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OPTIMIZATION BASED DETERMINATION OF THE SET OF CONTINUOUS-STATE COMPONENTS CRITICAL TO THE MECHANICAL SYSTEM STATE

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Abstract. *This paper proposes a new approach to determining the set of the most critical continuous-state components in continuous-state mechanical systems. Unlike traditional importance measures, which determine the criticality of individual components, our approach provides selection of the entire set of critical components. In addition, it enables determination of the minimal budget for achieving full or any required performance level of the mechanical system. We start from the continuum structure function and the system state definition through minimal cut sets. The problem defined in this paper is formulated as a determination of a set of components whose states maximize the system performance level under the budget constraint. Further, we prove that the solution of the formulated problem can be obtained based on the solution to a weighted minimal hitting set problem. The proposed approach is applied to a group of benchmark instances and the obtained results are compared to components' rankings obtained by using traditional importance measures.*

Key words: *Continuous-state, Criticality, Performance level, Importance measures*

1. INTRODUCTION

Identification of the critical system components has an important role in the mechanical system design, improvement, and maintenance. In the system design phase, it is used in deciding on the performance levels of the particular components included in the system [1]. When the system performance should be improved, the performances of the critical components are the ones that should be improved first. In the system maintenance planning they are used to effectively allocate maintenance resources [2]. In system engineering, critical components are the components whose performance mostly influence the system

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performance. Performance of a system and components can be expressed using different measures: reliability, availability, maintainability, safety, utility, obsolescence, detrition, fatigue, capacity, throughput etc. [3]. Regardless of the performance type, system's and components' performance levels are generally called system's and components' states.

Mechanical components' and system's states can be observed as:

- binary-states, where components and system can either be in a perfect functionality or a complete failure state,

- multi-states, with discrete degradation states between perfect functionality and complete failure,

- continuous-states, with continuous degradation states between perfect functionality and complete failure.

In the case of binary-states, only the so called "hard failure" appears. In multi-states and continuous-states cases, components can also have "soft failure" with different levels of degradation that do not exceed the predefined failure threshold [4].

In this paper we observe continuous-state components and systems. Factors that cause continuous degradation of components' states are processes such as wear, fatigue, and erosion [5]. Components and systems performance degrade continuously in many real-life mechanical systems: automobile tire whose performance degrades continuously as the tread wears; a coal-fired power plant consisting of boilers operating at fewer megavolts than their full capacity; gas turbine fueled by nozzles in which the flow of fuel can be reduces due to the nozzles blockages; internal combustion engine valves wear out after a large number of engine cycles; production systems where production capacity can deteriorate with time [6]; light emitting diode luminosity degradation [7]; the voltage degradation of membrane electrode assemblies in polymer electrolyte membrane fuel cell stack system [8] etc.

The approach used in reliability and risk theory to determine the most important (critical) system components is based on the importance measures (IM). The goal of IM is to rank the components according to their impact to make better system design or maintenance plan. In the system design phase, IM should be taken into consideration to determine the appropriate reliability of each of the components of the system being designed. In the maintenance process, IM are used to identify the component whose reliability should be improved to increase the system reliability, or to order spare parts [9], and/or in the component maintenance priority approach that prioritizes the components that need to be maintained during the repair of the failed component [10]. IMs can be used in diagnostics to rank components in terms of how likely they are to have caused a system failure [11]. In recent years, IM are used in the optimization of system resilience, which has become an important system performance [12, 13].

The first IM for binary systems was introduced by Birnbaum [14]. According to Birnbaum IM, the importance of a component is the difference between the reliability of the system when the component is fully functional and the reliability of the system when the component fails. Until now, more than sixty different binary IM were introduced [15]. A considerable number of IM for multi-state systems can also be found in literature [16-19].

However, there are only few IMs defined for continuous-state system and most of them reduce continuous-state to binary or multi-state by introducing threshold or partitioning approaches. Kim and Baxter [20] defined reliability importance of component j at system level α, $\alpha \in (0, 1)$, where $[0, \alpha)$ corresponds to the failure states, and $[\alpha, 1]$ corresponds to the operating states. To formulate a criticality measure model for continuous-state system, Lisnianski introduced discrete approximation [21, 22]. Consequently, the continuous-state system is represented by two multi-state systems that are then used as boundary points estimation for the given continuous-state system. Some of the continuous IM, such as Griffith IM [23] and mean absolute deviation IM [24], are obtained by adjustment of binary IM for continuous-states. De Jonge [25] reduced the continuous-state non-decreasing problem to a discrete-time discrete-state one using the probability of transition between states, and further modeled it using Markov chains. Chen, Qiu, and Zhao [26] used the Monte Carlo simulation method to determine optimal maintenance policies for continuous-state systems.

Most IMs rank individual components according to the calculated values of the selected IM and therefore cannot be applied to determine the simultaneous effect of a group of components on the system state [27]. This is one of the open issues on importance measures [28]. It was considered by few IMs, but only for pairs of binary or multi-state components [29, 30], module of three components [31] and for preselected group of components [32].

In this paper we propose an approach for simultaneous determination of the entire set of the most important continuous-state components of a continuous-state system. We formulate this new criticality measure as an optimization problem. The similar approach was already used in [33] for binary-state components and systems. It has been shown that investing in a set of critical components obtained using optimization approach provides higher reliability of the system than the reliability provided by investing in a set of components obtained using traditional IM.

The optimization problem formulation in this paper starts from definition of system state function based on minimal cut sets. The resulting problem is the global optimization problem with max-min-max objective. Therefore, we are about to solve them by solving the weighted minimal hitting set problem [34], also known as the minimal cost hitting set problem [35], which is complementary to the set covering problem.

The paper consists of five sections. The following section introduces the continuous-states systems, definition of the problem of simultaneous determination of the critical components and its mathematical programming formulation. Section 3 is devoted to solving the approach, where we prove the relationship between the solution of original max-min-max and weighted minimal hitting set problems. In Section 4, computational results of experiments on a group of benchmark instances are given. We compare the results of max-min-max and weighted minimal hitting set problems. Then we examine the influence of costs variances on the selection of critical components and the system state. Finally, we compare the state of the systems obtained by the proposed approach with the state obtained based on two cost related IM. Section 5 contains the conclusions and directions for further research.

2. Problem Definition and Mathematical Model Formulation

Let $K = \{1, 2, ..., n\}$ be a set of system's components and let $x_j \in [0, 1]$ be a state (performance level) of j-th component, $j \in K$. The vector $x = (x_1, x_2, ..., x_n)$ is called the state vector. The state of the continuous system is defined by the continuous structure function as follows [36].

Definition 1. A function γ: $[0, 1]^n \rightarrow [0, 1]$ which is nondecreasing in each coordinate of x is said to be a continuum structure function.

Continuum structure function expresses how the state of the coherent system depends on the components' states. One way of determining the system state is through the minimal cut sets [37, 38].

Definition 2. Cut set is a set of components whose failure causes the system failure. Minimum cut set (MCS) is a cut set that does not contain another cut set, that is, it is reduced to the minimum number of components whose failure causes system failure.

For the binary-state components and systems, the previous definition implies that disabling the failure of some component will eliminate all minimal cut sets containing that component as a potential cause of the system failure.

According to [36], continuum structure function for coherent systems can be defined as

$$\gamma(x) = \min_{i \in P} \max_{j \in C_i} x_j \tag{1}$$

where $P = \{1, 2, ..., m\}$ represents the set of MCS and C_i is the set of components in i-th MCS. Function γ is also called the system state function and it represents the state of the "best" component in the "worst" MCS.

Starting from the last interpretation of the system state, initial problem can be defined as: determining the minimal number of components whose improving of states (performance level) maximizes $\gamma(x)$, i.e., the state (performance level) of the system. Since the improving of the component's performance levels requires costs and considering that the maintenance budget is almost always limited, the final definition of the problem that is observed in this paper is:

Allocate the available budget to the components whose increasing of performance levels maximizes the system performance level.

The main assumptions of the defined problem are:

1. The continuous-state system is coherent.
2. The state space of components is $[0, 1]^n$ and the state space of the system is $[0, 1]$, where the state 1 represents the perfect functioning, while the state 0 represents total failure. The states in $(0, 1)$ correspond to different degradation levels, but not to complete failure.
3. All components' failures are statistically independent.
4. The cost of providing a component j with a certain performance level linearly depends on its performance level.
5. MCSs of a given system are already determined.

To formulate the mathematical model of the defined problem, we introduce the set of all components (K), the set of MCS (P), and the set of components in i-th MCS (C_i), such that $i \in P$, $\bigcup_{i \in P} C_i = K$. For each component $j \in K$ the cost of full performance level (c_j) is known, as well as the total budget for achieving performance level of the system (b). Decision variable that should be determined (x_j) represents the performance level of jth component, $j \in K$. Based on optimal values of variables, performance level of the system, $pl(x)$, such that $pl(x) = \gamma(x)$ can be obtained.

The mathematical model for maximization of the system performance level (MSPL) under the budget constraint, can now be formulated as follows:

$$
\begin{aligned}
&(\max)\, pl(x) = \min_{i\in P} \max_{j\in C_i} x_j \\
&\text{s.t.} \\
&\sum_{j\in K} \mathrm{c_j} x_j \le b \\
&0 \le x_j \le 1,\ j \in K
\end{aligned}
$$

The objective function represents the continuous system state defined in (1). Its maximum is the value of the ”best” component in the ”worst” MCS. Constraint in MSPL refers to the limited budget.

The formulated problem is the global optimization problem with max-min-max objective. The classes of max-min-max problems have been proven to be NP-hard [39]. It is possible to reformulate this problem in such a way that the optimal solution does not depend on the available budget. In the following section, weighted minimal hitting set formulation of the problem of determining the entire set of critical components will be presented.

3. Solving Approach

In order to formulate weighted minimal hitting set problem variant of critical components determination problem, the vector of binary variables $y = (y_j)$, $j \in K$ is introduced. A variable y_j is an indicator of criticality of the component j: if the component j is critical, then $y_j = 1$, otherwise $y_j = 0$. The remaining notation is the same as the notation used in MSPL model.

The mathematical model for minimization of the critical components’ cost (MCCC) is formulated as follows.

$$
\begin{aligned}
&(\min)\, fc(y) = \sum_{j\in K} c_j y_j \\
&\text{s.t.} \\
&\sum_{j\in C_i} y_j \ge 1,\ i \in P \\
&y_j \in \{0,1\},\ j \in K
\end{aligned}
$$

MCCC model corresponds to the weighted minimal hitting set problem [40]. The constraint ensures that each MCS is hit, i.e. each MCS contains at least one component $j \in K$ such that $y_j = 1$. The objective $fc\,(y)$ represents the cost of the full performance level for components $y = \{y_j \mid y_j\}$, since components in y hit all MCSs.

The following theorem determines the relationship between the MSPL and MCCC models, i.e., between the optimal solutions of these two problems.

Theorem 1. *If $y^* = \{y_j\}$, $j \in K$ is the optimal solution of MCCC problem then $x^*=\{x_j\}$, $j \in K$ is the optimal solution of MSPL problem, where*

$$
x_j = \min\left\{1, \frac{b y_j}{fc(y^*)}\right\},\ j \in K \tag{2}
$$

Proof. From (2) it follows $0 \le x_j \le 1$, for $j \in K$ and

$$\sum_{j\in K} c_j x_j \le \sum_{j\in K} c_j \frac{by_j}{fc(y^*)} = \frac{b}{fc(y^*)} \sum_{j\in K} c_j y_j = b$$

Therefore, $x^* = \{x_j\}, j \in K$ is a feasible solution of MSPL. Let now prove that x^* is the optimal solution of MSPL.

Since for each $i \in P$ there is $j \in C_i$ for which $y_j = 1$, it follows that

$$\max_{j\in C_i} x_j = \min\left\{1, \frac{b}{fc(y^*)}\right\}.$$

Moreover,

$$pl(x^*) = \min_{i\in P} \max_{j\in C_i} x_j = \min\left\{1, \frac{b}{fc(y^*)}\right\}.$$

In case of $fc(y^*) \le b$, x^* is the optimal solution of MSPL because $pl(x^*) = 1$.

In case of $fc(y^*) > b$, let us assume that x^* is not an optimal solution of MSPL. That means that there is a feasible solution $x = \{x_j\}, j \in K$ such that $pl(x) > pl(x^*) = \dfrac{b}{fc(y^*)}$.

Further, let

$$y_j = \begin{cases} 1, & x_j \ge pl(x) \\ 0, & x_j < pl(x) \end{cases}, \quad j \in K.$$

Given that $pl(x) = \min_{i\in P} \max_{j\in C_i} x_j$, it follows that for each $i \in P$ there is $i \in C_i$ for which $x_j \ge pl(x)$, i.e. for which $y_j = 1$. Hence, $y = \{y_j\}, j \in K$ is a feasible solution of MCCC. Furthermore, from the definition of y_j it follows that $y_j \le \dfrac{x_j}{pl(x)}$, from which we obtain

$$fc(y) = \sum_{j\in K} c_j y_j \le \sum_{j\in K} c_j \frac{x_j}{pl(x)} = \frac{1}{pl(x)} \sum_{j\in K} c_j x_j \le \frac{b}{pl(x)} < fc(y^*).$$

However, this is contrary to the fact that y^* is the optimal solution of MCCC, i.e. to the assumption that x^* is not an optimal solution of MSPL. Hence, x^* is the optimal solution of the MSPL problem, which has been proved.

Therefore, the optimal solution of the MSPL problem can be obtained from the optimal solution of the MCCC problem. Based on the equation (2), we conclude that there is an optimal solution $x^* = \{x_j\}, j \in K$ of MSPL such that all values of x_j are equal to 0 (when $y_j = 0$) or are equal to $spl = \min\left\{1, \dfrac{b}{fc(y^*)}\right\}$ (when $y_j = 1$).

If $fc(y^*) < b$, then the system can be provided with the full performance level ($spl = 1$) using less founds then the provided budget b.

If $fc(y^*) \geq b$, then the system can be provided with maximal performance level, in relation to the given budget b.

The performance level of the entire system $pl(x^*)$ is also equal to

$$spl = \min\left\{1, \frac{b}{fc(y^*)}\right\} \tag{3}$$

The MCCC model refers to the weighted minimal hitting set problem which is proven to be NP-hard. However, solving MCCC is much more efficient than solving MSPL for the problem of determining the critical components.

Moreover, in MCCC, the amount of available budget does not influence the optimal solution. Consequently, changes in the available budget do not require a restart of the optimization process, unlike the MSPL formulation. The new optimization should be done only if the cost c_j of some component $j \in K$ changes. In addition, MCCC can be used for determining the system states that can be reached with different values of available budget. When the optimal solution is obtained, it is enough to use the formula (3) to calculate the new state (performance level) of the system or to calculate minimal budget needed to achieve a required level of system performance.

4. Numerical Results

Three sets of experiments were conducted on 13 Benchmark Fault Trees (BFT) from [41], whose characteristics are given in Table 1. Columns C and MCS give the total number of components and the minimal cut sets of BFTs, respectively. Column R shows the range of MCSs, i.e. the minimal and maximal number of components in MCSs.

Table 1 Benchmark Fault Trees [41]

No	BFT	C	MCS	R
1	baobab1	61	46188	2-11
2	baobab2	32	4805	2-6
3	baobab3	80	24386	2-11
4	chineese	25	392	2-6
5	das9201	122	14217	2-7
6	das9202	49	27778	1-11
7	das9208	103	8060	2-6
8	edf9205	165	21308	1-8
9	ftr10	152	305	1-3
10	isp9603	91	3434	2-8
11	isp9605	32	5630	3-7
12	isp9606	89	1776	1-5
13	jbd9601	532	14007	1-7

All experiments were performed on a laptop computer equipped with Intel i7 CPU at 2.7GHz and 16 GB of RAM. Problems were solved using AMPL modeling environment and LGO solver for MSPL problems and CPLEX solver for MCCC problems. LGO solver is intended to solve global optimization problems with potentially many local optima. CPLEX is considered a state-of-the-art solver for linear and mixed integer programming.

Random instances were generated using the library of BFTs from Table 1 and assigning random values to the components' cost. The budget for improving the system performance level for each instance was determined as a constant percentage of the sum of all components' cost. The percentage was adjusted for each BFT in such a way that all instances have maximal performance level between 0.4 and 1.

Components' costs were generated as random integers that have Erlang's distribution. Mean and variance were adjusted differently in each set of experiments.

Considering that system performance level for all problems is in interval [0, 1] independently of the values of components' cost, we were allowed to aggregate experimental results and finally find some conclusion.

4.1 Comparison of MSPL and MCCC

In the first set of experiments, we were testing the usability of models MSPL and MCCC. MSPL is a global optimization problem that can have many local optima. MCCC is an NP hard binary programming problem.

For the test, we used BFTs and we generated 30 random instances for each of them. We solved them using both mathematical models. The average system performance levels (SPL) and time in seconds needed to solve the problems are presented in Table 2.

Table 2 MSPL and MCCC results

	MSPL		MCCC		Gap [%]
BFT	SPL	Time [sec.]	SPL	Time [sec.]	SPL
baobab1	0.6071	300.07*	0.7138	0.68	14.95
baobab2	0.6229	6.03	0.6966	0.28	10.58
baobab3	0.5438	300.03*	0.6621	0.55	17.87
chineese	0.5776	0.36	0.5788	0.01	0.21
das9201	0.1225	190.21	0.6584	0.13	81.39
das9202	0.5574	121.73	0.5821	0.38	4.24
das9208	0.3949	112.46	0.6313	0.06	37.45
edf9205	0.8181	300.02*	0.8212	0.38	0.38
ftr10	0.7258	6.96	0.7399	0.01	1.91
isp9603	0.4238	33.05	0.6055	0.05	30.01
isp9605	0.6071	8.42	0.6572	0.19	7.62
isp9606	0.6603	9.63	0.7386	0.03	10.60
jbd9601	0.7395	300.05*	0.8188	0.25	9.68
Average:	0.5693	129.92**	0.6849	0.23	16.89

* Optimization was interrupted after 5 minutes time limit.

** The average was calculated using the limited time.

It is obvious that LGO was not able to solve all of the problems to optimality, since the values in SPL column of MSPL are less than those in the SPL column of MCCC. The gap between obtained SPLs varied from 81.39% for "das9201" BFT to relatively modest 0.81% for "chineese". Moreover, for "das9201" the gap is very big even the solver "concluded" that the optimal solution was found. On the other hand, optimality of solutions obtained by MCCC is guaranteed.

Also, the difference in execution time is significantly better for MCCC model. Even relatively big instances like "baobab1" were solved in less than a second. After this practical confirmation of dominance of MCCC model over MSPL, further experiments were performed only using MCCC model.

4.2 Sensitivity of MCCC Solutions

The aim of the second set of experiments was to assess the sensitivity of solutions on the dispersion of components' cost. For each BFT, we generated three sets of 10 random instances with variances $4v_1 = 2v_2 = v_3$, respectively, but with the same mean. For each value of variance, we presented the average number of components in the solution (CS), and the average system performance level of the entire system in Table 3.

Table 3 MCCC results for different variance of cost values

	V1		V2		V3	
BFT	CS	SPL	CS	SPL	CS	SPL
baobab1	11.0	0.6451	11.6	0.6982	11.5	0.7981
baobab2	14.2	0.5214	14.1	0.5561	14.4	0.5943
baobab3	17.7	0.6167	17.4	0.6659	18.7	0.7037
chineese	5.4	0.5611	5.6	0.5847	5.8	0.5905
das9201	9.0	0.6604	9.0	0.6631	9.0	0.6517
das9202	8.0	0.5256	8.0	0.5348	8.2	0.6858
das9208	17.2	0.6144	17.4	0.6480	17.4	0.6315
edf9205	40.0	0.8349	40.0	0.8112	40.0	0.8176
ftr10	83.6	0.7317	83.2	0.7401	83.6	0.7477
isp9603	17.2	0.5899	17.7	0.5881	17.7	0.6385
isp9605	8.0	0.6179	8.0	0.6375	8.6	0.7161
isp9606	34.2	0.7140	35.3	0.7308	35.9	0.7709
jbd9601	270.0	0.8044	269.6	0.8163	275.5	0.8356
Average	41.2	0.6490	41.3	0.6673	42.0	0.7063

From the presented results, we can see that a better system performance level can be obtained if the components have more disperse costs. We explain that by the possibility that a set of cheaper components that hit all cuts can be found if there are more both cheap and expensive components, i.e., if variance is greater.

4.3 Comparison of MCCC and Traditional IMs

In the third set of experiments, we compared the MCCC model with traditional IMs. In the absence of a cost-based IM for continuous states in available literature, two cost binary IMs were used: Cost-based component importance (CBCI) and Cost-effective importance measure (CEIM). Although both IM are primarily developed for measuring the impact of the components' reliability on the overall system reliability, the terminology below will be generalized to the state, i.e., the level of performance of components and systems. The notation in the equations will be adjusted according to this generalization.

Importance measure CBCI was introduced in [42] and it is formulated as the Birnbaum IM extended by costs of components' improvements. CBCI of component *j* is defined as:

$$I_j^{CBCI} = -\frac{\partial C_j}{\partial pl_j} \tag{4}$$

where ∂C_j and ∂pl_j represent the increase of the system cost and the system state caused by state improvement of *j*th component. The components with lower values of I_j^{CBCI} are considered more important since they provide greater system state improvement with lower increase of costs.

Importance measure CEIM is defined as [43]:

$$I_j^{CEIM} = \frac{I_j^{GI}}{C_{f,j}} \tag{5}$$

where I^{GI} and $C_{f,j}$ represent the general importance (GI) and cost factor for component *j*, respectively. GI of component *j* is obtained as:

$$I_j^{GI} = \frac{\Delta pl_j}{pl} \tag{6}$$

where Δpl_j is the change in system state caused by the change in *j*th component state.

The cost factor for *j*th component is calculated as:

$$C_{f,j} = \frac{\sum_{j \in K} E(C_j)}{E(C_j)} \tag{7}$$

where $E(C_j)$ is the expected cost of state improvement for *j*-th component.

Components with higher value of I_j^{CEIM} will be considered the most important components, since they provide higher state at lower costs.

For each instance of BFTs from Table 1, values of I_j^{CBCI} and I_j^{CEIM} were calculated. Then, the components were ranked by obtained values of IM. For each instance the first *m* components that hit all MCS of a given BFT were selected. The total cost of the selected components represents the budget required to achieve the state of the system equal to 1. The ratio of total cost and the given budget for an instance gives the system state which can be achieved by investing in the selected components. These values are given in Table

4. The column Min gives the optimal solution of minimal hitting set problem where cost c_j is equal to 1 for all components $j \in K$. The columns CC show the average number of components obtained by solving MCCC and m selected component based on I_j^{CBCI} and I_j^{CEIM}, respectively. The columns SLP contain corresponding systems' performance levels.

Table 4 Results of MCCC and cost-based IMs

		MCCC		CBCI		CEIM	
BFT	Min	CC	SPL	CC	SPL	CC	SPL
baobab1	11	11.6	0.8097	37.5	0.2746	40	0.1433
baobab2	14	14	0.5728	25.2	0.3582	24.6	0.2381
baobab3	17	18.7	0.5656	61.6	0.1934	60.7	0.1278
chineese	5	5.6	0.6218	16	0.2829	12.7	0.1907
das9201	9	9	0.6380	105.7	0.0724	37.4	0.1344
das9202	8	8	0.7250	31.7	0.2071	24.4	0.1568
das9208	17	17.2	0.6489	91	0.1374	59.9	0.1549
edf9205	40	40	0.8251	160.1	0.2210	136.1	0.2379
ftr10	79	83.7	0.7419	150.8	0.4184	142.5	0.4220
isp9603	17	17	0.6559	71.3	0.1737	54	0.1666
isp9605	8	8	0.7845	22.1	0.3406	18.5	0.2656
isp9606	34	34.6	0.5890	84	0.2391	77.3	0.2224
jbd9601	268	273.2	0.8183	529.5	0.4087	523.8	0.4057
Average	40.5	41.6	0.6920	106.7	0.2560	93.2	0.2205

From the results in Table 4 we can see that the MCCC model outperforms I_j^{CBCI} and I_j^{CEIM} for all BFTs. Based on the results of MCCC, the number of components that hit all MCS is the same as in the minimal hitting set case for 6 of 13 BFT. The numbers of critical components obtained based on I_j^{CBCI} and I_j^{CEIM} are much larger, which means that they require much higher costs of selected components. Consequently, system performance levels that can be achieved using available budgets are significantly lower than levels obtained by MCCC model.

5. CONCLUSIONS

The goal of this paper was to propose a new approach for determination of the critical components, i.e., the components whose performance level has the biggest effect on the performance level of the mechanical system. Criticality of the components is usually defined by means of importance measures, but most of them are not directly applicable to the determination of the group of the critical components. To overcome this lack of IM, we have proposed an optimization approach that enables simultaneous determination of all of the most critical components. The approach was applied on continuous-state components of continuous-state system. The problem was defined as a problem of allocation of the available budget to the components whose increasing of performance levels maximizes the system performance level. Originally formulated as the global optimization problem, it was reformulated as the weighted minimal hitting set problem. The experiments that were conducted on a group of BFTs showed that the proposed approach outperforms those based on the chosen cost related IMs.

Besides determination of the critical components, the proposed approach enables calculation of the system states that can be reached with different values of available budget. Since the formulated problem is NP-hard, further research will be directed towards testing on large instances and developing suitable heuristics.

REFERENCES

1. Compare, M., Bellora, M., Zio, E., 2017, *Aggregation of importance measures for decision making in reliability engineering*, Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability, 231(3), pp. 242–254.
2. Barabady, J., Gao, X., Markeset, T., 2017, *Criticality analysis from maintainability point of view*, In Industrial Engineering and Engineering Management (IEEM), IEEE International Conference, pp. 2140–2143.
3. Kuo, W., Zhu, X., 2012, *Relations and generalizations of importance measures in reliability*, IEEE Transactions on Reliability, 61(3), pp. 659–674.
4. Chen, Y., Qiu, Q., Zhao, X., 2022, *Condition-based opportunistic maintenance policies with two-phase inspections for continuous-state systems*. Reliability Engineering & System Safety, *228*, 108767.
5. Shen, J., Elwany, A., Cui, I., 2018, *Reliability analysis for multi-component systems with degradation interaction and categorized shocks*, Applied Mathematical Modelling, 56, pp. 487–500.
6. Kuang, F., Dai, W., Zhao, Y., 2013, *Production reliability evaluation of continuum-state manufacturing system based on universal generating function*, In Industrial Engineering and Engineering Management (IEEM), 2013 IEEE International Conference, pp. 472–476.
7. Bae, S.J., Yuan, T., Kim, S.J., 2016, *Bayesian degradation modeling for reliability prediction of organic light-emitting diodes*, Journal of Computational Science, 17, pp. 117-125.
8. Yuan, T., Wu, X., Bae, S.J., Zhu, X., 2019, *Reliability assessment of a continuous-state fuel cell stack system with multiple degrading components*, Reliability Engineering & System Safety, 189, pp. 157-164.
9. De Jonge, B., Scarf, P.A., 2020, *A review on maintenance optimization*, European Journal of Operational Research, 285(3), pp. 805-824.
10. Dui, H., Wu, S., Zhao, J., 2021, *Some extensions of the component maintenance priority*, Reliability Engineering & System Safety, 214, 107729.
11. Huseby, A.B., Kalinowska, M., Abrahamsen, T., 2022, *Birnbaum criticality and importance measures for multistate systems with repairable components*, Probability in the Engineering and Informational Sciences, 36(1), pp. 66-86.
12. Dui, H., Liu, M., Song, J., Wu, S., 2023, I*mportance measure-based resilience management: Review, methodology and perspectives on maintenance*. Reliability Engineering & System Safety, 237, 109383.
13. Dui, H., Wang, X., Zhou, H., 2023, *Redundancy-based resilience optimization of multi-component systems*. Mathematics, 11(14), 3151.
14. Birnbaum, Z.W., 1969, *On the importance of different components in a multicomponent system*, In Multivariate Analysis, 2 (ed. Krishnaiah PR), Academic Press, New York, pp. 581–592.
15. Kuo W., Zhu, X., 2012, *Importance Measures in Reliability, Risk, and Optimization*, John Wiley&Sons, Chichester.
16. Levitin, G., Podofillini, L., Zio, E., 2003, *Generalised importance measures for multi-state elements based on performance level restrictions*, Reliability Engineering & System Safety, 82, pp. 287–298.
17. Ramirez-Marquez, J.E., Coit, D.W., 2007, *Multi-state component criticality analysis for reliability improvement in multi-state systems*, Reliability Engineering & System Safety, 94, 1608–1619.

18. Zaitseva, E., 2012, *Importance analysis of a multi-state system based on multiple-valued logic methods*, In A. Lisnianski and I. Frenkel (eds.), Recent Advances in System Reliability, Springer Series in Reliability Engineering, Springer-Verlag, London, pp. 113–134.
19. Xiahou, T., Liu, Y., Jiang, T., 2018, *Extended composite importance measures for multi-state systems with epistemic uncertainty of state assignment*, Mechanical Systems and Signal Processing, 109, pp. 305–329.
20. Kim, C., Baxter, L.A., 1987, *Reliability importance for continuum structure functions*, Journal of applied probability, 24(3), pp. 779–785.
21. Lisnianski, A., 2001, *Estimation of boundary points for continuum-state system reliability measures*, Reliability Engineering & System Safety, 74(1), pp. 81–88.
22. Lisnianski, A., 2002, *Continuous-state system reliability models as an extension of multi-state systems*, Proceedings of MMR, Mathematical Methods in Reliability, pp. 17–20.
23. Liu, Y., Si, S., Cui, L., Wang, Z., Sun, S., 2016, *A generalized Griffith im- portance measure for components with multiple state transitions*, IEEE Transactions on Reliability, 65(2), pp. 662–673.
24. Lin, Y.H., Li, Y.F., Zio, E., 2016, *Component importance measures for components with multiple dependent competing degradation processes and subject to maintenance*, IEEE Transactions on Reliability, 65(2), pp. 547–557.
25. de Jonge, B., 2019, *Discretizing continuous-time continuous-state deterioration processes, with an application to condition-based maintenance optimization*, Reliability Engineering & System Safety, 188, pp. 1-5.
26. Chen, Y., Qiu, Q., Zhao, X., 2022, *Condition-based opportunistic maintenance policies with two-phase inspections for continuous-state systems*, Reliability Engineering & System Safety, 228, 108767.
27. Makajić- Nikolić, D., Vujošević, M., Pavlović, P., 2018, *Importance Measures in Reliability and Maintenance*, Proceedings of 3rd Maintenance Forum 2018, Maintenance and Industry 4.0, Belgrade 24-26th May 2018, pp. 7-16.
28. Zio, E., 2011, *Risk importance measures*, In Safety and Risk Modeling and its Appliacations, Springer, London, pp. 151–196.
29. Chacko, V.M., Manoharan, M., 2011, *Joint importance measures for multi-state reliability systems*, Opsearch, 48(3), pp. 257–278.
30. Liu, M., Wang, D., Si, S., 2023, *Mixed reliability importance-based solving algorithm design for the cost-constrained reliability optimization model*, Reliability Engineering & System Safety, 237, 109363.
31. Chacko, V.M., Sania, A., 2024, *Joint module importance measures for multistate systems*, OPSEARCH, doi: 10.1007/s12597-023-00722-4
32. Borgonovo, E., Apostolakis, G.E., 2001, *A new importance measure for risk informed decision making*, Reliability Engineering & System Safety, 72, pp. 193–212.
33. Pavlovic, P., Makajic-Nikolic, D., Vujosevic, M., 2017, *A new approach for determining the most important system components and the budget-constrained system reliability improvement*, Maintenance and Reliability, 19(3), pp. 413-419.
34. Gonzalez, T.F., 2007, *Handbook of Approximation Algorithms and Metaheuristics*, CRC Press, Boca Raton.
35. Even, G., Rawitz, D., Shahar, S.M., 2005, *Hitting sets when the VC-dimension is small*, Information Processing Letters, 95(2), pp. 358–362.
36. Baxter, L.A., 1984, *Continuum structures*, I. Journal of Applied Probability, 21(4), pp. 802–815.
37. Ericson II, C.A., 2005, *Hazard Analysis Technique for System Safety*, John Wiley and Sons, New Jersey.
38. Vujošević, M., Makajić-Nikolić, D., Pavlović, P., 2017, *A new approach to determination of the most critical multi-state components in multi-state systems*, Journal of Applied Engineering Science, 15(4), pp. 401-405.
39. Aissi, H., Bazgan, C., Vanderpooten, D., 2009, *Minmax and minmax regret versions of combinatorial optimization problems: A survey*, European Journal of Operational Research, 197(2), pp. 427–438.
40. Chen, F., Liu, D., Zhang, Y., Su, J., 2010, *A scalable approach to analyzing network security using compact attack graphs*, Journal of Networks, 5(5), pp.543–550.
41. CLib: Configuration Benchmarks Library, 2017; Aralia Fault Trees available at https://www.itu.dk/research/cla/externals/clib/, accessed 29. September 2022.
42. Wu S., Coolen, F., 2013, *A cost-based importance measure for system components: An extension of the Birnbaum importance*, European Journal of Operational Research, 225(1), pp. 189–195.
43. Gupta, S., Bachttacharya, J., Barabady, J., Kumar, U., 2013, *Cost-effective importance measure: A new approach for resource prioritization in a production plant*, International Journal of Quality and Reliability Management, 30(4), pp. 379–386.