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LIMITING PROFILE OF AXISYMMETRIC INDENTER DUE TO THE INITIALLY DISPLACED DUAL-MOTION FRETTING WEAR

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Abstract. *Recently the final worn shape of elastic indenters due to fretting wear was analytically solved using the method of dimensionality reduction. In this paper we extend this model to dual-motion fretting wear and take into account that the indenter is initially pressed with constant indentation depth and moved horizontally with constant displacement. Two key parameters, the maximal indentation depth during oscillation and the stick area radius in the final state as well as the liming shape of indenter are analytically calculated. It is shown that the oscillation amplitudes and the initially indented or moved displacements have an influence on the final shaking-down shape.*

Key Words: *Fretting Wear, Dual-motion, Tangential Force, Oscillation*

1. INTRODUCTION

Fretting wear is a surface destruction process in the frictional contacts subjected to oscillating load with small amplitude [1]. This phenomenon occurs very often in the vibrating connections of mechanical elements, such as clamping devices, interference fit joints, gear or bearing contacts and electrical connectors, etc. [2-4]. Fretting leads to material loss, crack formation as well as fatigue failure [5]. In the last few decades many experimental and theoretical investigations have been intensively carried out to understand this process, for example, by using the finite element method or that of the boundary element [6, 7]. However, there are still some unsolved basic problems, especially under complicated loading [8]. Recently, a new method known as that of dimensionality reduction (MDR) was applied to analyzing the process of fretting wear as well as its final 'shake down' state for arbitrary axisymmetric shape of elastic or viscoelastic indenter [9-11]. In

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paper [9] a general theoretical solution of the limiting profile due to fretting wear was given for an arbitrary axisymmetric indenter. For the case of an elastic indenter under the tangential oscillation [10], a rapid numerical procedure based on the MDR was later developed to simulate the wear process, and its results for the final state of wear also verified the solution in [9]. Furthermore, a similar MDR-based procedure was suggested for a gross-slip wear problem and the results are exactly same to the solution obtained by the full FEM formulation, and it is for several orders of magnitude faster than the FEM [12]. Fretting wear of viscoelastic indenters was analyzed in papers [13] and [14], where the analytical solution of limiting profile due to dual-motion oscillation was presented in [13], and the numerical simulation of fretting wear under the tangential oscillation was carried out in [14]. These final worn shapes for spherical indenters under multiple-mode fretting conditions have been validated by experimental investigation [15]. Till now most work focuses on fretting wear only under the tangential oscillation and less on dual-mode fretting. In this paper, we consider the fretting wear of elastic indenter oscillating in both tangential and normal directions, and take into account the factor that the indenter has initially constant displacements in both normal and tangential direction.

2. Wear Criterion in Fretting Contact

This paper is an extension of the solutions in [9]; therefore, firstly we give a very brief discussion of wear condition in [9]. We consider a contact between a rigid axis-symmetrical body and an elastic half space. Under the normal load the indenter is pressed into the half space and then oscillates tangentially. It is known that, if the oscillation amplitude is small enough, there will be an annular slip-zone generated at the boundary of contact area and a circular stick-zone at the inner area, as illustrated in Fig.1.

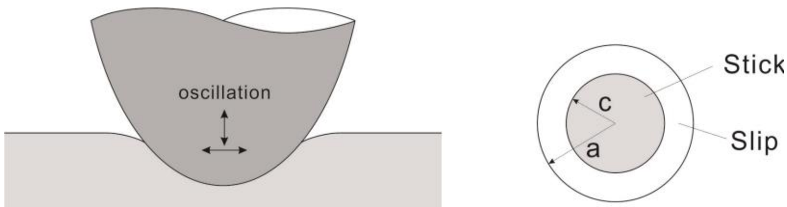

Fig. 1 Schematic representation of the stick-slip area in fretting contact

The stick and slip condition can be determined by the classic Amontons' law: if tangential stress τ is smaller than normal pressure p multiplied by a constant coefficient of friction μ, $\tau < \mu p$, the surfaces of contact bodies stick together, and in the slip region the tangential stress remains constant and equal to product μp:

$$\begin{cases} \tau < \mu p, & \text{in stick area} \\ \tau = \mu p, & \text{in slip area} \end{cases} \tag{1}$$

According to the Reye-Archard-Khrushchov wear law [16, 17], the wear volume is proportional to the normal force (or pressure), the relative tangential displacement and of contacting bodies and reversely proportional to the hardness. From this law, the wear in the local contact area vanishes when the normal pressure becomes zero or there is no relative displacement between two bodies. As described in [9], this no-wear condition can be written as:

$$\text{No wear condition: } \begin{cases} \text{either} & p = 0 \\ \text{or} & \Delta u_x = 0 \end{cases} \tag{2}$$

In the process of fretting wear, the surfaces in the stick area have no relative displacement, so that no wear occurs in this contact area during the whole process. Due to slip at the boundary wear occurs in this area, but the normal pressure will reduce to zero finally; therefore, there is no wear any more in this local contact area in the final state. In this paper we analyze this limiting profile of indenter.

3. Solution for Pre-Stressed Dual-Motion Problem

The analytical solution of limiting profile in [9] was obtained based on the method of dimensionality reduction (MDR). Using this method the three-dimensional normal and tangential contact problems for axis-symmetric bodies can be mapped into one-dimensional contact with a properly defined foundation [18-21]. According to the rules of the MDR, three-dimensional pressure distribution $p(r)$ can be calculated from the profile of one-dimensional indenter $g(x)$:

$$p(r) = \frac{E^*}{\pi} \int_r^\infty \frac{g'(x)}{\sqrt{x^2 - r^2}} \mathrm{d}x \,. \tag{3}$$

From no-wear conditions, Eq. (2), it follows that there are two parts in the contact areas in the final state: in the inner contact area with radius c no wear occurs because of no relative displacement Δu_x=0, so that the final profile keeps its initial form $g_\infty(x) = g_0(x)$ for $r < c$; at boundary $r > c$ the pressure in the final state reduces to zero, $p(r) = 0$. From Eq. (3), $p(r) = 0$ means that $g'(x) = 0$ and $g(x) = const$ for $c < x < a$ and the value of $const$ is equal to maximum indentation depth $d_{\max}$ achieved during the whole oscillation process. Thus, the one-dimensional MDR-transformed profile in the final shakedown state has the form

$$g_\infty(x) = \begin{cases} g_0(x), & \text{for } 0 < x < c \\ d_{\max}, & \text{for } c < x < a \end{cases} \tag{4}$$

According to the reverse transformation in the MDR, the three-dimensional limiting shape can be calculated as [9]

$$f_\infty(r) = \begin{cases} f_0(r), & \text{for } 0 < r < c \\ \dfrac{2}{\pi}\displaystyle\int_0^c \frac{g_0(x)}{\sqrt{r^2 - x^2}} dx + \frac{2}{\pi} d_{\max} \int_c^r \frac{1}{\sqrt{r^2 - x^2}} dx, & \text{for } c < r < a \end{cases} . \tag{5}$$

Eq. (5) gives the solution for limiting shake-down-state shape of the indenter. Given an initial three-dimensional profile of indenter, its limiting shape can be determined if the two parameters are known: radius c of the stick area in the limiting state and maximum indentation depth $d_{\max}$. In the following we discuss how these two governing parameters can be determined in our pre-stressed dual-motion problem.

Such a contact is taken into consideration. The indenter is pressed into an elastic half-space with an indentation depth d_0, and moved horizontally with a distance x_0, then oscillates harmonically according to

$$d = d_0 + u_z = d_0 + u_z^{(0)} \sin(\omega_z t + \phi) \tag{6}$$

in vertical direction and

$$u_x = x_0 + u_x^{(0)} \sin(\omega_x t) \tag{7}$$

in horizontal direction. ϕ is phase shift between normal and tangential oscillations. This movement is illustrated in Fig. 2. Here we consider small amplitude of oscillations under the assumption that $u_z^{(0)} < d_0$ and $u_x^{(0)} < x_0$, and all these four parameters are positive. Now we calculate the two important parameters.

(1) *The maximum indentation depth*. This one can be easily obtained by Eq. (6):

$$d_{\max} = \max_{(t)}\{d_0 + u_z^{(0)} \sin(\omega_z t + \phi)\} = d_0 + u_z^{(0)} . \tag{8}$$

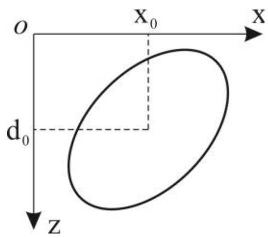

Fig. 2 Illustration of dual-motion of the indenter

(2) *Radius c of the stick area*. According to Eq.(1), radius c can be determined by the condition that tangential force $k_x u_x(c)$ of springs at each time moment is smaller than or equal to coefficient of friction μ multiplied by normal force $k_z u_z(x)$:

$$\left| G^* \Delta x \cdot (x_0 + u_x^{(0)} \sin(\omega_x t)) \right| \le \mu E^* \Delta x \cdot (d_0 + u_z^{(0)} \sin(\omega_z t + \phi) - g(c)) \tag{9}$$

Solving this inequality with respect to $g(c)$ gives

$$g(c) < d_0 + u_z^{(0)} \sin(\omega_z t + \phi) - \frac{G^*}{\mu E^*}(x_0 + u_x^{(0)} \sin(\omega_x t)) \tag{10}$$

or

$$g(c) = \min_{(t)}\left\{ d_0 + u_z^{(0)} \sin(\omega_z t + \phi) - \frac{G^*}{\mu E^*}(x_0 + u_x^{(0)} \sin(\omega_x t)) \right\} \tag{11}$$

From Eq.(11), it can be seen that the value of $g(c)$ is dependent on phase shift ϕ. If it is not fixed, that means phase shift ϕ is not constant but changes all the time, then $g(c)$ has a very simple and general form

$$g(c) = d_0 - u_z^{(0)} - \frac{G^*}{\mu E^*}(x_0 + u_x^{(0)}) . \tag{12}$$

However, if the phase shift is constant, the solution of Eq. (11) is not easy to calculate. Here we consider only a special case of same oscillation frequencies: $\omega_x = \omega_z = \omega$. Solving the Eq. (11) gives

$$g(c)_{\min} = d_0 - \frac{G^*}{\mu E^*} x_0 - \sqrt{u_z^{(0)2} - 2u_z^{(0)} u_x^{(0)} \frac{G^*}{\mu E^*} \cos\phi + \left(\frac{G^*}{\mu E^*} u_x^{(0)} \right)^2} \ . \qquad (13)$$

Now the two parameters, radius c of the stick area in the limiting state and maximum indentation depth $d_{\max}$ are obtained. Substitute Eqs. (8) and (13) to the limiting profile Eq. (5), then the three-dimensional limiting shape of the indenter can be calculated. It is seen that the two parameters as well as the limiting shape depend on oscillation amplitudes $u_z^{(0)}$ and $u_x^{(0)}$, phase shift ϕ between normal and tangential movements, and also the initial pre-indented and –displaced distance d_0 and x_0.

Radius c of the stick area is briefly discussed here. From Eq.(13), the smallest stick radius is given when phase shift $\phi = \pi$, and the value are the same to (12) in the case of no-fixed phase. The maximum stick radius (minimum wear volume) is realized at $\phi = 0$:

$$g(c) = d_0 - \frac{G^*}{\mu E^*} x_0 - \left| u_z^{(0)} - \frac{G^*}{\mu E^*} u_x^{(0)} \right| . \qquad (14)$$

If the phase $\phi = \pm\pi/2$, the stick area is given by

$$g(c) = d_0 - \frac{G^*}{\mu E^*} x_0 - \sqrt{u_z^{(0)2} + \left(\frac{G^*}{\mu E^*} \right)^2 u_x^{(0)2}} \ . \qquad (15)$$

It is noted that these results of the stick area as well as the related limiting profile are independent of the frequencies of normal and tangential oscillations.

With an example of parabolic indenter we show how the limiting profile can be calculated in the case of pre-displaced dual-motion. The one-dimensional profile of spherical indenter with radius R is given by $g(x) = x^2/R$ [18]. From Eq. (8) the maximal indentation depth is equal to $d_{\max} = d_0 + u_z^{(0)}$. If the phase shift between normal and tangential oscillations is $\phi = \pi$, then the stick radius is calculated by Eq. (12) as $c^2/R = d_0 - u_z^{(0)} - (x_0 + u_x^{(0)}) G^*/(\mu E^*)$. Substituting these two parameters $d_{\max}$ and c into basic solution, Eq. (5), the final profile in this case is then obtained. An example of this final shape is shown in Fig. 3. The worn shape of the indenter can be clearly seen, that is, this part lies in the slip area and the strongest wear is almost in the middle of the slip region.

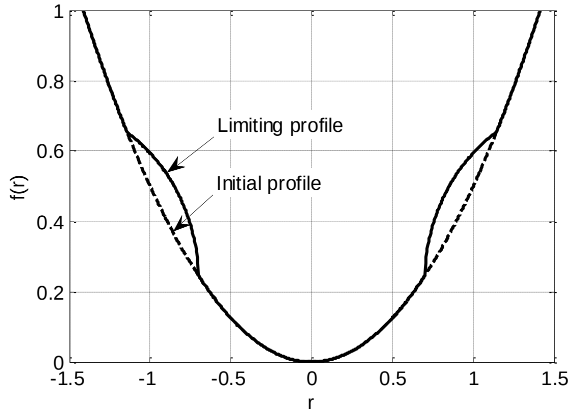

Fig. 3 An example of limiting shape of parabolic indenter

4. CONCLUSION

We extended the basic solution of limiting shape of axis-symmetric profiles due to fretting wear in paper [9] to the case of pre-stressed dual-motion fretting wear. It means that the indenter is pressed into the half space with initial indentation depth and initial tangential displacement; it oscillates in both vertical and horizontal directions. The emphasis of the analysis is placed on two parameters – the maximum indentation depth during the oscillation process and the radius of the stick area in the final state, which determine the limiting shape of worn profile according to basic analytical solution in [9]. For the particular case in this paper we derived and obtained the relation of these two parameters, and it is shown that they depend on the oscillation amplitudes, the phase shift between normal and tangential movements, as well as on the initially indented and displaced distance. Especially the different phase shift between normal and tangential oscillations for the same frequency will result in a different size of the stick area as well as a different limiting profile. With an example of parabolic indenter oscillating on a half space, we present its final worn shape. The worn area is clearly observed and the volume of material loss can be further calculated by comparison with the initial shape of profile.

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