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# BENCHMARK STUDY OF RE-IDENTIFICATION METHODS BASED ON STOCHASTIC FUZZY NORMALIZATION AND THEIR APPLICATION TO DECISION-MAKING PROBLEMS IN ENGINEERING

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**Abstract**. *In Multi-Criteria Decision Analysis (MCDA), data normalization is essential for ensuring the comparability of heterogeneous and often conflicting evaluation criteria. Conventional normalization techniques, although methodologically straightforward, are predominantly tailored for monotonic criteria, rendering them ineffective for non-monotonic criteria characterized by extrema within the interval rather than at its boundaries. This limitation significantly undermines their applicability in the re-identification of decision models, as they fail to adequately account for the complexity and variability inherent in non-monotonic evaluation approaches. This paper presents a study on the application of stochastic fuzzy normalization (STFN) in combination with popular MCDA methods such as VIKOR, TOPSIS, and MABAC in addressing engineering problems. The study evaluates the effectiveness of this approach in re-identifying decision models, emphasizing its capability to manage nonlinearities and nonmonotonic criteria, mitigate rank reversal phenomena, and adapt to dynamic decision-making scenarios. In this work, the Fuzzy Reference Model (FRM) is leveraged as a robust simulation framework to evaluate the performance of STFN in re-identifying decision models, enabling comprehensive benchmarking of MCDA techniques by providing detailed preference information for each decision option. Through a practical case study involving the selection of an optimal energy source for an industrial plant, the study illustrates how fuzzy normalization supports reliable re-identification of decision models. These comparative analyses reveal potential outcomes and highlight notable differences when STFN is applied in conjunction with various MCDA methods, demonstrating the value of this approach in decision-making contexts.*

**Key words**: *MCDA, Re-identification, STFN, TOPSIS, VIKOR, MABAC*

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## 1. INTRODUCTION

Multi-Criteria Decision Analysis (MCDA) is a comprehensive decision-support methodology instrumental in situations where multiple, often opposing, criteria must be considered [1]. MCDA methods enable systematic and transparent decision-making by integrating a variety of criteria and decision-maker preferences, allowing a more accurate understanding of the compromises between different options. MCDA is widely used in many fields, such as management [2], engineering [3], environmental protection [4], urban planning [5], and public health [6]. In management and business, for example, MCDA supports companies in evaluating and selecting strategies, investments, and suppliers, considering both financial and non-financial aspects [7,8].

Dealing with problems that involve multiple opposing criteria is problematic. Therefore, techniques are constantly being developed to facilitate such decisions. One of the critical elements of MCDA is data normalization, which makes it possible to compare different criteria by converting them to a standard scale [9]. With normalization, criteria become comparable, greatly facilitating the analysis and selection of optimal solutions. In addition, normalization can immune MCDA methods to rank reversal paradox and allow modeling of more favorable subjective preferences of decision makers.

Normalization, a pivotal process in the re-identification of MCDA models, becomes even more significant when expert knowledge is lacking or when decision-making models need updating. In this article, we present a study that not only compares the effectiveness of various normalization methods, including the stochastic fuzzy normalization (STFN) based on triangular fuzzy numbers method [10], in the context of MCDA model re-identification, but also provides practical insights into how the STFN method performs in re-identifying models such as VIseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR), Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) and Multi-Attributive Border Approximation area Comparison (MABAC). These findings are of direct relevance to researchers and practitioners in the field of MCDA.

Indeed, our study makes an essential contribution to the field of MCDA by providing insights into when and what normalization method to use to achieve decision models that closely approximate real-world conditions. The primary novelty of this work lies in its detailed comparative analysis of normalization methods for MCDA model re-identification, addressing a gap in the existing literature. Specifically, the application of STFN to re-identify models such as VIKOR, TOPSIS, and MABAC distinguishes this study from prior research. The study evaluates the conditions under which STFN excels and demonstrates its potential to mitigate challenges like rank reversal and the absence of expert input.

Moreover, the sensitivity analyses conducted in this study represent a significant advancement by systematically assessing the robustness of the re-identified models against variations in core values and boundary extrapolation. These analyses provide nuanced insights into the stability and adaptability of STFN-based approaches, offering valuable guidance for applications where decision models must contend with uncertainty and fluctuating data conditions. The practical case of selecting an optimal energy source for an industrial plant further highlights the real-world applicability of these methods. By demonstrating how STFN can effectively balance robustness and adaptability in complex, multidimensional decision-making scenarios, this work bridges the gap between theoretical development and practical implementation.

Through these contributions, this study advances the state-of-the-art in MCDA, providing practical tools and strategies to enhance decision model robustness and adaptability. The results have significant implications for improving the quality and efficiency of decision-making processes across various domains, particularly those characterized by complexity and uncertainty.

The rest of the paper is organized as follows. Section 2 presents a review of the literature related to the normalization approaches used and their comparison methods. Section 3 discusses preliminary assumptions related to fuzzy sets, fuzzy reference models, MCDA methods, and the STFN re-identification approach. Section 4 conducts a comparative study of re-identification methods. Section 5 introduces a practical example related to an engineering problem, specifically the selection of an optimal energy source for an industrial plant, demonstrating the application of STFN-based methods in a real-world context. Section 6 presents a discussion of the compared methods for re-identification. Finally, Section 7 concludes the paper and provides directions for future research.

## 2. Literature Review

In the MCDA literature, many normalization methods are used to compare and evaluate different criteria. Normalization is a critical step in the MCDA process, as it allows different criterion scales to be converted to a standard scale for direct comparison [9]. The most commonly used normalization methods include:

- Min-Max Normalization: Converts criteria values to the interval [0,1] based on the minimum and maximum value of a given criterion [11].
- Z-Score Normalization: Based on the mean and standard deviation, transforming values in a way that eliminates units of measurement [12].
- Vector Normalization: Transforms values by dividing each value by the norm of the vector, which is particularly useful in cases where the units of measure are different [11].
- Sum Normalization: Involves transforming the values of the criteria by dividing each value by the sum of all the values of the criterion [13].

In the context of MCDA, different approaches use different standardization methods to address specific requirements and challenges in the decision-making process. An example of a classic normalization method is the TOPSIS approach [14], which uses vector normalization. Another classic method using min-max normalization is the VIKOR method [15]. However, in addition to these traditional approaches, new decision-making methods also use normalization, among which are Combined Compromise Solution (COCOSO) - which uses min-max normalization [16], Combinative Distance-based Assessment (CODAS) - which is based on linear normalization [17], Multi-Attributive Border Approximation Area Comparison (MABAC) - which uses min-max normalization [18], and Multi-Attributive Ideal-Real Comparative Analysis (MAIRCA) - also based on min-max normalization [19].

Min-max normalization is widespread because of the numerous studies associated with it and its ability to deal with the phenomenon of reverse rankings. Therefore, approaches immune to this phenomenon often use similar functions for normalization. An example of such an approach is Stable Preference Ordering Towards Ideal Solution (SPOTIS) [20], which is based on normalization based on boundary values (i.e., min-max). Another example is the Reference Ideal Method (RIM) [21], which uses a similar mechanism based on cutoff values for criteria for normalization. Due to their robustness against the rank reversal paradox, these approaches are essential in decision-making when there is a risk of such a phenomenon.

The growing complexity of decision-making problems has significantly contributed to the development of MCDA methods, where normalization plays a crucial role in enabling robust decision processes. Recent advancements highlight the role of fuzzy-based methods in addressing uncertainty and complexity in data. Eti et al. [22] developed an innovative fuzzy decision-making framework to enhance electric vehicle charging infrastructure, showcasing how fuzzy modeling can address nonlinearities in data. Similarly, Kizielewicz and Sałabun [23] introduced the Stochastic Identification of Weights (SITW) method, re-identifying multi-criteria weights to dynamically adjust models to evolving decision-making conditions. These developments emphasize the importance of normalization in adapting MCDA methods to dynamic environments where data variability and problem multidimensionality are critical.

Moreover, extensions of fuzzy methods have enabled their integration into intelligent decision support systems. Hussain and Ullah [24] demonstrated the application of spherical Sugeno-Weber operators to enhance the practical usability of fuzzy models, while Narang et al. [25] proposed a fuzzy extension of the MEthod based on the Removal Effects of Criteria (MEREC) using parabolic measures, improving precision in criterion classification. These innovations underline the potential of combining traditional MCDA approaches with fuzzy frameworks to address complex problems in business and technical contexts.

Applications of fuzzy methods extend beyond logistics and infrastructure. Tešić and Marinković [26] employed Fermatean fuzzy weight operators to select combat systems based on efficiency, illustrating the potential of these methods in defense-related decision-making. Similarly, Kannan et al. [27] developed the Linear Diophantine Fuzzy CODAS method to improve specialist selection processes in logistics, while Asif et al. [28] applied Hamacher operators to Pythagorean fuzzy sets, broadening their applicability to multi-attribute decision-making. Gazi et al. [29] employed the Pentagonal Fuzzy DEcision Making Trial and Evaluation Laboratory (DEMATEL) methodology to identify key criteria in empowering women in sports, highlighting the societal benefits of fuzzy MCDA approaches.

Further, Kara et al. [30] applied a hybrid MEREC- Weighted Euclidean Distance-Based Approach (WEDBA) methodology to evaluate the performance of Turkish universities, and Kurtay [31] utilized the fuzzy Evaluation based on Distance from Average Solution (EDAS) method to select military vehicles, emphasizing the need for domain-specific adaptations of MCDA techniques. Mifdal and Saracoglu [32] conducted a classification analysis using Analytic Hierarchy Process (AHP) and Activity-Based Costing (ABC) to optimize inventory management, while Yushuo and Ling [33] proposed a prospect theory-based model to evaluate logistics enterprises' safety standardization performance, contributing to operational management.

The importance of normalization techniques has also been highlighted in various comparative studies. For instance, [11] explored the impact of normalization techniques on decision rankings, illustrating how different methods can lead to varied outcomes. Similarly, [34] examined normalization techniques in MCDM, emphasizing factors like decision-maker preferences and data structures. In [35], researchers identified suitable normalization methods for integration with the Preference Selection Index (PSI) method, while [36] proposed an assessment framework enriched with metrics. This framework, employing the Simple Additive Weighting (SAW) method, offers a structured approach to evaluating and selecting normalization techniques, providing practical guidance for ensuring reliable decision-making across diverse domains.

## 3. PRELIMINARIES

### 3.1 Fuzzy Reference Model

The Fuzzy Reference Model (FRM) is a multi-criteria decision maker's preference function that has information about the preferences of each decision option. Sałabun proposed this approach to estimate the accuracy of TOPSIS normalization methods [37]. Using a fuzzy reference model, multi-criteria decision-making techniques can be compared. In order to create the FRM model, the following steps should be followed [37]:

- Step 1. Select the number and monotonicity of criteria, where criteria can have monotonicity such as profit, cost, or be non-monotonic.
- Step 2. Create a membership function for each criterion, where the function values should be in the domain [0, 1].
- Step 3. Provide an evaluation value for each combination of information grains. This can be a random value but must be consistent with the previous assumptions (criterion type).
- Step 4. Create a rule base for the FRM model based on the Modus Ponens tautology [38].

The fuzzy reference model is visualized in Fig. 1.

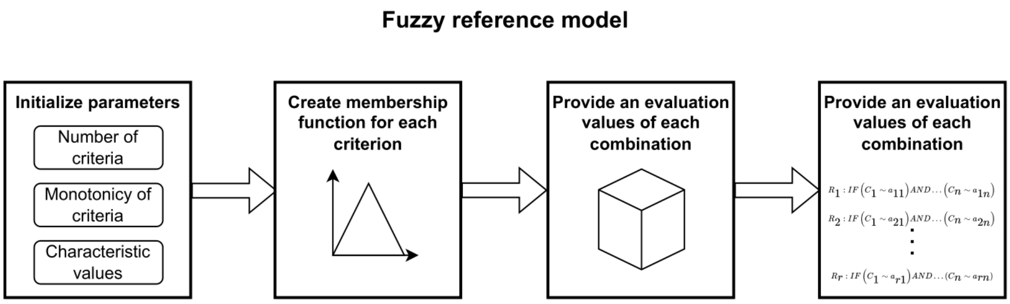


**Fig. 1** Fuzzy reference model

### 3.2. TOPSIS

The Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) operates within a structured framework centered on reference points and employs a systematic approach to evaluate alternatives [39]. Fundamental to this method are two pivotal reference points: the Positive Ideal Solution (PIS) and the Negative Ideal Solution (NIS). TOPSIS assesses alternatives by measuring their proximity to these reference points. The complete TOPSIS procedure can be represented by Fig. 2.

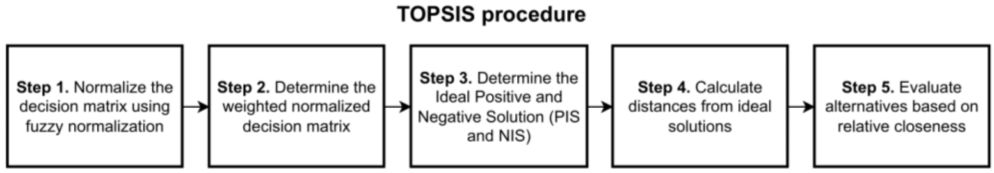


**Fig. 2** TOPSIS procedure

### 3.3 VIKOR

The VIekriterijumsko KOmpromisno Rangiranje (VIKOR) method was developed to address discrete problems involving conflicting criteria [15]. The core concept of this approach is to identify compromise solutions, rank the decision alternatives, and select the optimal alternative. In the VIKOR technique, compromise ranking is achieved by assessing the proximity measure relative to the ideal alternative. The steps involved in the VIKOR method can be represented by Fig. 3.

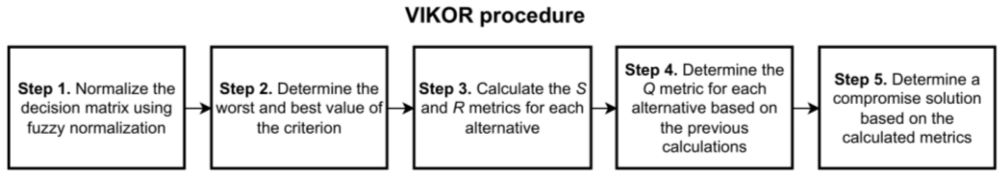


**Fig. 3** VIKOR procedure

### 3.4 MABAC

The Multi-Attributive Border Approximation Area Comparison (MABAC) method, introduced by [18], addresses practical multi-criteria decision-making challenges. Its notable feature is the ability to manage conflicting criteria and diverse data units through an embedded normalization algorithm. The method's mathematical basis involves assessing alternatives' distances from the boundary approximation area (*G*). Its simplicity contributes to its widespread adoption in decision-making contexts, as evidenced by [40]. Moreover, the method offers extensions to handle uncertain data. The MABAC procedure in combination with fuzzy normalisation can be represented by Fig. 4.

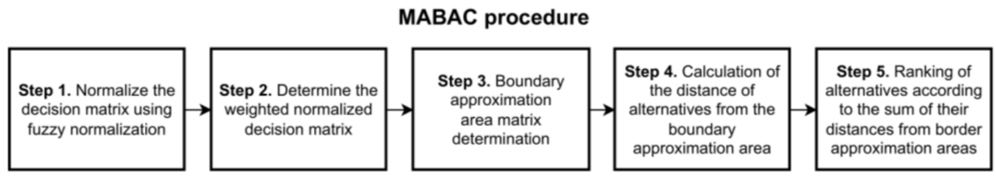


**Fig. 4** MABAC procedure

### 3.1 Stochastic Fuzzy Normalization Based on Triangular Fuzzy Numbers

This section will introduce a method for re-identification that utilizes fuzzy normalization, termed STochastic Fuzzy Normalization (STFN) based on triangular fuzzy numbers. Initially proposed by Kizielewicz and Dobryakova in their paper [10], this method employs STFN to re-identify a continuous TOPSIS model. The core concept involves utilizing a stochastic optimization algorithm to search for Triangular Fuzzy Number (TFN). In the context of this study, the stochastic Differential Evolution (DE) method was employed. Moreover, this study will delve into a broader spectrum of MCDA approaches, including TOPSIS, VIKOR, and MABAC. The entire re-identification process employing the STFN approach can be delineated into the following steps:

- Step 1. Select a dataset. The dataset should contain criteria vectors ($C$), a criteria weights vector ($W$), and a ranking vector ($R$).
- Step 2. Select a stochastic optimization method. In this step, choose a stochastic method for solving the optimization problem and select its parameters.
- Step 3. Model training. Training the model is done using the stochastic optimization algorithm and the fitness function, which can be represented Fig. 5.

**Algorithm: Fitness Function (STFN)**

```
procedure FITNESS(solutions):
    preferences ← base(C,W,solutions)
    return rw(base.rank(preference), R)
end procedure
```

**Fig. 5** STFN fitness function

## 4. Comparative Study of Re-Identification Methods

This study will compare the effectiveness of multi-criteria model re-identification methods. This area is significant in decision-making in various fields, such as management, engineering, and economics. We will use synthetically generated random data for this purpose, allowing us to control the experiment conditions and obtain reliable results. In order to present the re-identification problem more comprehensively, we will focus on comparing two decision criteria, which often occur in real decision-making scenarios. This way, we can study how different re-identification methods deal with various criteria. The study framework is detailed in Fig. 6, which shows the critical steps of the experiment. The first will divide the decision matrix into two parts: a training matrix, used to teach re-identification models, and a test matrix, used to evaluate these models. An essential part of this process will be ensuring that there are enough alternatives in both matrices to ensure that the data is representative. After the split, both matrices will be evaluated using the FRM, allowing us to assess the re-identification methods' effectiveness objectively.

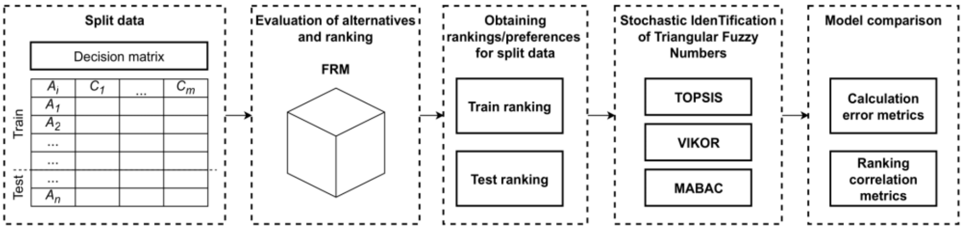


**Fig. 6** Framework for the comparative study of methods of re-identification of decision-making models

The scores obtained from the FRM will then be converted into rankings: a training ranking and a test ranking. This step will allow us to compare the results of different re-identification methods. This study will focus on three main re-identification methods: STFN-TOPSIS, STFN-VIKOR and STFN-MABAC. Each of these methods will be trained using a stochastic evolutionary difference method, and we will select the appropriate parameters based on a literature review and preliminary experiments. For the purposes of this study, the implementation was carried out using the pymcdm library along with the pymcdm-reidentify extension [41, 42]. After training, we will proceed to evaluate the performance of each of these methods by comparing their results with a reference model. In addition to evaluating based on rankings, we will also analyze the FRM characteristic objects evaluated by the re-identification models. This analysis will allow us to understand better what decision-making aspects are most relevant to each method and how well they reflect decision-makers preferences.

For this reason, metrics such as the weighted Spearman rank correlation coefficient and error metrics were utilized. Weighted Spearman correlation coefficient was primarily employed because the output results were rankings, making it a natural choice for evaluating the consistency of rank order between re-identification methods and the FRM. Additionally, error metrics such as Mean Squared Error (MSE) and Mean Absolute Error (MAE) were used to analyze how STFN-based methods, including TOPSIS, VIKOR, and MABAC, evaluate characteristic objects and to quantify the differences between their assessments and the FRM. This combination of metrics provides insights into both ranking alignment and numerical discrepancies, ensuring a comprehensive evaluation of the methods.

However, our study used a FRM, a nonlinear evaluation function for the two criteria under consideration. This model is crucial because it allows us to account for the nonlinear relationships between the alternatives and the decision criteria. The decision plane, as evaluated by the FRM, is shown in Fig. 7, which allows the complexity of the decision space to be visualized. In order to create the FRM model for the two criteria, 14 linguistic values were taken and evenly distributed in the value space. Based on these values, 196 characteristic objects were generated, which were used to evaluate the decision matrices. The evaluation process followed the FRM methodology, similar to Characteristic Object METhod (COMET), where each alternative was evaluated concerning the nearest characteristic objects. This evaluation was carried out using the Mamdani model, which allows uncertainty and ambiguity to be taken into account in the decision evaluation.

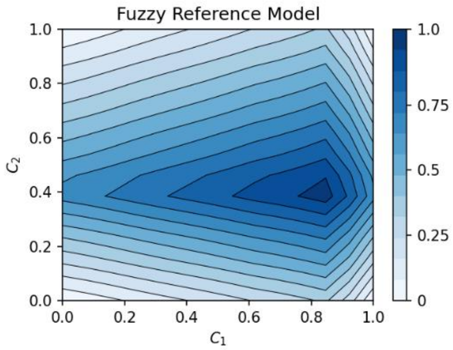


**Fig. 7** Decision plane for fuzzy reference model used in the study.

After obtaining preferences for the selected decision matrix, training the re-identification methods, i.e., STFN-TOPSIS, STFN-VIKOR, and STFN-MABAC, was started. The stochastic optimization method used to find the boundary values of TFN for normalization was based on the evolutionary difference method. The values of the decision matrix, drawn from the range [0, 1] for both criteria from uniform distribution, were also used as boundary values for TFN. The learning process lasted 1000 epochs, during which, for re-identification by the STFN-TOPSIS method, similarity was obtained on the training set with a value of $r_w$ equal to 0.93722, for STFN-VIKOR the value of $r_w$ was 0.88446, and for STFN-MABAC it reached 0.92212.

The decision grids obtained from training the models are shown with the help of Fig. 8. It is noticeable that in the case of the STFN-VIKOR approach, the formation of the function is slightly more distorted than in the case of STFN-TOPSIS and STFN-MABAC. This results from a more complex decision function based on two components. In addition, unlike the STFN-TOPSIS and STNF-MABAC methods, the STFN-VIKOR approach uses the $v$ parameter, which also affects the shape of the decision grid. In the case under study, a parameter $v$ of 0.5 was used, and $Q$ values were used to carry out the ranking process. The decision lattices obtained using the STFN-TOPSIS and STFN-MABAC approaches are closer to the reference model, the FRM.

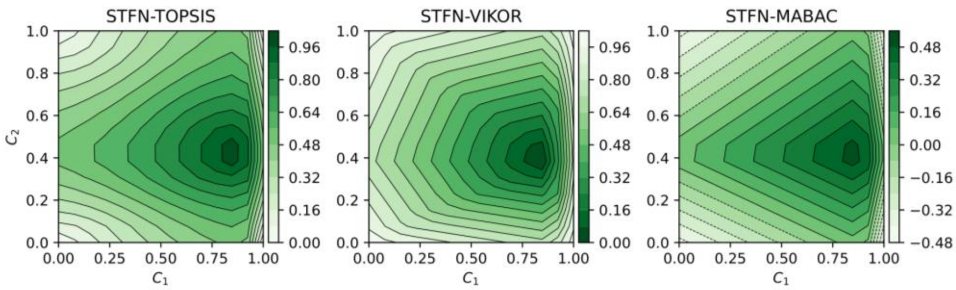


**Fig. 8** Obtained decision grids by selected re-identification methods learned on the training matrix

The resulting core values for the re-identification approaches for the triangular fuzzy numbers used for normalization are presented using a Table 1. This table shows the core values for each re-identification approach for the two decision criteria. It is worth noting that the differences between the obtained core values and the reference core are relatively small. For example, for criterion $C_1$, all re-identification approaches achieved core values close to 0.9, indicating a high degree of similarity to the reference model. However, for criterion $C_2$, some differences in the core values between the approaches can be seen, which may suggest differences in the effectiveness of the reference model mapping in the context of this criterion.

**Table 1** Obtained cores for triangular fuzzy numbers intended to normalize criteria in re-identification models

| Approach | $C_1$ | $C_2$ |
|---|---|---|
| STFN-TOPSIS | 0.903885 | 0.417369 |
| STFN-VIKOR | 0.903885 | 0.396207 |
| STFN-MABAC | 0.903885 | 0.413124 |

After obtaining the STFN-TOPSIS, STFN-VIKOR, and STFN-MABAC, re-identification models were tested on a test matrix of 20 alternatives. The first step in this testing was to compare the rankings obtained for the decision test matrix. For this purpose, each alternative was evaluated by all three re-identification approaches, and the evaluations were then ranked according to the selected criteria. For the STFN-TOPSIS and STFN-MABAC methods, the higher the evaluation value, the higher the ranking, while STFN-VIKOR reversed the situation. The resulting rankings are shown in Fig. 9. Analysis of these rankings showed that the ranking positions for the test set differ slightly between the STFN-TOPSIS and STFN-MABAC approaches. There are only two positions of difference for alternative $A_2$ and alternative $A_{10}$. However, the most significant differences appeared in the comparison between STFN-TOPSIS and STFN-MABAC and the STFN-VIKOR approach. For 11 ranking alternatives, differences were observed between the methods.

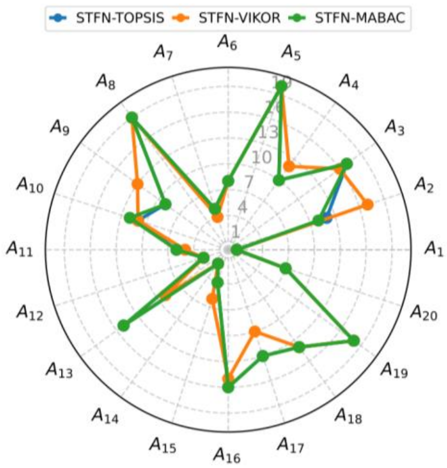


**Fig. 9** Comparison of the rankings of the test decision matrix by the obtained re-identification models

After obtaining the rankings for the test decision matrix, a comparison was made between these rankings and the ranking derived from the FRM model. This comparison is shown in Fig. 10. The comparative analysis considered two leading indicators: weighted Spearman correlation coefficient ($r_w$) and the similarity coefficient of the rankings. A weighted Spearman correlation coefficient of 0.91106 was obtained for the STFN-TOPSIS method, indicating a high degree of agreement with the FRM model. In addition, the similarity coefficient of the rankings was 0.95431, which further confirms the effectiveness of this method in mapping the preferences of the reference model. For the STFN-VIKOR method, weighted Spearman correlation coefficient was 0.89424, indicating a slightly lower degree of agreement with the FRM model than STFN-TOPSIS. However, the similarity coefficient of the rankings reached 0.96000, suggesting high agreement in the order of the alternatives' ratings. The STFN-MABAC method obtained a weighted Spearman correlation coefficient of 0.90304 and a ranking similarity coefficient of 0.95408. These results indicate good agreement with the FRM model and high stability in evaluating the rankings.

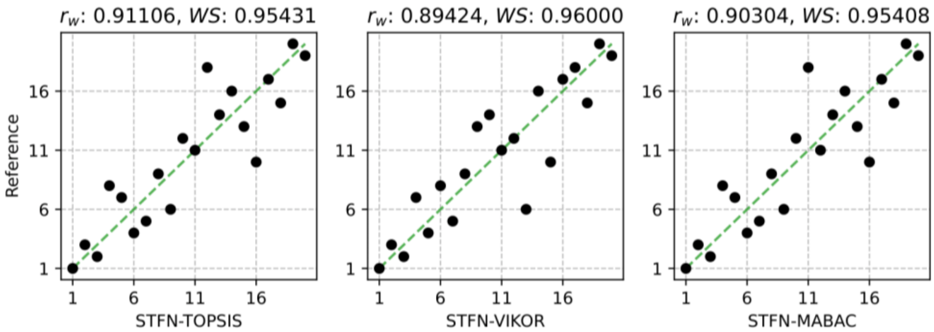


**Fig. 10** Comparison of rankings derived from redientfication models with a reference ranking (test decision matrix)

The analysis of these results allows us to conclude that all three reidentification methods showed high efficiency in reproducing the preferences of the reference model. At the same time, the differences in the results between the methods may suggest their different properties and the degree of adaptation to the specifics of the decision-making problem under study. The main difference is shaping the decision grid by functions, which MCDA methods have within themselves. Therefore, STFN-VIKOR has less similarity with the reference ranking than the STFN-TOPSIS and STFN-MABAC approaches.

This study also compared preferences for characteristic objects. These objects are the main base of the FRM model, which plays a reference role. Their ratings are crucial to the re-identification process, so we focused on analyzing the preferences of characteristic objects, which play an essential role in our study. However, due to the different rating scales used by the re-identification methods and the consequent differences in the visualization of the decision grid, we decided to normalize the preference values of characteristic objects using a min-max approach. After normalizing the reference preference vectors, we calculated their differences and the preferences of characteristic objects obtained by re-identification methods. The results of this comparison are shown in the Table 2.

**Table 2** Comparison of normalized preferences of characteristic objects derived from re-identification approaches with the preferences of FRM characteristic objects

| Metric | STFN-TOPSIS | STFN-VIKOR | STFN-MABAC |
|---|---|---|---|
| MSE | 0.008166 | 0.019175 | 0.007460 |
| MAE | 0.069528 | 0.093929 | 0.069160 |
| R2 | 0.855731 | 0.661245 | 0.868212 |

Analysis of these values allows us to assess the degree of correspondence between the preferences of characteristic objects obtained by the re-identification methods and the reference FRM model. Higher MSE and MAE indices and lower values of R2 may suggest more significant discrepancies between the preferences calculated by the re-identification methods and the reference preferences. The STFN-MABAC method shows the lowest MSE and R2 values and the highest MAE value, suggesting that it is least consistent with the reference model. The STFN-TOPSIS method, on the other hand, achieves the highest MSE and R2 values and the lowest MAE value, suggesting that it is most consistent with the reference model. Based on these results, it is possible to direct the selection to the STFN-TOPSIS method as the best fit for the FRM model in the analyzed context.

Fig. 11 presents a comparative analysis of TFN cores obtained from 1,000 optimization runs for the STFN-TOPSIS, STFN-VIKOR, and STFN-MABAC methods.

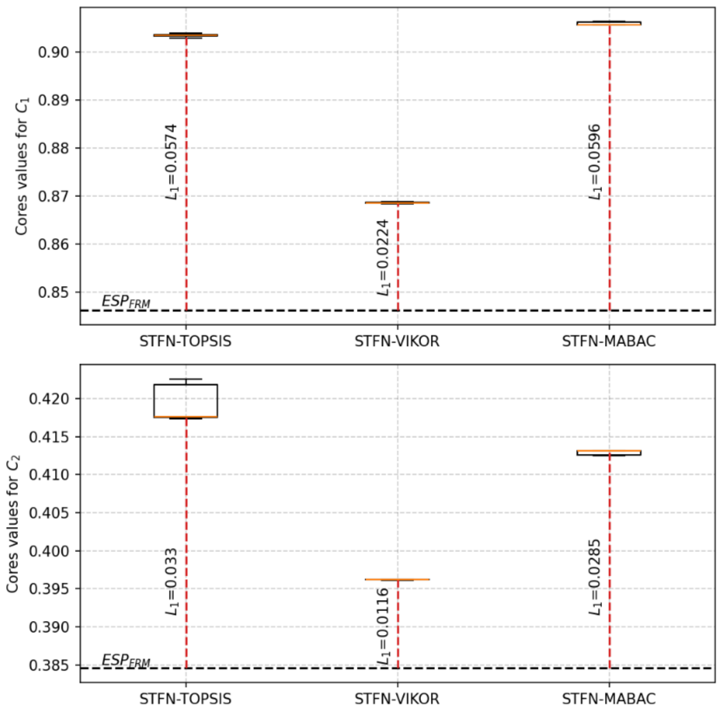


**Fig. 11** Boxplots of TFN core values for $C_1$ (top) and $C_2$ (bottom) across 1,000 runs of STFN-TOPSIS, STFN-VIKOR, and STFN-MABAC, with Manhattan distances to the Expected Solution Point derived from the FRM ($ESP_{FRM}$)

The medians of the core values for the criteria ($C_1$ and $C_2$) and their deviations from the Expected Solution Point (ESP) derived from the FRM model are highlighted. For criterion $C_1$, the median core values were 0.9035 for STFN-TOPSIS, 0.8686 for STFN-VIKOR, and 0.9057 for STFN-MABAC, with Manhattan distances from the ESP measured at 0.0574, 0.0224, and 0.0596, respectively. Criterion $C_2$ showed median core values of 0.4176, 0.3962, and 0.4131 for the same methods, with respective Manhattan distances of 0.033, 0.0116, and 0.0285. The results indicate that STFN-VIKOR demonstrates the closest alignment to the FRM model for both criteria, reflecting its excellent consistency in identifying cores closer to the expected reference point. While all approaches produced cores with minor discrepancies, the STFN-TOPSIS method exhibited a noticeable spread of values for the second criterion, with a standard deviation of 0.0021.

After a comparative analysis of STFN-based approaches as to the reference model, sensitivity analyses were conducted. Fig. 12 illustrates the sensitivity analysis of the STFN-TOPSIS, STFN-VIKOR, and STFN-MABAC methods with respect to changes in TFN cores ($C_1^{core}$ and $C_2^{core}$) across the range [0,1].

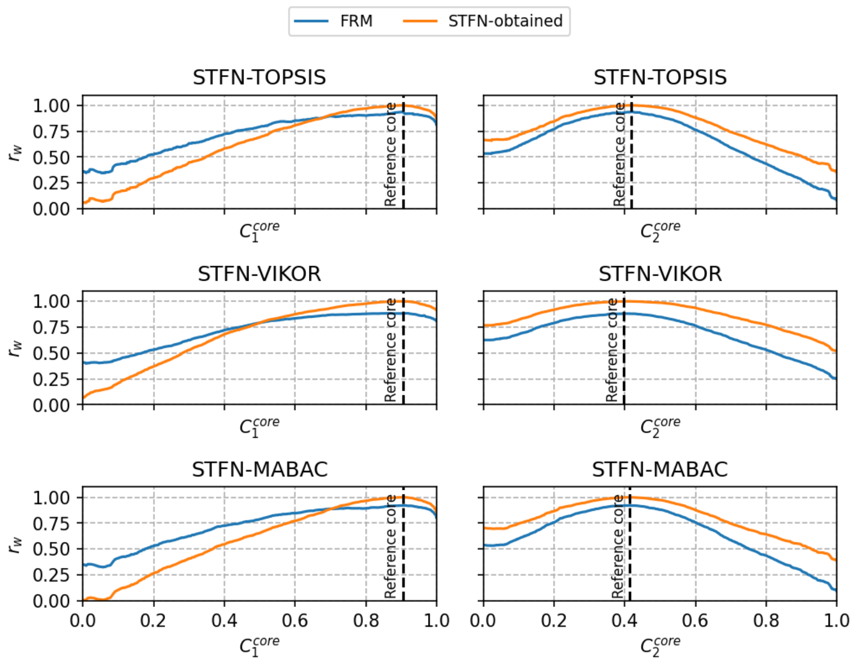


**Fig. 12** Comparison of Sensitivity Analysis for STFN-TOPSIS, STFN-VIKOR, and STFN-MABAC: Correlation with FRM and STFN-Obtained Rankings Across Core Values ($C_1^{core}$ and $C_2^{core}$).

For $C_1^{core}$, STFN-TOPSIS demonstrates the highest correlation with the FRM model at optimal configurations (Max $r_w$ = 0.9370) but exhibits greater variability overall, as reflected by its higher standard deviation (SD = 0.1873) and lower mean correlation (Mean $r_w$ = 0.7250). Similarly, STFN-MABAC achieves a comparable Mean $r_w$ = 0.7210, though it shows slightly better robustness with a lower standard deviation (SD = 0.1890). STFN-

VIKOR, while maintaining a consistent performance (Mean $r_w$ = 0.7180, Max $r_w$ = 0.8840), is marginally less aligned with the FRM for $C_1^{core}$ compared to the other methods. For $C_2^{core}$, STFN-VIKOR exhibits the highest robustness, with the lowest variability (SD = 0.1719) and a relatively high Mean $r_w$ = 0.6850. In contrast, STFN-MABAC and STFN-TOPSIS show greater sensitivity, with STFN-MABAC displaying a slightly lower mean correlation (Mean $r_w$ = 0.6420) and higher variability (SD = 0.2280). Notably, STFN-TOPSIS achieves the highest peak correlation with the FRM model (Max $r_w$ = 0.9370) but also has significant deviations at non-optimal points, indicating lower robustness overall.

The algorithm presented in Fig. 13 demonstrates a systematic approach to conducting a sensitivity analysis for the extrapolation of TFN boundaries in STFN-TOPSIS, STFN-VIKOR, and STFN-MABAC models, focusing on rank reversal phenomena.

**Listing: Sensitivity analysis in terms of extrapolation**

```
def sesitivity_extrapolation(stfn_topsis, stfn_vikor, stfn_mabac):
    dct = defaultdict()
    for n_samples in [5, 10, 25]:
        for method in [stfn_topsis, stfn_vikor, stfn_mabac]:
            lst = list()
            for c1, c2 in product(range(-0.1, 1.11, 0.01)):
                if 1 >= c1 >= 0 and 1 >= c2 >= 0:
                    continue
                for i in range(1000):
                    reference_set = generate_reference_samples(50)
                    additional_set = generate_additional_samples(n_samples, c1, c2)
                    reference_rank = method.ref(refernce_set)
                    method.boundary(c1, c2)
                    extrapolation_rank = method.ext([reference_set, additional_set])
                    lst.append(rw(reference_rank, extrapolation_rank))
            dct[n_samples][method].append([lst, c1, c2])
    return dct
```

**Fig. 13** Python implementation of sensitivity analysis for extrapolated boundaries in STFN-TOPSIS, STFN-VIKOR, and STFN-MABAC models

The procedure begins with generating reference samples uniformly distributed within the original TFN boundary interval [0,1] for both criteria ($C_1$ and $C_2$). Subsequently, the TFN boundaries are extrapolated beyond their original range using a grid of values from [-0.1, 1.11] for both criteria. However, extrapolated samples falling within the original boundary region ($1 \geq C_1 \geq 0$ and $1 \geq C_2 \geq 0$) are excluded, as rank reversal is not observed in this domain. New samples are randomly generated within the extended boundaries, adhering to specific conditions based on the position of $C_1$ and $C_2$ relative to zero. The reference samples are then combined with these additional samples to form augmented datasets, and preference rankings are computed for the reference samples only. Weighted Spearman rank correlation coefficient ($r_w$) is calculated between rankings derived from the original and extrapolated TFNs, with the process repeated 1,000 times for robustness. These experiments are conducted for extended datasets of varying sizes (5, 10, and 25 alternatives), allowing for a comprehensive evaluation of the sensitivity of each method to boundary extrapolation. This sensitivity analysis for the extrapolation of TFN boundaries highlights the robustness of the methods in handling deviations beyond the modeled boundary conditions and provides critical insights into the stability and reliability of rankings under such perturbations.

The graphs in Fig. 14 illustrate the sensitivity analysis for boundary extrapolation of TFNs in STFN-TOPSIS, STFN-VIKOR, and STFN-MABAC models across varying numbers of added alternatives (5, 10, and 25). The axes represent the extrapolated boundaries of criteria $C_1$ and $C_2$, while the color gradient indicates mean weighted Spearman rank correlation coefficient ($r_w$) between rankings derived from original and extrapolated TFNs. Higher mean $r_w$ values (lighter regions) signify robustness to boundary changes, while lower mean $r_w$ values (darker regions) highlight rank reversal sensitivity. The striped diagonal region corresponds to the excluded domain where extrapolated boundaries overlap the original TFN boundaries, ensuring stability as rank reversal does not occur. STFN-TOPSIS and STFN-MABAC demonstrate greater robustness in the extrapolated regions, maintaining lighter color gradients and higher $r_w$ values across configurations. In contrast, STFN-VIKOR exhibits lower $r_w$ values in the extrapolated regions (closer to darker blue), indicating greater sensitivity to boundary manipulations. These findings emphasize the relative stability of STFN-TOPSIS and STFN-MABAC under boundary extrapolation and highlight the increased rank instability of STFN-VIKOR in such scenarios.

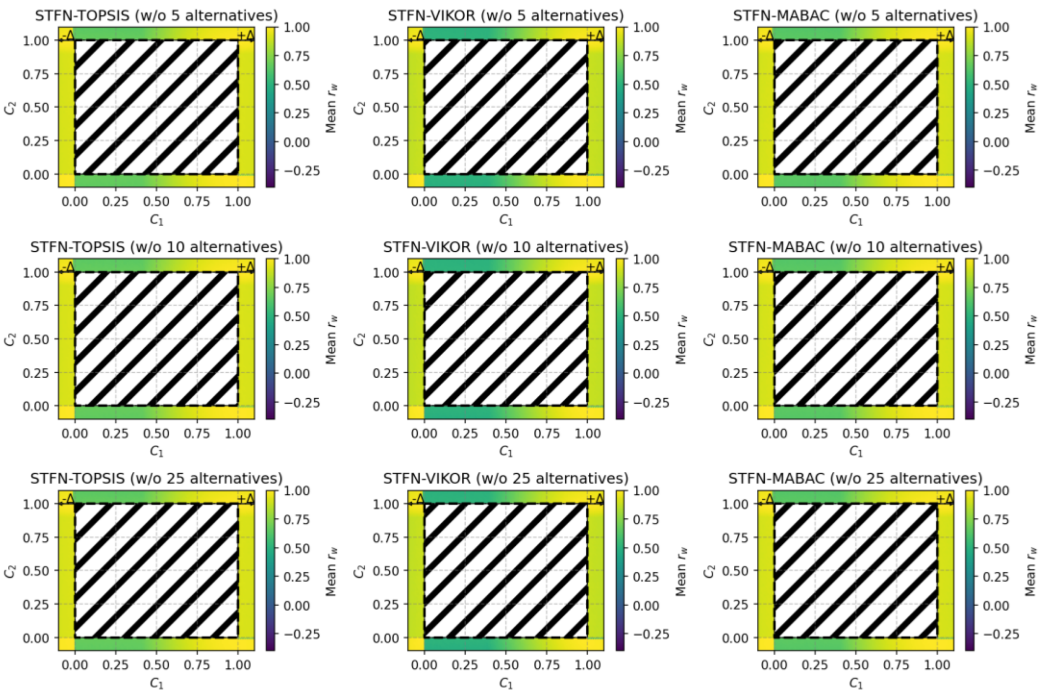


**Fig. 14** Sensitivity analysis of STFN-TOPSIS, STFN-VIKOR, and STFN-MABAC methods for extrapolated triangular fuzzy number boundaries

The presented graphs in Fig. 15 illustrate the standard deviation of weighted Spearman rank correlation coefficient ($r_w$) across extrapolated TFN boundaries ($C_1$ and $C_2$) for STFN-TOPSIS, STFN-VIKOR, and STFN-MABAC methods under varying numbers of added alternatives (5, 10, and 25). The color gradient represents standard deviation magnitude, where darker regions indicate more excellent stability, and lighter, yellow regions reflect higher variability. STFN-TOPSIS and STFN-MABAC demonstrate similar patterns of robust ranking stability with low standard deviation values across most extrapolated regions, showing minimal sensitivity to boundary changes. STFN-VIKOR, while exhibiting slightly higher variability, maintains consistent sensitivity patterns across the tested scenarios. Increasing the number of alternatives from 5 to 25 does not lead to significant changes in standard deviation distributions, suggesting that the methods remain stable regardless of dataset size. These results emphasize the resilience of STFN-TOPSIS and STFN-MABAC under boundary extrapolation and highlight that STFN-VIKOR, while slightly more sensitive, also maintains stable behavior across varying dataset sizes.

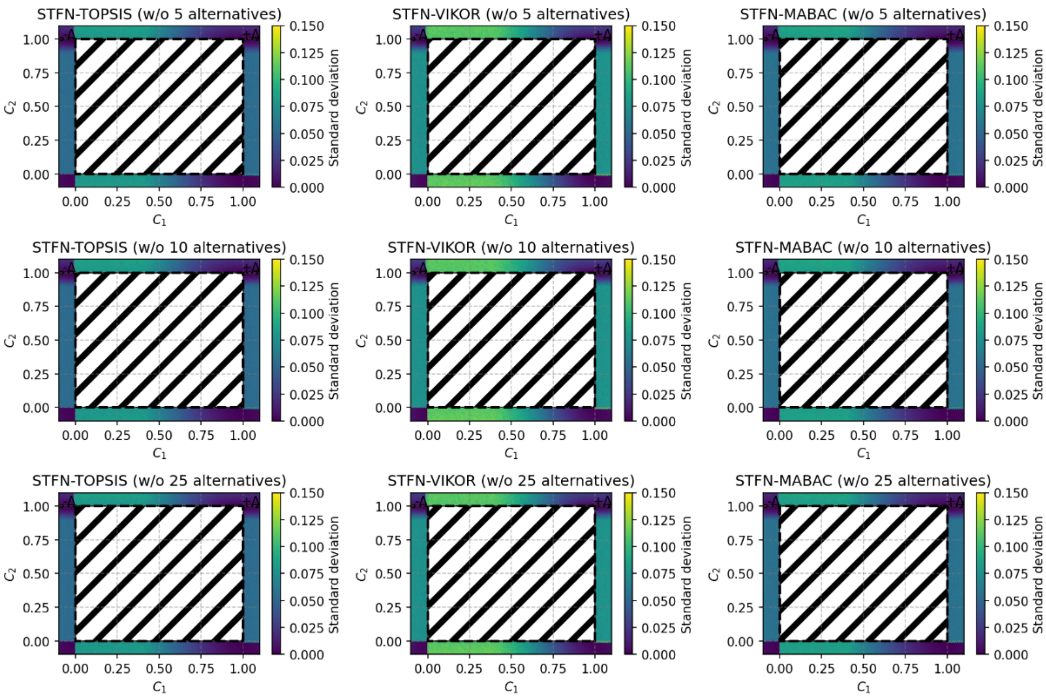


**Fig. 15** Standard deviation analysis of weighted Spearman rank correlation coefficient ($r_w$) for STFN-TOPSIS, STFN-VIKOR, and STFN-MABAC across extrapolated TFN boundaries

## 5. PRACTICAL EXAMPLE

This section presents a practical application of decision support methodologies to solve a complex engineering problem: reidentifying the model for selecting the optimal energy source for an industrial plant. Using synthetic data inspired by studies on energy sources [43,44], the analysis evaluates seven alternative energy options based on six key criteria. These criteria include: $C_1$ (operating cost, \$/MWh), $C_2$ (CO2 emissions, kg CO2/MWh), $C_3$ (installation cost, \$/MW), $C_4$ (energy supply stability, %), $C_5$ (environmental impact beyond CO2 emissions, measured on a scale of 1 to 10, where 10 indicates minimal impact), and $C_6$ (installation lifespan, years). The decision matrix, presented in Table 3, provides a comprehensive comparative framework for these alternatives, enabling a systematic evaluation of their relative performance across economic, environmental, and operational dimensions.

**Table 3** Decision matrix for evaluating energy source alternatives for an industrial plant

| Alternative | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ | $C_6$ |
|---|---|---|---|---|---|---|
| Natural gas ($A_1$) | 70 | 400 | 900000 | 90 | 5 | 25 |
| Wind energy ($A_2$) | 50 | 20 | 1500000 | 70 | 8 | 20 |
| Nuclear energy ($A_3$) | 100 | 5 | 5000000 | 99 | 6 | 60 |
| Solar energy ($A_4$) | 60 | 30 | 1200000 | 50 | 9 | 25 |
| Biomass energy ($A_5$) | 80 | 10 | 1800000 | 85 | 6 | 30 |
| Coal ($A_6$) | 40 | 1000 | 800000 | 95 | 3 | 30 |
| Hydroelectric energy ($A_7$) | 55 | 10 | 3000000 | 98 | 10 | 50 |

After applying the subjective model to evaluate the energy source alternatives, the ranking obtained was $A_5 > A_2 > A_7 > A_4 > A_6 > A_3 > A_1$. Using this ranking, the model was reidentified through the STFN-TOPSIS, STFN-VIKOR, and STFN-MABAC approaches. In this study, equal weights were assigned to all criteria, ensuring that the sum of the weights equaled 1. Regarding the nature of the criteria, it would typically be necessary to specify whether each criterion represents a profit or a cost. However, in this case, the use of STFN-based approaches eliminates this requirement. This is because the TFNs inherently capture the non-linear trends associated with each criterion, rendering the explicit declaration of profit or cost unnecessary. The reidentification process employed the evolutionary differential algorithm for core identification, as in the previous study. For the TFNs used in normalization, the boundary values were set according to the smallest and largest values present in the decision matrix. The identified cores obtained for each method are displayed in Table 4.

The identified cores reveal notable differences and similarities among the methods. STFN-TOPSIS assigns the highest core values for operating cost ($C_1$ = 100) and CO2 emissions ($C_2$ = 1000), emphasizing these criteria more strongly than the other methods. In contrast, STFN-VIKOR places greater importance on installation cost ($C_3$ = 2780147), while STFN-MABAC provides a balanced approach, with the highest emphasis on energy supply stability ($C_4$ = 74.90) and comparable values for environmental impact beyond CO2 ($C_5$ = 9.93) and lifespan ($C_6$ = 49.90). Despite differences in prioritization, all methods show alignment in valuing sustainability, particularly in terms of minimal environmental impact ($C_5$) and long-lasting installations ($C_6$), as evidenced by their relatively similar cores in these categories.

**Table 4** Identified cores for energy source selection using STFN-based methods

| Method | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ | $C_6$ |
|---|---|---|---|---|---|---|
| STFN-TOPSIS | 100 | 1000 | 1883156 | 71.97 | 9.99 | 50.18 |
| STFN-VIKOR | 40 | 48 | 2780147 | 62.50 | 8.93 | 48.06 |
| STFN-MABAC | 52 | 109 | 2170837 | 74.90 | 9.93 | 49.90 |

From the models reidentified based on the decision matrix, rankings were obtained and presented in Fig. 16. The figure illustrates the weighted Spearman rank correlation ($r_w$) between the reference ranking and the rankings produced by the STFN-TOPSIS, STFN-VIKOR, and STFN-MABAC methods, along with the positional shifts in alternative rankings. STFN-TOPSIS achieves the highest alignment with the reference ranking ($r_w$ = 0.987), indicating strong consistency, while STFN-VIKOR and STFN-MABAC show slightly lower alignment ($r_w$ = 0.946), suggesting minor deviations. Alternatives $A_5$, $A_2$, and $A_7$ maintain consistent positions across all methods, reflecting stability and agreement with the reference. In contrast, significant positional shifts are observed for $A_3$ (e.g., moving from position 3 in the reference ranking to position 7 in STFN-TOPSIS) and $A_6$ (e.g., shifting from position 6 in the reference to position 3 in STFN-TOPSIS). Alternatives $A_1$ and $A_4$ remain stable across all methods. Overall, STFN-TOPSIS demonstrates the closest alignment with the reference ranking, with fewer positional deviations, while STFN-VIKOR and STFN-MABAC exhibit similar performance but slightly greater variability in rankings.

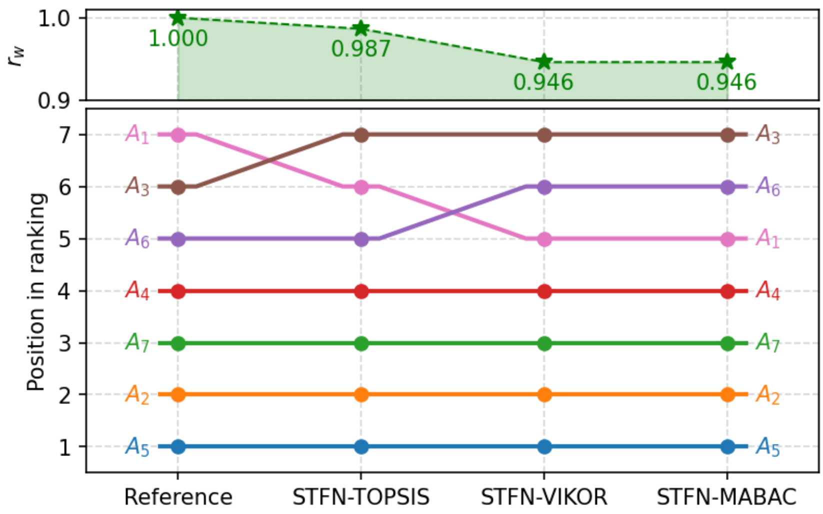


**Fig. 16** Weighted Spearman rank correlation and positional changes between the reidentified models and the reference ranking

## 6. DISCUSSION

The analysis results highlight the significant role of reidentification using the stochastic fuzzy normalization (STFN) based on triangular fuzzy numbers method in improving decision-making processes. STFN application enables precise modeling of complex decision models, mainly when expert knowledge is unavailable or when existing models must be adjusted to changing conditions or decision-maker preferences. Reidentification through STFN has proven to be an effective tool in MCDA methods such as TOPSIS, VIKOR, and MABAC, enhancing the accuracy of preference representation and the hierarchy of alternatives. The results confirm a high level of alignment between models reidentified with STFN and reference models, making this process an essential element of decision support in dynamic environments.

The resilience of STFN-based methods to rank reversal and their ability to maintain ranking stability during boundary extrapolation further underscores their value in dynamic decision-making scenarios. Both STFN-TOPSIS and STFN-MABAC exhibit consistent performance across various conditions, particularly excelling in scenarios with stable datasets. However, STFN-VIKOR, while effective, displays higher sensitivity to changes in boundary conditions, suggesting it may be more suited for decision-making problems that require fine-tuned preference modeling rather than broad adaptability. Importantly, the flexibility of the STFN reidentification process allows for iterative updates to decision models, enabling quick adaptations to evolving data or decision-maker priorities without requiring a full model redesign. This adaptability, combined with the reidentification process's inherent resilience to rank-reversal phenomena, significantly enhances the stability and reliability of multi-criteria analysis. As a result, STFN-based approaches are particularly appealing in rapidly changing domains like environmental policy, urban planning, or supply chain management, where decision criteria and conditions frequently shift, emphasizing their critical role in enabling robust and adaptable decision-making solutions.

The specific case of energy source selection for an industrial plant further exemplifies the efficacy of STFN reidentification. The rankings derived from the decision matrix revealed strong alignment between the reference model and the reidentified models using STFN-TOPSIS, STFN-VIKOR, and STFN-MABAC. STFN-TOPSIS demonstrated the closest correlation with the reference ranking, while STFN-VIKOR and STFN-MABAC maintained slightly lower but comparable alignment. These methods effectively preserved stability in ranking critical alternatives such as $A_5$, $A_2$, and $A_7$, while also capturing meaningful deviations for alternatives like $A_3$ and $A_6$. This case highlights the practical utility of STFN methods in balancing robustness and adaptability, even under conditions of significant variability in criteria or preferences.

Despite its many advantages, reidentification with STFN faces some challenges. Firstly, the effectiveness of this process depends on a clearly defined normalization process within MCDA methods. Methods that lack an explicit normalization step may require adaptation or additional mechanisms to enable STFN application. Secondly, using triangular fuzzy numbers limits the system's ability to model more complex uncertainty structures. Expanding reidentification to include other types of fuzzy numbers, such as trapezoidal fuzzy numbers or more advanced membership functions, could significantly broaden its applicability.

It is also worth noting that the reidentification process with STFN can be computationally demanding, especially for large datasets. Proper parameter selection and accurate interpretation of results are crucial for achieving high effectiveness and precision. Despite these challenges, STFN demonstrates exceptional potential in adapting and improving decision models, as evidenced by high correlation coefficients with reference models and the stability of rankings.

## 7. CONCLUSIONS

This study presents a comprehensive comparison of re-identification methods for MCDA models, including STFN-TOPSIS, STFN-VIKOR, and STFN-MABAC, in relation to the FRM. The findings confirm that these methods exhibit commendable performance in accurately reflecting the preferences of the FRM, with the application of stochastic fuzzy normalization (STFN) based on triangular fuzzy numbers proving highly effective in handling nonlinearities inherent in decision models. STFN-TOPSIS and STFN-MABAC demonstrate similar performance, achieving high robustness and stability across various scenarios, particularly in maintaining rank stability during boundary extrapolation. In contrast, STFN-VIKOR, while effective, shows greater sensitivity to boundary changes and rank reversal, making it less consistent in conditions of significant uncertainty or extrapolated data. Sensitivity analyses reveal that all methods maintain stability regardless of dataset size (number of added alternatives: 5, 10, or 25), though rank instability slightly increases near boundary extremes, with STFN-TOPSIS and STFN-MABAC showing greater resilience compared to the more variable STFN-VIKOR. These findings highlight the capability of STFN-based methods to enable robust normalization, mitigate rank reversal phenomena, and ensure reliable decision-making outcomes, particularly in scenarios where expert knowledge is unavailable or dynamic adjustments are required.

The practical application of these methods in the selection of optimal energy sources for an industrial plant reinforces their effectiveness in real-world scenarios. The reidentification process highlighted strong alignment between reidentified models and the reference model, particularly with STFN-TOPSIS, which demonstrated the highest correlation and stability. This case study further underscores the utility of STFN-based approaches in balancing robustness and adaptability, even under conditions of significant variability in criteria or preferences, showcasing their value in addressing complex, multidimensional decision problems.

In future works, a more extensive comparison of re-identification techniques across diverse decision scenarios could enhance the versatility and efficacy of decision-making models. Expanding the repertoire of fuzzy numbers, including moving beyond triangular fuzzy numbers to other types, could better capture nonlinearity and improve accuracy. Additionally, adapting the STFN approach to effectively handle uncertain and high-dimensional data is crucial, as real-world decision-making often involves imprecise information and complex datasets. Incorporating probabilistic methods, interval-based fuzzy numbers, or dimensionality reduction techniques (e.g., Principal component analysis (PCA)) could enhance robustness and computational efficiency. Advanced machine learning approaches may further refine STFN, enabling it to address complex decision spaces and broaden its applicability to domains like finance and healthcare, ultimately facilitating more informed and adaptive decision-making processes.

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