FACTA UNIVERSITATIS
Series: **Mechanical Engineering** Vol. 13, N° 2, 2015, pp. 181 - 191

Original scientific paper

NON-LINEAR MATHEMATICAL MODELS IN THE THEORY OF EXPERIMENTAL DESIGN: APPLICATION IN THE MANUFACTURING PROCESSES

UDC 621.7+519.2

Anđela Lazarević, Dragoljub Lazarević

Faculty of Mechanical Engineering, University of Niš, Serbia

Abstract. *Since real processes usually are not linear, different non-linear mathematical models are usually applied for studying their behavior: high order polynomials, power, exponential and other functions. In this paper, the application of multiple power functions is presented, with and without interactions between the influential factors, in modelling various technological problems. The modeling results show that the proper mathematical model is selected, one that guarantees high accuracy in the entire experimental space. Furthermore, an important conclusion is reached, stating that interactions between the influential factors are not of importance in such mathematical models; thus they can be neglected. Therefore, it is shown that it is more practical to use the basic mathematical model (without interactions) than the expanded mathematical model (with interactions)*

Key Words: *Experimental Design, Non-linear Mathematical Models, Plasma Cutting, Face Milling*

1. INTRODUCTION

The theory of experimental design represents a qualitatively new approach to the theoretical-experimental analysis and optimization of complex processes/systems, with universal application and range of advantages in comparison to the concept and practice of the one-factor-at-a-time method [1-2]. The theory of experimental design addresses management of an experiment, i.e., its preparation and physical realization as well as processing and analysis of the experimental results according to the previously determined plan, which enables the variations of the influential factors simultaneously on various levels, in each of the following series of trials.

The experimental design was proven as successful method in various fields, which is especially evident in complex research objects with a large number of influential factors.

Received June 4, 2015 / Accepted July 20, 2015
Corresponding author: Anđela Lazarević
Affiliation: Faculty of Mechanical Engineering, University of Niš, Aleksandra Medvedeva 14, 18000 Niš
E-mail: andjela.lazarevic@gmail.com

The basic characteristics and advantages of the theory of experimental design are as follows [1,5,6]: number of trials, successive experiment management, i.e., in stages (step by step), from simpler to more complex designs, a simple statistical (regression and dispersion) analysis of experimental results, a possibility of qualitative and quantitative assessment of effects of each of the influential factors and their interactions on the target function, easy optimization of a process/system which is the subject of research, on the basis of obtained empirical (regression) model of the target function that encompasses the complete experimental space, minimum time and material losses (expenses) for experiment realization, elimination of the subjective influence of the researcher, etc. [7].

The outline of recommended procedure in the theory of experimental design, as a statistical approach in designing and analysing an experiment, include [1-4]: recognition and statement of the problem, choice of factors, levels, and range, selection of the response variable, choice of experimental design, performing the experiment, statistical analysis of the data, and conclusions and recommendations.

The selection of mathematical model (regression equation), which is used to establish a connection between influential factors and target function, depends on the research objective, the complexity of the phenomenon under study, the selected experimental design, as well as quantity and quality of available information.

Both theory and practice have shown that in most cases the best choice is a mathematical model in the form of polynomials (linear, quasi-linear, square, etc.). More complex mathematical models ensure higher accuracy in the prediction of researched system behaviour in the selected hyperspace. However, such models also imply a more complex experimental procedures, higher expenses, time-consuming analysis and interpretation of experimental results etc. On the other hand, the selection of the simplest linear mathematical model does not enable an analysis of the factor interactions' effects; various examples undoubtedly show that the factor interactions' effect on the target function can be more significant than that of separate factors. It should also be emphasized that it is the case of lower-order factor interactions, since the higher-order factor interactions can be neglected. Namely, interactions of many factors, as a rule, do not influence the accuracy of the selected mathematical model.

Various experiments and practical knowledge point to the fact that the mathematical models used for the analysis and optimisation of complex processes are those in the form of multiple power, exponential, or some other function instead of polynomials. Such mathematical models are easily transformed into linear functions (first order polynomials). Therefore, the synergy of a complex non-linear mathematical model and simple processing and analysis of experimental data is established [8].

Upon linearization of the aforementioned functions, a basic mathematical model without interactions is obtained. As it was mentioned, the influence of interactions should not be rejected *a priori*. The choice of appropriate criterion for building the mathematical model is not often obvious [9-10]. The accuracy of basic non-linear models without interactions and expanded non-linear mathematical models with interactions are comparatively analyzed on examples given in this paper.

2. COMPLEX POWER FUNCTIONS AS MATHEMATICAL MODELS

Numerous experiments have shown that a successful approximation can be made for modelling most diverse technological processes by using a complex power equation [11-16] in the following form:

$$F_c = C \cdot X_1^{p_1} \cdot X_2^{p_2} \cdot X_3^{p_3} \cdots X_i^{p_i} \cdots X_k^{p_k} \tag{1}$$

where X_i (i=1,2,3…k) - influential factors (natural coordinates), C, p_i - constants to be determined, k - number of factors.

By using logarithmic form for the Eq. (1) it could be rewritten in a reduced linear form:

$$y = b_0 + b_1 x_1 + \cdots + b_i x_i + \cdots + b_k x_k \tag{2}$$

where $y = \ln F_c$, $b_0 = \ln C$, $b_i = p_i$, $x_i = \ln X_i$

Eq. (2) is represented in coded coordinates. The connection between natural and coded coordinates is established through the following transformation equations:

$$x_i = 1 + 2\frac{\ln X_i - \ln X_{i\max}}{\ln X_{i\max} - \ln X_{i\min}};\ i = 1,2,3\ldots k \tag{3}$$

According to Eq. (3), coded coordinates take integer values (x_i=+1 ∨ 0 ∨ -1).

For the purposes of dispersion analysis it is necessary to repeat trials at certain points in the experimental hyperspace. The systems of trial repetition are as follows:

1. Repetition for n_0 times only in the central point of the experimental design (x_i=0);
2. Uniform repetition for n times in each vertex of an experimental hypercube (x_i=±1);
3. Non-uniform repetition for n_u times in certain vertices of an experimental hypercube, or possibly, only in one point.

It should be emphasised that for such mathematical model the determination of basic levels of influential factors is conducted by applying the following relations:

$$X_{i0}^2 = X_{i\max} \cdot X_{i\min} \tag{4}$$

After determining the coefficients of linear regression Eq. (2), the unknown constants of target function Eq. (1) are calculated using the following formulas:

$$p_i = \frac{2b_i}{\ln(X_{i\max} / X_{i\min})}\ ;\ C = \exp\left(\sum_{i=0}^{k} b_i - \sum_{i=1}^{k} p_i \ln X_{i\max}\right) \tag{5}$$

The mathematical model can be expanded by introducing interactions of influential factors, when Eq. (2) turns into the quasi-linear form. By introducing transformation equations into this equation, one gets the expanded form of power function Eq. (1) in the form:

$$\begin{aligned} K = C \cdot X_1^{p_1} \cdot X_2^{p_2} \ldots \exp\ (p_{12} \ln X_1 \cdot \ln X_2 + \ldots + p_{123} \ln X_1 \cdot \ln X_2 \cdot \ln X_3 + \ldots + \\ p_{ijl\cdots m} \ln X_i \cdot \ln X_j \cdot \ln X_l + \ldots + p_{ijlm\cdots\cdots k} \ln X_i \cdot \ln X_j \ldots \ln X_k) \end{aligned} \tag{6}$$

2.1 Selected examples of mathematical models application

Example I: Modelling of the kerf in plasma arc cutting

The objective of the plasma cutting process is to concentrate a large amount of energy on a small surface of a workpiece which leads to intensive heating of the material surface. The source of energy is high temperature and high speed ionized gas. The gas is ionized by means of a direct current passing between the cathode (inside the nozzle) and the anode (workpiece). The plasma jet cuts the material by releasing the energy spent for the plasma gas ionization upon hitting the workpiece surface. The removal of the melted material from the cutting zone is done by the plasma jet kinetic energy.

Even though it is the case of a complex process, which is characterised by a large number of influential factors, the previous analysis has shown that this number can be reduced to three influential factors (input values, independent variables): cutting current (*I*), cutting speed (*V*) and material thickness (*s*) (Table 1). As the target function (output value, dependent variable) one of the basic characteristics of the cutting quality is selected – kerf (*W*). Influential factors were varied on two levels.

Table 1 Cutting factors and their levels

Cutting factor	Symbol	Unit	Factor levels	
			Level 1 (Low)	Level 2 (High)
Cutting current	*I*	A	45	80
Material thickness	*s*	mm	4	6
Cutting speed	*V*	m/min	0.9	1.3

For the purpose of this research, a number of experiments were done [17]. The entire experiment consists of 25 trials, while 8 measurement results are used for regression analysis (Table 2). The main experimental matrix (used for all the examples in this paper) is represented by the full factorial design of type 2^3. Such a design enables the application of mathematical models including independent assessment of the main effects, as well as all influential factor interactions.

Straight-line cuts are made in the performed experiment by varying input process factors, according to Table 1. Test samples are made of stainless steel X10CrNiMn-16-10-2 EN designation (EN10025). The experimental research of plasma cutting process is conducted on the CNC machine for plasma cutting, type HPm Steel Max 6.25. The given data on the kerf represent mean values from obtained 3 measurements.

For the purposes of using Eqs. (1-6), the following substitution of variables was introduced: $F_c=W$; $X_1=I$; $X_2=V$; $X_3=s$. The need to determine the unknown constant *C* and exponents p_1; p_2; p_3 in this case arises from the basic Eq. (1).

The regression analysis is conducted, coefficients of the linear mathematical model are generated, and thus the regression equation in coded coordinates obtains the following form:

$$y = 0.62338 + 0.03463 \quad x_1 - 0.04263 \quad x_2 + 0.008125 \quad x_3 \tag{7a}$$

This equation is easily transformed, in the afore-described manner, into non-linear regression equation in natural coordinates:

$$W = 1.08073 I^{0.12038} V^{-0.23186} s^{0.04008} \tag{7b}$$

The kerf increases with the increase in cutting current and material thickness since the exponents I and s are positive, whereas the exponent of cutting speed is negative, indicating decreasing tendency of kerf, with increasing cutting speed. Eq. (7b) is suitable for application in engineering practice.

By introducing interactions into the Eq. (7a), the quasi-linear regression equation is obtained in the form:

$$\begin{aligned} y = 0.62338 + 0.03463 x_1 - 0.04263 x_2 + 0.008125 x_3 + 0.001125 x_1 x_2 + \\ + 0.007375 x_1 x_3 + 0.006125 x_2 x_3 - 0.008125 x_1 x_2 x_3 \end{aligned} \tag{8a}$$

In the natural coordinates, this regression equation has the form:

$$\begin{aligned} W = 3.75 I^{-0.17675_1} V^{-5.50963} s^{-0.73409} \exp(1.225267 \ln I \\ \cdot \ln V + 0.185933 \ln I \cdot \ln s + 3.266585 \ln V \cdot \ln s - 0.7577 \ln I \cdot \ln V \cdot \ln s) \end{aligned} \tag{8b}$$

For the purposes of verification of the generated mathematical models, the results from 17 trials are used, otherwise not used for the purposes of regression analysis, as shown in Table 3. This experiment confirmation, which contains a great number of trails, should contribute to higher reliability of conclusions from the following analysis.

Table 2 Main experiment

	I	V	s	W_{exp}	W_{cal}	δ	Δ
1	80 (+1)	1.3 (+1)	6 (+1)	1.88	1.8655 / 1.8776	- 0.7713 / - 0.1277	0.7713 / 0.1277
2	45 (-1)	1.3 (+1)	6 (+1)	1.75	1.7406 / 1.7507	- 0.5371 / 0.0400	0.5371 / 0.0400
3	80 (+1)	0.9 (-1)	6 (+1)	2.05	2.0315 / 2.0483	-0.9024 / - 0.0829	0.9024 / 0.0829
4	45 (-1)	0.9 (-1)	6 (+1)	1.86	1.8955 / 1.8571	1.9086 / - 0.1559	1.9086 / 0.1559
5	80 (+1)	1.3 (+1)	4 (-1)	1.83	1.8354 / 1.8276	0.2951 / - 0.1311	0.2951 / 0.1311
6	45 (-1)	1.3 (+1)	4 (-1)	1.70	1.7126 / 1.6989	0.7412 / - 0.0647	0.7412 / 0.0647
7	80 (+1)	0.9 (-1)	4 (-1)	1.98	1.9987 / 1.9779	0.9444 / - 0.1061	0.9444 / 0.1061
8	45 (-1)	0.9 (-1)	4 (-1)	1.91	1,8650 / 1.9079	-2.3560 / - 0.1099	-2.3560 / - 0.1099

Note: Results in the numerator are related to the basic mathematical model, while in the denominator is related to the expanded mathematical model

Into Table 2 and Table 3, the calculation results obtained by applying Eq. (7b) and Eq. (8b) are also inserted, as well as corresponding errors.

The following relations were used for the calculation of errors:

- For the percentage error (δ) and mean percentage error ($\bar{\delta}$),

$$\delta = \frac{y_{cal} - y_{\exp}}{y_{\exp}} 100(\%) \; ; \; \bar{\delta} = \frac{\sum \delta}{n} \tag{9a}$$

- For absolute percentage error (**Δ**) and mean absolute percentage error ($\bar{\Delta}$),

$$\Delta = |\delta| \quad ; \quad \bar{\Delta} = \frac{\sum \Delta}{n} \tag{9b}$$

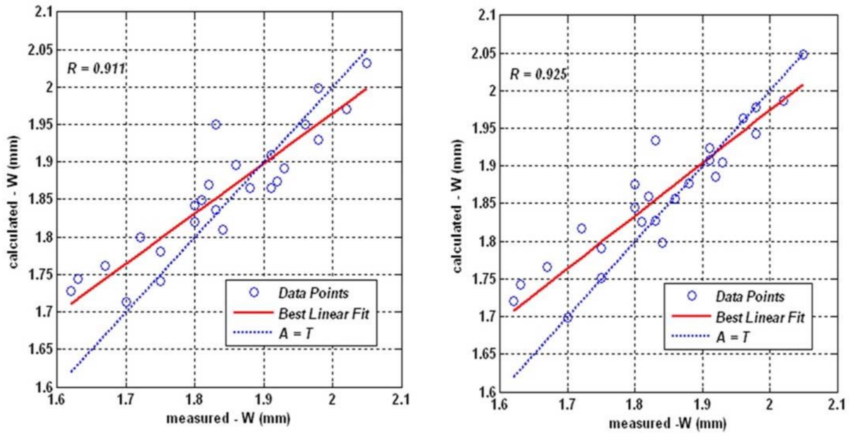
where *n* - the number of trials in the experiment.

Table 3 Confirmation experiment

I	V	s	W_{exp}	W_{cal}	δ	Δ
80	1.025	6	2.02	1.9711 / 1.9862	-2.4208 / -1.6733	2.4208 / 1.6733
80	1.075	6	1.96	1.9495 / 1.9640	-0.5357 / 0.2041	0.5357 / 0.2041
80	1.125	6	1.98	1.9291 / 1.9430	-2.5707 / -1.8687	2.5707 / 1.8687
80	1.175	6	1.91	1.9097 / 1.9231	-0.0157 / 0.6859	0.0157 / 0.6859
80	1.225	6	1.93	1.8913 / 1.9042	-2.0052 / -1.3368	2.0052 / 1.3368
80	1.275	6	1.92	1.8739 / 1.8863	-2.4010 / -1.7552	2.4010 / 1.7552
80	1.0	4	1.83	1.9505 / 1.9336	6.5847 / 5.6612	6.5847 / 5.6612
80	1.2	4	1.82	1.8698 / 1.8593	2.7363 / 2.1593	2.7363 / 2.1593
45	1.0	6	1.81	1.8498 / 1.8259	2.1989 / 0.8785	2.1989 / 0.8785
45	1.1	6	1.84	1.8094 / 1.7982	-1.6630 / - 2.2717	1.6630 / 2.2717
45	0.95	4	1.80	1.8418 / 1.8756	2.3222 / 4.2000	2.3222 / 4.2000
45	1.0	4	1.80	1.8200 / 1.8455	1.1111 / 2.5278	1.1111 / 2.5278
45	1.05	4	1.72	1.7995 / 1.8173	4.6221 / 5.6570	4.6221 / 5.6570
45	1.1	4	1.75	1.7802 / 1.7909	1.7257 / 2.3371	1.7257 / 2.3371
45	1.15	4	1.67	1.7619 / 1.7659	5.5030 / 5.7425	5.5030 / 5.7425
45	1.20	4	1.63	1.7446 / 1.7424	7.0307 / 6.8957	7.0307 / 6.8957
45	1.25	4	1.62	1.7282 / 1.7201	6.6790 / 6.1790	6.6790 / 6.1790

Note: Results in the numerator are related to the basic mathematical model, while in the denominator is related to the expanded mathematical model

Based on the data from Table 2, one can conclude that the calculation errors are negligible, regardless whether the basic regression Eq. (7b) or the expanded regression Eq. (8b) is applied. As it could have been expected, calculation errors are larger in relation to confirmation experiment results (Table 3).



a) b)

Fig. 1 Correlations between the experimental results and results predicted by the generated model for the kerf calculation: a.) for basic model, b) for expanded model

The conformance assessment of the mathematical model prediction with the experimental results can also be conducted using the correlation coefficient (*R*).

Based on the Fig. 1 it could be concluded that both mathematical models ensure high levels of correlation, which are very similar, although not identical. The perfect prediction implies that all points should lie on a straight line passing through the origin and inclined at 45º. From Fig. 1 one can observe that there is a relatively small deviation of the line of regression (which represents the best linear approximation of the data) from the ideal line.

Table 4 Adequacy assessment criteria of the mathematical models related for the plasma cutting process

Experiment	Basic mathematical model			Expanded mathematical model		
	Δ_{max}	$\overline{\Delta}$	*R*	Δ_{max}	$\overline{\Delta}$	*R*
Main experiment	2.36	1.06	0.911	0.16	0.1	0.925
Confirm.experiment	7.03	3.07		6.9	3.06	

Taking three criteria (Δmax, $\overline{\Delta}$, R) into consideration results in the fact that an expanded mathematical model cannot guarantee better prediction for randomly selected points within the chosen experimental space (Table 4).

Example II: Modelling of the thermal stress in the face milling

Face milling process has periodical characteristics, since the number of teeth in contact with material change periodically. Periodical heating and cooling of the teeth is very unfavourable, since it itself represents a thermal process.

In this investigation three influential factors are chosen for modelling the face milling process: cutting speed (*V*), unit feed rate (f_z) and depth of cut (*a*) (Table 5).

Table 5 Cutting factors and their levels

Cutting factor	Symbol	Unit	Factor levels	
			Level 1 (Low)	Level 2 (High)
Cutting speed	V	m/s	2.32	3.67
Unit feed rate	f_z	mm/tooth	0.178	0.280
Depth of cut	a	mm	1.00	2.25

Thermo-electric current (U) is chosen as the target function. Influential factors are varied on two levels. A sample of 16 trials is extracted from the literature [18]. One half of trials are used for conducting the regression analysis (Table 6), while another half is used for the selected mathematical models verification (Table 7).

Table 6 Main experiment

V	f_z	a	U_{exp}	U_{cal}	δ	Δ
3.67 (+1)	0.28 (+1)	2.25 (+1)	14.8	$\frac{14.8188}{14.8000}$	$\frac{0.1270}{0.0000}$	$\frac{0.1270}{0.0000}$
2.32 (-1)	0.28 (+1)	2.25 (+1)	13.4	$\frac{13.3733}{13.3998}$	$\frac{-0.1993}{-0.0015}$	$\frac{0.1993}{0.0015}$
3.67 (+1)	0.18 (-1)	2.25 (+1)	14.0	$\frac{14.0413}{14.0000}$	$\frac{0.2950}{0.0000}$	$\frac{0.2950}{0.0000}$
2.32 (-1)	0.18 (-1)	2.25 (+1)	12.7	$\frac{12.6716}{12.6998}$	$\frac{-0.2236}{-0.0016}$	$\frac{0.2236}{0.0016}$
3.67 (+1)	0.28 (+1)	1.00 (-1)	14.2	$\frac{14.1413}{14.2000}$	$\frac{-0.4134}{0.0000}$	$\frac{0.4134}{0.0000}$
2.32 (-1)	0.28 (+1)	1.00 (-1)	12.7	$\frac{12.7618}{12.7000}$	$\frac{0.4866}{0.0000}$	$\frac{0.4866}{0.0000}$
3.67 (+1)	0.18 (-1)	1.00 (-1)	13.4	$\frac{13.3993}{13.4001}$	$\frac{-0.0052}{0.0007}$	$\frac{0.0052}{0.0007}$
2.32 (-1)	0.18 (-1)	1.00 (-1)	12.1	$\frac{12.0922}{12.1000}$	$\frac{-0.0645}{0.0000}$	$\frac{0.0645}{0.0000}$

Note: Results in the numerator are related to the basic mathematical model, while in the denominator is related to the expanded mathematical model

Test samples are made of steel C 60 DIN designation. The experiment is done on the milling machine FS-GVK-3 (Prvomajska). Cutting head JAL G-750 (ϕ 125 mm) with hard material inserts SPAN 12 03 ER is used. The metering of the thermo-electric current (in mV) is done using thermocouples and heatmeter (Digital Multi-thermometer). The metering is done in a few seconds after the beginning of the cutting process, namely in the stationary phase of the process.

It should be mentioned that in the confirmation experiment the first two trials are related to the central point of the experimental design. The other trails are related to the experimental points located on the coordinate axes of the experimental design, therefore, both models generate the same results.

Table 7 Confirmation experiment

V	f_z	a	U_{exp}	U_{cal}	δ	Δ
2.95	0.223	1.0	13.4	$\frac{13.3863}{13.3863}$	$\frac{-0.1022}{-0.1022}$	$\frac{0.1022}{0.1022}$
2.95	0.223	1.0	13.7	$\frac{13.3863}{13.3863}$	$\frac{-2.2898}{-2.2898}$	$\frac{2.2898}{2.2898}$
1.83	0.223	1.5	12.1	$\frac{12.0805}{12.0805}$	$\frac{-0.1612}{-0.1612}$	$\frac{0.1612}{0.1612}$
4.65	0.223	1.5	14.9	$\frac{14.8332}{14.8332}$	$\frac{-0.4483}{-0.4483}$	$\frac{0.4483}{0.4483}$
2.95	0.142	1.5	12.6	$\frac{12.6839}{12.6839}$	$\frac{0.6659}{0.6659}$	$\frac{0.6659}{0.6659}$
2.95	0.351	1.5	14.1	$\frac{14.1276}{14.1276}$	$\frac{0.1957}{0.1957}$	$\frac{0.1957}{0.1957}$
2.95	0.223	0.67	12.8	$\frac{12.7742}{12.7742}$	$\frac{-0.2016}{-0.2016}$	$\frac{0.2016}{0.2016}$
2.95	0.223	3.37	15.0	$\frac{14.0276}{14.0276}$	$\frac{-6.4827}{-6.4827}$	$\frac{6.4827}{6.4827}$

Note: Results in the numerator are related to the basic mathematical model, while in the denominator is related to the expanded mathematical model

Applying the same methodology as in the previous examples, the following regresion equations are generated:

- for the mathematical model without interactions:

$$y = 2.59423 + 0.05132 x_1 + 0.02695 x_2 + 0.02340 x_3 \tag{10a}$$

$$U = 12.29976 V^{0.22380} f_z^{0.11898} a^{0.05771} \tag{10b}$$

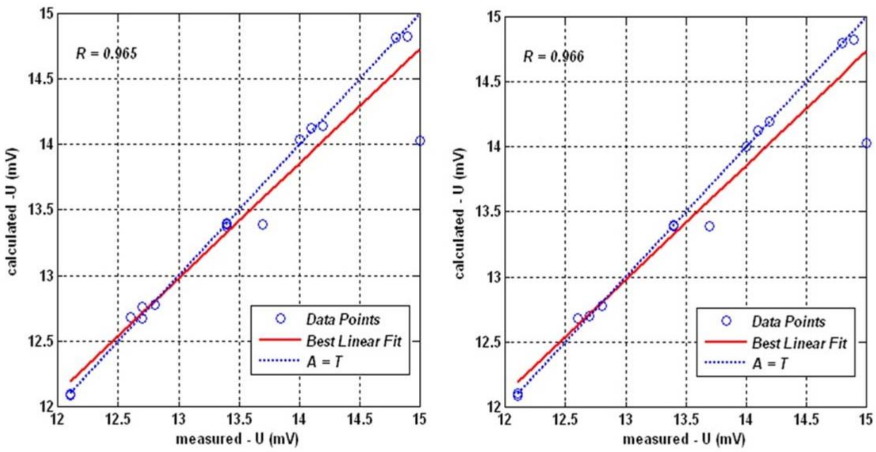
- for the mathematical model with interactions:

$$y = 2.59423 + 0.05132 x_1 + 0.02695 x_2 + 0.02340 x_3 + 0.001439 x_1 x_2 - \\ - 0.002106 x_1 x_3 + 0.000356 x_2 x_3 - 0.000959 x_1 x_2 x_3 \tag{11a}$$

$$U = 12.1562 V^{0.23298} f_z^{0.11741} a^{-0.04102} \exp(0.07980 \ln V \cdot \ln a - \\ - 0.06930 \ln f_z \cdot \ln a + 0.06834 \ln V \cdot \ln f_z \cdot \ln a) \tag{11b}$$

From Eqs. (10) and (11) it is obvious that the thermo-electric current increases with the increase of cutting speed, unit feed rate and depth of cut, since exponents V, f_z and a are positive. The regression equation in the form of Eq. (10b) is suitable for application in engineering practice.

Based on Fig. 2 (which relates to the entire experiment) one can conclude that both mathematical models ensure high levels of correlations, which are almost identical.



a) b)

Fig. 2 Correlations between the experimental results and results predicted by the generated model for the face milling process: a) for basic model, b) for expanded model

Taking all criteria into consideration results in the fact that an expanded mathematical model cannot guarantee a better prediction for randomly selected points within the entire experimental hyperspace (Table 8).

Table 8 Adequacy assessment criteria of the mathematical models related to the face milling process

Experiment	Basic mathematical model			Expanded mathematical model		
	Δ_{max}	$\bar{\Delta}$	R	Δ_{max}	$\bar{\Delta}$	R
Main experiment	0.49	0.23	0.965	0.02	0.005	0.966
Confirm. experiment	6.48	1.32		6.48	1.32	

3. Conclusions

This paper presents the application of non-linear mathematical models for different technological process modelling. It is possible to draw the following conclusions from the analysis of described example:

1. The classical theory of experimental design shows that non-linear mathematical models in the form of higher-order polynomials ensure a higher accuracy than the linear ones. These models, especially models including factors' interactions, make intricate the analysis and interpretation of modeling results. However, non-linear mathematical models in the form of complex power functions with interactions do not guarantee higher accuracy than the same models without interactions. This is the reason why the application of such mathematical models can be justified only in specific cases.

2. Although non-linear mathematical models without factors' interactions show lower accuracy in the points of experiment than expanded mathematical models, the mean error of both models within the entire experimental space is practically identical.

3. It is shown that it is more practical and purposeful to use the basic mathematical model (without interactions) than the expanded mathematical model (with interactions).

4. The accuracy of selected mathematical models in this paper is significantly larger in experimental points than in other points of experimental hyperspace.

5. The analogous analysis should indicate whether the same conclusions can be reached for other mathematical models of this type.

REFERENCES

1. Montgomery, D. C., 2001, *Design and analysis of experiments*, John Wiley & Sons, Inc., New York.
2. Kim, W. K., Kim, H. T., Ryu, G. M., Kim, G. H., 2013, *Study on high-speed cutting characteristics using design of experiments*, International Journal of Precision Engineering and Manufacturing, 14(10), pp. 1869-1872.
3. Dersjö, T., Olsson, M., 2012, *Efficient design of experiments for structural optimization using significance screening*, Structural and Multidisciplinary Optimization, 4(2), pp. 185-196.
4. Cauffriez, L., Loslever, P., Caouder, N., Turgis, F., Copin, R., 2013, *Robustness study and reliability growth based on exploratory design of experiments and statistical analysis: a case study using a train door test bench*, The International Journal of Advanced Manufacturing Technology, 66(1-4), pp. 27-44.
5. Czitrom, V., 1999, *One-factor-at-a-time versus designed experiments*, The American Statistician, 53(2), pp. 126-131.
6. Novik, F. S., Arsov, J. B., 1980, *Optimization of technological processes using methods of experimental design*, Mashinostoenie, Moscow.
7. Lazarevic, A., Marinkovic, V., Lazarevic, D., 2010, *Expanded non-linear mathematical models in the theory of experimental design: A case study*, Proceedings of the 10^{th} International Conference Research and Development in Mechanical Industry, pp. 304-310.
8. Peixoto, J. L., 1990, *A property of well-formulated polynomial regression models*, The American Statistician, 44(1), pp. 26-30.
9. Montgomery, D. C., Myers, R. H., Carter, W. H., Vining, G. G., 2005, *The hierarchy principle in designed industrial experiments*, Quality and reliability engineering international, 21, pp. 197-201.
10. Heller, C., Schmoeckel, D., 1988, *Umformen von Aluminium-blechen bei erhöhten Temperaturen*, Aluminium, 64(4), pp. 398-405.
11. Choudhury, I. A., El-Baradie, M. A., 1997, *Surface roughness prediction in the turning of high-strength steel by factorial design of experiment*, Journal of Material Processing Technology, 67,pp. 55-61.
12. Choudhury, I. A., El-Baradie, M. A., 1998, *Tool-life prediction model by design of experiment for turning high strength steel (290 BHN)*, Journal of Material Processing Technology, 77, pp. 319-326.
13. Shunmungam, M. S., Bhaskara Reddy, S. V., Narendan, T. T., 2000, *Selection of optimal conditions in multi-pass face-milling using a genetic algorithm*, International Journal of Machine Tool & Manufacture, 40, pp. 401-414.
14. Dasgupta, R., Thakur, R., Govindrjan, B., 2002, *Regression analysis of factors affecting high stress abrasive wear behaviour*, Practical Failure Analysis, 2(2), pp. 65-68.
15. Tosun, N., Ozlar, L., 2002, *A study of tool life in hot machining using artificial neural networks and regression analysis method*, Journal of Materials Processing Technology, 124, pp. 99-104.
16. Samanta, B., Erevelles, W., Omurtag, Y., 2008, *Prediction of workpiece surface roughness using soft computing*, Proc, IMechE, Part B: J. Engineering Manufacture, 222, pp.1221-1232.
17. Lazarević, A., 2010, *Modelling of the correlations between the parameters of the plasma arc cutting and heat balance analysis using the method of artificial intelligence*, (in Serbian) PhD Thesis, Faculty of Mechanical Engineering, University of Nis.
18. Kovač, P., Milikić, D., 2006, *Predicting equation of thermo-electric currents and cutting conditions generated in face milling*, University of Novi Sad, Faculty of Technical Sciences, Novi Sad.