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# METAHEURISTIC-BASED TUNING OF PROPORTIONAL-DERIVATIVE LEARNING RULES FOR PROPORTIONAL-INTEGRAL FUZZY CONTROLLERS IN TOWER CRANE SYSTEM PAYLOAD POSITION CONTROL

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**Abstract**. *This paper proposes the use of metaheuristic optimization algorithms to tune the Proportional-Derivative (PD) learning rules within the framework of Iterative Learning Control applied to low-cost Takagi-Sugeno Proportional-Integral (PI)-fuzzy controllers for tower crane system payload position control. Four PD learning rules are considered: direct rule with current (in the iteration domain) control error, direct rule with previous control error, indirect rule, and open-closed-loop rule. The fuzzy controllers are tuned by the Extended Symmetrical Optimum method applied to the linear PI controllers, and then by the modal equivalence principle. Set-point filters are included for overshoot reduction. A unified design approach is formulated for all four PD learning rules in terms of optimally computing the gains in the iteration domain using metaheuristic optimization algorithms that solve optimization problems with objective functions expressed as the sum of the squared control error multiplied by time, where the two variables are the parameters of the PD learning rules. Seven popular metaheuristic optimization algorithms are implemented. Real-time experimental results from ten iterations of these optimization algorithms support the performance comparison of the fuzzy control systems.*

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## 1. INTRODUCTION

Iterative Learning Control (ILC) performance improvement of the control system by performing repeated experiments (iterations or trials or cycles). According to [1], the collected data from previous experiments is used in an iterative way. A systematic way to design ILC structures is the optimal tuning [2]. The Proportional-Derivative (PD) learning rule is a representative component in several ILC structures, with recent applications given in [3] and [4].

For better readability, the nomenclature of all symbols used in this paper is included at the beginning of the supplementary appendix given in [5] and organized in Table 1, which describes the abbreviations, variables, and parameters.

A classification of optimal ILC is given in [6], several data-driven ILC techniques are reviewed in [7], and recent applications to wafer stages, ball screw feed-drive systems, turntable systems and a combination with immune deep reinforcement learning are reported in [8], [9], [10] and [11], respectively. General classifications of ILC are given in [10] and [12] as direct ILC (i) and indirect ILC (ii), which combine ILC with feedback control loops: (i) ILC computes the control signals or improves their computation strategy; (ii) the controllers generate the control signals and ILC updates some parameter gains and/or variables of the control loops including reference inputs (or set-points) and controller tuning parameters.

The fusion of fuzzy control and ILC was coined by the authors back in 2006 [13] and continued in [14] to exploit the advantages of both control strategies. This has resulted in several low-cost Proportional-Integral (PI)-fuzzy controllers, and fuzzy logic has been incorporated into the learning rules specific to ILC. Another way of mixing fuzzy control with ILC is the approximation of the unknown dynamics of the controlled processes, considering fuzzy models to cope with the nonlinear mechanisms of the processes, and further the inclusion in adaptive ILC structures. The most recent applications of adaptive structures deal with high-speed trains [15], permanent magnet synchronous motors [16], multi-agent systems [17], [18], and treating the sampling period as a parameter to compensate for its influence on the control system performance using the optimization-based design [19].

Building upon authors' recent approaches to the performance improvement of fuzzy control systems for tower crane (TC) systems, using ILC with (a) direct PD learning rule with current (in the iteration domain) control error and the two parameters (i.e. the gains) optimally tuned using the metaheuristic Grey Wolf Optimizer (GWO) given in [20], (b) direct PD learning rule with previous control error using the metaheuristic Slime Mould Algorithm (SMA) [21] and (c) indirect PD learning rule using SMA [22], the new contributions of this paper are:

- A new ILC-based fuzzy control system structure, with the notation (d), derived from the open-closed-loop ILC with direct PD learning rule formulated in [23] for linear systems and a continuous-time derivative term.

- The systematic performance improvement of ILC-based fuzzy control systems for TC systems focusing on payload position control is proposed using the four PD learning rules (a), (b), (c) and (d) specified above, with seven popular metaheuristic optimization algorithms involved in tuning the two PD gains: GWO, SMA, Particle Swarm Optimization (PSO) [24], Gravitational Search Algorithm (GSA) [25], Charged System Search (CSS) [26], Whale Optimization Algorithm (WOA) [27], and African Vultures Optimization Algorithm (AVOA) [28]. These learning rules are combined with Takagi-Sugeno PI-fuzzy controllers leading to a new generation of low-cost fuzzy controllers.
- A novel unified design approach for all four PD learning rules. This approach optimally computes the gains of each learning rule in the iteration (or the experiment) domain using metaheuristic optimization algorithms that solve optimization problems with objective functions expressed as the sums of squared control errors multiplied by time and the variables represented by the two gains of the PD learning rules.
- The control system performance comparison based on real-time experimental results after a few iterations (namely, ten ones) of the metaheuristic optimization algorithms on the TC system laboratory equipment.

The main advantage of these new contributions with respect to the state-of-the-art is the fact that the novel design approach does not require any complicated computations for the optimal ILC in order to guarantee convergence. These computations become even more complicated as PI-fuzzy controllers are involved, whose models must be considered in the design, and they are avoided in the proposed design approach.

The Takagi-Sugeno PI-fuzzy controllers are designed by the fuzzification of the linear PI controllers tuned by the Extended Symmetrical Optimum (ESO) method [29], [30], applied to a simplified process model using the modal equivalence principle [31]. The ESO method is advantageous as it offers a tradeoff to the empirical performance indices of the control systems using only one design parameter, and set-point filters are included for overshoot reduction. The stability of the fuzzy control system is ensured by using modeling based on fuzzy basis functions.

Although not shown in the paper, the proposed controllers reject constant load type disturbances. The presence of the integral block in the fuzzy controller structure helps in this regard. However, after rejecting the disturbances, the ranking of the ILC-based fuzzy controllers will not remain similar.

The paper treats the following topics: the control system structures and the design approach are presented in the next section. The TC system and its model are briefly described in Section 3. The experimental results are given in Section 4, and the conclusions are highlighted in Section 5.

## 2. ILC-Based Fuzzy Control System Structures and Unified Design Approach

### 2.1 Control System Structures

The Single Input-Single Output (SISO) fuzzy control system structure with direct PD learning rule with current control error is shown in Fig. 1 (a), where: $r$ – the reference input or the set-point, assumed to be repetitive, $u$ – the control signal, $d$ – the disturbance input also assumed to be repetitive, $e$ – the control error, $y$ – the controlled output, $F$ – the set-point filter, $C$ – the fuzzy controller, $P$ – the controlled process (namely, a SISO sub-system of the TC system), M – the memory block, $k_p$ and $k_d$ – the proportional gain and the derivative gain of the PD learning rule, $k$ – the index of the current sampling interval or the discrete time index, $q^{-1}$ – the backward shift operator in the iteration domain, and $j$ – the subscript that indicates the current iteration (or cycle or trial or experiment). The expression of the PD learning rule used in Fig. 1 (a) is expressed as follows after adapting the PD learning rules given in [13] and [20]:

$$u_{[j]}(k) = u_{[j-1]}(k) + k_p e_{[j]}(k) + k_d [e_{[j]}(k) - e_{[j-1]}(k)], \tag{1}$$

where the control error is

$$e_{[j]}(k) = r(k) - y_{[j]}(k). \tag{2}$$

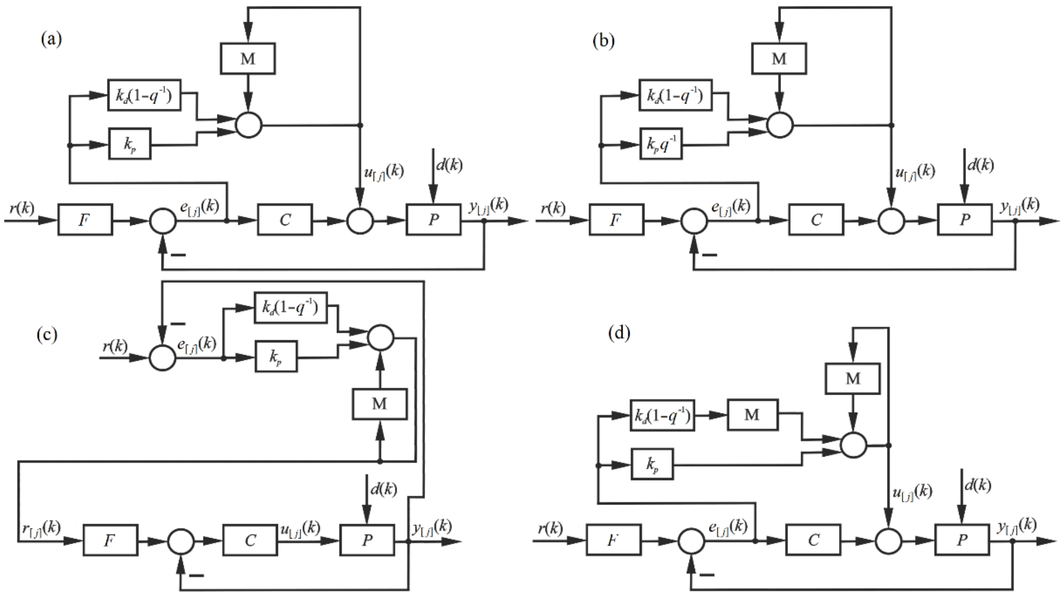


**Fig. 1** Structures of fuzzy control systems with: direct PD learning rule with current control error (a) (adapted from [20]), direct PD learning rule with previous control error (b) (adapted from [21]), indirect PD learning rule (c) (adapted from [22]), and open-closed-loop PD learning rule (d)

The structure of SISO fuzzy control system with direct PD learning rule with previous control error is shown in Fig. 1 (b), with the following rule obtained by adapting the PD learning rules given in [13] and [21]:

$$u_{[j]}(k) = u_{[j-1]}(k) + k_p e_{[j-1]}(k) + k_d [e_{[j]}(k) - e_{[j-1]}(k)]. \tag{3}$$

The structure of SISO fuzzy control system with indirect PD learning rule is shown in Fig. 1 (c), with the following rule obtained by adapting the PD learning rule given in [22]:

$$r_{[j]}(k) = r_{[j-1]}(k) + k_p e_{[j]}(k) + k_d [e_{[j]}(k) - e_{[j-1]}(k)], \quad (4)$$

and it outlines a second notation for the set-point, namely $r_{[j]}(k)$ indicating the set-point produced by the ILC algorithm and applied to the control loop, different to $r(k)$, which indicates the set-point of the control system.

The structure of SISO fuzzy control system with open-closed-loop PD learning rule is shown in Fig. 1 (d), where the following new PD learning rule is obtained by adapting the PD learning rule given in [23] to a discrete time formulation:

$$u_{[j]}(k) = u_{[j-1]}(k) + k_p e_{[j]}(k) + k_d [e_{[j-1]}(k) - e_{[j-2]}(k)]. \quad (5)$$

As specified in the previous section, $C$ is a low-cost Takagi-Sugeno PI-fuzzy controller, and its design and tuning start with the tuning of the linear PI controller. A simplified model of the controlled process ($P$) with the following transfer function with respect to the control signal is used:

$$P(s) = k_P / [s(1 + T_\Sigma s)], \quad (6)$$

with the parameters $k_P$ (the process gain) and $T_\Sigma$ (the small or parasitic time constant) partly known in the process model, and partly obtained, as in [22], by least-squares identification making use of input-output data obtained after real-time experiments conducted on the real process. Using these control system structures and models, the unified design approach is described in the next subsection.

### 2.2 Unified design approach

PI controllers are recommended in [29] and [30] to control the processes with transfer functions of type (6). The transfer function of the (linear) PI controller is

$$C(s) = k_C [1 + 1/(T_i s)], \quad (7)$$

where $k_C$ is the controller gain and $T_i$ is the controller integral time constant.

As mentioned in the previous section, the ESO method [29], [30] is successful in tuning the PI controller parameters because it ensures a convenient tradeoff to a set of empirical control system performance indices. The tradeoff is achieved using a single design parameter, $\beta$, within the range $1 < \beta \le 20$, as shown in Fig. 2, which illustrates the empirical performance indices $\sigma_1$ – overshoot, $\hat{t}_r = t_r / T_\Sigma$ – normalized rise time, $\hat{t}_s = t_s / T_\Sigma$ – normalized settling time, both times defined in the unit step modification of $r$, and $\varphi_m$ – phase margin.

The PI tuning conditions specific to the ESO method are [30]

$$k_C = 1/(\sqrt{\beta}\, k_P\, T_\Sigma), T_i = \beta\, T_\Sigma. \quad (8)$$

and the transfer function of one simple version of set-point filter $F$, which improves the performance by the cancellation of a zero in the closed-loop transfer function with respect to $r$, is [30]

$$F(s) = 1/(1 + \beta\, T_\Sigma s). \quad (9)$$

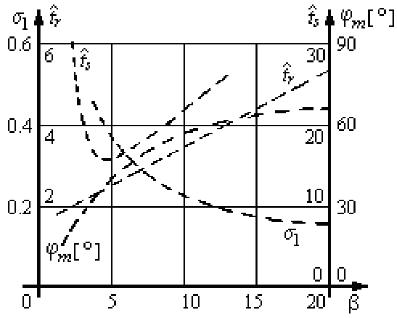


**Fig. 2** Empirical performance indices versus design parameter β [30]

The control system performance is next improved by replacing the linear PI controller with the Takagi-Sugeno PI-fuzzy controller illustrated in Fig. 3 (structure and input membership functions), with: $z^{-1}$ – the backward shift operator in the discrete time domain, TISO-FB – the Two Inputs-Single Output fuzzy block, $e(k)$ – the control error, $u(k)$ – the control signal, $\Delta e(k)$ – the increment of the control error, and $\Delta u(k)$ – the increment of the control signal. The subscript [$j$] is dropped out in Fig. 3 for the sake of simplicity.

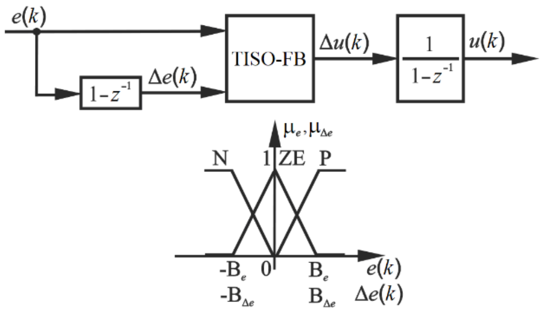


**Fig. 3** Low-cost Takagi-Sugeno PI-fuzzy controller structure and input membership functions (adapted from [22])

The recurrent expression of the incremental discrete-time PI controller is obtained using Tustin's discretization method

$$\Delta u(k) = K_P[\Delta e(k) + \mu\, e(k)], \tag{10}$$

with the parameters [14], [32]

$$K_P = k_C(1 - 0.5T_s / T_i),\ \mu = 2T_s /(2T_i - T_s), \tag{11}$$

where $T_s > 0$ is the sampling period, which is set to meet the requirements of quasi-continuous digital control.

TISO-FB uses the weighted sum defuzzification method, and the inference engine uses the MAX and MIN operators. The TISO-FB rule base consists of two rules, R1 and R2, which are defined as follows to enable the low-cost design and implementation of the fuzzy controller [22], [32]:

$$\begin{aligned}
&\text{R1: IF } (e(k) \text{ IS N AND } \Delta e(k) \text{ IS N) OR } (e(k) \text{ IS P AND } \Delta e(k) \text{ IS P)} \\
&\quad \text{THEN } \Delta u(k) = \eta\, K_P [\Delta e(k) + \mu\, e(k)], \\
&\text{R2: IF } (e(k) \text{ IS ZE) OR } (e(k) \text{ IS N AND } \Delta e(k) \text{ IS ZE) OR } (e(k) \text{ IS N AND } \Delta e(k) \text{ IS P)} \\
&\quad \text{OR } (e(k) \text{ IS P AND } \Delta e(k) \text{ IS ZE) OR } (e(k) \text{ IS P AND } \Delta e(k) \text{ IS N) THEN} \\
&\quad \Delta u(k) = K_P [\Delta e(k) + \mu\, e(k)],
\end{aligned} \tag{12}$$

where the parameter $\eta$, $0.25 \le \eta \le 0.75$, is inserted to further diminish the overshoot.

The modal equivalence principle [31] is next applied leading to the following tuning condition [32]:

$$B_{\Delta e} = \mu\, B_e. \tag{13}$$

To summarize, the parameters of the low-cost PI-fuzzy controller are $\beta$, which is set to ensure the trade-off to the empirical performance indices of the linear control system, $\eta$, which is set in relation to the overshoot alleviation, and $B_e$, which is set in relation to Fig. 3 to ensure the firing of both rules, possibly taking into account stability constraints [14], or optimally tuned [32].

After showing in Section 2 in [5] that it is not possible to guarantee the stability of the fuzzy control system due to the saturation and dead zone static nonlinearities placed on the process inputs, a Lyapunov-based stability analysis is offered, using the Takagi-Sugeno fuzzy model of the process and fuzzy basis functions. This analysis is important because it avoids the popular parallel distributed compensation based on solving linear matrix inequalities. The stability analysis, supported by [33] and [34], is easy for the user and is included in one of the steps of the design approach.

As pointed out in Section 1, the gains $k_p$ and $k_d$ of the PD learning rules in the four ILC-based fuzzy control system structures are obtained as the solutions to the following optimization problem, which is processed using [20], [21] and [22]:

$$\boldsymbol{\rho}^* = \arg \min_{\boldsymbol{\rho} \in D_{\boldsymbol{\rho}}} J(\boldsymbol{\rho}),\ J(\boldsymbol{\rho}) = \frac{1}{N} \sum_{k=1}^{N_s} ke_{[j]}^2(k, \boldsymbol{\rho}), \tag{14}$$

where $D_{\rho} \subset \Re^2$ is the feasible domain of the parameter vector $\boldsymbol{\rho}$, $\boldsymbol{\rho}^*$ is the optimal parameter vector, i.e. the vector solution to the optimization problem, with the expressions

$$\boldsymbol{\rho} = [k_p \quad k_d]^T,\ \boldsymbol{\rho}^* = [k_p^* \quad k_d^*]^T, \tag{15}$$

$T$ indicates matrix transposition, $J(\boldsymbol{\rho})$ is the objective function, and $N_s$ is the number of data samples, which sets the width of the time horizon, $N_s T_s$.

The optimization problem expressed in (14) is solved in this paper using seven metaheuristic optimization algorithms. An important issue in this regard is to map the optimization algorithms onto the optimization problem. These algorithms operate with a total number of $N$ agents, and each agent is assigned to a position vector $\mathbf{X}_{i[j]}$

$$\mathbf{X}_{i[j]} = [x_{i[j]}^1 \ldots x_{i[j]}^f \ldots x_{i[j]}^q]^T \in D_{\boldsymbol{\rho}} \subset \Re^q, i = 1 \ldots N, \tag{16}$$

where: $x_{i[j]}^f$ – the position of $i^{\text{th}}$ agent in $f^{\text{th}}$ dimension, $f$=1…$q$, $q$=2 in this particular case, $j$ – the index of the current iteration in both ILC (Fig. 1) and the optimization algorithms,

$j$=1…$j_{\max}$, and $j_{\max}$ – the maximum number of iterations. PSO is mapped onto (14) in terms of

$$\mathbf{X}_{i[j]} = \boldsymbol{\rho}, S_{i[j]} = J(\boldsymbol{\rho}), i = 1...N, \mathbf{P}_{g,Best} = \boldsymbol{\rho}^*, \tag{17}$$

where $S_{i[j]}$ is the fitness function specific to PSO, and $\mathbf{P}_{g,Best}$ is the best swarm position vector. GSA, CSS, GWO, WOA and SMA are mapped onto (14) in terms of

$$\mathbf{X}_{i[j]} = \boldsymbol{\rho}, S_{i[j]} = J(\boldsymbol{\rho}), i = 1...N, \arg \min_{i=1...N} J(\mathbf{X}_{i[j_{\max}]}) = \boldsymbol{\rho}^*, \tag{18}$$

where $S_{i[j]}$ is the fitness function specific to GSA, CSS, GWO, WOA and SMA. AVOA is mapped onto (14) in terms of

$$\mathbf{X}_{i[j]} = \boldsymbol{\rho}, S_{i[j]} = J(\boldsymbol{\rho}), i = 1...N, \mathbf{X}^{BV1}_{[j_{\max}]} = \boldsymbol{\rho}^*, \tag{19}$$

where $S_{i[j]}$ is the fitness function specific to AVOA, and $\mathbf{X}^{BV1}_{[j_{\max}]}$ is the first best vector solution obtained at the iteration $j_{\max}$. In all algorithms, the search domain is the feasible domain $D_{\rho} \subset \Re^2$ in (14).

The unified design approach applied to the ILC-based fuzzy control systems with the four PD learning rules consists of the following steps, which are obtained after organizing the steps given in [20], [21] and [22]:

*Step 1*. The sampling period $T_s$ is set. The performance specifications imposed to the fuzzy control system are expressed in terms of the optimization problem expressed in (14). The associated dynamic regime is defined in order to assess the objective function values through experiments conducted on the real-world process.

*Step 2*. The value of $j_{\max}$ is set.

*Step 3*. The values of the parameters $\beta$, $\eta$ and $B_e$ of the Takagi-Sugeno PI-fuzzy controller are set. The tuning condition (13) is applied to obtain the value of $B_{\Delta e}$. An experiment is conducted to check the stability condition of the fuzzy control system given in Theorem 1 in [5], and only those controller parameters that satisfy the stability condition are used next.

*Step 4*. The parameters specific to the metaheuristic optimization algorithms are set, and the algorithms are applied to compute the values of the gains $k_p^*$ and $k_d^*$.

The above steps are actually a quick and useful guide to easily tuning the controllers. Steps 1 to 4 are applied to the TC system in order to validate the proposed design approach in a payload position control example. Details of the TC system equipment are given in the next section.

## 3. TOWER CRANE SYSTEM

The laboratory equipment involved in conducting the experiments and assessing the efficiency of the fuzzy controllers and design approach proposed in this paper is built around the TC system with technical information given in [35], and it is presented in detail in Section 1 of the supplementary appendix given in [5]. A photo of the laboratory equipment is illustrated in Fig. 4.

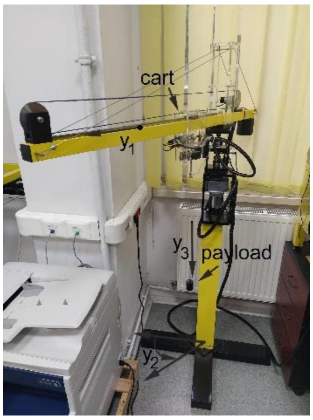


**Fig. 4** Photo of tower crane system laboratory equipment [36]

The TC system equipment is a Multi Input-Multi Output system focused on controlling the cart position, the arm angular position, and the payload position. A relatively simple control approach is to separately design three SISO control systems to control each position (output). To keep the paper to a reasonable length, only the results for the payload position control are presented; moreover, the payload position control is the most challenging out of the three SISO ones. A diagram with details of the equipment and its communication with a computer are presented in Fig. 1 given in [5].

The description of the TC system equipment and its modeling carried out in [36], with hardware and software details given in [37], is supported by [38], which was used in the derivation of the state equation of payload position dynamics. Using the information given in this section, in [5], and the theoretical results proposed in Section 2, the experimental validation is carried out in the next section.

## 4. Experimental Results and Comparison

The real-time experiments were conducted on the TC system experimental setup described in the previous section and [5] to apply the novel unified design approach formulated in Section 3 to the four ILC-based fuzzy control systems with PD learning rules. As mentioned in the previous sections, the payload position control is considered in this paper, therefore the controlled output $y$ in Section 2 is actually the payload position $y_3$ ($m$) = $x_9$ ($m$) of the TC system as described in [5].

### 4.1 Application of Design Approach

Aspects concerning the steps of the design approach are highlighted as follows. The dynamic regime used to evaluate the value of the objective function defined in (14) is characterized by: zero initial conditions, no disturbances applied, but arbitrary disturbances can appear, and the reference input is next given as a reference trajectory along with the TC system responses. The objective function is evaluated using the sampling period set to $T_s$ = 0.01 s, the width of the time horizon is set to 70 s, corresponding to a number of $N_s$ = 70/0.01 + 1 = 7001 data samples.

Applying least-squares identification, the parameters of the process transfer function in (7) obtain the values $k_P$ = 0.2505 and $T_\Sigma$ = 0.0533 s. Setting the design parameter $\beta$=9, the tuning conditions specific to the ESO method given in (8) lead to the parameters of the linear PI controller $k_C$ = 24.99 and $T_i$ = 0.48 s. Applying (11), the parameters of the discrete-time PI controller are $K_P$ = 11.8531 and $\mu$ = 0.0211. Setting the parameter of the PI-fuzzy controller $B_e$ = 0.04, the other parameter results from (13), $B_{\Delta e} = 8.4\cdot 10^{-4}$. The third parameter of the PI-fuzzy controller is set to $\eta$ = 0.2550. These parameter values guarantee, via the stability analysis in [5], the stability of the fuzzy control system.

The feasible domain of $\boldsymbol{\rho}$ is set by modifying the domains used in [20-22] to obtain the unified domain

$$D_{\boldsymbol{\rho}} = \{k_p \mid -0.1 \le k_p \le 0.1\} \times \{k_d \mid -0.1 \le k_d \le 0.1\}. \tag{20}$$

The initial parameters of the four PD learning rules are different to those used in [20-22]:

$$k_{p[0]} = 0, k_{d[0]} = 0, \tag{21}$$

where the subscript 0 is forced to capture the situation without ILC, namely without the PD learning rule in Fig. 1. This ensures a fair comparison of the control system structures, the PD learning rules, and the optimization algorithms on actually 1 + $j_{\max}$ iterations. The maximum number of iterations is set to $j_{\max}$ = 10 to ensure the low-cost implementation, and for the same reason, the number of agents for all seven metaheuristic optimization algorithms is set to $N$ = 10.

In the context of ensuring the full transparency of the experimental validation and comparison results, the non-random parameters specific to the seven optimization algorithms are set as follows: the weight parameters of PSO: $c_1 = c_2 = 0.3$, a linear decrease of the inertia weight parameter $w_{[j]}$ of PSO with the bounds $w_{\max}$ = 0.9 and $w_{\min}$ = 0.4:

$$w_{[j]} = w_{\max} - j(w_{\max} - w_{\min}) j_{\max}, \tag{22}$$

the exponential decrease law of the gravitational constant $g_{[j]}$ of GSA:

$$g_{[j]} = g_0 e^{-j\zeta / j_{\max}}, \tag{23}$$

with the initial value $g_0$ = 100, the exponent parameter $\zeta$ = 8.5 and the parameter at the denominator of the force $\varepsilon_{\mathrm{GSA}} = 10^{-4}$ to avoid possible divisions by zero, each agent in CSS is considered as a charged sphere of uniform volume charge density with radius $a$ = 1, the CSS acceleration and velocity parameters $k_{a[j]}$ and $k_{v[j]}$, respectively:

$$k_{a[j]} = 2(1 - j / j_{\max}), k_{v[j]} = 2(1 + j / j_{\max}), \tag{24}$$

and the parameter at the denominator of the separation distance $\varepsilon_{\mathrm{CSS}} = 10^{-4}$ to avoid possible divisions by zero, the threshold in the position update equation specific to SMA $z$ = 0.03, and the parameter in the fitness weight of each slime mould specific to SMA $\varepsilon_{\mathrm{SMA}} = 10^{-3}$ to avoid possible divisions by zero, and the controlling parameters specific to AVOA set to: $P_1$ = 0.6, $P_2$ = 0.4, $P_3$ = 0.6, $\alpha_{\mathrm{AVOA}}$ = 0.8, $\beta_{\mathrm{AVOA}}$ = 0.2, and $\gamma_{\mathrm{AVOA}}$ = 2.5.

The results of the comparison may be different for parameter settings other than the values specified above. Additional details on the implementation of the optimization algorithms are given in [32] and [39-41], where it is also highlighted that these parameter settings ensure a good trade-off to exploration and exploitation. In the same context of ensuring full transparency, the data processed in the statistical analysis conducted to compare the performance of all four control system structures and seven optimization algorithms is freely available in the *Data_FUME_2.m* Matlab file [42].

### 4.2 Experimental Results, Comparison and Discussion

The optimization algorithms were run 30 times. All results are presented in averaged values. The results obtained after conducting the variance (ANOVA) test of the minimum objective function $J_{\min}$ evaluated after running these seven optimization algorithms on the four ILC-based fuzzy control system structures are presented in Fig. 5, with the general notation M-Q, where M indicates the PD learning rule, namely M $\in$ {C, P, I, O}, C – direct rule with current control error (Fig. 1 (a)), P – direct rule with previous control error (Fig. 1 (b)), I – indirect rule (Fig. 1 (c)), and O – open-closed-loop rule (Fig. 1 (d)), and Q indicates the optimization algorithm, namely Q $\in$ {PSO, GSA, CSS, GWO, WOA, SMA, AVOA}.

Fig. 5 and Table 2 given in [5] show that the best $J_{\min}$ is achieved by the indirect PD learning rule and five optimization algorithms for this process and this fuzzy controller, in the following descending order, with small differences: I-WOA, I-GSA, I-CSS, I-GWO and I-PSO.

The average values of the optimal parameters of the four PD learning rules obtained after running the seven optimization algorithms are presented in Table 2 given in [5]. Typical responses of the fuzzy control system before and after the application of the four PD learning rules and an optimization algorithm are shown in Figs. 2 to 5 in [5]. The performance improvement offered by ILC combined with metaheuristic optimization algorithms is clearly highlighted.

The main results of the non-parametric statistical tests conducted on the 28 cases (four ILC-based fuzzy control system structures multiplied by seven optimization algorithms) are presented in Tables 3 and 4 given in [5]. These results show that the best performance is exhibited by the indirect PD learning rule and three optimization algorithms, namely I-CSS, I-GWO and I-WOA, followed by the same rule and other two optimization algorithms, namely I-GSA and I-PSO, again with small differences.

The following performance indices were considered for comparison: rise time, settling time, percent overshoot, steady-state error, and mean sum of squared control signals. Accordingly, experiments were conducted to compare the performance of the control system structures and the results are summarized in Table 5 given in [5] both initially and after 10 iterations.

The results show significant improvement of the rise time, settling time, overshoot and mean sum of squared control signals after ten iterations over the same indices calculated initially. A numerical comparison and a discussion of the experimental results are given in Section 3 of [5]. The comparison and the discussion provide suggestive evidence of the advantages and disadvantages of the four controllers whose PD learning rules are tuned using different metaheuristic optimization algorithms; they also show which one performs better or best.

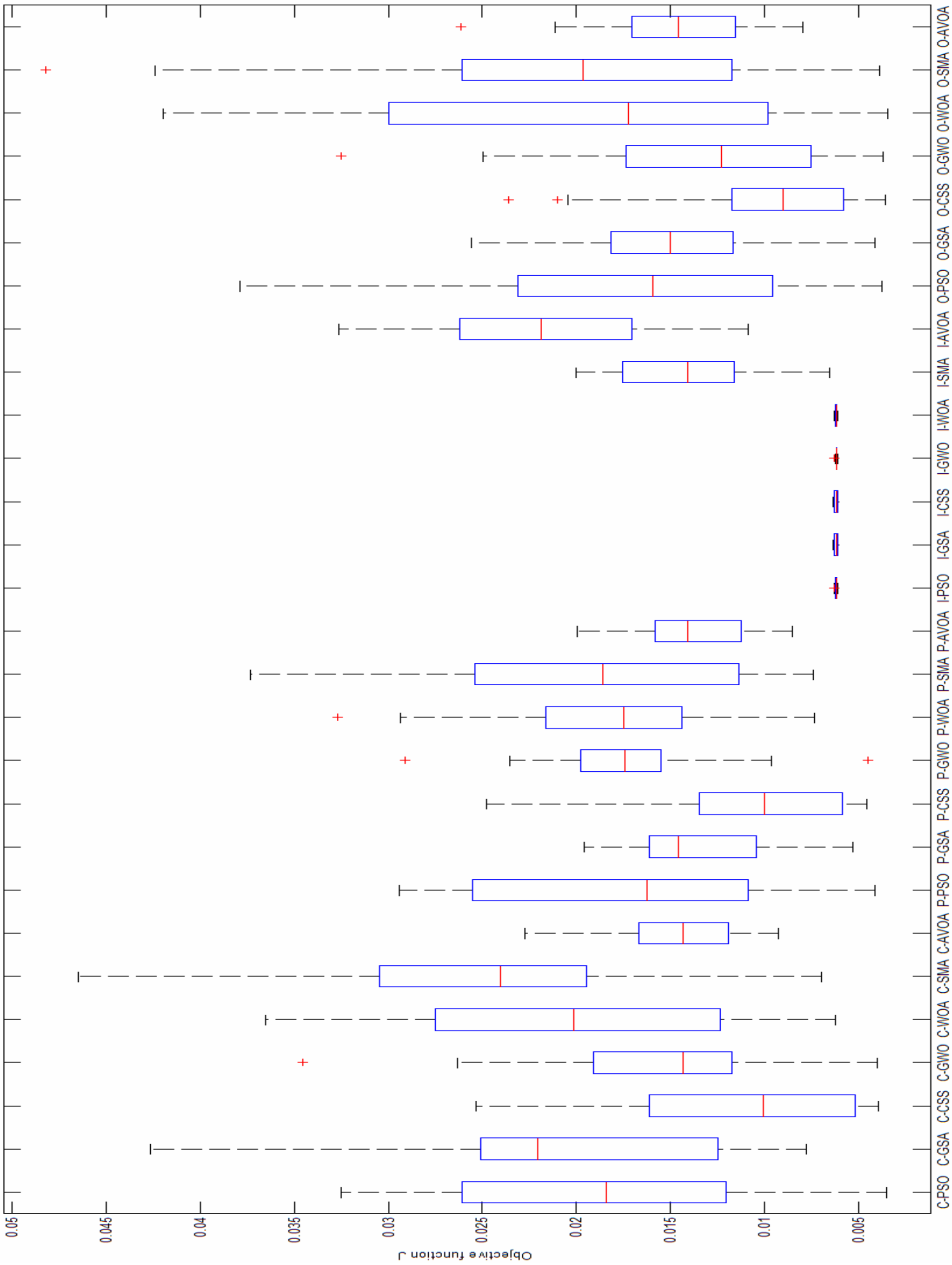


**Fig. 5** ANOVA test of minimum objective function *J* for all four ILC-based fuzzy control system structures and seven optimization algorithms

The results of the comparison carried out in [5] cannot be generalized to any controlled process. However, experience in controlling certain processes can be used and incorporated into the fuzzy control rules. Such representative processes include the finite element formulation for piezoelectric active laminated shells [43, 44], biomedical systems including image processing [45], natural language processing [46], spiking neural P systems [47], bed combustion processes [48], human well-being and resilience [49], complex planetary gearboxes [50], bin packing [51], telesurgical robotic systems [52], VANETs [53], robust evolving cloud-based controllers [54], cloud computing systems for traffic management [55], cognition processes [56], and friction compensation in haptic interfaces [57].

## 5. Conclusions

This paper proposed the performance improvement of low-cost fuzzy control systems with four Proportional-Derivative (PD) learning rules in the context of Iterative Learning Control (ILC) in terms of optimally tuning the two PD parameters (or gains) using seven metaheuristic algorithms. A unified design approach is formulated, and the performance improvement is proved and compared by real-time experimental results that correspond to payload position control of tower crane systems.

As specified in Section 1, the first advantage of the design approach proposed in this paper is the avoidance of complicated computations specific to the classical optimal ILC approaches in relation to the need to guarantee convergence.

As shown in Fig. 3 given in [5], the average objective function changes quickly as the iterations go by (fast decrease of the objective function $J$). This is especially true for the ILC-based fuzzy control system structure with an indirect PD learning rule, optimized by five algorithms, which ensure the average $J_{\min} = 0.0062$. The value of $J$ remains practically constant after iteration 1. Although the use of metaheuristic algorithms usually requires many evaluations of $J$ and thus is generally a limitation, these results clearly show the fast decrease of $J$ after only iteration 1, which is the second advantage of the design approach. The same conclusion is drawn after examining the evolution of the average values of the parameters $k_p$ and $k_d$ of the PD learning rules in Fig. 4 [5].

The stability analysis of the four control systems performed in [5] can also be performed in a different approach in the context of data-driven control, since ILC can also be considered as a data-driven control technique. This can be seen as related to the optimization problem solved by the authors for optimal reference input tracking, and the results given in the book [36] can be used as a starting point for different approaches to stability analysis.

Future research will be focused on the further reduction of the number of evaluations of the objective function. Incorporating gradient information into data-driven control is expected to be a viable solution and suggests another avenue for future study.

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