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ON COMMUTATIVE HYPER *BE*-ALGEBRAS

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Abstract. In this paper, we introduce commutative hyper *BE*-algebra and study it in detail. We show that every commutative (row diagonal, column row, very thin) hyper *BE*-algebra is a *BE*-algebra.

1. Introduction and Preliminaries

H. S. Kim and Y. H. Kim introduced the notion of a *BE*-algebra as a generalization of a dual *BCK*-algebra [4]. S. S. Ahn and et al. introduced the notions of terminal sections of a *BE*-algebras and gave some characterization of commutative *BE*-algebras in terms of lattices order relations and terminal sections [1]. A. Rezaei and et al. show that a commutative implicative *BE*-algebra is equivalent to the commutative self-distributive *BE*-algebra. Also, they proved that every Hilbert algebra is a self-distributive *BE*-algebra and commutative self-distributive *BE*-algebra is a Hilbert algebra and show that cannot remove the conditions commutativity and self-distributivity [7].

The hyper algebraic structure theory was introduced in 1934, by F. Marty at the 8th congress of Scandinavian Mathematicians [5]. Hyper-structures have many applications to several sectors of both pure and applied sciences. Y. B. Jun and et al. applied the hyper-structures to *BCK*-algebras and introduced the notion of a hyper *BCK*-algebra which is a generalization of *BCK*-algebra and investigated some related properties [3].

Recently, A. Radfar and et al. introduced the notion of hyper *BE*-algebra and defined some types of hyper-filters in hyper *BE*-algebras. They showed that under special condition hyper *BE*-algebras are equivalent to dual hyper *K*-algebras [6].

In this paper we characterize the relation between dual hyper *K*-algebras and commutative hyper *BE*-algebras and some types of commutative hyper *BE*-algebras and characterization of $RD/CR/V$-hypercommutative *BE*-algebras are state. We show that every commutative *RD*-hyper *BE*-algebra of order 3 is a commutative *BE*-algebra.

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Definition 1.1. [2] Let H be a nonempty set and $\circ : H \times H \to P^*(H)$ be a hyperoperation. Then $(H; \circ, 0)$ is called a hyper K-algebra, if it satisfies the following axioms:

(HK_1) $(x \circ z) \circ (y \circ z) < x \circ y$,

(HK_2) $(x \circ y) \circ z = (x \circ z) \circ y$,

(HK_3) $x < x$,

(HK_4) $x < y$ and $y < x$ imply that $x = y$,

(HK_5) $0 < x$, for all $x, y, z \in H$.

Where $x < y$ is defined by $0 \in x \circ y$.

Theorem 1.1. *[2] Let H be a hyper K-algebra. Then*

(i) $x \in x \circ 0$,

(ii) $0 \in 0 \circ x$, *for all* $x \in H$.

Definition 1.2. [6] Let H be a nonempty set and $\circ : H \times H \to P^*(H)$ be a hyperoperation. Then $(H; \circ, 1)$ is called a hyper BE-algebra, if it satisfies the following axioms:

(HBE_1) $x < 1$ and $x < x$,

(HBE_2) $x \circ (y \circ z) = y \circ (x \circ z)$,

(HBE_3) $x \in 1 \circ x$,

(HBE_4) $1 < x$ implies $x = 1$, for all $x, y, z \in H$.

A hyper-BE-algebra is said to be

(i) row hyper BE-algebra (for short, R-hyper BE-algebra), if $1 \circ x = \{x\}$, for all $x \in H$,

(ii) column hyper BE-algebra (for short, C-hyper BE-algebra), if $x \circ 1 = \{1\}$, for all $x \in H$,

(iii) diagonal hyper BE-algebra (for short, D-hyper BE-algebra), if $x \circ x = \{1\}$, for all $x \in H$,

(iv) thin hyper BE-algebra (for short, T-hyper BE-algebra), if that is RC-hyper BE-algebra,

(v) very thin hyper BE-algebra (for short, V-hyper BE-algebra), if that is RCD-hyper BE-algebra,

$(H;\circ,1)$ is called a dual hyper K-algebra if satisfies (HBE_1), (HBE_2) and the following axioms:

(DHK_1) $x \circ y < (y \circ z) \circ (x \circ z)$,

(DHK_4) $x < y$ and $y < x$ imply that $x = y$, for all $x, y, z \in H$.

Where the relation " $<$ " is defined by $x < y \Leftrightarrow 1 \in x \circ y$. For any two nonempty subsets A and B of H, we define $A < B$ if and only if there exist $a \in A$ and $b \in B$ such that $a < b$ and

$$A \circ B = \bigcup_{a \in A, b \in B} a \circ b.$$

Theorem 1.2. [6] *Let H be a hyper BE-algebra. Then*

(*i*) $A \circ (B \circ C) = B \circ (A \circ C)$,

(*ii*) $A < A$,

(*iii*) $1 < A$ *implies* $1 \in A$,

(*iv*) $x < y \circ x$,

(*v*) $x < y \circ z$ *implies* $y < x \circ z$,

(*vi*) $x < (x \circ y) \circ y$,

(*vii*) $z \in x \circ y$ *implies* $x < z \circ y$,

(*viii*) $y \in 1 \circ x$ *implies* $y < x$, *for all* $x, y, z \in H$ *and* $A, B, C \subseteq H$.

Corollary 1.1. *[6] Every dual hyper K-algebra is a hyper BE-algebra.*

Theorem 1.3. *[6] Let H be a CD-hyper BE-algebra. Then*

(*i*) $x \circ (y \circ x) = \{1\}$,

(*ii*) $z \in x \circ y$ *implies* $y \circ z = \{1\}$,

for all $x, y, z \in H$.

Theorem 1.4. [8] *Let X be a commutative BE-algebra. Then $x \leq y$ and $y \leq x$ implies $x = y$, for all $x, y \in X$.*

2. On commutative hyper *BE*-algebras

Definition 2.1. A hyper *BE*-algebra (dual hyper *K*-algebra) H is said to be commutative if $(x \circ y) \circ y = (y \circ x) \circ x$, for all $x, y \in H$.

Example 2.1. (i). Let $H = \{1, a, b\}$. Define the hyper-operations " $\circ_1$ " as follows:

$\circ_1$	1	a	b
1	$\{1\}$	$\{a\}$	$\{b\}$
a	$\{1, b\}$	$\{1, a, b\}$	$\{1, a\}$
b	$\{1, a, b\}$	$\{a\}$	$\{1, a, b\}$

Then $(H, \circ_1)$ is a commutative hyper *BE*-algebra.

(ii). Define the hyper operation " $\circ$ " on $\mathbb{R}$ as follows:

$$x \circ y = \begin{cases} \{y\} & \text{if } x = 1 \\ \mathbb{R} & \text{otherwise} \end{cases}$$

Then $(\mathbb{R}; \circ, 1)$ is a commutative hyper *BE*-algebra.

Lemma 2.1. *Let H be a commutative hyper BE-algebra. Then H satisfies in (DHK_1).*

Proof. Let H be a commutative hyper *BE*-algebra and $x, y, z \in H$. Then

$$\begin{aligned} (x \circ y) \circ ((y \circ z) \circ (x \circ z)) &= (x \circ y) \circ (x \circ ((y \circ z) \circ z) \\ &= (x \circ y) \circ (x \circ ((z \circ y) \circ y)) \\ &= (x \circ y) \circ ((z \circ y) \circ (x \circ y)) \\ &= (z \circ y) \circ ((x \circ y) \circ (x \circ y)). \end{aligned}$$

Now, using (HBE_1), we have $1 \in (x \circ y) \circ (x \circ y)$.
Also, by (HBE_2),

$$1 \in (z \circ y) \circ ((x \circ y) \circ (x \circ y)).$$

Thus

$$1 \in (x \circ y) \circ ((y \circ z) \circ (x \circ z)).$$

Therefore (DHK_1) holds. □

Theorem 2.1. *H is a commutative dual hyper K-algebra if and only if H is a commutative hyper BE-algebra and satisfies in (DHK_2).*

Proof. Let H be a commutative dual hyper K-algebra. Using Corollary 1.1, H is a commutative hyper *BE*-algebra.

Conversely, let H be a commutative hyper *BE*-algebra, satisfies in condition (DHK_2) and $x, y, z \in H$. Then by Lemma 2.1, (DHK_1) holds. Therefore H is a dual hyper K-algebra. □

The following example show that condition (DHK_2) in Theorem 2.1, is necessary.

Example 2.2. Let $H = \{1, a, b\}$. Define the hyper-operation " $\circ$ " as follows:

$\circ$	1	a	b
1	$\{1\}$	$\{a\}$	$\{b\}$
a	$\{1\}$	$\{1\}$	$\{1, a, b\}$
b	$\{1, a, b\}$	$\{1, b\}$	$\{1, a, b\}$

Then H is a commutative hyper BE-algebra. Since $a < b$ and $b < a$, H is not a hyper K-algebra.

Proposition 2.1. *Let H be a commutative R-hyper BE-algebra. Then $x \circ y = y \circ x = \{1\}$ implies $x = y$.*

Proof. Let $x \circ y = y \circ x = \{1\}$. Since H is a commutative R-hyper BE-algebra,

$$\{x\} = 1 \circ x = (y \circ x) \circ x = (x \circ y) \circ y = 1 \circ y = \{y\}.$$

Therefore $x = y$. $\square$

Lemma 2.2. *Let H be a commutative C-hyper BE-algebra. Then*

(i) *H is a commutative CD-hyper BE-algebra,*

(ii) *$y \in 1 \circ x$ implies $1 \circ x \subseteq 1 \circ y$,*

(iii) *$y \in 1 \circ x$ if and only if $x \in 1 \circ y$,*

for all $x, y \in H$.

Proof. (i). Let $x \in H$. Then by commutativity, $x \circ x \subseteq (1 \circ x) \circ x = (x \circ 1) \circ 1 = 1 \circ 1 = \{1\}$. Thus H is a commutative D-hyper BE–algebra.

(ii). Let $y \in 1 \circ x$. Using Theorem 1.2 ($viii$), $y < x$ and so $1 \in y \circ x$. Since $y \in 1 \circ x$, by Theorem 1.3(ii), $x \circ y = \{1\}$ and by commutativity,

$$1 \circ x \subseteq (y \circ x) \circ x = (x \circ y) \circ y = 1 \circ y.$$

Hence $1 \circ x \subseteq 1 \circ y$.

(iii). Let $y \in 1 \circ x$. By (ii), $1 \circ x \subseteq 1 \circ y$. By ($HBE_3$), $x \in 1 \circ x \subseteq 1 \circ y$. Thus $x \in 1 \circ y$. Similarly, $x \in 1 \circ y$ implies $y \in 1 \circ x$. $\square$

Corollary 2.1. *H is a commutative C-hyper BE-algebra if and only if H is a commutative CD-hyper BE-algebra.*

Lemma 2.3. *Let H be a commutative CR-hyper BE-algebra. Then*

(i) *H is a commutative V-hyper BE-algebra,* (*ii*) *If* $x \circ y = \{1\}$, *then* $(y \circ x) \circ x = \{y\}$,

(*iii*) *If* $x \circ y = \{1\}$, *then* $y \circ x = \{z\}$ *and* $y \circ z = \{y\}$, *for some* $z \in H$,

for all $x, y \in H$.

Proof. (*i*). By Corollary 2.1, H is a commutative CRD-hyper BE-algebra and so it is a commutative V-hyper BE-algebra.

(*ii*). If $x \circ y = \{1\}$, then $(y \circ x) \circ x = (x \circ y) \circ y = \{1\} \circ y = \{y\}$.

(*iii*). Let $x \circ y = \{1\}$. At first we prove that $y \circ x$ is a singleton set. Let $a, b \in y \circ x$. By (*ii*), $a \circ x \subseteq (y \circ x) \circ x = \{y\}$. Thus $a \circ x = \{y\}$. By a similar way, $b \circ x = \{y\}$. By (*i*), H is a V-hyper BE-algebra and so it is a CD-hyper BE-algebra. Now, using Theorem 1.3 (*ii*) and since $a \in y \circ x$, we have $x \circ a = x \circ b = \{1\}$. By (*ii*), $(a \circ x) \circ x = \{a\}$. Thus

$$\{a, b\} \subseteq y \circ x = (a \circ x) \circ x = \{a\}.$$

Hence $a = b$ and $y \circ x$ is a singleton set. Now, let $y \circ x = \{z\}$. By using (*ii*), $y \circ z = y \circ (y \circ x) = \{y\}$. □

Theorem 2.2. *Every commutative CR-hyper BE-algebra is a commutative BE-algebra.*

Proof. Let H be a commutative CR-hyper BE-algebra. Let $a, b \in x \circ y$. By Lemma 2.3 (*i*), H is a V-hyper BE-algebra and so is a CD-hyper BE-algebra. By Theorem 1.3 (*ii*) and since $a, b \in x \circ y$, we can see that $y \circ a = y \circ b = \{1\}$. Now, using Lemma 2.3 (*iii*),

$$a \circ y = \{c\},\ c \circ y = \{a\},\ b \circ y = \{d\}, \text{ and } d \circ y = \{b\} \text{ for some } c, d \in H.$$

Since $d = b \circ y \subseteq (x \circ y) \circ y = (y \circ x) \circ x$, there is $t_1 \in y \circ x$, such that $d \in t_1 \circ x$. By Theorem 1.3 (*ii*), we imply that $x \circ d = \{1\}$. In a similar way, $c = a \circ y \subseteq (x \circ y) \circ y = (y \circ x) \circ x$, there is $t_2 \in y \circ x$, such that $c \in t_2 \circ x$. By Theorem 1.3 (*ii*), we get $x \circ c = \{1\}$. By (HBE_2),

$$b \circ a \subseteq b \circ (x \circ y) = x \circ (b \circ y) = x \circ d = \{1\}.$$

$$a \circ b \subseteq a \circ (x \circ y) = x \circ (a \circ y) = x \circ c = \{1\}.$$

Thus $a \circ b = b \circ a = \{1\}$. Now, using Proposition 2.1, $a = b$. Therefore $x \circ y$ is a singleton set for every $x, y \in H$ and so H is a commutative BE-algebra. □

3. Characterization of commutative hyper *BE*-algebra of order 3

From now on, H is a commutative hyper BE-algebra of order 3.

Lemma 3.1. *Let H be a hyper BE-algebra. Then*

(*i*) $1 \notin 1 \circ x$, *for all* $1 \neq x \in H$, (*ii*) $1 \in x \circ 1$, *for all* $x \in H$.

Proof. (*i*). Let $x \neq 1$ and $x \in H$. $1 \in 1 \circ x$ implies $1 < x$. By (HBE_4), $x = 1$, which is a contradiction.

(*ii*). The proof is clear by using (HBE_1). □

Theorem 3.1. *Let H be a commutative D-hyper BE-algebra of order 3. Then H is a commutative CD-hyper BE-algebra of order 3.*

Proof. Let $H = \{1, a, b\}$ be a commutative D-hyper BE-algebra and it is not a C-hyper BE-algebra. Then $a \circ 1 \neq \{1\}$ or $b \circ 1 \neq \{1\}$. Without loss of generality, let $a \circ 1 \neq \{1\}$. By (HBE_1), $1 \in a \circ 1$. Thus $a \circ 1 = \{1, a\}$, $\{1, b\}$ or $\{1, a, b\}$ and so we have three cases:

Case 1: If $a \circ 1 = \{1, a\}$, then

$$1 \circ a \subseteq 1 \circ (a \circ 1) = a \circ (1 \circ 1) = a \circ 1 = \{1, a\}.$$

By Lemma 3.1, $1 \notin 1 \circ a$ and so $1 \circ a = \{a\}$. Now, by commutativity,

$$\{1\} = a \circ a = (1 \circ a) \circ a = (a \circ 1) \circ 1 = \{1, a\} \circ 1 = \{1, a\},$$

which is a contradiction. Thus $a \circ 1 \neq \{1, a\}$.

Case 2: If $a \circ 1 = \{1, b\}$, then by Lemma 3.1(*ii*) and (HBE_2),

$$\{1, b\} = a \circ 1 \subseteq a \circ (b \circ 1) = b \circ (a \circ 1) = b \circ \{1, b\} = b \circ 1 \cup b \circ b = 1 \cup b \circ 1.$$

Hence $b \circ 1 = \{1, b\}$. Which is a contradiction.

Case 3: If $a \circ 1 = \{1, a, b\}$, then by commutativity,

$$(1 \circ a) \circ a = (a \circ 1) \circ 1 = \{1, a, b\} \circ 1 \supseteq a \circ 1 = \{1, a, b\}.$$

Now, by Lemma 3.1, $1 \circ a = \{a\}$ or $\{a, b\}$. Since $1 \circ a = \{a\}$, we have

$$(1 \circ a) \circ a = a \circ a = \{1\}.$$

Thus $1 \circ a \neq \{a\}$ and so $1 \circ a = \{a, b\}$. Also,

$$\{1, a, b\} = (1 \circ a) \circ a = \{a, b\} \circ a = a \circ a \cup b \circ a = 1 \cup b \circ a.$$

Thus $\{a, b\} \subseteq b \circ a$. Also, since $1 \circ a = \{a, b\}$, by Theorem 1.2 (*viii*), $b < a$ and so $1 \in b \circ a$. Hence $b \circ a = \{1, a, b\}$. Also, by $1 \circ a = \{a, b\}$ and Lemma 2.2(*iii*), we imply that $a \in 1 \circ b$ and so $1 \circ b = \{a, b\}$. Now, since H is a D—-hyper BE—-algebra and by (HBE_2),

$$b \circ a \subseteq b \circ (1 \circ b) = 1 \circ (b \circ b) = 1 \circ 1 = \{1\}.$$

Thus $b \circ a = \{1\}$, which is a contradiction. □

Corollary 3.1. *Three concepts - commutative C-hyper BE-algebra, commutative D-hyper BE-algebra and commutative CD-hyper BE-algebra - coincide.*

Corollary 3.2. *Every commutative RD-hyper BE-algebra of order 3 is a commutative BE-algebra.*

Theorem 3.2. *There exist two commutative V-hyper BE-algebra of order 3 up to isomorphism.*

Proof. Let $H = \{1, a, b\}$ be a commutative V-hyper BE-algebra. Then by Theorem 2.2, H is a BE-algebra and so,

$$1 \circ 1 = \{1\},\ 1 \circ a = \{a\},\ 1 \circ b = \{b\}$$

and

$$a \circ 1 = b \circ 1 = a \circ a = b \circ b = \{1\}.$$

By Theorem 1.4, a and b are comparable, hence $a < b$ or $b < a$. Case $a < b$ and case $b < a$ are isomorphic. So, without lose of generality let $a < b$. Then $a \circ b = \{1\}$. By commutativity, we have,

$$(b \circ a) \circ a = (a \circ b) \circ b = 1 \circ b = \{b\}.$$

If $b \circ a = \{1\}$, then $(b \circ a) \circ a = 1 \circ a = \{a\}$, which is a contradiction.
If $b \circ a = \{a\}$, then $(b \circ a) \circ a = a \circ a = \{1\}$, which is a contradiction.
Since H is a BE-algebra, $b \circ a = \{b\}$. Therefore H is a commutative V-hyper BE-algebra.

$\circ$	1	a	b
1	$\{1\}$	$\{a\}$	$\{b\}$
a	$\{1\}$	$\{1\}$	$\{1\}$
b	$\{1\}$	$\{b\}$	$\{1\}$

Now, if a and b are comparable, then $a \circ b = \{a\}$ or $\{b\}$. If $a \circ b = \{a\}$, then by Theorem 1.3(*ii*), $b \circ a = \{1\}$ and so $b < a$, which is a contradiction. Hence $a \circ b = \{b\}$. By a similar way $b \circ a = \{a\}$. So, H is a commutative V-algebra.

$\circ$	1	a	b
1	$\{1\}$	$\{a\}$	$\{b\}$
a	$\{1\}$	$\{1\}$	$\{b\}$
b	$\{1\}$	$\{a\}$	$\{1\}$

□

Theorem 3.3. *There exist three commutative D-hyper BE-algebra of order 3 up to isomorphism.*

Proof. By Theorem 3.1, H is a CD-hyper BE-algebra and so,

$$1 \circ 1 = 1 \circ a = a \circ a = b \circ 1 = b \circ b = \{1\}.$$

By Lemma 3.1(i), $1 \notin 1 \circ a$. Also, by (HBE_3), $a \in 1 \circ a$. Thus $1 \circ a = \{a\}$ or $\{a, b\}$. By a similar way, $1 \circ b = \{b\}$ or $\{a, b\}$. If $1 \circ a = \{a\}$, then $1 \circ b = \{b\}$ (Since $1 \circ b = \{a, b\}$ we have $b \in 1 \circ a$, which is a contradiction). Thus H is a commutative V-hyper BE-algebra. By Theorem 3.2, two commutative V-hyper BE-algebra exist.

Now, if $1 \circ a = \{a, b\}$, then by Lemma 2.2(iii), $a \in 1 \circ b$ and so $1 \circ b = \{a, b\}$. By Theorem 1.3 (ii), $1 \circ a = \{a, b\}$ implies $b \circ a = \{1\}$ and $1 \circ b = \{a, b\}$ implies $a \circ b = \{1\}$. Therefore H is a commutative hyper BE-algebra in the following table.

$\circ$	1	a	b
1	$\{1\}$	$\{a, b\}$	$\{a, b\}$
a	$\{1\}$	$\{1\}$	$\{1\}$
b	$\{1\}$	$\{1\}$	$\{1\}$

□

Theorem 3.4. (i). *There exist 889 commutative hyper BE-algebras of order 3 up to isomorphism.*

(ii). *There exist 68 commutative R-hyper BE-algebras of order 3 up to isomorphism.*

4. Conclusion and future work

Now, in the following table we summarize the results of this paper and show that the number of all kinds of commutative hyper BE-algebras of order 3. We note that by Corollary 3.1, the three concepts - commutative D-hyper BE-algebra, commutative C-hyper BE-algebra and commutative CD-hyper BE-algebra - coincide. Also, by Theorem 2.2, Corollaries 3.1 and 3.2, every commutative $RD/CR/V$-hyper BE-algebra is a BE-algebra.

Type	Condition	Number
commutative hyper BE-algebra	$a < b$ and $b \nless a$	325
commutative hyper BE-algebra	$a < b$ and $b < a$	495
commutative hyper BE-algebra	$a \nless b$ and $b \nless a$	69
commutative hyper BE-algebra		889
commutative R-hyper BE-algebra	$a < b$ and $b \nless a$	30
commutative R-hyper BE-algebra	$a < b$ and $b < a$	24
commutative R-hyper BE-algebra	$a \nless b$ and $b \nless a$	14
commutative R-hyper BE-algebra		68
commutative D-hyper BE-algebra	$a < b$ and $b \nless a$	1
commutative D-hyper BE-algebra	$a < b$ and $b < a$	1
commutative D-hyper BE-algebra	$a \nless b$ and $b \nless a$	1
commutative D-hyper BE-algebra		3
(commutative RD-hyer)commutative BE-algebra	$a < b$ and $b \nless a$	1
(commutative RD-hyer)commutative BE-algebra	$a < b$ and $b < a$	0
(commutative RD-hyer)commutative BE-algebra	$a \nless b$ and $b \nless a$	1
(commutative RD-hyer)commutative BE-algebra		2

In the future work we will try to get some results on another type of commutative hyper *BE*-algebras and state some properties on this structure and investigate some relationships between them.

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