} \odot (FR) \int_0^1 t \odot \varphi'(tx + (1-t)\rho_a)dt, \widetilde{0}\right)$$

$$+ D\left(\frac{(\rho_b - x)^2}{\rho_b - \rho_a} \odot (FR) \int_0^1 t \odot \varphi'(tx + (1-t)\rho_b)dt, \widetilde{0}\right),$$

$$= \frac{(x-\rho_a)^2}{\rho_b-\rho_a} D\left((FR)\int_0^1 t\odot\varphi'(tx+(1-t)\rho_a)dt, \widetilde{0}\right)$$
$$+\frac{(\rho_b-x)^2}{\rho_b-\rho_a} D\left((FR)\int_0^1 t\odot\varphi'(tx+(1-t)\rho_b)dt, \widetilde{0}\right),$$

$$\leq \frac{(x-\rho_a)^2}{\rho_b-\rho_a}\int_0^1 tD\left(\varphi'(tx+(1-t)\rho_a), \widetilde{0}\right)dt$$
$$+\frac{(\rho_b-x)^2}{\rho_b-\rho_a}\int_0^1 tD\left(\varphi'(tx+(1-t)\rho_b), \widetilde{0}\right)dt, \tag{2.2}$$

Since $D(\varphi', \widetilde{0})$ be $(A,B,G,D)-$convex function and $D(\varphi'(x), \widetilde{0}) \leq M$, we have

$$\begin{aligned} D\left(\varphi'(tx+(1-t)\rho_a), \widetilde{0}\right) &\leq t^{AG}D\left(\varphi'(x), \widetilde{0}\right) + \left(1-t^{BG}\right)^D D\left(\varphi'(\rho_a), \widetilde{0}\right) \\ &\leq M\left[t^{AG} + \left(1-t^{BG}\right)^D\right] \end{aligned} \tag{2.3}$$

$$\begin{aligned} D\left(\varphi'(tx+(1-t)\rho_b), \widetilde{0}\right) &\leq t^{AG}D\left(\varphi'(x), \widetilde{0}\right) + \left(1-t^{BG}\right)^D D\left(\varphi'(\rho_b), \widetilde{0}\right) \\ &\leq M\left[t^{AG} + \left(1-t^{BG}\right)^D\right] \end{aligned} \tag{2.4}$$

Now using (2.3) and (2.4) in (2.2) we get (2.1). □

Corollary 2.1. *In Theorem* 2.1, *one can see the following.*

1. *If one takes* $G = D = 1$, $A \in [0,1]$ *and* $B \in (0,1]$, *in* (2.1), *one has the Fuzzy Ostrowski inequality for* $(A,B)-$*convex functions in* 1^{st} *kind:*

$$D\left(\varphi(x), \frac{1}{\rho_b-\rho_a}\odot(FR)\int_{\rho_a}^{\rho_b}\varphi(t)dt\right) \leq M\left(\frac{1}{A+2} + \frac{B\left(\frac{2}{B},2\right)}{B}\right)I(x).$$

2. *If one takes* $B = G = 1$, $A \in [0,1]$ *and* $D \in [0,1]$, *in* (2.1), *then one has the Fuzzy Ostrowski inequality for* $(A,D)-$*convex functions in* 2^{nd} *kind:*

$$D\left(\varphi(x), \frac{1}{\rho_b-\rho_a}\odot(FR)\int_{\rho_a}^{\rho_b}\varphi(t)dt\right) \leq M\left(\frac{1}{A+2} + \frac{1}{(D+1)(D+2)}\right)I(x).$$

3. *If one takes* $A = D = s$, $B = 1, G = r$, *where* $s \in [0,1]$ *and* $r \in (0,1]$ *in* (2.1), *then one has the Fuzzy Ostrowski inequality for* $(s,r)-$*convex functions in mixed kind:*

$$D\left(\varphi(x), \frac{1}{\rho_b-\rho_a}\odot(FR)\int_{\rho_a}^{\rho_b}\varphi(t)dt\right) \leq M\left(\frac{1}{rs+2} + \frac{B\left(\frac{2}{r},s+1\right)}{r}\right)I(x).$$

4. *If one takes $A = B = r$ and $G = D = 1$, where $r \in (0, 1]$ in* (2.1)*, then one has the Fuzzy Ostrowski inequality for $r-$convex functions in 1^{st} kind:*

$$D\left(\varphi(x), \frac{1}{\rho_b - \rho_a} \odot (FR) \int_{\rho_a}^{\rho_b} \varphi(t)dt\right) \leq M\left(\frac{1}{r+2} + \frac{B\left(\frac{2}{r}, 2\right)}{r}\right) I(x).$$

5. *If one takes $B = G = 1$, $A = D = s$ where $s \in [0, 1]$, in* (2.1)*, then one has the Fuzzy Ostrowski inequality for $s-$convex functions in 2^{nd} kind:*

$$D\left(\varphi(x), \frac{1}{\rho_b - \rho_a} \odot (FR) \int_{\rho_a}^{\rho_b} \varphi(t)dt\right) \leq M\left(\frac{1}{s+1}\right) I(x).$$

6. *If one takes $A = D = 0$ and $B = G = 1$ in* (2.1)*, then one has the Fuzzy Ostrowski inequality for $P-$convex functions:*

$$D\left(\varphi(x), \frac{1}{\rho_b - \rho_a} \odot (FR) \int_{\rho_a}^{\rho_b} \varphi(t)dt\right) \leq MI(x).$$

7. *If one takes $A = B = G = D = 1$, in* (2.1)*, then one has the Fuzzy Ostrowski inequality for convex functions:*

$$D\left(\varphi(x), \frac{1}{\rho_b - \rho_a} \odot (FR) \int_{\rho_a}^{\rho_b} \varphi(t)dt\right) \leq \frac{M}{2} I(x).$$

Theorem 2.2. *Suppose all the assumptions of Lemma* 1.1 *hold. Additionally, assume that $[D(\varphi', \widetilde{0})]^q$ is $(A, B, G, D)-$convex function on $[\rho_a, \rho_b], q \geq 1$ and $D(\varphi'(x), \widetilde{0}) \leq M$. Then for each $x \in (\rho_a, \rho_b)$ the following inequality holds:*

$$\begin{aligned} & D\left(\varphi(x), \frac{1}{\rho_b - \rho_a} \odot (FR) \int_{\rho_a}^{\rho_b} \varphi(t)dt\right) \\ & \leq \frac{M}{(2)^{1-\frac{1}{q}}} \left(\frac{1}{AG+2} + \frac{B\left(\frac{2}{BG}, D+1\right)}{BG}\right)^{\frac{1}{q}} I(x). \end{aligned} \tag{2.5}$$

Proof. From the Inequality (2.2) and power mean inequality [26]

$$\begin{aligned} & D\left(\varphi(x), \frac{1}{\rho_b - \rho_a} \odot (FR) \int_{\rho_a}^{\rho_b} \varphi(t)dt\right) \\ & \leq \frac{(x-\rho_a)^2}{\rho_b - \rho_a} \left(\int_0^1 tdt\right)^{1-\frac{1}{q}} \left(\int_0^1 t\left[D\left(\varphi'(tx + (1-t)\rho_a), \widetilde{0}\right)\right]^q dt\right)^{\frac{1}{q}} \\ & + \frac{(\rho_b - x)^2}{\rho_b - \rho_a} \left(\int_0^1 tdt\right)^{1-\frac{1}{q}} \left(\int_0^1 t\left[D\left(\varphi'(tx + (1-t)\rho_b), \widetilde{0}\right)\right]^q dt\right)^{\frac{1}{q}}. \end{aligned} \tag{2.6}$$

Since $[D(\varphi',\widetilde{0})]^q$ be $(A,B,G,D)-$convex function and $D(\varphi'(x),\widetilde{0})\leq M$, we have

$$\begin{aligned}
&\left[D\left(\varphi'(tx+(1-t)\rho_a),\widetilde{0}\right)\right]^q\\
\leq\;& t^{AG}\left[D\left(\varphi'(x),\widetilde{0}\right)\right]^q+\left(1-t^{BG}\right)^D\left[D\left(\varphi'(\rho_a),\widetilde{0}\right)\right]^q\\
\leq\;& M^q\left[t^{AG}+\left(1-t^{BG}\right)^D\right], \qquad (2.7)
\end{aligned}$$

$$\begin{aligned}
&\left[D\left(\varphi'(tx+(1-t)\rho_b),\widetilde{0}\right)\right]^q\\
\leq\;& t^{AG}\left[D\left(\varphi'(x),\widetilde{0}\right)\right]^q+\left(1-t^{BG}\right)^D\left[D\left(\varphi'(\rho_b),\widetilde{0}\right)\right]^q\\
\leq\;& M^q\left[t^{AG}+\left(1-t^{BG}\right)^D\right]. \qquad (2.8)
\end{aligned}$$

Now using (2.7) and (2.8) in (2.6) we get (2.5). □

Corollary 2.2. *In Theorem* 2.2, *one can see the following.*

1. *If one takes* $q=1$, *one has the Theorem* 2.1.
2. *If one takes* $G=D=1$, $A\in[0,1]$ *and* $B\in(0,1]$, *in* (2.5), *one has the Fuzzy Ostrowski inequality for* $(A,B)-$*convex functions in* 1^{st} *kind:*
$$\begin{aligned}
&D\left(\varphi(x),\frac{1}{\rho_b-\rho_a}\odot(FR)\int_{\rho_a}^{\rho_b}\varphi(t)dt\right)\\
\leq\;&\frac{M}{(2)^{1-\frac{1}{q}}}\left(\frac{1}{A+2}+\frac{B\left(\frac{2}{B},2\right)}{B}\right)^{\frac{1}{q}}I(x).
\end{aligned}$$
3. *If one takes* $B=G=1$, $A\in[0,1]$ *and* $D\in[0,1]$, *in* (2.5), *then one has the Fuzzy Ostrowski inequality for* $(A,D)-$*convex functions in* 2^{nd} *kind:*
$$\begin{aligned}
&D\left(\varphi(x),\frac{1}{\rho_b-\rho_a}\odot(FR)\int_{\rho_a}^{\rho_b}\varphi(t)dt\right)\\
\leq\;&\frac{M}{(2)^{1-\frac{1}{q}}}\left(\frac{1}{(A+2)}+\frac{1}{(D+1)(D+2)}\right)^{\frac{1}{q}}I(x).
\end{aligned}$$
4. *If one takes* $A=D=s$, $B=1, G=r$, *where* $s\in[0,1]$ *and* $r\in(0,1]$ *in* (2.5), *then one has the Fuzzy Ostrowski inequality for* $(s,r)-$*convex functions in mixed kinds:*
$$\begin{aligned}
&D\left(\varphi(x),\frac{1}{\rho_b-\rho_a}\odot(FR)\int_{\rho_a}^{\rho_b}\varphi(t)dt\right)\\
\leq\;&\frac{M}{(2)^{1-\frac{1}{q}}}\left(\frac{1}{rs+2}+\frac{B\left(\frac{2}{r},s+1\right)}{r}\right)^{\frac{1}{q}}I(x).
\end{aligned}$$

5. *If one takes* $A = B = r$ *and* $G = D = 1$, *where* $r \in (0, 1]$ *in* (2.5), *then one has the Fuzzy Ostrowski inequality for* $r-$*convex functions in* 1^{st} *kind:*

$$D\left(\varphi(x), \frac{1}{\rho_b - \rho_a} \odot (FR) \int_{\rho_a}^{\rho_b} \varphi(t)dt\right) \leq \frac{M}{(2)^{1-\frac{1}{q}}} \left(\frac{1}{r+2} + \frac{B\left(\frac{2}{r}, 2\right)}{r}\right)^{\frac{1}{q}} I(x).$$

6. *If one takes* $B = G = 1$, $A = D = s$ *where* $s \in [0, 1]$, *in* (2.5), *then one has the Fuzzy Ostrowski inequality for* $r-$*convex functions in* 2^{nd} *kind:*

$$D\left(\varphi(x), \frac{1}{\rho_b - \rho_a} \odot (FR) \int_{\rho_a}^{\rho_b} \varphi(t)dt\right) \leq \frac{M}{(2)^{1-\frac{1}{q}}} \left(\frac{1}{s+1}\right)^{\frac{1}{q}} I(x).$$

7. *If one takes* $A = D = 0$ *and* $B = G = 1$ *in* (2.5), *then one has the Fuzzy Ostrowski inequality for* $P-$*convex functions:*

$$D\left(\varphi(x), \frac{1}{\rho_b - \rho_a} \odot (FR) \int_{\rho_a}^{\rho_b} \varphi(t)dt\right) \leq \frac{M}{(2)^{1-\frac{1}{q}}} I(x).$$

8. *If one takes* $A = B = G = D = 1$, *in* (2.5), *then one has the Fuzzy Ostrowski inequality for convex functions:*

$$D\left(\varphi(x), \frac{1}{\rho_b - \rho_a} \odot (FR) \int_{\rho_a}^{\rho_b} \varphi(t)dt\right) \leq \frac{M}{2} I(x).$$

Remark 2.1. In Theorem 2.2, one can see the following.

1. If one takes $x = \frac{\rho_a + \rho_b}{2}$ in (2.5), one has the Fuzzy Ostrowski Midpoint inequality for $(A, B, G, D)-$ convex functions in Mixed kinds:

$$D\left(\varphi\left(\frac{\rho_a + \rho_b}{2}\right), \frac{1}{\rho_b - \rho_a} \odot (FR) \int_{\rho_a}^{\rho_b} \varphi(t)dt\right) \leq \frac{M\left(\rho_b - \rho_a\right)}{(2)^{2-\frac{1}{q}}} \left(\frac{1}{AG+2} + \frac{B\left(\frac{2}{BG}, D+1\right)}{BG}\right)^{\frac{1}{q}}.$$

2. If one takes $x = \frac{\rho_a + \rho_b}{2}$, $G = D = 1$, $A \in [0, 1]$ and $B \in (0, 1]$ in (2.5), one has the Fuzzy Ostrowski Midpoint inequality for $(A, B)-$convex functions in 1^{st} kind:

$$D\left(\varphi\left(\frac{\rho_a + \rho_b}{2}\right), \frac{1}{\rho_b - \rho_a} \odot (FR) \int_{\rho_a}^{\rho_b} \varphi(t)dt\right) \leq \frac{M\left(\rho_b - \rho_a\right)}{(2)^{2-\frac{1}{q}}} \left(\frac{1}{A+2} + \frac{B\left(\frac{2}{B}, 2\right)}{B}\right)^{\frac{1}{q}}.$$

3. If one takes $x = \frac{\rho_a+\rho_b}{2}$, $B = G = 1$, $A \in [0,1]$ and $D \in [0,1]$ in (2.5), then one has the Fuzzy Ostrowski Midpoint inequality for $(A, D)-$convex functions in 2^{nd} kind:
$$\begin{aligned} & D\left(\varphi\left(\frac{\rho_a+\rho_b}{2}\right), \frac{1}{\rho_b-\rho_a} \odot (FR)\int_{\rho_a}^{\rho_b} \varphi(t)dt\right) \\ \leq\ & \frac{M(\rho_b-\rho_a)}{(2)^{2-\frac{1}{q}}}\left(\frac{1}{(A+2)}+\frac{1}{(D+1)(D+2)}\right)^{\frac{1}{q}}. \end{aligned}$$

4. If one takes $x = \frac{\rho_a+\rho_b}{2}$, $A = D = s$, $B = 1, G = r$, where $s \in [0,1]$ and $r \in (0,1]$ in (2.5), then one has the Fuzzy Ostrowski Midpoint inequality for $(s,r)-$convex functions in mixed kinds:
$$\begin{aligned} & D\left(\varphi\left(\frac{\rho_a+\rho_b}{2}\right), \frac{1}{\rho_b-\rho_a} \odot (FR)\int_{\rho_a}^{\rho_b} \varphi(t)dt\right) \\ \leq\ & \frac{M(\rho_b-\rho_a)}{(2)^{2-\frac{1}{q}}}\left(\frac{1}{rs+2}+\frac{B\left(\frac{2}{r}, s+1\right)}{r}\right)^{\frac{1}{q}}. \end{aligned}$$

5. If one takes $x = \frac{\rho_a+\rho_b}{2}$, $A = B = r$ and $G = D = 1$, where $r \in (0,1]$ in (2.5), then one has the Fuzzy Ostrowski Midpoint inequality for $r-$convex functions in 1^{st} kind:
$$\begin{aligned} & D\left(\varphi\left(\frac{\rho_a+\rho_b}{2}\right), \frac{1}{\rho_b-\rho_a} \odot (FR)\int_{\rho_a}^{\rho_b} \varphi(t)dt\right) \\ \leq\ & \frac{M(\rho_b-\rho_a)}{(2)^{2-\frac{1}{q}}}\left(\frac{1}{r+2}+\frac{B\left(\frac{2}{r}, 2\right)}{r}\right)^{\frac{1}{q}}. \end{aligned}$$

6. If one takes $x = \frac{\rho_a+\rho_b}{2}$, $B = G = 1$, $A = D = s$ where $s \in [0,1]$, in (2.5), then one has the Fuzzy Ostrowski Midpoint inequality for $r-$convex functions in 2^{nd} kind:
$$D\left(\varphi\left(\frac{\rho_a+\rho_b}{2}\right), \frac{1}{\rho_b-\rho_a} \odot (FR)\int_{\rho_a}^{\rho_b} \varphi(t)dt\right) \leq \frac{M(\rho_b-\rho_a)}{(2)^{2-\frac{1}{q}}}\left(\frac{1}{s+1}\right)^{\frac{1}{q}}.$$

7. If one takes $x = \frac{\rho_a+\rho_b}{2}$, $A = D = 0$ and $B = G = 1$ in (2.5), then one has the Fuzzy Ostrowski Midpoint inequality for $P-$convex functions:
$$D\left(\varphi\left(\frac{\rho_a+\rho_b}{2}\right), \frac{1}{\rho_b-\rho_a} \odot (FR)\int_{\rho_a}^{\rho_b} \varphi(t)dt\right) \leq \frac{M(\rho_b-\rho_a)}{(2)^{2-\frac{1}{q}}}.$$

8. If one takes $x = \frac{\rho_a+\rho_b}{2}$, $A = B = G = D = 1$, in (2.5), then one has the Fuzzy Ostrowski Midpoint inequality for convex functions:
$$D\left(\varphi\left(\frac{\rho_a+\rho_b}{2}\right), \frac{1}{\rho_b-\rho_a} \odot (FR)\int_{\rho_a}^{\rho_b} \varphi(t)dt\right) \leq \frac{M(\rho_b-\rho_a)}{4}.$$

Theorem 2.3. *Suppose all the assumptions of Lemma* 1.1 *hold. Additionally, assume that* $[D(\varphi', \widetilde{0})]^q$ *is* $(A, B, G, D)-$*convex function on* $[\rho_a, \rho_b], q > 1$ *and*

$D(\varphi'(x), \widetilde{0}) \leq M$. *Then for each* $x \in (\rho_a, \rho_b)$, *the following inequality holds:*

$$D\left(\varphi(x), \frac{1}{\rho_b - \rho_a} \odot (FR) \int_{\rho_a}^{\rho_b} \varphi(t)dt\right)$$

$$\leq \frac{M}{(p+1)^{\frac{1}{p}}} \left(\frac{1}{AG+1} + \frac{B\left(\frac{1}{BG}, D+1\right)}{BG}\right)^{\frac{1}{q}} I(x), \tag{2.9}$$

where $p^{-1} + q^{-1} = 1$.

Proof. From the Inequality (2.2) and Hölder's inequality [27]

$$D\left(\varphi(x), \frac{1}{\rho_b - \rho_a} \odot (FR) \int_{\rho_a}^{\rho_b} \varphi(t)dt\right)$$

$$\leq \frac{(x-\rho_a)^2}{\rho_b - \rho_a} \left(\int_0^1 t^p dt\right)^{\frac{1}{p}} \left(\int_0^1 \left[D\left(\varphi'(tx + (1-t)\rho_a), \widetilde{0}\right)\right]^q dt\right)^{\frac{1}{q}}$$

$$+\frac{(\rho_b - x)^2}{\rho_b - \rho_a} \left(\int_0^1 t^p dt\right)^{\frac{1}{p}} \left(\int_0^1 \left[D\left(\varphi'(tx + (1-t)\rho_b), \widetilde{0}\right)\right]^q dt\right)^{\frac{1}{q}}. \tag{2.10}$$

Since $[D(\varphi', \widetilde{0})]^q$ be $(A, B, G, D)-$convex function and $D(\varphi'(x), \widetilde{0}) \leq M$, we have

$$\left[D\left(\varphi'(tx + (1-t)\rho_a), \widetilde{0}\right)\right]^q$$

$$\leq t^{AG}\left[D\left(\varphi'(x), \widetilde{0}\right)\right]^q + \left(1 - t^{BG}\right)^D \left[D\left(\varphi'(\rho_a), \widetilde{0}\right)\right]^q$$

$$\leq M^q \left[t^{AG} + \left(1 - t^{BG}\right)^D\right], \tag{2.11}$$

$$\left[D\left(\varphi'(tx + (1-t)\rho_b), \widetilde{0}\right)\right]^q$$

$$\leq t^{AG}\left[D\left(\varphi'(x), \widetilde{0}\right)\right]^q + \left(1 - t^{BG}\right)^D \left[D\left(\varphi'(\rho_b), \widetilde{0}\right)\right]^q$$

$$\leq M^q \left[t^{AG} + \left(1 - t^{BG}\right)^D\right]. \tag{2.12}$$

Now using (2.11) and (2.12) in (2.10) we get (2.9). □

Corollary 2.3. *In Theorem* 2.3, *one can see the following.*

1. *If one takes* $G = D = 1$, $A \in [0,1]$ *and* $B \in (0,1]$, *in* (2.9), *one has the Fuzzy Ostrowski inequality for* $(A, B)-$*convex functions in* 1^{st} *kind:*

$$D\left(\varphi(x), \frac{1}{\rho_b - \rho_a} \odot (FR) \int_{\rho_a}^{\rho_b} \varphi(t)dt\right)$$

$$\leq \frac{M}{(p+1)^{\frac{1}{p}}} \left(\frac{1}{A+1} + \frac{B\left(\frac{1}{B}, 2\right)}{B}\right)^{\frac{1}{q}} I(x).$$

2. *If one takes $B=G=1$, $A\in[0,1]$ and $D\in[0,1]$, in (2.9), then one has the Fuzzy Ostrowski inequality for $(A,D)-$convex functions in 2^{nd} kind:*

$$\begin{aligned} &D\left(\varphi(x), \frac{1}{\rho_b-\rho_a}\odot(FR)\int_{\rho_a}^{\rho_b}\varphi(t)dt\right)\\ \leq\ &\frac{M}{(p+1)^{\frac{1}{p}}}\left(\frac{1}{A+1}+\frac{1}{D+1}\right)^{\frac{1}{q}}I(x). \end{aligned}$$

3. *If one takes $A=D=s$, $B=1, G=r$, where $s\in[0,1]$ and $r\in(0,1]$ in (2.9), then one has the Fuzzy Ostrowski inequality for $(s,r)-$convex functions in mixed kinds:*

$$\begin{aligned} &D\left(\varphi(x), \frac{1}{\rho_b-\rho_a}\odot(FR)\int_{\rho_a}^{\rho_b}\varphi(t)dt\right)\\ \leq\ &\frac{M}{(p+1)^{\frac{1}{p}}}\left(\frac{1}{rs+1}+\frac{B\left(\frac{1}{r},s+1\right)}{r}\right)^{\frac{1}{q}}I(x). \end{aligned}$$

4. *If one takes $A=B=r$ and $G=D=1$, where $r\in(0,1]$ in (2.9), then one has the Fuzzy Ostrowski inequality for $r-$convex functions in 1^{st} kind:*

$$\begin{aligned} &D\left(\varphi(x), \frac{1}{\rho_b-\rho_a}\odot(FR)\int_{\rho_a}^{\rho_b}\varphi(t)dt\right)\\ \leq\ &\frac{M}{(p+1)^{\frac{1}{p}}}\left(\frac{1}{r+1}+\frac{B\left(\frac{1}{r},2\right)}{r}\right)^{\frac{1}{q}}I(x). \end{aligned}$$

5. *if one takes $B=G=1$, $A=D=s$ where $s\in[0,1]$ in (2.9), then one has the Fuzzy Ostrowski inequality for $s-$convex functions in 2^{nd} kind:*

$$D\left(\varphi(x), \frac{1}{\rho_b-\rho_a}\odot(FR)\int_{\rho_a}^{\rho_b}\varphi(t)dt\right)\leq\frac{M}{(p+1)^{\frac{1}{p}}}\left(\frac{2}{s+1}\right)^{\frac{1}{q}}I(x).$$

6. *If one takes $A=D=0$ and $B=G=1$ in (2.9), then one has the Fuzzy Ostrowski inequality for $P-$convex functions:*

$$D\left(\varphi(x), \frac{1}{\rho_b-\rho_a}\odot(FR)\int_{\rho_a}^{\rho_b}\varphi(t)dt\right)\leq\frac{(2)^{\frac{1}{q}}M}{(p+1)^{\frac{1}{p}}}I(x).$$

7. *If one takes $A=B=G=D=1$ in (2.9), then one has the Fuzzy Ostrowski inequality for convex functions:*

$$D\left(\varphi(x), \frac{1}{\rho_b-\rho_a}\odot(FR)\int_{\rho_a}^{\rho_b}\varphi(t)dt\right)\leq\frac{M}{(p+1)^{\frac{1}{p}}}I(x).$$

Remark 2.2. In Theorem 2.3, one can see the following.

1. If one takes $x = \frac{\rho_a+\rho_b}{2}$ in (2.9), one has the Fuzzy Ostrowski Midpoint inequality for $(A, B, G, D)-$ convex functions in Mixed kinds:

$$D\left(\varphi\left(\frac{\rho_a+\rho_b}{2}\right), \frac{1}{\rho_b-\rho_a} \odot (FR)\int_{\rho_a}^{\rho_b} \varphi(t)dt\right) \leq \frac{M(\rho_b-\rho_a)}{2(p+1)^{\frac{1}{p}}}\left(\frac{1}{AG+1} + \frac{B\left(\frac{1}{BG}, D+1\right)}{BG}\right)^{\frac{1}{q}}.$$

2. If one takes $x = \frac{\rho_a+\rho_b}{2}$, $G = D = 1$, $A \in [0, 1]$ and $B \in (0, 1]$, in (2.9), one has the Fuzzy Ostrowski Midpoint inequality for $(A, B)-$convex functions in 1^{st} kind:

$$D\left(\varphi\left(\frac{\rho_a+\rho_b}{2}\right), \frac{1}{\rho_b-\rho_a} \odot (FR)\int_{\rho_a}^{\rho_b} \varphi(t)dt\right)$$
$$\leq \frac{M(\rho_b-\rho_a)}{2(p+1)^{\frac{1}{p}}}\left(\frac{1}{A+1} + \frac{B\left(\frac{1}{B}, 2\right)}{B}\right)^{\frac{1}{q}}.$$

3. If one takes $x = \frac{\rho_a+\rho_b}{2}$, $B = G = 1$, $A \in [0, 1]$ and $D \in [0, 1]$, in (2.9), then one has the Fuzzy Ostrowski Midpoint inequality for $(A, D)-$convex functions in 2^{nd} kind:

$$D\left(\varphi\left(\frac{\rho_a+\rho_b}{2}\right), \frac{1}{\rho_b-\rho_a} \odot (FR)\int_{\rho_a}^{\rho_b} \varphi(t)dt\right)$$
$$\leq \frac{M(\rho_b-\rho_a)}{2(p+1)^{\frac{1}{p}}}\left(\frac{1}{A+1} + \frac{1}{D+1}\right)^{\frac{1}{q}}.$$

4. If one takes $x = \frac{\rho_a+\rho_b}{2}$, $A = D = s$, $B = 1, G = r$, where $s \in [0, 1]$ and $r \in (0, 1]$ in (2.9), then one has the Fuzzy Ostrowski Midpoint inequality for $(s, r)-$convex functions in mixed kinds:

$$D\left(\varphi\left(\frac{\rho_a+\rho_b}{2}\right), \frac{1}{\rho_b-\rho_a} \odot (FR)\int_{\rho_a}^{\rho_b} \varphi(t)dt\right)$$
$$\leq \frac{M(\rho_b-\rho_a)}{2(p+1)^{\frac{1}{p}}}\left(\frac{1}{rs+1} + \frac{B\left(\frac{1}{r}, s+1\right)}{r}\right)^{\frac{1}{q}}.$$

5. If one takes $x = \frac{\rho_a+\rho_b}{2}$, $A = B = r$ and $G = D = 1$, where $r \in (0, 1]$ in (2.9), then one has the Fuzzy Ostrowski Midpoint inequality for $r-$convex functions in 1^{st} kind:

$$D\left(\varphi\left(\frac{\rho_a+\rho_b}{2}\right), \frac{1}{\rho_b-\rho_a} \odot (FR)\int_{\rho_a}^{\rho_b} \varphi(t)dt\right)$$
$$\leq \frac{M(\rho_b-\rho_a)}{2(p+1)^{\frac{1}{p}}}\left(\frac{1}{r+1} + \frac{B\left(\frac{1}{r}, 2\right)}{r}\right)^{\frac{1}{q}}.$$

6. if one takes $x = \frac{\rho_a+\rho_b}{2}$, $B = G = 1$, $A = D = s$ where $s \in [0, 1]$ in (2.9), then one has the Fuzzy Ostrowski Midpoint inequality for $s-$convex functions in 2^{nd} kind:

$$D\left(\varphi\left(\frac{\rho_a+\rho_b}{2}\right), \frac{1}{\rho_b-\rho_a} \odot (FR)\int_{\rho_a}^{\rho_b} \varphi(t)dt\right)$$
$$\leq \frac{(2)^{\frac{1}{q}-1}M(\rho_b-\rho_a)}{(p+1)^{\frac{1}{p}}}\left(\frac{1}{s+1}\right)^{\frac{1}{q}}.$$

7. If one takes $x = \frac{\rho_a+\rho_b}{2}$, $A = D = 0$ and $B = G = 1$ in (2.9), then one has the Fuzzy Ostrowski Midpoint inequality for $P-$convex functions:

$$D\left(\varphi\left(\frac{\rho_a+\rho_b}{2}\right), \frac{1}{\rho_b-\rho_a} \odot (FR)\int_{\rho_a}^{\rho_b}\varphi(t)dt\right) \leq \frac{(2)^{\frac{1}{q}-1}M\left(\rho_b-\rho_a\right)}{(p+1)^{\frac{1}{p}}}.$$

8. If one takes $x = \frac{\rho_a+\rho_b}{2}$ and $A = B = G = D = 1$ in (2.9), then one has the Fuzzy Ostrowski Midpoint inequality for convex functions:

$$D\left(\varphi\left(\frac{\rho_a+\rho_b}{2}\right), \frac{1}{\rho_b-\rho_a} \odot (FR)\int_{\rho_a}^{\rho_b}\varphi(t)dt\right) \leq \frac{M\left(\rho_b-\rho_a\right)}{2\,(p+1)^{\frac{1}{p}}}.$$

3. Conclusion

Ostrowski inequality is one of the most celebrated inequalities. In this paper, we presented the generalized notion of $(A, B, G, D)-$convex (concave) functions in mixed kinds. This class of functions contains many important classes, including the class of $(A, B)-$convex (concave) functions in 1^{st} and 2^{nd} kind [15], $(s, r)-$convex (concave) functions in mixed kinds [2], $s-$convex (concave) functions in 1^{st} and 2^{nd} kind [4], $P-$convex (concave) functions [14], quasi convex(concave) functions [16] and the class of convex (concave) functions[3]). We have stated our first main result in section 2, the generalization of the Ostrowski inequality [23] via Fuzzy Riemann integrals with $(A, B, G, D)-$convex (concave) functions in mixed kinds. Further, we used different techniques including Hölder's inequality[27] and power mean inequality[26] for the generalization of the Fuzzy Ostrowski inequality.

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