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**Original scientific paper**

# GECKO-INSPIRED FRACTAL BUFFER FOR PASSENGER ELEVATOR

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**Abstract**. *Elevator accidents are common occurrences and can cause serious injuries and property damages. If an elevator falls from a tall building and hits the ground at high speed, passengers in the car have little chance of survival. We designed a fractal buffer with hierarchical structure, which is inspired by the gecko's pad system, to minimize the damage. A fractal-fractional oscillator is established to show the frequency-amplitude relationship of the fractal buffer using He's frequency formulation. The fractal-structured vibration-absorbing metamaterial in low frequency contributes to the elevator safety. This paper opens a new window for designing safe and reliable buffers, and provides entrepreneurs with new ideas for the next generation of elevators.*

**Key words**: *Metamaterials, Passenger elevator, Elevator crash, Energy absorption, Fractal theory, Hierarchical structure, Duffing oscillator, He's frequency formulation*

## 1. INTRODUCTION

The first safety elevator in history was exhibited at the 1854 New York World's Fair by an American mechanical engineer Elisha Graves Otis. Subsequently, as the technology developing, the traction drive passenger elevator was used. Due to the defects during elevator construction, or lacking regular maintenance or faulty operation, accidents happen easily. Elevator accidents include failure of the elevator's door which may trap passengers, failure of the brake system which may lead to elevator crash such as rushing to the top or falling to the ground. Among all of these, elevator crash is the most serious case. Assuming that an elevator car falls from the 10$^{th}$ floor (35 meters) and if other systems fail, it will reach the speed of 94.29 km/h when it hits the ground. In a traction drive elevator system,

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a buffer is installed at the bottom of the elevator shaft. There are two types of buffers, one is for energy accumulation, such as polyurethane buffers and steel spring buffers. The other is for energy dissipation, such as oil buffers [1].

A buffer is used to absorb the enormous kinetic energy to prevent the car from falling. Usually, the buffer is made of polyurethane, alloy or other materials, which have the advantage of high elasticity and effective energy absorption.

To avoid elevator accidents, Yao et al. proposed an elevator fault monitoring system based on the Internet of Things technology [2]. Tome et al. designed an automatic speed measurement system applied to elevator overspeed governors [3]. Lozzi et al. investigated the failure models of an inclined elevator [4]. Zheng et al. recommended a soft-landing system for a full-load elevator [5].

However, there are still many things need to be improved in an elevator buffer system. At present, elevator buffers are mainly made of a single material with simple structure, which cannot fully absorb the energy within a short period of time. If the kinetic energy of the falling car cannot be fully absorbed by the buffer, passengers will get injured easily.

In this paper, inspired by the unique structure of the gecko's pad [6,7], we propose a fractal buffer with a hierarchical structure which can drastically absorb the kinetic energy brought by the elevator car in a short distance, so that passengers are better protected and have greater chance of survival. The two-scale fractal theory [8,9,10] is used in the design of the elevator buffer. This study provides a theoretical insight into the innovative elevator design and sheds light on advanced elevator manufacturing.

## 2. GECKO'S PAD AND ITS FRACTAL STRUCTURE

Fractal geometry is a valuable mathematical tool for the description of surface microstructures [11,12] and the quantification of complex networks [13,14]. The surface fractality and the network fractality provide a straightforward yet effective approach to predicting the complexity, simplifying theoretical analysis and enhancing its reliability.

The gecko is famous for its ability of fast movement, strong adhesiveness, and smooth landing. When a gecko falls from a high place to the ground, it hardly gets hurt. Many attempts were made to discover the mechanisms of the gecko's pad and many gecko-inspired systems appeared. For example, gecko-like dry adhesive surfaces [15], gecko-inspired adhesives [16,17,18], gecko-inspired gripping pads [19], switchable dry/wet adhesives [20], soft robotic grippers [21] and other bio-inspired materials [22].

Mathematically, the microstructure of a gecko's pad can be approximated by fractal geometry, which is characterized by self-similarity. The microstructure of a gecko's pad is shown in Fig. 1.

There is a hierarchical structure in the gecko's pad system. When a gecko falls onto the ground, the kinetic energy can be completely absorbed by the pad in a very short time, therefore the structure of the gecko's pad has a super energy absorption capability.

To understand better the relationship between the structure of a gecko's pad and the energy absorption ability, we use fractal theory as a mathematical tool. Fractal theory is good at describing unsmooth boundaries, porous media, discontinuous problems, etc. In the 1960s, Mandelbrot raised a question about measuring the length of the coastline of Britain, and he found that as the measurement scale decreased to a very small value, the length of the coastline tended to infinite large. This work, published in Science, ushered in the era of fractals [23].

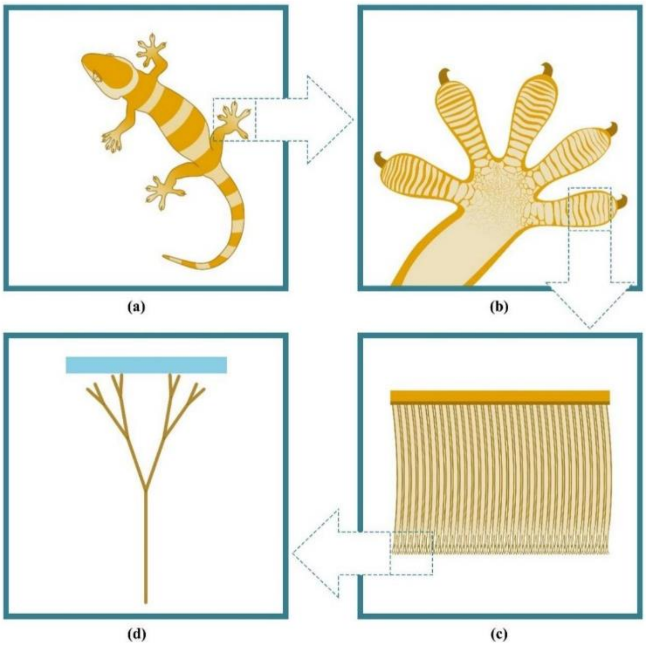


**Fig. 1** Gecko's pad system

Currently, Hausdorff fractal dimension is widely used in engineering, and it requires self-similarity at all scales. A hierarchical structure with three or four cascades is often used, but an additional cascade with self-similarity requires high cost. Therefore, in this paper we allow the two-scale fractal dimensions to mimic the fractal nature of the gecko's pad system.

A gecko's pad has multiple cascades from centimeters to nanometers, a soft landing induces only the first nanoscale cascade, and a hard landing puts the whole pad system into work in a very short time. This property is very useful for designing a fractal buffer.

## 3. Fractal Buffer System

Inspired by the gecko's pad, a fractal buffer can be made of some metamaterials with a hierarchical structure. The main idea is to build a gecko-inspired system that can completely absorb the kinetic energy of the fast-falling elevator car, so that the car ends up with soft landing in a short time. The speed of the elevator car should reduce gradually so as to avoid any injury to the passengers.

Xu et al. proposed a fractal cushioning using the Hilbert fractal structure and investigated its quasi-static property [24]. Traditionally, the buffer is made of alloy or polyurethane, which can absorb energy to some extent. However, in an elevator crash, it is of little help to the passengers. To improve the property of the buffer, we start by changing its material. By using metamaterials [25], we can obtain a wider range of elastic coefficients.

Fractal metamaterials have been employed extensively in noise and sound absorption [26] and wave absorption [27]. They exhibit distinctive mechanical properties [28] and damping characteristics [29]. A fractal buffer made of metamaterials is proposed, as shown in Fig. 2. The top of the metamaterials has the smallest elastic coefficient, while the bottom has the largest elastic coefficient. Several layers of different materials are placed according to different elasticity coefficients. By taking into account the weight of the elevator car (including passengers) and the fall distance, the choice of buffer metamaterials and the corresponding thickness can be determined. Metamaterials can be fabricated by 3D printing technology [30,31]. Generally, 3D printed mechanical metamaterials have negative Poisson's ratio, steady-state structures, and origami structures [32], these properties offer a total new road to design smart programmable mechanical materials, e.g., metamaterial bones [33] and smart fabrics [34].

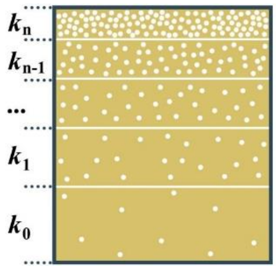


**Fig. 2** Hierarchical metamaterial with different properties at different cascades

The gecko's pad is made of proteins, a type of soft and high molecular weight material. Similarly, man-made soft materials such as metamaterials [25] and auxetic materials [35,36] with hierarchical structure can be applied to the design of fractal buffers.

The best candidate for metamaterials or auxetic materials with hierarchical structure is the fractal-like porous structure, such as Menger's sponge. Different porosity leads to different property, and the fractal dimensions are the key factor influencing the property. The different porosities for different cascades in a fractal buffer result in different elastic coefficients and can be adjusted during the manufacturing process.

## 4. Theoretical Analysis

The fractal buffer system consists of a cascade of rigidities. The top layer has the lowest stiffness, while the bottom layer has the highest stiffness. The equivalent coefficient of elasticity can be calculated as

$$\frac{1}{k} = \frac{N_0}{k_0} + \frac{aN_0}{k_1} + ... + \frac{a^n N_0}{k_n}, \tag{1}$$

where $k$ is the equivalent elastic coefficient, $k_i$ ($i$ = 0 ~ $n$) is the elastic coefficient of the buffer in the $i$-th layer, and $N_0$ is number of the first layer, $a$ is the self-similarity index, generally, we choose $a$=2 or $a$=3. The equivalent elastic coefficient is a function of the displacement of the fractal buffer:

$$k = k(h) = b\exp(ch), \tag{2}$$

where $b$ and $c$ are positive constants, $h$ is the displacement. Eq. (2) implies that when the fractal buffer is hit, it can adjust its elastic coefficient according to the displacement, so that the elevator car can land smoothly and passengers can be prevented from injury in elevator accidents.

If we view the elevator car as a mass point, its total potential energy is

$$E = mgH \,, \tag{3}$$

where $m$ is the mass of the car, $g$ the gravity acceleration, $H$ is height. The initial velocity is:

$$u = \sqrt{2gH} \,. \tag{4}$$

The governing equation of the fractal buffer can be expressed as

$$m\frac{d^2u}{dt^{2\alpha}} + ku + \varepsilon u^3 = 0, u(0) = 0, \frac{du}{dt^\alpha}(0) = \sqrt{2gH} \,, \tag{5}$$

where $\varepsilon u^3$ is the nonlinear term, and $\varepsilon$ can be negative for metamaterials, $du/dt^\alpha$ is the two-scale fractal derivative [10], $\alpha$ is the two-scale fractal dimension, which can be calculated by He-Liu fractal dimension formulation [8]. Furthermore, the genetic programming methodology [13] offers an alternative approach to the accurate determination of fractality. As demonstrated in Ref. [13], the accuracy of this approach is as high as 98%.

The approximate solution of Eq. (5) reads [37]

$$u = A\sin\omega t^\alpha \,, \tag{6}$$

where $A$ is the amplitude and $\omega$ is the frequency. According to He's frequency formulation [38], we have

$$\omega^2 = \left.\frac{df(u)}{du}\right|_{u=A/2} \,. \tag{7}$$

Here $f(u)$ is given as

$$f(u) = \frac{ku + \varepsilon u^3}{m} \,. \tag{8}$$

So we obtain

$$\omega^2 = \frac{k}{m} + \frac{3\varepsilon A^2}{4m} \,. \tag{9}$$

According to Eq. (6), we have

$$\frac{du}{dt^\alpha} = A\omega\cos\omega t^\alpha \,. \tag{10}$$

That means

$$\frac{du}{dt^\alpha}(0) = A\omega \,. \tag{11}$$

According to the initial conditions, we, therefore, have

$$A\omega = \sqrt{2gH} \,. \tag{12}$$

Combining Eqs. (9) and (12), we have

$$\omega^2 = \frac{k}{m} + \frac{3\varepsilon gH}{2m\omega^2} . \tag{13}$$

Solving $\omega$ from Eq. (13) yields

$$\omega = \sqrt{\frac{2k + \sqrt{4k^2 + 24\varepsilon mgH}}{4m}} . \tag{14}$$

From Eq. (12), the amplitude can be solved as

$$A = \sqrt{\frac{8mgH}{2k + \sqrt{4k^2 + 24\varepsilon mgH}}} , \tag{15}$$

and the period reads

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{4m}{2k + \sqrt{4k^2 + 24\varepsilon mgH}}} . \tag{16}$$

We assume that the total energy is absorbed by the fractal buffer system within one quarter period, and the landing velocity is zero. The falling time is

$$t = \frac{\Delta}{v} , \tag{17}$$

where $v$ is average velocity of the falling car, $\Delta$ is the maximal amplitude. It requires that

$$A \le \Delta . \tag{18}$$

According to Eqs. (2) and (15), the amplitude can be solved from the following transcendental equation

$$A = \sqrt{\frac{8mgH}{2b\exp(cA) + \sqrt{4b^2\exp(2cA) + 24\varepsilon mgH}}} . \tag{19}$$

The average velocity can be calculated as

$$v = \frac{\sqrt{2gH}}{2} . \tag{20}$$

So the falling time is

$$t = \frac{\Delta}{v} = \frac{\sqrt{2}\Delta}{\sqrt{gH}} . \tag{21}$$

It should meet the following requirement

$$t \le \frac{T}{4} . \tag{22}$$

That is

$$\frac{\sqrt{2}\Delta}{\sqrt{gH}} \le \pi\sqrt{\frac{m}{2k+\sqrt{4k^2+24\varepsilon mgH}}}\ . \tag{23}$$

In view of Eq. (2), we write Eq. (23) in the form

$$\frac{2\Delta}{gH} \le \frac{m\pi^2}{2b\exp(c\Delta)+\sqrt{4b^2\exp(2c\Delta)+24\varepsilon mgH}}\ . \tag{24}$$

Eq. (24) is the criterion for designing the needed fractal buffer.

## 5. Fractal Buffer's Maximal Displacement

We have obtained the criterion given in Eq. (24) for smooth landing during elevator accidents, but it is an inexplicit form. An explicit form for the maximal displacement of the fractal buffer's motion is much needed. We consider the following equation

$$R(\Delta) = \frac{2\Delta}{gH} - \frac{m\pi^2}{2b\exp(c\Delta)+\sqrt{4b^2\exp(2c\Delta)+24\varepsilon mgH}} = 0\ . \tag{25}$$

In order to solve $\Delta$ from Eq. (25), we apply Chun-Hui He's iteration method [39-41]. During experiment, its value can be approximately estimated, for example,

$$\Delta_1 = \lambda/2\ , \tag{26}$$

$$\Delta_2 = \lambda/4\ , \tag{27}$$

where $\lambda$ is the height of the fractal buffer. According to Ref. [41], we have

$$R_1 = \frac{2\Delta_1}{gH} - \frac{m\pi^2}{2b\exp(c\Delta_1)+\sqrt{4b^2\exp(2c\Delta_1)+24\varepsilon mgH}}\ , \tag{28}$$

and

$$R_2 = \frac{2\Delta_2}{gH} - \frac{m\pi^2}{2b\exp(c\Delta_2)+\sqrt{4b^2\exp(2c\Delta_2)+24\varepsilon mgH}}\ . \tag{29}$$

So an explicit form for the maximal displacement of the fractal buffer's motion is obtained

$$\Delta = \frac{R_1\Delta_2 - R_2\Delta_1}{R_1 - R_2}\ . \tag{30}$$

The iteration can be continued for a better explicit form. When the buffer reaches its maximum displacement, a much longer time is needed, which can be guaranteed by the fractal vibration system. Fig. 3 shows the dynamic property of the fractal buffer for different fractal dimensions. It is obvious that a smaller value of the fractal dimension leads to a larger half period. When the half period tends to infinity, the fractal buffer becomes extremely stable and the passengers inside the car can be protected. This extended period property is also called the low frequency property [42,43].

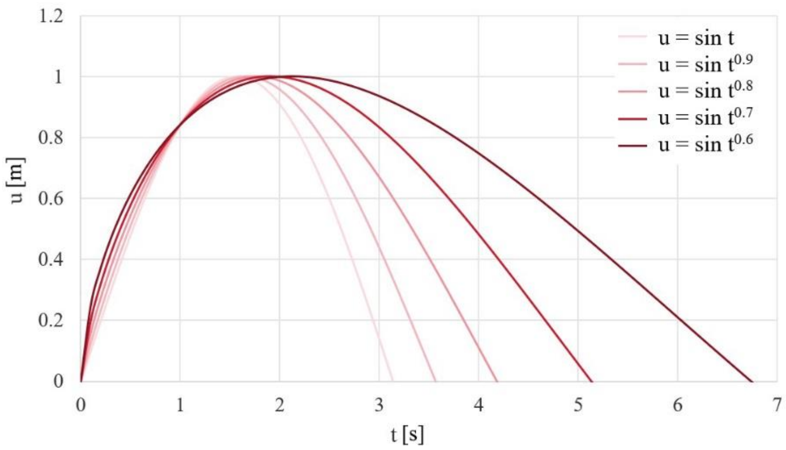


**Fig. 3** Dynamical property of the fractal buffer with different fractal dimensions

## 6. Conclusion

In order to prevent injury to passengers in an elevator crash, a fractal buffer system, which can completely absorb the kinetic energy of the falling elevator, is proposed, and the criterion for designing the needed fractal buffer is discussed. The system exhibits a low equivalent frequency, which can be adjusted by the fractality, and the maximal displacement should be at the quarter period. These two properties guarantee the system's safety and reliability during an accident.

The metamaterials or auxetic materials with special property, as given by Eq. (2), can be fabricated with ease by 3D printing technology [32,33] and by spinning technology [44,45]. The 3D-printed metamaterials exhibit a negative Poisson's ratio, which can be adjusted during the printing process. Furthermore, the fractality can be precisely determined, allowing for the smart fabrication and AI optimization of the fractal buffer system.

The amplitude-frequency of the fractal buffer's vibration can be readily and efficaciously revealed by He's frequency formulation [38,46]. In a forthcoming article, we will discuss the effect of the negative Poisson's ratio on the periodic property of the fractal buffer system.

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