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# RISK PRIORITY EVALUATION OF POWER TRANSFORMER PARTS BASED ON HYBRID FMEA FRAMEWORK UNDER HESITANT FUZZY ENVIRONMENT

**Bin Zhou[1], Jing Chen[1], Qun Wu[2], Dragan Pamučar[3], Weizhong Wang[4], Ligang Zhou[5]**

[1]Anhui Electrical Engineering Professional Technique College, Hefei, Anhui, China
[2] School of Economics and Management, Southeast University, Nanjing, Jiangsu, China
[3] Department of Logistics, Military Academy, University of Defence in Belgrade, Belgrade, Serbia
[4] School of Economics and Management, Anhui Normal University, Wuhu, Anhui, China
[5] School of Mathematical Sciences, Anhui University, Hefei, Anhui, China

**Abstract**. *The power transformer is one of the most critical facilities in the power system, and its running status directly impacts the power system's security. It is essential to research the risk priority evaluation of the power transformer parts. Failure mode and effects analysis (FMEA) is a methodology for analyzing the potential failure modes (FMs) within a system in various industrial devices. This study puts forward a hybrid FMEA framework integrating novel hesitant fuzzy aggregation tools and CRITIC (Criteria Importance Through Inter-criteria Correlation) method. In this framework, the hesitant fuzzy sets (HFSs) are used to depict the uncertainty in risk evaluation. Then, an improved HFWA (hesitant fuzzy weighted averaging) operator is adopted to fuse risk evaluation for FMEA experts. This aggregation manner can consider different lengths of HFSs and the support degrees among the FMEA experts. Next, the novel HFWGA (hesitant fuzzy weighted geometric averaging) operator with CRITIC weights is developed to determine the risk priority of each FM. This method can satisfy the multiplicative characteristic of the RPN (risk priority number) method of the conventional FMEA model and reflect the correlations between risk indicators. Finally, a real example of the risk priority evaluation of power transformer parts is given to show the applicability and feasibility of the proposed hybrid FMEA framework. Comparison and sensitivity studies are also offered to verify the effectiveness of the improved risk assessment approach.*

**Key words**: *Risk priority evaluation, Power transformer parts, Aggregation operator, CRITIC method, Hesitant fuzzy sets*

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**Corresponding authors**: Jing Chen and Qun Wu
Anhui Electrical Engineering Professional Technique College, Hefei, Anhui, 230051, China, and School of Economics and Management, Southeast University, Nanjing, Jiangsu, 211189, China
E-mails: chenjingdz024@163.com (J. Chen) and qunw_ahu@126.com (Q. Wu)

## 1. INTRODUCTION

The power transformer is one of the most important and expensive facility in the power system [1]. It holds the balance in guaranteeing the secure operation of the power system. Usually, power transformers in the operating process are often subject to various factors kinds of stresses as electrical, mechanical, chemical, and thermal stress from internal and external environments. If some accidents happen in the power transformer, it will destroy the normal running of the power systems, even thorough disruption of power systems. The power transformer is a complicated structure that involves several parts such as iron core, tap switch, cooling system, non-electric quantity protection, winding, bush, and body part [2]. The running failure risk of the power transformer is closely correlated with the risks of the power transformer parts. Through decomposing the power transformer into several parts and achieving the quantitative risk assessment and priority evaluation of these parts, one can identify the weak link of the system and determine the risk factor which does highly affect the equipment reliability [3]. The risk priority evaluation of power transformer parts significantly enhances the pertinence on transformer repair and maintenance and prevents and reduces failure risk in electricity-management systems. Consequently, the study on risk priority evaluation of power transformer parts has essential significance in both theory and practice [4].

The FMEA (failure mode and effect analysis) method, known as one of the most effective risk analysis tools, has been widely applied in prioritizing the risk failure modes (FMs) in electro-medic industry [5], supply chain system [6], and the chemical industry [7]. Diverse disciplines and fields like evidential reasoning method [8], prospect theory [9], probabilistic graphic [10], and logistic regression [11] have also been performed to boost the development of the theoretical researches on FMEA. As for the traditional FMEA, experts employ a 1-10 numerical scale to offer the risk priority number (RPN) to measure the risk value and the overall risk prioritization. A large value implies a high-risk degree. For more details about the FMEA implementation process, one can read [12, 13]. Nevertheless, the RPN ignores uncertainty when experts present their reference opinions. Owing to the uncertain and complex nature of risk, one cannot accurately estimate the risk priority evaluation in real life. Plenty of uncertain tools have been set to overcome the deficiencies of RPN in classical FMEA [14, 15]. The fuzzy technique is one of the most active and the broadest fields of application research in improving FMEA [16-18].

The fuzzy technique is a flexible means of representing subjective uncertainty and fuzziness when experts declare their opinions. With the scholars increasingly focusing attention on fuzzy sets (FSs) theory, there are various types of FSs supporting the applicability and convenience of uncertain FMEA for risk evaluation and prioritization. Some famous FSs such as the intuitionistic fuzzy sets (IFSs) [19], Z-numbers [20], and Pythagorean fuzzy sets [21] were used in the FMEA successfully, which improved the robustness of the risk analysis procedure. However, there often exist conditions that experts are hesitant to offer a precise risk value. Then, the classical FSs will be inadequate and incapable of describing such hesitant evaluation information. As an expansion of the traditional FSs and IFSs, the hesitant fuzzy sets (HFSs) has several possible membership values, making it more flexible and comprehensive in representing fuzzy information [22]. The HFSs also has the simple mathematical expression form and numerical computation [23]. It has already successfully been used in various decision-making fields like green sustainable airport design [24], supplier selection [25], and E-learning website evaluation [26]. This paper uses the HFSs to appraise the fuzzy evaluation of risk indicators under each FM to better express the complicated vagueness and hesitancy in the risk assessment.

The risk assessment in the FMEA framework could be thought of as a multi-criteria group decision-making (MCGDM) problem [27, 28]. Under the MCGDM term for FMEA, the FMEA experts' weights and the risk indicators' weights play crucial roles in deciding the group risk evaluation and the final collective risk values of the FMs. In the FMEA team, the FMEA experts might own different perspectives of the risk assessments as they have diverse professional backgrounds and specialized skills. Concerning experts' weights, because the risk analysis process needs to cooperate mutually, the principle of the support degrees among their opinions provides a new angle for fusing their risk evaluations. Regarding the weights of risk indicators, existing objective weighting methods like the entropy method [9] and the maximizing deviation method [29] think mostly about the differences among the risk indicators. They do not take the possible correlations among risk indicators into account. Hence the objective weighting method that can consider the correlations among risk indicators when operating the risk analysis is welcome. Meanwhile, the subjective weights cannot be negligible when processing the risk evaluation. As a result, it could be imperative to incorporate both subjective and objective weights for FMEA experts and risk indicators to achieve a comprehensive risk evaluation result.

Even though the classical and improved FMEA framework for risk evaluation and prioritization have made fruitful achievements. As the current risk analysis environment gets more complex, FMEA still holds some space for its self-exploitation in mathematical modeling. In summary, the following three research motivations will increase the feasibility and practicability of the FMEA problems:

- The precise numerical risky evaluations are unavailable to the FMEA experts. Even though the FSs have numerous domains to FMEA problems, few studies paid attention to the HFSs for FMEA risk evaluation. It is meaningful to research the FMEA disposing of uncertain information under the HFSs environment.
- Despite many successful application fields using the HFSs, there is still a gap in the theoretical research results of the HFSs, especially in dealing with HFSs of different lengths. It is beneficial to develop a solution that can not only ensure the integrity of the HFSs but also offer a computational model and some information measures for the HFSs with different lengths.
- Most of the existing FMEA frameworks for risk evaluation support that mutual independence exists among the risk indicators. However, there are often some correlations among risk indicators in actual risk analysis. Thus, it is necessary to integrate the objective weights for risk indicators with the FMEA approach.

This paper will develop a risk priority evaluation model using a hybrid FMEA approach under the HFSs environment, and the main contributions of our paper include:

- The HFSs representation model is an efficient tool for expressing the hesitant fuzzy evaluation offered by the FMEA experts. Using the least common multiple rule to deal with HFSs of different lengths and utilizing the novel hesitant fuzzy aggregation operators and distance measures for hesitant fuzzy modeling and calculation can enrich the theoretical research results of the HFSs.
- Consider the evaluation similarity between experts and the correlations among risk indicators. The novel HFWA operator with the support degree is developed to fuse the individual risk evaluation matrix. Then the novel HFWGA operator with CRITIC weights is applied to decide the risk priorities of FMs. The subjective and objective combined weights will help get a complete risk evaluation result.

- To illustrate the validity of our proposed novel FMEA methodology, a case study of risk priority evaluation of power transformer parts is provided. Then the comparative analysis with some existing FMEA models is implemented, which also vastly guarantees the persuasive power.

The rest of our paper is: Some literature is reviewed in Section 2. Section 3 are some basic theoretical concepts about the HFSs, including the definition, new operational laws and aggregation operators, and the distance measure. Section 4 constructs a hybrid FMEA framework with hesitant fuzzy aggregation tools and combined weights for FMEA experts and risk indicators. Section 5 employs the proposed model for the risk prioritization of power transformer parts. In Section 6, sensitivity and comparative analyses are performed to verify the validity of the proposed model. Some conclusions are drawn in Section 7.

## 2. Literature Review

### 2.1. FMEA Approach Using FSs Theory

FMEA is one of the most powerful tools for analyzing system reliability and safety. Research showed that the FMEA plays a significant role in maintenance management strategies in risk analysis. With the relentless march of technological progress, the mathematic model, MCDM approach, and artificial intelligence are adopted more and more when solving FMEA problems. The risk priority evaluation involves high complexity and various angles, and the FMEA method is proven to be valid to assist experts in making a reasonable decision. In recent years, many scholars have focused on research in the FMEA area using fuzzy techniques and achieved encouraging results [30, 31]. Zhang et al. [32] developed a linguistic distribution opinion evolution-based social network consensus model to help FMEA experts reach a consensus. Using an extended COPRAS method to solve the risk evaluation issues and determine the weights with the Kemeny Median and SWARA methods, Shen and Liu [33] proposed a novel risk assessment model in a free double hierarchy hesitant fuzzy linguistic term set environment. Jin et al. [34] ranked FMs using interval-valued q-rung orthoptic fuzzy sets (IVq-ROFSs). They developed a scientific risk evaluation model combining the IVq-ROF-deviation maximization method with the IVq-ROF-additive ratio assessment method. Huang and Xiao [35] developed a novel improved FMEA framework for risk evaluation with a novel concept named the interval-valued intuitionistic fuzzy clouds.

The risk analysis in FMEA is uncertain, complex, and ambiguous. It would be an excellent choice for the experts to use the FSs tools to represent their vagueness in the FMEA execution. This way can preserve information integrity and improve the accuracy of quantitative analyzes.

### 2.2. Decision-Making Evaluation Model with HFSs

Due to the subjective and objective complexity in real-world risk analysis, experts would have difficulties giving their perspectives precisely. By contrast, experts often feel more comfortable hesitantly giving their opinions. The HFSs expressed by several possible membership values have proven solid in modeling the fuzzy evaluation.

In recent years, many researchers have modelled the uncertain information using the HFSs [36–40]. With the early screening of lung cancer is becoming exceedingly significant for the effective treatment of lung cancer, Liao et al. [36] constructed a framework uses the double normalization-based multi-aggregation (DNMA) and Delphi methods with HFSs to solve the lung cancer screening problem. Mishra et al. [37] established a hybrid framework containing the ARAS method and a novel divergence measure for HFSs, to find the best antiviral therapy which can be reliable for COVID-19 patients with mild symptoms. Liao et al. [38] put forward a Choquet integral-based hesitant fuzzy gained and lost dominance score method considering the interactions among criteria and the experts' preference characteristics in MCGDM problems and then applied their model to the higher business education evaluation. Considering that energy storage technologies (ESTs) enable coping with the intermittency of energy sources by storing excess energy for use when needed, Colak and Kaya [39] established an integrated MCDM model consolidating the Delphi, AHP, and VIKOR methods with HFSs to evaluate ESTs for Turkey.

As reviewed beforehand, it is regularly hard for FMEA experts to give their appraisal utilizing exact value on a mathematical size of 1-10. Compared with other fuzzy tools, the HFSs is more successful in displaying the hesitant uncertainty in real applications. Hence, HFSs are applied to manage the ambiguous risk evaluations in this paper.

### 2.3. The Objective CRITIC Weighting Approach

When computing the risk priority of FMs, the final evaluation and ranking are highly dependent on the weights of the risk indicators. The weighting determination methods usually incorporate subjective and objective weighting strategies. The subjective weights mainly rely on the expert's judgments, which are impacted by the experts' professional knowledge structure and expertise and skills in the related areas. The objective weighting technique decides the weights with the given assessment data by addressing mathematical models. These objective weights are beneficial in cases where the subjective weights given by the experts are inconsistent so that one can achieve more objective results.

The well-known objective CRITIC weighting method was initiated by Diakoulaki et al. [41] in 1995. It relies on the analysis of the assessment matrix for collecting all preference information included in the assessment criteria. The calculation of acquiring the objective weight contains the standard deviation of the criteria and the correlation between the criteria. CRITIC method has been successfully utilized in accomplishing various decision-making scenarios [42–46]. Lai and Liao [42] focused on solving the blockchain platform evaluation issues by incorporating linguistic D numbers (LDNs), DNMA method, and CRITIC method. In their model, the CRITIC technique is combined with the LDN-based DNMA approach to mirror the correlations between criteria in the blockchain platform assessment course. As for the sustainable supply chain risk management (SSCRM), to characterize the principal risk factors and rank the business alternatives based on these factors, Abdel-Basset and Mohamed [43] developed a joined pathogenic methodology dependent on the TOPSIS-CRITIC strategy to gauge the SSCRM factors. Peng and Huang [44] presented a novel q-ROF financial risk decision-making method based on CoCoSo with CRITIC methods. They applied the combined weight method based on CRITIC and the linear weighted comprehensive process to simultaneously consider subjective and objective information. Wu et al. [45] offered a practical model for site determination of photovoltaic hydrogen production project based on the MCDM method. Their model determines the subjective and objective weights by applying BWM and CRITIC, respectively. Then the collective weights are derived by using the game theory.

The execution of FMEA is a run of the MCGDM issue, and the weights of both FMEA experts and risk indicators ought to be fundamental components in FMEA. In a MCGDM, experts' opinions will be supported by each other, and there are often correlations between criteria in actual FMEA implementation courses. Thus, it is necessary to consider the similarity measure among FMEA experts and the amount of information contained in the risk indicators to offer objective weights for real-world FMEA problems.

## 3. Methodology

### 3.1. Hesitant Fuzzy Sets

**Definition 1.** [23] *Let X be a nonempty set. A HFSs H on X is a function that maps each element of X to a subset of* [0,1] *such that*

$$H = \{< x, h(x) > | \, x \in X\} \tag{1}$$

*where* $h(x)$ *is a set of possible values in* [0,1], *denoting the possible membership degrees of an element* $x \in X$ *to the set H.*

Xia and Xu [23] called $h(x)=\{\gamma_1,\ldots,\gamma_{\#h}\}$ a hesitant fuzzy element (HFE), and let Ω be the set of all HFEs. $\gamma_l$ ($l$=1,…,$\#h$) and $\#h$ are the possible elements and the elements number of $h$, respectively. Without loss of generality, the elements in $h(x)$ are in increasing order, and $\#h$ is called the length of $h$.

**Definition 2.** [23] *For a HFE h,* $s(h) = \frac{1}{\#h}\sum_{l=1}^{\#h}\gamma_l$ *is called the score function of h. For two HFEs* $h_1$ *and* $h_2$*, if* $s(h_1)>s(h_2)$*, then* $h_1$ *is superior to* $h_2$*, denoted by* $h_1 \succ h_2$*; if* $s(h_1)=s(h_2)$*, then* $h_1$ *is indifferent to* $h_2$*, denoted by* $h_1 \sim h_2$*.*

### 3.2. New Operational Laws and Two Novel Aggregation Operators for HFSs

One can find that different HFEs may have different lengths, leading to difficulties in making operations on HFEs. The least common multiple (LCM) rule is widely used in number theory and statistics. It gives valuable references and ideas in studying the multi-fuzzy sets. Inspired by [47], the concept of r-HFE is:

**Definition 3.** *Let* $h=\{\gamma_1,\gamma_2,\ldots,\gamma_{\#h}\}$ *be a HFE, and #h be its length, then* $h^r$ *is a bag* (*or multiset*) *with* $r\cdot\#h$ *elements in* $h^r$ *such that*

$$h^r = \{\underbrace{\gamma_1,\gamma_1,\ldots,\gamma_1}_{r\ times},\underbrace{\gamma_2,\gamma_2,\ldots,\gamma_2}_{r\ times},\ldots,\underbrace{\gamma_{\#h},\gamma_{\#h},\ldots,\gamma_{\#h}}_{r\ times}\}. \tag{2}$$

**Example 1**. Let $h_1$={0.2,0.3} and $h_2$={0.5,0.6,0.7} be two HFEs. As the lengths of $h_1$ and $h_2$ are different, we use the proposed LCM principle to normalize the two HFEs. The lcm of 2 and 3 is 6, then $h_1^3$={0.2,0.2,0.2,0.3,0.3,0.3} and $h_2^2$={0.5,0.5,0.6,0.6,0.7,0.7}.

According to Theorems 2-5 in [48], it has shown that some significant information measures of a HFE stay unaltered under the LCM rule, so next we propose some new operational laws and two novel aggregation operators for HFSs based on the LCM rule

**Definition 4**. *Let h,* $h_1$*, and* $h_2$ *be three HFEs,* $\#h$*,* $\#h_1$ *and* $\#h_2$ *are their lengths, respectively,* $L=lcm(\#h_1,\#h_2)$*, and* $\lambda$ *be a real number. Then*

(1) *Addition:*

$$h_1 \oplus h_2 = \{\underbrace{\gamma_1^1,...,\gamma_1^1}_{L/\#h_1\ times}, \underbrace{\gamma_2^1,...,\gamma_2^1}_{L/\#h_1\ times},..., \underbrace{\gamma_{\#h_1}^1,...,\gamma_{\#h_1}^1}_{L/\#h_1\ times}\} \oplus \{\underbrace{\gamma_1^2,...,\gamma_1^2}_{L/\#h_2\ times}, \underbrace{\gamma_2^2,...,\gamma_2^2}_{L/\#h_2\ times},..., \underbrace{\gamma_{\#h_2}^2,...,\gamma_{\#h_2}^2}_{L/\#h_2\ times}\}$$
$$= \bigcup_{l=1,...,L} \left\{\gamma_{\sigma(l)}^1 + \gamma_{\sigma(l)}^2 - \gamma_{\sigma(l)}^1 \cdot \gamma_{\sigma(l)}^2\right\}.$$

(2) *Multiplication:*

$$h_1 \otimes h_2 = \{\underbrace{\gamma_1^1,...,\gamma_1^1}_{L/\#h_1\ times}, \underbrace{\gamma_2^1,...,\gamma_2^1}_{L/\#h_1\ times},..., \underbrace{\gamma_{\#h_1}^1,...,\gamma_{\#h_1}^1}_{L/\#h_1\ times}\} \otimes \{\underbrace{\gamma_1^2,...,\gamma_1^2}_{L/\#h_2\ times}, \underbrace{\gamma_2^2,...,\gamma_2^2}_{L/\#h_2\ times},..., \underbrace{\gamma_{\#h_2}^2,...,\gamma_{\#h_2}^2}_{L/\#h_2\ times}\}$$
$$= \bigcup_{l=1,...,L} \{\gamma_{\sigma(l)}^1 \cdot \gamma_{\sigma(l)}^2\}.$$

(3) *Scalar-multiplication:* $\lambda h = \bigcup_{\gamma \in h} \{1-(1-\gamma)^\lambda\}$.

(4) *Power operation:* $h^\lambda = \bigcup_{\gamma \in h} \{\gamma^\lambda\}$.

**Property 1**. *Let $h$, $h_1$, and $h_2$ be three HFEs, $\lambda$, $\lambda_1$, and $\lambda_2$ be three real numbers, then*

(1) $h_1 \oplus h_2 = h_2 \oplus h_1$;

(2) $h_1 \otimes h_2 = h_2 \otimes h_1$;

(3) $\lambda(h_1 \oplus h_2) = \lambda h_1 \oplus \lambda h_2$;

(4) $(h_1 \otimes h_2)^\lambda = h_1^\lambda \otimes h_2^\lambda$;

(5) $\lambda_1 h \oplus \lambda_2 h = (\lambda_1 + \lambda_2)h$;

(6) $h^{\lambda 1} \otimes h^{\lambda 2} = h^{\lambda 1 + \lambda 2}$.

**Definition 5.** *Let $h_j$ ($j$=1,2,…,$n$) be a collection of HFEs, $\#h_j$ ($j$=1,2,…,$n$) are the lengths of these HFLEs. L is the LCM of $\#h_j$ ($j$=1,2,…,$n$), and $w=(w_1,w_2,\ldots,w_n)^T$ is the associated weighting vector satisfying $w_j \geq 0$ and $\sum_j w_j = 1$. Then a novel hesitant fuzzy weighted averaging operator is a mapping HFWA: $\Omega^n \rightarrow \Omega$ such that:*

$$HFWA_w(h_1, h_2, ..., h_n) = \overset{n}{\underset{j=1}{\oplus}} w_j h_j = \bigcup_{l=1,2,...,L} \left\{1 - \prod_{j=1}^{n} (1-\gamma_{\sigma(l)}^j)^{w_j} \mid \gamma_{\sigma(l)}^j \in h_j^{L/\#h_j}\right\}. \tag{3}$$

**Definition 6.** *Let $h_j$ ($j$=1,2,…,$n$) be a collection of HFEs, $\#h_j$ ($j$=1,2,…,$n$)* and $L$ be defined before, a novel hesitant fuzzy geometric weighted averaging operator is a mapping *HFGWA:* $\Omega^n \rightarrow \Omega$ such that:

$$HFWGA_w(h_1, h_2, ..., h_n) = \overset{n}{\underset{j=1}{\otimes}} (h_j)^{w_j} = \bigcup_{l=1,2,...,L} \left\{\prod_{j=1}^{n} (\gamma_{\sigma(l)}^j)^{w_j} \mid \gamma_{\sigma(l)}^j \in h_j^{L/\#h_j}\right\}. \tag{4}$$

### 3.3. Novel Distances Between HFSs

**Definition 7.** *Let $h_1$ and $h_2$ be two HFEs, L be the lcm of $\#h_1$ and $\#h_2$, then the normalized generalized distance measure between $h_1$ and $h_2$ is defined as*

$$GD(h_1, h_2) = \left(\frac{1}{L} \sum_{l=1}^{L} \left|\gamma_{\sigma(l)}^1 - \gamma_{\sigma(l)}^2\right|^\kappa\right)^{1/\kappa}, \tag{5}$$

*where $\gamma_{\sigma(l)}^1$ and $\gamma_{\sigma(l)}^2$ are the lth elements in $h_1^{L/\#h1}$ and $h_2^{L/\#h2}$, respectively, and $\kappa \neq 0$ is the generalized distance parameter.*

In particular, if $\kappa$=1 and $\kappa$=2, then Eq. (5) becomes the Hamming distance and the Euclidean distance, respectively. Similarly, the Hausdorff distance can be also extended to HFSs environment. The Hausdorff distance between any two HFEs $h_1$ and $h_2$ is:

$$HauD(h_1,h_2)=\max_l\{|\gamma^1_{\sigma(l)}-\gamma^2_{\sigma(l)}|\}, l=1,2,...,L. \tag{6}$$

In addition, we can obtain some hybrid distance measures combining the above distance measures, such as the generalized hybrid distance between $h_1$ and $h_2$:

$$GHD(h_1,h_2)=\left(\frac{1}{2}\left(\frac{1}{L}\sum_{l=1}^{L}|\gamma^1_{\sigma(l)}-\gamma^2_{\sigma(l)}|^\kappa+\max_l\{|\gamma^1_{\sigma(l)}-\gamma^1_{\sigma(l)}|^\kappa\}\right)\right)^{1/\kappa}. \tag{7}$$

**Property 2**. *The above distance measure $d(h_1,h_2)$ between any two HFSs $h_1$ and $h_2$ satisfies the following properties:*
***P(1)***. *Reflexivity: $d(h_1,h_2)=0$ if and only if $h_1=h_2$;*
***P(2)***. *Boundedness: $0\le d(h_1,h_2)\le 1$;*
***P(3)***. *Symmetry: $d(h_1,h_2)=d(h_2,h_1)$;*
***P(4)***. *Triangle inequality: For any HFSs $h_1$, $h_2$ and $h_3$, $d(h_1,h_3)\le d(h_1,h_2)+d(h_2,h_3)$.*

## 4. Proposed Hybrid FMEA Framework for Risk Assessment under HFSs Environment

### 4.1. Problem Description

The risk evaluation and prioritization in the FMEA framework can be naturally view as a MCGDM problem, i.e., the $m$ risk failure modes $FM_i$ ($i$=1,2,…,$m$) are alternatives, the $n$ risk factors $C_j$ ($j$=1,2,…,$n$) are criteria, and there are $g$ FMEA experts $E_k$ ($k$=1,2,…,$g$) joining in the risk evaluation. The criteria and experts' weighting vectors $w=(w_1,w_2,…,w_n)^T$ and $\lambda=(\lambda_1,\lambda_2,…,\lambda_g)^T$ satisfying $w_j$, $\lambda_k\ge 0$, $\sum_j w_j=1$ and $\sum_k\lambda_k=1$. Let $R^k=(r^k_{ij})_{m\times n}$ ($k$=1,2,…,$g$) be the risk evaluation matrix given by $E_k$ (see Table 1), where $r^k_{ij}$ implies the risk value for the $FM_i$ with respect to criterion $C_j$ offered by expert $E_k$.

**Table 1** Risk priority evaluation in the FMEA framework based on MCGDM method

| | $E_1$ | | | | $E_2$ | | | | … | $E_g$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $C_1$ | $C_2$ | … | $C_n$ | $C_1$ | $C_2$ | … | $C_n$ | … | $C_1$ | $C_2$ | … | $C_n$ |
| $FM_1$ | $r^1_{11}$ | $r^1_{12}$ | … | $r^1_{1n}$ | $r^2_{11}$ | $r^2_{12}$ | … | $r^2_{1n}$ | ⋯ | $r^g_{11}$ | $r^g_{12}$ | … | $r^g_{1n}$ |
| $FM_2$ | $r^1_{21}$ | $r^1_{22}$ | … | $r^1_{2n}$ | $r^2_{21}$ | $r^2_{22}$ | … | $r^2_{2n}$ | ⋯ | $r^g_{21}$ | $r^g_{22}$ | … | $r^g_{2n}$ |
| … | … | … | … | … | … | … | … | … | … | … | … | … | … |
| $FM_m$ | $r^1_{m1}$ | $r^1_{m2}$ | … | $r^1_{mn}$ | $r^2_{m1}$ | $r^2_{m2}$ | … | $r^2_{mn}$ | ⋯ | $r^g_{m1}$ | $r^g_{m2}$ | … | $r^g_{mn}$ |

The hybrid FMEA framework under the HFSs environment with the MCGDM technique contains three major stages (also see the flowchart exhibited in Fig. 1):

1. Risk evaluation of each FM: The HFSs are utilized to represent the risk evaluation.
2. Aggregation of the individual risk evaluation: The new HFWA operator with combined weights is utilized to fuse the individual risk evaluation matrix.
3. Risk prioritization of each FM: The novel HFWGA operator with the CRITIC weights is applied to determine the risk priorities of FMs.

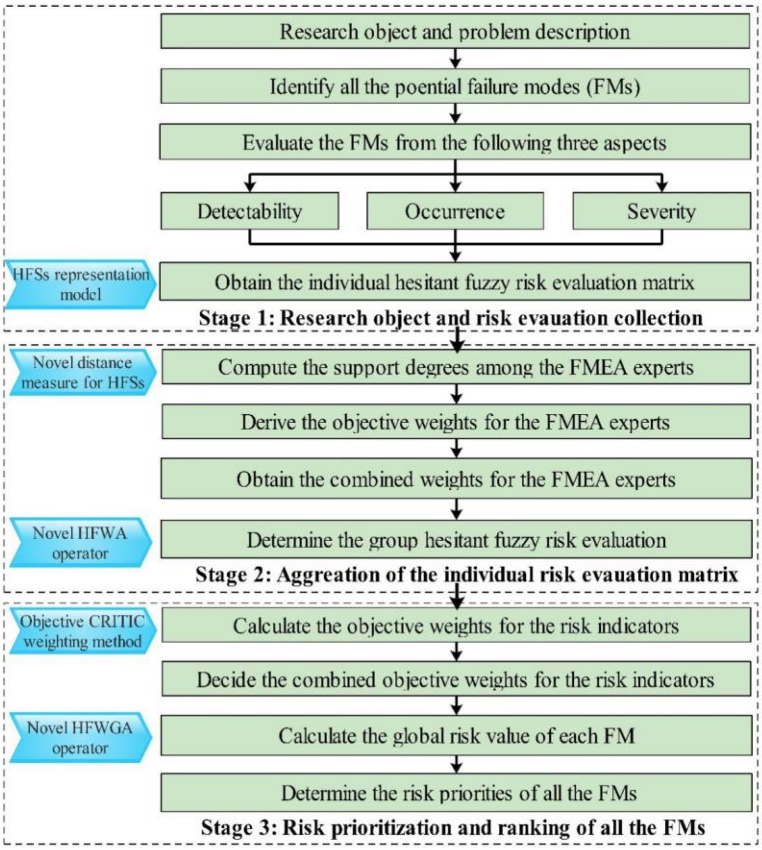


**Fig. 1** The flow chart of the proposed decision-making process

### 4.2. Risk Evaluation Procedure

**Step 2.1**: Determine all the potential FMs

First, invite the FMEA experts to carry on the discussion on the risk analysis. According to the expertise and experience of the FMEA experts, different experts can complement each other with their advantages to identify and determine all the potential FMs.

**Step 2.2**: Form the hesitant fuzzy risk evaluation matrix

After identifying and selecting all the potential FMs, each FMEA expert is asked to offer their opinions on the risk levels towards the risk indicators O, S, D using several possible membership values concerning the linguistic term 'Very high'. Then, all the hesitant fuzzy risk values of $FM_i$ on risk indicator $C_j$ offered by expert $E_k$ can be recorded as $r_{ij}^k$ ($i$=1,…,$m$;$j$=1,…,$n$;$k$=1,…,$g$). Finally, all the $r_{ij}^k$ values form $g$ decision matrix $R^k=(r_{ij}^k)_{m\times n}$ ($k$=1,2,…,$g$) corresponding to the FMEA expert $E_k$ ($k$=1,2,…,$g$).

### 4.3. Risk Evaluation Information Aggregation

When it comes to aggregating the group risk evaluation information, in most existing FMEA analyses, the weights of FMEA experts are often subjectively given in advance. However, this will lead to the collective risk evaluation being more dependent on experts with larger weights. Using objective weights for FMEA experts, on the one hand, can eliminate the extreme aggregation mode and, on the other hand, may also achieve more objective aggregation results. Based on the subjective trust relationship and the objective similarity measure, we propose a combined weighting method to determine the weights of FMEA experts. Then, we employ the newly developed HFWA operator to fuse the individual risk evaluation information under HFSs environment.

**Step 3.1**: Compute the support degree among the risk evaluation values

As for the risk values $r_{ij}^k$ representing $FM_i$ on risk indicator $C_j$ offered by FMEA expert $E_k$, if it is close to the risk evaluation $r_{ij}^{k'}$ ($k'\neq k$) given by the other FMEA experts, then it should have higher support degree. Specifically, the support degree between $r_{ij}^k$ and $r_{ij}^{k'}$ ($k'\neq k$) can be formulated as:

$$\sup(r_{ij}^k, r_{ij}^{k'}) = 1 - d(r_{ij}^k, r_{ij}^{k'}). \tag{8}$$

Obviously, when $d(r_{ij}^k, r_{ij}^{k'})=0$, meaning that expert $E_k$ and expert $E_{k'}$ have the same evaluation result concerning the $FM_i$ on risk indicator $C_j$. They give the largest mutual support to each other. Then, the support degree between $r_{ij}^k$ and all the others experts is

$$\sup(r_{ij}^k) = \frac{1}{g-1}\sum_{k'\neq k}\sup(r_{ij}^k, r_{ij}^{k'}) = 1 - \frac{\sum_{k'\neq k} d(r_{ij}^k, r_{ij}^k)}{g-1}. \tag{9}$$

**Step 3.2**: Calculate the weight of expert $E_k$ concerning the $FM_i$ on risk indicator $C_j$

Based on the support degrees defined in Eqs. (8) and (9), the weight of expert $E_k$ towards the $FM_i$ on risk indicator $C_j$ can be obtained directly by using each $\sup(r_{ij}^k)$ divide the sum of $\sup(r_{ij}^k)$ such that

$$\lambda_{ij}^k = \frac{\sup(r_{ij}^k)}{\sum_k \sup(r_{ij}^k)} = \frac{\sum_{k'\neq k}\sup(r_{ij}^k, r_{ij}^{k'})}{\sum_k\sum_{k'\neq k}\sup(r_{ij}^k, r_{ij}^{k'})} = \frac{g-1-\sum_{k'\neq k} d(r_{ij}^k, r_{ij}^k)}{k(g-1)-\sum_k\sum_{k'\neq k} d(r_{ij}^k, r_{ij}^k)}. \tag{10}$$

So an objective weight can be set for each FMEA expert. Assume the subjective weighting vector for the experts is $\lambda=(\lambda_1,\lambda_2,\ldots,\lambda_g)^T$. Consequently, the combined weight for expert $E_k$ is

$$\tilde{\lambda}_{ij}^k = \alpha\lambda_{ij}^k + (1-\alpha)\lambda_k, \tag{11}$$

where $\alpha$ is a balancing parameter between the objective weight and subjective weight.

**Step 3.3**: Form the group risk evaluation matrix

Applying the weights derived in **Step 3.2**, the group risk evaluation matrix $R^c=(r_{ij}^c)_{m\times n}$ can be computed by using the new HFWA operator such that

$$r_{ij}^c = HFWA_\lambda(r_{ij}^1, r_{ij}^2, \ldots, r_{ij}^g) = \bigcup_{l_{ij}=1,2,\ldots,L_{ij}}\left\{1-\prod_{k=1}^{g}(1-\gamma_{\sigma(l_{ij})}^k)^{\tilde{\lambda}_{ij}^k} \mid \gamma_{\sigma(l_{ij})}^k \in (r_{ij}^k)^{L_{ij}/\#r_{ij}^k}\right\}, \tag{12}$$

where $L_{ij}$ is the LCM of $\#r_{ij}^k$ ($k$=1,2,…,$g$).

### 4.4. Risk Priority Calculation

The conventional FMEA method often sets the equal weight or the subjective known weights for the risk indicators. They cannot tackle the issue with risk indicators interaction and unknown weights. CRITIC is one of the most effective MCDM methods that spotlights the objective criteria weight. This method determines the objective weight dependent on two elements of data, including the distinctions and correlations among various criteria all the while. The first is the contrast intensity using the standard deviation to illustrate each criterion separately. The second dimension is the conflict between criteria, the linear correlation coefficient between criteria measures the conflict. To solve the risky preference indicated by HFSs, we investigate this strategy in the HFSs environment as:

**Step 4.1**: Decide the objective weights of the risk indicators: CRITIC method

As for group risk evaluation matrix $R^c=(r^c_{ij})_{m\times n}$ derived in Section 4.3, compute the score $s^c_{ij}$ of each hesitant fuzzy risk evaluation value $r^c_{ij}$, then we form a score matrix $S^c=(s^c_{ij})_{m\times n}$ as

$$S^c = \begin{pmatrix} s^c_{11} & s^c_{12} & \cdots & s^c_{1n} \\ s^c_{21} & s^c_{22} & \cdots & s^c_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ s^c_{m1} & s^c_{m2} & \cdots & s^c_{mn} \end{pmatrix}_{m\times n}. \tag{13}$$

The standard deviation $\sigma_j$ of the $j$th criterion, which is used to measure the contrast intensities of criteria and can be calculated by

$$\sigma_j = \sqrt{\frac{\sum_{i=1}^{m}(s^c_{ij} - \bar{s}^c_j)}{m}} \text{ with } \bar{s}^c_j = \frac{\sum_{i=1}^{m} s^c_{ij}}{m}. \tag{14}$$

The correlation coefficient between the $j$th and $l$th criteria, which is used to measure the conflicts between criteria is calculated as

$$\rho_{jl} = \frac{\sum_{i=1}^{m}(s^c_{ij} - \bar{s}^c_j)(s^c_{il} - \bar{s}^c_l)}{\sqrt{\sum_{i=1}^{m}(s^c_{ij} - \bar{s}^c_j)^2 \sum_{i=1}^{m}(s^c_{il} - \bar{s}^c_l)^2}}. \tag{15}$$

The quantity of information $q_j$ contained in the criterion $C_j$ is computed as

$$q_j = \sigma_j \sum_{l=1}^{n}(1 - \rho_{jl}). \tag{16}$$

Then the objective weighting vector of the risk indicators can be derived as

$$w^O_j = \frac{q_j}{\sum_j q_j}. \tag{17}$$

So an objective weight can be set for each risk indicator. Assume the subjective weighting vector for the risk indicator is $w^S=(w^S_1, w^S_2, \dots, w^S_n)^T$. Consequently, the combined weight for risk indicator $C_j$ is

$$w_j = \beta w^O_j + (1-\beta) w^S_j, \tag{18}$$

where $\beta$ is a balancing parameter between the objective and subjective weights.

**Step 4.2**: Calculate the global risk value of each FM

Combining the weights obtained in **Step 4.1** with the group risk matrix $R^c=(r^c_{ij})_{m\times n}$, applying the novel HFWGA operator to fuse each risk indicator to calculate the global risk value of each FM such that:

$$r_i^c = HFWGA_w(r_{i1}^c, r_{i2}^c, \ldots, r_{in}^c) = \bigcup_{l_i=1,2,\ldots,L_i} \left\{ \prod_{j=1}^{n} (\gamma_{\sigma(l_i)}^j)^{w_j} \mid \gamma_{\sigma(l_i)}^j \in (r_{ij}^c)^{L_i/\#r_{ij}^c} \right\}, \tag{19}$$

where $L_i$ is the LCM of $\#r^c_{ij}$ ($j$=1,2,…,$n$).

**Step 4.3:** Determine the risk priorities of all the FMs

Concerning the global risk value $r_i^c$, calculate their score values. Then the risk priority ranking order of each FM can be determined according to their score values. That is, the bigger $r_i^c$ is, the higher risk priority of $FM_i$ is.

## 5. Illustrative Example

This section gives a case study on the risk priority evaluation of the power transformer parts to illustrate the applicability and effectiveness of the hybrid FMEA framework for risk analysis. Besides, the sensitivity analysis is conducted to expound the robustness of our model. Furthermore, the comparative analysis is also implemented to verify the benefits of our hybrid FMEA framework for risk evaluation and prioritization.

### 5.1. Research Object and Problem Description

Power transformers are critical equipment of substations, power plants, and electric management departments, the running conditions of which will affect the safety and stability of the system. The correct and in-time diagnosis of their faults and operation maintenance is crucial. Based on the gross structure and constituent of a power transformer, the FMEA experts unify all the potential FMs as winding, iron core, bush, body part, non-electric quantity protection, tap switch, cooling. They are denoted by $FM_i$ ($i$=1,…,7). Fig. 2 gives the statistical failure data of power transformers parts of one specific region in the past 10 years. Then we employ the proposed hybrid FMEA framework to evaluate and prioritize the identified FMs of the power transformer parts.

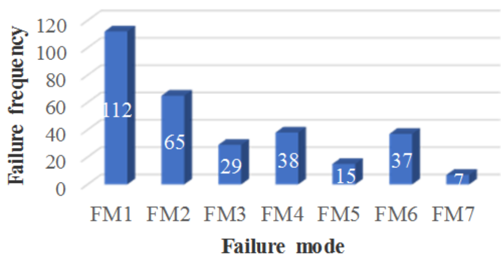


**Fig. 2** Fault statistics of transformer parts

### 5.2. The Decision-Making Steps

*5.2.1 Risk Evaluation of Each FM*

Concerning the above selected FMs, a FMEA group including three experts $E_k$ ($k$=1,2,3) are asked to offer their opinions on the risk evaluations of FMs using the HFSs. In terms of multi-dimensional risk evaluation, each FM is evaluated under three risk indicators occurrence (O), severity (S), and detection (D). The HFSs of the seven FMs under the three risk indicators provided by all the FMEA experts are put together in Tables 2-4.

**Table 2** The hesitant fuzzy evaluation provided by FMEA expert $E_1$

| | O | S | D |
|---|---|---|---|
| $FM_1$ | {0.4,0.5} | {0.8,0.9} | {0.6,0.7} |
| $FM_2$ | {0.3,0.4} | {0.7,0.8} | {0.7,0.8} |
| $FM_3$ | {0.2,0.3} | {0.7,0.8} | {0.3,0.4} |
| $FM_4$ | {0.1,0.2} | {0.7,0.8,0.9} | {0.4,0.5} |
| $FM_5$ | {0.1} | {0.6,0.7} | {0.7,0.8} |
| $FM_6$ | {0.1,0.2} | {0.5,0.6} | {0.4} |
| $FM_7$ | {0.1} | {0.3,0.4} | {0.4,0.5} |

**Table 3** The hesitant fuzzy evaluation provided by FMEA expert $E_2$

| | O | S | D |
|---|---|---|---|
| $FM_1$ | {0.3,0.4} | {0.9,1} | {0.7,0.8} |
| $FM_2$ | {0.2} | {0.8,0.9} | {0.7,0.8} |
| $FM_3$ | {0.1,0.2} | {0.6,0.7} | {0.4,0.5} |
| $FM_4$ | {0.2,0.3} | {0.7,0.8} | {0.4,0.5,0.6} |
| $FM_5$ | {0.1,0.2} | {0.5,0.6,0.7} | {0.7,0.8,0.9} |
| $FM_6$ | {0.1} | {0.6,0.7} | {0.2,0.3,0.4} |
| $FM_7$ | {0.1,0.2} | {0.2,0.3,0.4} | {0.2,0.3} |

**Table 4** The hesitant fuzzy evaluation provided by FMEA expert $E_3$

| | O | S | D |
|---|---|---|---|
| $FM_1$ | {0.5} | {0.9} | {0.6,0.7,0.8} |
| $FM_2$ | {0.1,0.2} | {0.8,0.9,1} | {0.6,0.7,0.8} |
| $FM_3$ | {0.1} | {0.6,0.7,0.8} | {0.4,0.5,0.6} |
| $FM_4$ | {0.2,0.3} | {0.7,0.8,0.9} | {0.3,0.4,0.5} |
| $FM_5$ | {0.2} | {0.5,0.6,0.7} | {0.5,0.6,0.7} |
| $FM_6$ | {0.2,0.3} | {0.5,0.6} | {0.3,0.4} |
| $FM_7$ | {0.2,0.3} | {0.3,0.4,0.5} | {0.2,0.3,0.4} |

*5.2.2. Risk Evaluation Information Aggregation*

In this stage, we first need to derive the weights of the expert. On the one hand, the objective weights can be obtained based on the improved hesitant fuzzy Hamming distance measure ($\kappa$=1 in Eq. (5)) and the support degrees Eqs. (8) and (9). Specifically, the total support degrees of all the risk values constitute a support matrix, as shown in Table 5. Combining the support matrix and Eq. (12), the objective weights of FMEA experts can be determined, as shown in Table 6.

**Table 5** The total support degrees of all the risk values

| | $E_1$ | | | $E_2$ | | | $E_3$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | O | S | D | O | S | D | O | S | D |
| $FM_1$ | 0.925 | 0.925 | 0.925 | 0.875 | 0.925 | 0.925 | 0.900 | 0.950 | 0.950 |
| $FM_2$ | 0.825 | 0.875 | 0.975 | 0.900 | 0.925 | 0.975 | 0.875 | 0.900 | 0.950 |
| $FM_3$ | 0.875 | 0.925 | 0.875 | 0.925 | 0.925 | 0.925 | 0.900 | 0.950 | 0.900 |
| $FM_4$ | 0.900 | 0.975 | 0.950 | 0.950 | 0.950 | 0.925 | 0.950 | 0.975 | 0.925 |
| $FM_5$ | 0.925 | 0.950 | 0.900 | 0.950 | 0.975 | 0.875 | 0.925 | 0.975 | 0.825 |
| $FM_6$ | 0.925 | 0.950 | 0.925 | 0.900 | 0.900 | 0.925 | 0.875 | 0.950 | 0.950 |
| $FM_7$ | 0.900 | 0.950 | 0.825 | 0.925 | 0.925 | 0.875 | 0.875 | 0.925 | 0.900 |

**Table 6** The objective weights of the FMEA experts

| | $E_1$ | | | $E_2$ | | | $E_3$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | O | S | D | O | S | D | O | S | D |
| $FM_1$ | 0.343 | 0.330 | 0.330 | 0.324 | 0.330 | 0.330 | 0.333 | 0.339 | 0.339 |
| $FM_2$ | 0.317 | 0.324 | 0.336 | 0.346 | 0.343 | 0.336 | 0.337 | 0.333 | 0.328 |
| $FM_3$ | 0.324 | 0.330 | 0.324 | 0.343 | 0.330 | 0.343 | 0.333 | 0.339 | 0.333 |
| $FM_4$ | 0.321 | 0.336 | 0.339 | 0.339 | 0.328 | 0.330 | 0.339 | 0.336 | 0.330 |
| $FM_5$ | 0.330 | 0.328 | 0.346 | 0.339 | 0.336 | 0.337 | 0.330 | 0.336 | 0.317 |
| $FM_6$ | 0.343 | 0.339 | 0.330 | 0.333 | 0.321 | 0.330 | 0.324 | 0.339 | 0.339 |
| $FM_7$ | 0.333 | 0.339 | 0.317 | 0.343 | 0.330 | 0.337 | 0.324 | 0.330 | 0.346 |

Assume the subjective weights for the experts are $\lambda$=(0.2,0.3,0.5)$^T$. By using Eq. (11), and let $\alpha$=0.5, referring to the equal importance between the objective and subjective weights. Consequently, Table 7 provides the combined weight for expert $E_k$.

**Table 7** The combined weights of the FMEA experts

| | $E_1$ | | | $E_2$ | | | $E_3$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | O | S | D | O | S | D | O | S | D |
| $FM_1$ | 0.271 | 0.265 | 0.265 | 0.312 | 0.315 | 0.315 | 0.417 | 0.420 | 0.420 |
| $FM_2$ | 0.259 | 0.262 | 0.268 | 0.323 | 0.321 | 0.318 | 0.418 | 0.417 | 0.414 |
| $FM_3$ | 0.262 | 0.265 | 0.262 | 0.321 | 0.315 | 0.321 | 0.417 | 0.420 | 0.417 |
| $FM_4$ | 0.261 | 0.268 | 0.270 | 0.320 | 0.314 | 0.315 | 0.420 | 0.418 | 0.415 |
| $FM_5$ | 0.265 | 0.264 | 0.273 | 0.320 | 0.318 | 0.318 | 0.415 | 0.418 | 0.409 |
| $FM_6$ | 0.271 | 0.270 | 0.265 | 0.317 | 0.311 | 0.315 | 0.412 | 0.420 | 0.420 |
| $FM_7$ | 0.267 | 0.270 | 0.259 | 0.321 | 0.315 | 0.318 | 0.412 | 0.415 | 0.423 |

Based on the combined FMEA experts' weights, one can apply the newly introduced HFWA aggregation operator Eq. (12) to fuse the individual risk evaluation matrix into a group matrix $R^c=(r^c_{ij})_{7\times3}$. Table 8 displays the collective risk evaluation information.

**Table 8** The group risk evaluation matrix

| | O | S | D |
|---|---|---|---|
| $FM_1$ | {0.416,0.471} | {0.880,1} | {0.635,0.635,0.676,0.736,0.777,0.777} |
| $FM_2$ | {0.188,0.257} | {0.778,0.778,0.833,0.88,1,1} | {0.662,0.662,0.7,0.764,0.8,0.8} |
| $FM_3$ | {0.127,0.189} | {0.629,0.629,0.672,0.731,0.773,0.773} | {0.375,0.375,0.421,0.476,0.522,0.522} |
| $FM_4$ | {0.175,0.275} | {0.7,0.7,0.773,0.8,0.876,0.876} | {0.36,0.36,0.434,0.461,0.534,0.534} |
| $FM_5$ | {0.143,0.175} | {0.529,0.529,0.6,0.629,0.7,0.7} | {0.63,0.63,0.703,0.735,0.811,0.811} |
| $FM_6$ | {0.143,0.214} | {0.534,0.534} | {0.299,0.299,0.328,0.370,0.4,0.4} |
| $FM_7$ | {0.143,0.219} | {0.267,0.267,0.343,0.37,0.444,0.444} | {0.257,0.257,0.298,0.358,0.399,0.399} |

*5.2.3. Calculate the Risk Priorities of FMs*

Regarding the group risk evaluation matrix Table 8, we first utilize the CRITIC weighting method to derive the objective weights. Based on the score of HFSs and Eq. (14), the standard deviation of each risk indicator can be calculated as $\sigma_1$=0.0932, $\sigma_2$=0.1833, $\sigma_3$=0.1666. Using Eq. (15), one can obtain the correlation coefficient matrix $\rho=(\rho_{jl})_{3\times3}$ among the risk indicators. Then, by using Eqs. (16) and (17), the quantity of information contained in each risk indicator and the objective weights of the risk indicators can be derived as $q_1$=0.9016, $q_2$=0.6685, $q_3$=0.8785 and $w_1^O$=0.2380, $w_2^O$=0.3472, $w_3^O$=0.4147. Combining with the equal weights of the conventional FMEA model and using Eq. (18) with $\beta$=0.5 (implying the equal importance between the objective and subjective weights). Then the combined weights for risk indicators are $w_1$=0.2857, $w_2$=0.3403, $w_3$=0.3740.

**Table 9** The global hesitant fuzzy risk evaluation of each FM

| | Global hesitant fuzzy risk evaluation | Score | Ranking |
|---|---|---|---|
| $FM_1$ | {0.6289,0.6289,0.6439,0.7190,0.7338,0.7338} | 0.681 | 1 |
| $FM_2$ | {0.4882,0.4882,0.5103,0.5873,0.6242,0.6242} | 0.554 | 2 |
| $FM_3$ | {0.3286,0.3286,0.3507,0.4336,0.4461,0.4461} | 0.387 | 5 |
| $FM_4$ | {0.3675.0.3675.0.4073.0.4798.0.5229.0.5229} | 0.445 | 3 |
| $FM_5$ | {0.3886,0.3886,0.4227,0.4623,0.4974,0.4974} | 0.443 | 4 |
| $FM_6$ | {0.2948,0.2948,0.3051,0.3802,0.3914,0.3914} | 0.343 | 6 |
| $FM_7$ | {0.2210,0.2210,0.2534,0.3146,0.3483,0.3483} | 0.284 | 7 |

Finally, based on Eq. (19), one can obtain the global hesitant fuzzy risk evaluation of each FM as shown in Table 9. Based on the score of global hesitant fuzzy risk value of each FM, then the risk priority ranking order are determined also shown in Table 9.

As a result, we can now rank the FMs in light of the scores values, and the risk priority ranking of all the FMs is $FM_1 \succ FM_2 \succ FM_4 \succ FM_5 \succ FM_3 \succ FM_6 \succ FM_7$. Apparently, *FM*1 winding owns the highest risk level, which deserves the greatest attention.

## 6. Discussions

### 6.1. Sensitivity Analysis

Next, a sensitivity analysis is conducted to explore the effect of the related parameters in our model on the risk calculation and rankings. $\alpha$ and $\beta$ indicate the importance degree of the objective weights on experts and risk indicators, respectively. Larger $\alpha$ and $\beta$ imply a larger preference towards the objective weights. Here, $\alpha$ and $\beta$ are from 0 to 1 to analyze the sensitivity. The fixed $\alpha$ and $\beta$ are $\alpha$=$\beta$=0.5. Fig. 3 and Fig. 4 indicate how different single $\alpha$ and $\beta$ affects the final score values of the 7 FMs, respectively.

One can see from Fig. 3 and Fig. 4, changing a single parameter will affect the final score values of FMs. The parameters $\alpha$ and $\beta$ play certain roles in deciding the combined weights for the experts and the risk indicators and then affecting the final score values of FMs. Specifically, the scores of FMs decrease monotonically with the increase of $\beta$, but the trends of the scores of FMs are not the same with $\alpha$ increases. These are mainly because the parameter $\beta$ directly affects the global weights of the risk indicators, while the parameter $\alpha$ only impacts the local weights of the experts. Furthermore, one can draw the following conclusions based on Fig. 3 and Fig. 4.

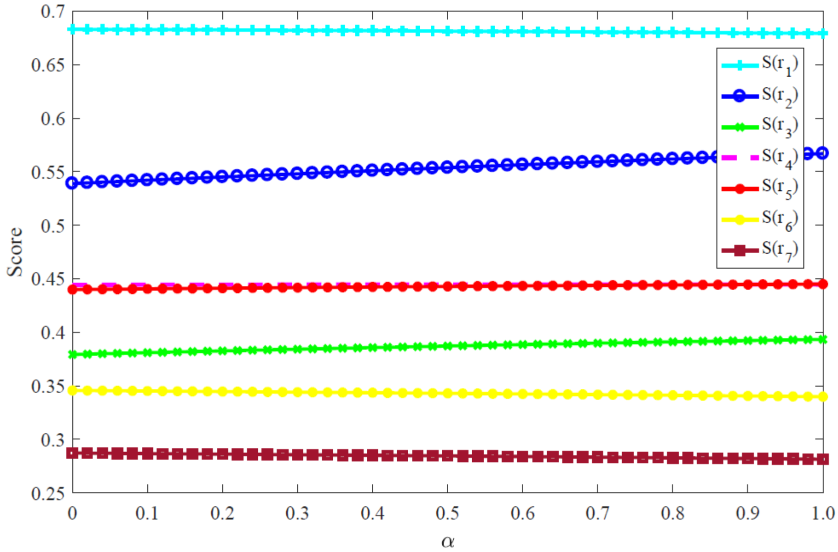


**Fig. 3** Variation of the score values of the FMs with different $\alpha$

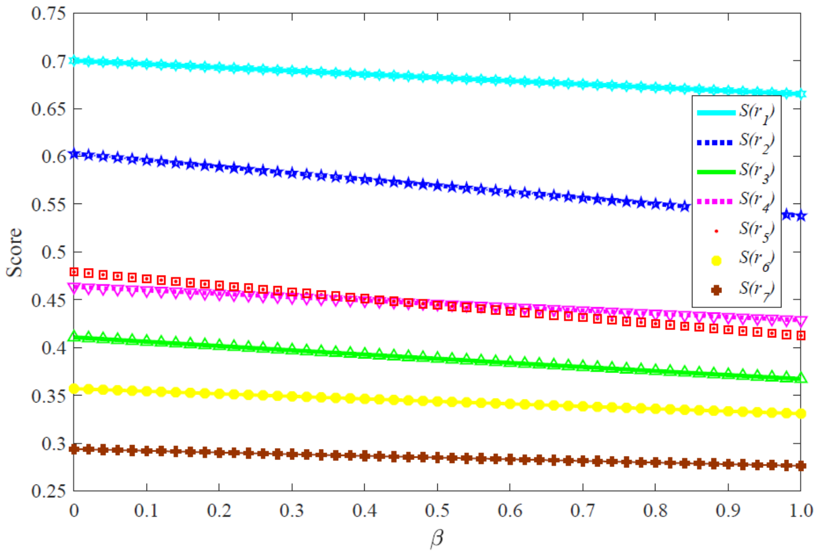


**Fig. 4** Variation of the score values of the FMs with different $\beta$

- The effect on the final score values of FMs of $\alpha$ is less than that of $\beta$. The score values present a stable status as the single parameter $\alpha$ changes, while the score values decrease with an extensive variation range when a single parameter $\beta$ changes. In essence, the objective weights for risk indicators have more influence on the final score values than the objective weights for experts.

- Changing either single $\alpha$ and $\beta$ will make a different ranking for the FMs, especially for $FM_4$ and $FM_5$. The ranking of $FM_5$ increases with the increase of $\alpha$, and the ranking of $FM_4$ increases with the rise of $\beta$. Although the other FMs' ranking stays the same, these dynamic changes cannot be negligible since they directly relate to the maintenance and inspection costs.
- The final score values affected by the two combined weighting methods with two parameters $\alpha$ and $\beta$ can constitute a three-dimensional variation pattern. Here are two two-dimensional images that present the changing trend of final score values to facilitate sensitivity analysis directly.

According to the above calculations and analysis, it is apparent that the combined weights for risk assessment are flexible and easy to operate, and experts can select suitable parameter values to control the balance between the objective and subjective weights. Due to the natural uncertainty and complexity in risk analysis, taking these factors into account is necessary and meaningful.

### 6.2. Comparative Analysis

To verify the effectiveness of our hybrid FMEA framework for risk assessment, this section we compare our method with some existing FMEA models under the same case study. These comparative methods contain the classical RPN method [49], the HF-VIKOR method [50], the HF-TOPSIS method [51], and the generalized TODIM method [52].

Specifically, we first transform the HFSs into precise numerical values in [0,10] based on their scores, and then we utilize the classical FMEA method [49] to determine the RPN of each FM. Next, according to the distance measure between the HFSs [53] and the objective weights for risk indicators using the maximizing deviation method, we apply the HF-VIKOR [50], and HF-TOPSIS [51] to derive the compromise solution and the closeness coefficient of each FM, respectively. Finally, the generalized TODIM method [52] is extended into the hesitant fuzzy environment, and an extended generalized TODIM is developed for the risk prioritization.

**Table 10** The comparisons of different methods

| | Classical RPN [49] | | HF-VIKOR [50] | | | | HF-TOPSIS [51] | | | | Generalized TODIM [52] | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | RPN | Order | $S_i$ | $R_i$ | $Q_i$ | Order | $D_i^-$ | $D_i^+$ | $CCI_i$ | Order | $\pi_i$ | Order |
| $FM_1$ | 299.78 | 1 | 0.139 | 0.074 | 0.106 | 1 | 0.078 | 0.497 | 0.864 | 1 | 1 | 1 |
| $FM_2$ | 134.35 | 2 | 0.354 | 0.221 | 0.287 | 2 | 0.163 | 0.417 | 0.719 | 2 | 0.803 | 2 |
| $FM_3$ | 47.19 | 5 | 0.645 | 0.272 | 0.459 | 5 | 0.328 | 0.240 | 0.423 | 5 | 0.346 | 5 |
| $FM_4$ | 80.48 | 3 | 0.551 | 0.241 | 0.396 | 4 | 0.283 | 0.291 | 0.507 | 4 | 0.526 | 3 |
| $FM_5$ | 71.73 | 4 | 0.521 | 0.255 | 0.388 | 3 | 0.269 | 0.304 | 0.530 | 3 | 0.477 | 4 |
| $FM_6$ | 38.00 | 6 | 0.736 | 0.301 | 0.518 | 6 | 0.394 | 0.171 | 0.302 | 6 | 0.179 | 6 |
| $FM_7$ | 22.27 | 7 | 0.856 | 0.320 | 0.588 | 7 | 0.480 | 0.100 | 0.172 | 7 | 0 | 7 |

The computed target values and the corresponding ranking orders of the FMs' risk prioritization gained by these methods are listed in Table 10. From Table 10, one can find some ponderable results.

- First, $FM_1$ always has the highest level of risk priority, and $FM_7$ always has the lowest risk priority in all the five risk evaluation methods. The most crucial and least severe FMs are always the same under the five risk evaluation methods. This result coincides with the fault statistics of transformer parts concerning $FM_1$ and $FM_7$ shown in Fig. 2.

- Second, our risk priority ranking results of FMs are consistent with the classical RPN method and the generalized TODIM method for FMEA, which reveal a relatively high homogeneity between our proposed FMEA approach and the other two methods. This also implies that our proposed hybrid FMEA framework for risk assessment is valid to evaluate and rank the risk prioritization of FMs.
- Finally, regarding the HF-VIKOR and HF-TOPSIS methods, $FM_5$ has a higher risk priority than $FM_4$, which is opposite with our approach and the other two methods. But on the whole, there are slight differences between the overall risk values of $FM_5$ and $FM_4$. The different decision-making mechanisms cause the ranking difference.

More specifically, compared with the above four comparative models, our method has the following differences and advantages:

- First, the classical FMEA [49] is based on the RPN method that employs precise values as the risk evaluation information and treats the risk indicators as equally important. Compared with the classical FMEA method, our proposed hybrid FMEA model can successfully deal with uncertainty representation (HFSs), uncertain information fusion (hesitant fuzzy aggregation techniques), and combined weights effects (combined weighting method). The proposed FMEA framework for risk analysis is more comprehensive and objective.
- Second, the HF-VIKOR [50], and HF-TOPSIS [51] methods for FMEA are based on the reference level models. The selection of the reference points has a strong influence on the final risk evaluation results. Our FMEA model directly fuses the risk evaluation, and it has a different decision-making mechanism with them. The maximizing deviation method is used to derive the objective weights for risk indicators. It is based on the contrast intensity of each risk indicator and does not consider the correlations between risk indicators. Hence, our hybrid FMEA model is more straightforward and produces more flexible weights, creating more objective and practical risk analysis results.
- Third, the generalized TODIM method for FMEA [52] is based on the dominant comparative model, and it can consider several parameters and embody diverse experts' risk attitudes. Nevertheless, certain limitations of this method will exist concerning the selection of related parameters in practice. Our model produces consistent results with the generalized TODIM model. Still, the parameters in our combined weights are more acceptable, and the decision-making mechanism also corresponds with the multiplicative property of the classical FMEA.

By providing a novel hybrid FMEA framework integrating the hesitant fuzzy aggregation technique with the combined weights approach, the developed FMEA model is forthright and flexible in handling risk assessments issues containing multiple experts and risk indicators. The combined weights help cover the group experts' consensus and correlations between risk indicators. It is more reasonable and practicable than the FMEA models that direct aggregate individual risk evaluation information and assume that the risk indicators are independent of each other in practical risk analysis.

## 7. Conclusions, Limitations, and Future Studies

The FMEA approach served as a practical risk analysis framework is widely adopted in risk assessment and management with remarkable results. With the increasing complexity of the engineering system, the uncertain tool is also being accepted by many experts when they express their risk evaluations. The HFSs has been a very efficient representation model to describe the hesitant and fuzzy assessments. This paper develops a hybrid FMEA framework for risk evaluation and prioritization with a novel hesitant fuzzy computational model and CRITIC weighting method. Our model efficaciously and comprehensively deals with information representation, information fusion, and risk evaluation. The novel hesitant fuzzy computational representation model can enrich the theoretical research results of the HFSs. The combined weights can obtain a more accurate result as it considers both the support degrees among experts and the subjective power of FMEA experts. The HFWGA operator with CRITIC weights can nicely satisfy the multiplicative property of the classical FMEA model and consider the correlations between risk indicators. Finally, the risk priority evaluation of power transformer parts with sensitivity and comparative analyses reveals that our model is efficient in providing risk analysis support.

This paper has vital significance in theory and practical risk analysis. However, our model still has deficiencies, which point the way for future study. First, the risk indicators used to evaluate the FMs might remain inadequate. In the FMEA framework, only three risk indicators are considered. More aspects and risk indicators should help improve the risk assessment's comprehensiveness. Second, two common aggregation tools, including the weighted averaging and geometric weighted averaging operators, are utilized to fuse the hesitant fuzzy evaluation information. Some more advanced aggregation techniques such as the Choquet average operator [54] and geometric Bonferroni mean [55] can also improve the interactive effect of the interaction relationships among risk factors. Finally, although the CRITIC weighting method can determine objective weights considering the differences and correlations among various risk indicators simultaneously, the inherent relation between the risk indicators is worthy of further investigation and exploitation.

The research results offer a reference for the FMEA and risk assessment. One can get a reasonable risk prioritization if the decision process is implemented correctly. The hesitant fuzzy aggregation technique is involved in these two stages to fuse the individual risk evaluation matrix and decide the total risk value. Some MCDM methods like the VIKOR [56] and the generalized TODIM [57] can also be used in the second stage. The proposed FMEA model uses HFSs as the evaluation representations. Some other uncertain or fuzzy techniques [58, 59] will also be feasible when expressing subjective uncertainties. Finally, the proposed FMEA model is applied to the risk prioritization of the power transformer parts. In other areas like the logistics management [60], sustainable vehicle facilities [61], emergency situation [62], and railway management [63, 64], the application will also be beneficial.

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